Early Performance Evaluation of Conceptual Flight Vehicles with Multiple Types of Effectors

Background

Designs of conceptual flight vehicles must meet, among many other things, controllability, stability, and maneuverability performance requirements in order to be certified for operation. However, early in the design process, a flight vehicle's ability to meet these requirements is often limited by performance, stability, or control authority availability. Thus, it is essential for designers to evaluate the overall performance and control authority of candidate concepts early in the design phase. Designers normally consider numerous possible vehicle configurations before stability and control system groups begin their analysis and design. There is a need, therefore, for a brief and methodical evaluation of new concepts to enable a much more efficient vehicular design process from start to finish. However, there is no existing framework for conducting a sophisticated and systematic analysis of early designs controlled with multiple types of effectors. The methodology presented here in this document is intended to bridge this gap and to simplify the flight control systems engineering effort in evaluating new vehicle concepts in terms of satisfying mission and performance goals.

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Introduction

Flight vehicles must meet controllability and maneuverability performance requirements in order to be certified for operations. A vehicle's ability to meet these requirements is often limited by the amount of control authority available. Thus, it is essential for designers to evaluate the control authority of candidate concepts early in the conceptual design phase. Normally the designer considers numerous possible configurations before the stability and control group begin their analysis. An early evaluation by the designer before a detailed control system design begins makes the design process much more efficient. In this document we are presenting a methodology for rapid evaluation of performance and control effectiveness of conceptual vehicle designs against design requirements imposed by the vehicle purpose and its mission at critical flight conditions.

In the early phase of a new flight vehicle the design team must determine the purpose, the missions, and hardware configuration of the vehicle. This defines its initial shape and the aerodynamic lift and drag coefficients. From the lift and drag coefficients trajectories are created by means of point mass or 6-dof simulations that achieve the mission goals and the critical flight conditions are identified. This design cycle, however, is an iterative process and there is a need for rapid re-evaluation of vehicle performance in terms of stability, maneuverability and effector trimming capability in order to balance the moments and forces which are defined in the trajectory environment. This static evaluation of vehicle concept is performed prior to any linear analysis, flight control system design, and time domain simulations which are considerably more time consuming.

Sometimes you take a look at flight vehicle design and you instinctively suspect that there is something wrong with it and wished that you had a quick way of proving it. The shape tells you something about its aerodynamics, the weight distribution about stability, the effectors location and size about maneuverability. Sometimes you suspect that the CG is either too far back or too close to the front or that the effectors are too small or too big and you are not sure if the engines or aero-surfaces provide the forces and moments required to maneuver it. There are also situations where you would like to trade control authority in one direction against other directions by repositioning the effectors, or trying other types of effectors in different locations, or that you may simply want to size the effectors capability in a given environment, or to trade control authority among effectors. There are also times when the designer would like to know the consequences of losing an aero-surface, RCS jets, thrust or an actuator from a TVC engine, or to evaluate the effects of CG variations. An optimized trajectory does not provide any information about the vehicle performance and its ability to survive the trajectory environment. Mainly because it is missing the effector information. The vehicle may be unable to follow a desired trajectory, either because it does not have sufficient control authority to trim or it may be too divergent or too stable and unable to maneuver. An analysis process is needed between trajectory optimization and control design that predicts the vehicle performance and its capability to track the proposed trajectory.

All these capabilities and more are addressed by the methodology presented in this document that provides a systematic approach and the necessary tools to rapidly evaluate an early vehicle concept. It uses aerodynamic data, trajectory, propulsion, and mass properties data to evaluate if the flight vehicle concept possesses adequate control power, static stability and maneuverability qualities to satisfy the requirements defined at critical flight conditions along a trajectory, without getting involved in simulations, dynamic modeling and control design. In addition, the method provides guidelines on how to take a corrective action in case the vehicle concept does not meet the required performance. This methodology is implemented in a software tool that is integrated as an option within the Flixan program. We call it "Trim" for short, although it performs a lot more functions other than trimming the control effectors. Trim is an interactive, user friendly, Windows based program that uses graphics and dialogs to rapidly assess the overall performance and controllability of conceptual vehicle designs. It is not limited to only aircraft or rockets but it can be applied to any flight vehicle configuration that uses control surfaces, gimbaling engines, thrust varying engines, reaction control jets, or a combination of the above. It is not intended to replace the stability and control analysis but rather to improve and to simplify the evaluation of initial designs by helping the analyst to decide which concepts should be rejected or pursued further. In fact, it integrates nicely with other Flixan tools by generating input data for flight vehicle dynamic modeling at fixed flight conditions, and for combining multiple types of effectors.

The main functions of the Trim program is to effectively combine multiple types of effectors together, calculate the effector trim angles and thrusts along a trajectory, and also to evaluate the vehicle performance in terms of some critical performance parameters which are described in chapter 3. The performance parameters are calculated as a function of time by processing the vehicle data along the trajectory. Trim also provides interactive tools for visually analyzing vehicle maneuverability against wind-shear disturbances at selected flight conditions by means of vector diagrams. Contour plots are also used for visualizing vehicle stability and controllability over the entire Mach versus alpha envelope. Graphic utilities are included for plotting and comparing various trajectories and performance parameters against time. There are also options for generating dynamic models and effector mixing matrices at selected flight conditions along the trajectory. The Trim algorithm is very versatile because it allows the designer to perform a thorough and systematic analysis in different off-nominal situations, which include: aerodynamic uncertainties, parameter variations, "what if" analysis that help improve the design by graphically modifying some vehicle parameters and evaluating its robustness to trajectory alterations, modified trimming conditions, or by introducing external disturbances. All these features lead to designs that satisfy mission requirements in adverse situations.

The Trim program is not intended to be used only on aircraft but it is designed for multiple types of atmospheric vehicle concepts with blended features, including missiles, launch vehicles, rocket-planes, or re-entry type vehicles. It combines and generalizes various stability and performance criteria developed separately for aircraft and for launch vehicles and extends them to multiple generic vehicle configurations. Almost any type of flight vehicle can be synthesized in a short time, analyze its performance, evaluate the effects of parameter dispersions, perform disturbance analysis, determine the type of effectors, their location, sizing, etc. The flight vehicles are controlled either by aero surfaces, thrust vectoring engines, thrust varying engines, reaction control jets, or any combination of the above effectors. The performance criteria are calculated as a function of the combined effectors system and not separately, such as a function of the aero-surfaces alone or the TVC engines alone. The Trim algorithm, therefore, uses an effective method for combining the various types of effectors together which attempts to optimize performance in the controlled directions. Finally, the Trim program also provides a good starting point for a detailed control design and dynamic analysis work by coordinating with the Flixan Vehicle Modeling program and preparing input data for generating dynamic models at selected flight conditions along the trajectory.

1. Methodology Overview

The design of a new flight vehicle from the flight control point of view undergoes through various phases of design and analysis, as shown in Figure (1.1). The designer begins by collecting the vehicle data base, which initially, is poor and some of the numbers may have to be roughly estimated or guessed. As the design progresses, however, and new numbers become available the database gradually becomes more refined. This is, obviously, an iterative process.

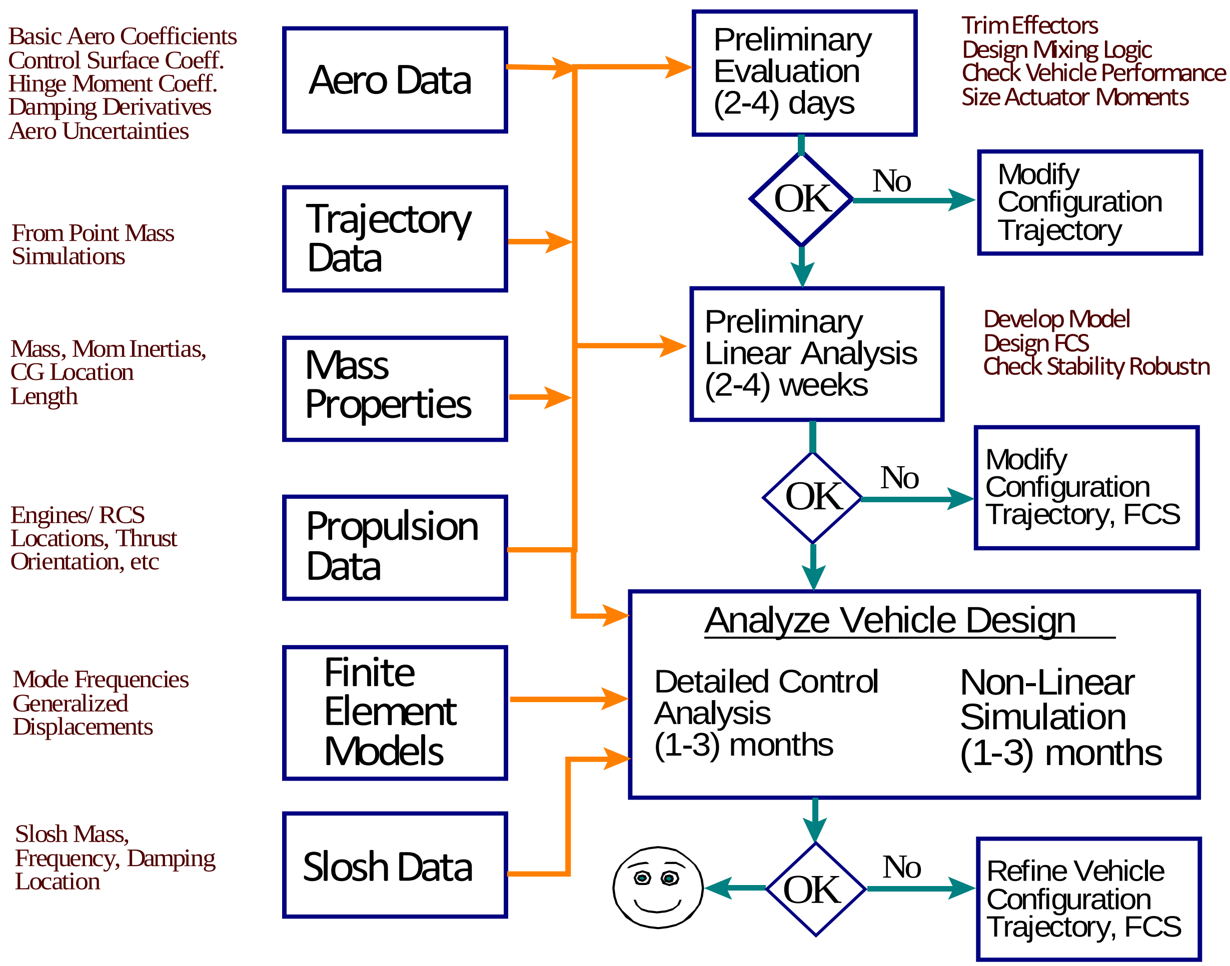


Figure 1.1 Three Phase of Flight Vehicle Design from the Controls Point of View.

The control analyst begins by collecting the vehicle mass properties: weights, inertias, center of mass as a function of vehicle weight, geometry, locations of the vehicle sensors, engines, jets, control surfaces, etc. Then he or she must request aero data from the aerodynamics group consisting of aerodynamic coefficients for the base body, control surface increment coefficients, aero uncertainties, damping, and hinge moments coefficients. Propulsion data for the engines and the reaction control jets are also needed, such as thrusts, directions, thruster locations, etc. A preliminary point-mass trajectory is also needed, such as one created by the POST program optimized with respect to some performance criteria, such as maximizing payload or minimizing heating. The trajectories consist of several variables as a function of time, such as: alpha, beta, velocity, Mach, acceleration, dynamic pressure, thrust, etc. The trajectory variables define the vehicle mission and the environment that it must endure during flight. A trajectory, however, does not include the vehicle rotational dynamics and its interaction with the control system. The Trim program is used to trim the effectors, evaluate controllability and the overall performance. The analysis is static, although it predicts some dynamic characteristics of the vehicle, such as: the time to double amplitude, short period resonances, maneuverability, control authority, and its robustness to uncertainties and disturbances. The next step (or second phase) in this iterative process is to create dynamic models at critical flight conditions, design a preliminary control system and to analyze dynamic stability, robustness to uncertainties, and dynamic performance in response to gusts and guidance commands. In the next phase, a six-dof simulation is created that includes the preliminary control laws interpolated between the design cases. It includes also the vehicle rotational dynamics and it can generate more efficient trajectories than POST for Trim analysis and control system redesign. The linear analysis models are augmented with more detail dynamics, such as tail-wags-dog, fuel sloshing, and flexibility. This time the control laws are better refined and more details may be included, such as lead-lag, low-pass, flex filters, logic, etc. The 6-dof simulation is also continuously refined. This modeling/ design/ analysis, process is repeated several times until the FCS design and the trajectories converge (possibly multiple trajectories). The user is referred to read the control design and simulation examples presented in Section 10.

In the preliminary phase of the Trim analysis, however, we are not yet concerned with the flight control system and simulations. In fact we don't even need an FCS, because all we are doing is finding out how stable or unstable the vehicle is and if the proposed effectors are strong enough to control it in the trajectory environment. An FCS design for dynamic analysis is performed in the second phase. If a system is stabilizable and maneuverable then it will also be easy to design a control system that meets some reasonable performance requirements. You can learn a lot about the vehicle dynamic behavior and maneuverability from static analysis before getting into the dynamics, control design, and simulations. If a vehicle does not have a satisfactory stability and maneuverability properties, then, even if you are able to design a control system that stabilizes it, it will probably be susceptible to noise and not robust to variations and uncertainties. So this preliminary process helps the designer to converge towards a well behaved airframe and mission environment without getting involved in detailed dynamic analysis. This is important because at an early phase some key decisions have to be made regarding the placement of sensors and the location and size of the vehicle effectors, typically: the control surfaces, TVC engines, and the reaction control jets, which determine the control authority. With increasing demand for agility and use of advanced FCS with relaxed static stability, consideration of control power has become an important issue in modern flight designs. Excessive control authority can translate into increased weight and drag, while inadequate control power can result in a failed design. Thus, the designer's goal when sizing and placing thrusters or control surfaces is to provide sufficient, yet not excessive, control power to meet the required controllability and maneuvering performance

requirements. Having, therefore, a methodology for analyzing performance and to properly size the control power is essential for optimizing conceptual flight vehicle configurations.
To summarize this overview, the Trim program is an important tool for a quick phase-1 type performance evaluation of a new flight vehicle concept without requiring flight control system design and a 6-dof simulation. A simple static performance analysis is created after trimming to predict the vehicle performance and its dynamic characteristics in presence of disturbances, uncertainties and parameter variations, and to provide corrective actions in the event where the requirements are not satisfied. In the second and third phases of the design a preliminary rigid-body dynamic analysis of the flight vehicle is performed at critical flight conditions. The analyst selects some critical flight conditions to create dynamic models along the trajectory, and the Trim program generates the input data which are used by the Flight Vehicle Modeling Program (FVMP) to generate state-space systems at the selected flight conditions. The dynamic models are then used to design control laws, design an effector combination logic, perform linear analysis, and to eventually develop 6-dof simulations.

Figure (1.2) shows the inputs and outputs of the Trim program which initially calculates the positions of the vehicle effectors (called trimming). The effectors must rotate at a certain angle or to vary their thrust in order to produce moments and forces which balance the base vehicle moments and forces. The input files to the program are: a trajectory file, aero data consisting of basic vehicle aero plus aero-surface increments, damping and hinge moment coefficients, mass properties consisting of inertias and CG location as a function of weight, and slosh data which are optional and they are only used when there is fuel sloshing in order to generate inputs for the FVMP. The data files in Trim must be shaped in standard formats in order to be accessible by the program and this may take a few hours to complete. Otherwise, the analysis process is straightforward and it should not take more than a few minutes to analyze and to evaluate performance characteristics of new vehicle concepts and to provide some recommendations for improvements, as it is demonstrated in the examples. In addition to trimming the effectors the program performs several other functions which are essential for preliminary analysis of a flight vehicle.

1. Evaluates the overall capability of the conceptual vehicle with its effectors to meet the mission requirements by calculating some critical performance parameters along the flight trajectory that characterize stability, maneuverability and controllability in the presence of wind disturbances. These parameters are described in section (3).

2. Plots of the trajectory data as a function of time and provides the graphic capability for the analyst to modify graphically some of the trajectory variables, such as: the angle of attack, sideslip, CG location, etc, by means of dialogs and interactive graphics. The user re-trims and reevaluates the vehicle performance using the modified trajectory. This is useful for analyzing parameter dispersions, CG shifts, and other "what if" type of studies.

3. When the vehicle has multiple effectors, and multiple types of effectors, after trimming the user can graphically reshape the deflection angles or the thrusts of some effectors. This is done by constraining their deflections and, therefore, trading the deflections of some effectors against others. The constrains are adjusted graphically by means of dialogs and interactive graphics. It can be used, for example, in trading elevon versus body-flap and speed-brake deflections, since they all affect pitching moment and longitudinal forces.
4. It provides graphical utilities for visualizing the basic aerodynamics and the aero-surface coefficients by plotting the coefficients or their derivatives as a function of alpha, beta, Mach number, and surface deflection (δ).
5. It uses Contour plots to display some of the important performance parameters, such as, pitch and lateral stability parameters, the LCDP and control authority, as a function of Mach versus alpha. Contour plots provide a wider perspective of performance in the entire Mach versus alpha range (rather than in the vicinity of the trajectory). They help locating undesirable or favorable flight conditions and they provide direction on how to reshape the trajectory in order to improve performance and to avoid undesirable flight conditions.
6. Creates an effector mixing logic matrix for combining multiple types of effectors. The mixing matrix receives the flight control acceleration demands and calculates the effector positions that will produce the demanded acceleration changes. It uses pseudo-inversion based on geometry and the individual control authority of each effector. It optimizes controllability because it allocates control authority proportionally to the individual effector capability in the demanded directions. It also provides some form of open-loop decoupling between the control axes. The mixing logic matrix is used for analyzing the static performance characteristics of a vehicle. It is either constant, calculated at a fixed flight condition, or time-varying calculated at every time point along the trajectory.
7. Generates dynamic models for control design and simulations at critical points along the trajectory. The user selects the analysis points and the Trim program generates the input data at the selected flight conditions. The Flixan "*Flight Vehicle Modeling*" program generates various types of state-space systems and matrices required for control analysis and simulations at the selected trajectory points, as it is shown in the examples section 10.
8. Generates Vector Diagrams which are used for analyzing vehicle controllability against wind disturbances at specific flight conditions. It compares the control moments or forces in two directions against the disturbance moments and forces due to an alpha or beta dispersions from trim. There are four types of vector diagrams, presented in Section 8, for analyzing moments, forces, accelerations, and partials.
9. Calculates the control moments at the hinges of the control surfaces as a function of the trajectory parameters and the control surface trim angles. This option is only available when a hinge moment coefficients file (.HMco) is included in the database.
10. Provides additional data visualization utilities for plotting and overlaying previously calculated data, such as: effector trim positions, performance parameters, multiple trajectories and aero-surface hinge moments.

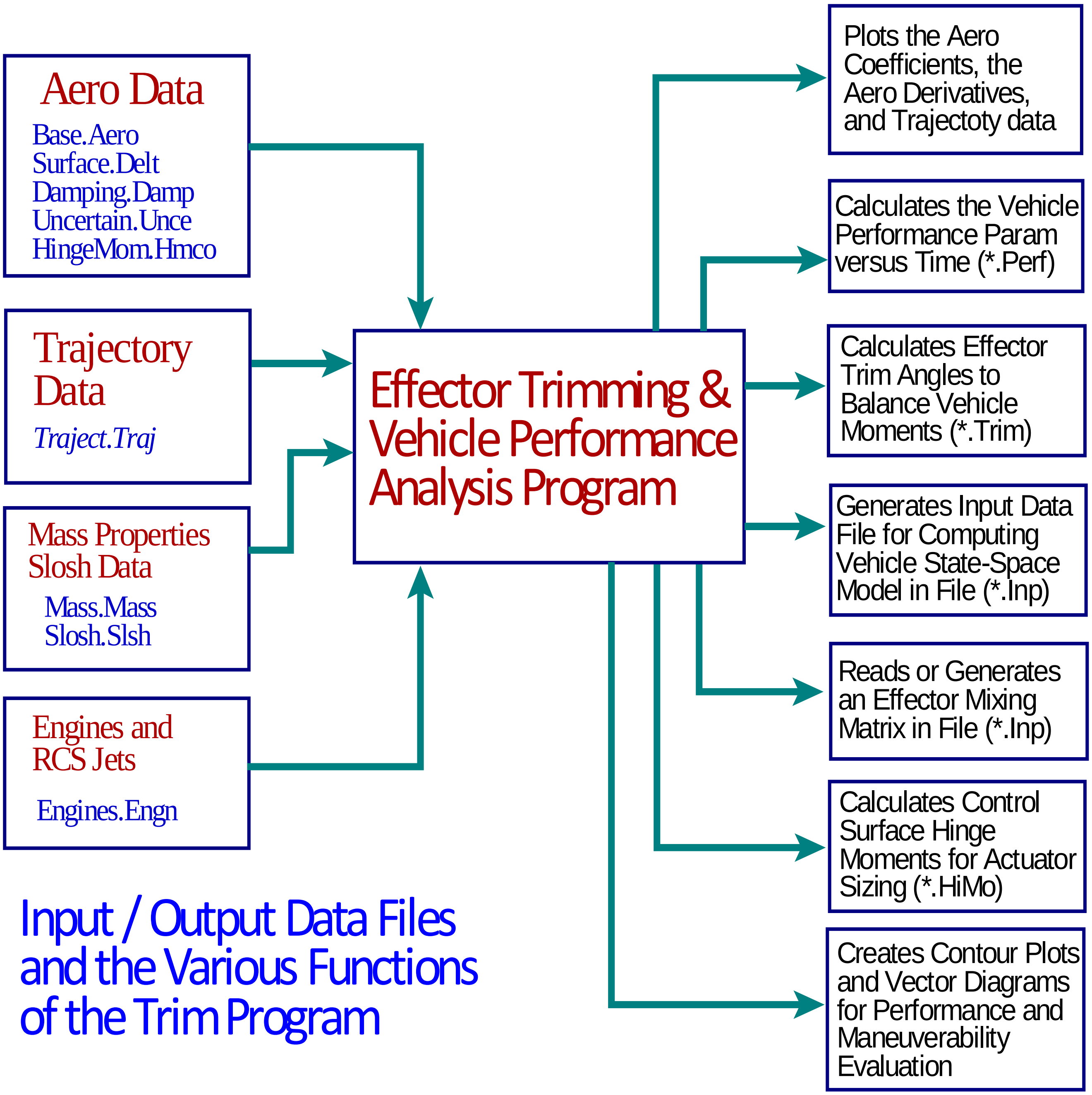


Figure 1.2 Inputs and Outputs of the Trim Program

1.1 Control Effectors

The effectors are the devices that provide the "muscle" power to maneuver the vehicle. The Trim program provides the capability for the user to select between four different types of effectors: (a) thrust vector control (TVC) consisting of engines that pivot either in two directions (pitch and yaw) or in a single (skewed) direction (γ), (b) engines of variable thrust (throttling), (c) reaction control jets (RCS), and (d) control aero-surfaces that rotate about a constant hinge. A throttling engine has a nominal thrust Te, and it provides control forces on the vehicle by varying its thrust by a certain amount above and below nominal. The amount of thrust variation above and below Te is defined by the throttle parameter in the engine data file ($0<T_h<1$), which must be between zero and less than one. The actual thrust during trimming is determined by the trimming algorithm which calculates the engine throttle control input $\delta_{Thro}(t)$ required to balance the acceleration, and it varies between -1 to +1. The actual engine thrust at any moment is $Te(1+T_h*\delta_{Thro})$. When the throttle control is zero it corresponds to the nominal thrust value Te and the thrust remains always positive. An engine may be defined to be both: gimbaling and throttling.

The RCS jets in Trim are also variable thrust devices but they are defined slightly different. They are continuous thrusters with zero nominal thrust. They can generate either positive or negative forces at a location on the vehicle along a specified direction. They represent a pair of back-to-back firing jets that can generate positive or negative forces. In the propulsion data file an RCS jet is defined by its maximum thrust T_{max} and by the throttle parameter which is set exactly to 1. This is the parameter that differentiates between a thrust varying propulsion engine and an RCS jet. The actual thrust during trimming is defined by the jet throttle control input δ_{Thro} which varies between -1 to +1 and corresponds to jet thrust variation between $-T_{max}$ to $+T_{max}$. Aircraft engines, for example, and reaction control jets are defined as thrust varying engines. A re-entry glider vehicle can be controlled by control surfaces and RCS jets. Commercial aircraft use both, control surfaces and engine throttling. Launch vehicles use mainly thrust vector control (TVC) engines and sometimes in combination with differential throttling and RCS jets. Notice that in Trim the RCS jets are treated as analog and not as "on-off" devices because the purpose of the Trim program is for sizing the jets and not for simulations. "On-off" RCS jet firing is typically implemented in closed-loop simulations using dynamic models generated by the Flixan Vehicle Modeling program (FVMP). For further details see the examples in Section 10.

1.2 Dynamic Modeling and Effector Mixing Logic

When the initial vehicle evaluation is complete, in terms of calculating the effector trim angles, determining the vehicle stability, performance, maneuverability, calculating the actuator hinge moments, etc, and if the initial analysis is acceptable, the next step is to start generating dynamic models for analyzing dynamic performance at critical flight conditions. These models are used for designing flight control laws, and for performing dynamic analysis to evaluate the control system stability, closed-loop performance, and robustness to parameter uncertainties. The Trim program generates input data at specific flight conditions that can be processed by the FVMP. The initial models created by Trim are simple rigid-bodies but later they can be augmented by the user in the FVMP environment by including additional features, such as structural bending, tail-wags-dog, fuel sloshing, dynamic coupling with actuator models, etc. The Trim program also generates an effector mixing logic matrix at specific flight conditions. The mixing logic converts the flight control demands (mainly 3 rotational plus some translational accelerations) to actuator commands. It is included in the flight control system logic.

2. Trimming the Effectors for Balancing the Flight Vehicle Moments and Forces

Trimming is the process of balancing the base vehicle moments and forces produced due to the angles of attack α and sideslip β as it flies along a trajectory, with moments and forces produced by the vehicle effectors. We are mostly interested in trimming the three moments (roll, pitch and yaw). Sometimes we also include the axial and normal (Ax and Az) accelerations. We rarely include the side acceleration (Ay) in trimming. The data needed for trimming are: the basic aero coefficients, aero-surface (AS) increment coefficients, thrust vector control (TVC) engine data, and a trajectory. A trajectory is a table of flight parameters as a function of time. It is typically an Excel format of column data starting with time in the leftmost column and consisting of a list of flight parameters, such as: altitude, vehicle mass, angle of attack, sideslip, dynamic pressure, Mach number, velocity, acceleration, thrust etc. A trajectory captures the flight vehicle mission, its flight environment, and the maneuvering requirements. It is initially obtained from point mass 3-dof trajectory optimization programs such as “POST” which neglects the rotational dynamics. Some of the criteria for shaping a trajectory are: heating, fuel efficiency, payload weight maximization, and structural loading. Trajectories are typically created by trajectory specialists, but if we don't have a trajectory we can begin by creating our own at flight conditions near the critical missions. The primary concern of the flight control analyst is to make sure that the vehicle has the effector control authority to maneuver along the required mission and that the vehicle stability (or instability) is within acceptable limits, in both static and dynamic sense. In this document, however, we are dealing with static stability and in having the control authority to produce the required accelerations against the expected aerodynamic disturbances.

The aerodynamic moments and forces acting on the base vehicle are generated by the angles of attack and sideslip specified in the trajectory. They must be balanced with the control moments and forces generated by the vehicle effectors (gimbaling or throttling) which must have the control authority to trim and also to retain some authority for other functions. If the effectors do not have the required authority to trim, then either the trajectory has to be modified, or the effectors, or the vehicle aerodynamics, or all of the above must be modified until an acceptable trimming condition can be achieved. This is usually an iterative process that requires several attempts and perhaps changes in the vehicle shape and size of the surfaces or the TVC. As a guideline, the control authority required for trimming should not exceed half of the maximum control capability of each effector. There should be some control capability reserved for maneuvering the vehicle and for reacting against wind gusts and other disturbances. Also, the uncertainties in the vehicle parameters and CG location may cause further uncertainty in the trim angles. Balancing the three vehicle moments is usually the main objective when trimming the vehicle along the trajectory. Sometimes it is also necessary for the effectors system to have the control authority to trim not only along the 3 moments but also along some of the linear accelerations, mainly along the x and z directions. For example, it may be necessary to balance the normal acceleration independently of the pitch moment during landing or when separating from another vehicle by using elevon, body-flap and vertical thrusters. This will prevent over-pitching the vehicle. It is also possible to trim the axial acceleration by means of a speed brake or by varying the engine thrust independently of pitch. Trimming is also important for sizing the control surfaces, the TVC max deflections, the engine thrusts, throttling capability, and the RCS jet thrusts. It can also be used to determine the max CG variations, the installation angles for the engines or the positions of the aero-surfaces.

In this section we are presenting a method for calculating the effector trim angles along a trajectory. This method is based on pseudo-inversion of a matrix and it attempts to allocate control authority to effectors according to their individual control capability in specific directions. It calculates the trim position of each effector as a function of time at each point along the trajectory. For a perfectly trimmed flight along a given trajectory the moments and forces applied on the vehicle due to propulsion and the aerodynamics (angles of attack and sideslip), should generate the accelerations which are defined in the trajectory. Otherwise, the control effectors must be used in order to provide the additional moments and forces required to match the required accelerations. The idea of trimming is to adjust the aero-surfaces, the TVC deflection angles, and thrusts, as necessary in order to provide the additional aerodynamic and propulsion forces and moments on the vehicle which are needed in order to match the angular and linear accelerations defined in the trajectory.

Equation (2.1) is a “Force=Mass x Acceleration” type of equation. On its right hand side there is a 6-dimentional vector consisting of 3 rotational and 3 translational accelerations extracted from the trajectory and they are multiplied by the vehicle mass and moments of inertia, as shown, to convert to moments and forces. The moments and forces on the left side of this equation are due the aero-dynamics, propulsion, and disturbance forces, and they must match the right hand side at each point along the trajectory. Note that in equation (2.1) the moments and forces on the LHS consist of both: base vehicle plus effector moments and forces.

$$\begin{bmatrix} M(aero) \\ F(aero) \end{bmatrix} + \begin{bmatrix} M(thrust) \\ F(thrust) \end{bmatrix} + \begin{bmatrix} M(disturb) \\ F(disturb) \end{bmatrix} = \begin{bmatrix} Inertia \times \dot{\omega}(traject) \\ Mass \times Accelerat(traject) \end{bmatrix} \tag{2.1}$$

In order to solve this equation we must separate the base terms from the effector terms on the LHS of equation (2.1). The base terms cannot be modified by the effectors because they are caused by the aero forces due to the angles of attack and sideslip. They are caused by the engine thrusts acting on the base body (not throttling and at zero deflection). The only moments and forces that can be adjusted in this equation, in order to match the RHS with the LHS terms are the contributions from the effector deflections and thrust variations. The trim positions are the control surface or TVC engine deflections from their installation (zero) positions, or thrust variations, which are required in order to balance equation (2.1). The effectors must not only provide sufficient control authority to balance the equation but they must also have additional control authority for maneuverability and to overcome unexpected disturbances, such as gusts, etc. In a typical trajectory derived from a point mass simulation the angular accelerations in the trajectory are usually zero because it assumes that the vehicle moments are perfectly balanced and the effector trimming boils down to zeroing the moments on the LHS of equation 2.1. When the rotational accelerations are available, however, either from a 6-dof simulation or test data, they can be used to provide a more accurate trimming capability. The accelerations are multiplied by the vehicle moments of inertia matrix to calculate the total vehicle moments (roll, pitch, yaw), as shown on the RHS of equation (2.1). Similarly, the translational accelerations are multiplied by the vehicle mass to calculate the total force on the vehicle along x, y, and z. The linear accelerations (Ax and Az) in a typical trajectory are not zero. It is important to be able to trim along those two directions because the axial acceleration affects the range and the normal acceleration affects the altitude. During trimming they must be matched by the aerodynamic and the propulsion forces on the vehicle.

In this section we shall develop a trimming algorithm for multiple effector types that employs as many effectors as they are available on the vehicle. The number of effectors must be greater than or equal to the number of degrees of freedom (dof) that must be balanced, plus all directions to be trimmed and eventually controlled should be accessible by at least one effector. When the number of effectors exceed the number of degrees-of-freedom (dofs) the solution is over-determined. The more effectors the better the controllability because they can be combined more efficiently to control the directions they can influence. Having an abundance of effectors is also good for redundancy. The present trimming logic uses a pseudo-inverse algorithm that favors the effectors which are able to produce more "muscle" in the demanded direction by allocating bigger control authority and hence increased activity, than the effectors which are less capable and they become less active in that direction. All four types of effectors can be combined and used by the trim algorithm: gimbaling engines (pitch and yaw), throttling engines, RCS jets, and aero-surfaces.

The orientation of each engine with respect to the vehicle is defined in the engine data file. The direction of the engine thrust is defined by two angles (elevation and azimuth or simply pitch and yaw) which are measured from the -x axis. The engine is either mounted at a fixed position relative to the vehicle or it can be gimbaled in the pitch and yaw directions with respect to its mounting position and to provide thrust vectored control (TVC). Its thrust is either constant or it can be modulated up and down the nominal thrust to provide throttle control. It may also do both: gimbaling and throttling simultaneously. Aircraft engines, for example, are modeled as throttling engines having a nominal thrust (Te), with a certain amount of thrust variation about Te. Reaction control jets (RCS) are also considered to be throttling engines. They are mounted at fixed angles relative to the -x direction and their thrust can vary continuously between zero and ±Tmax. In Trim the RCS jets are not considered to be "on/off" devices, but they are continuous (analog) thrusters and negative thrusting is permitted. A single thruster is used to model a pair of back-to-back firing jets producing positive or negative forces as a function of the throttle control input. It is not necessary for all surfaces to be used for trimming and for flight control. Some of the aero surfaces can be defined in the aero-surfaces data file as fixed with zero max and min deflections and at a non-zero bias position (such as for example a fixed body flap).

In order to solve the trimming equations numerically we must rewrite equation (2.1) by separating the moments and forces produced by each effector as consisting of two parts: a fixed part, and an adjustable part. For this reason in the equations that follow we will express in detail the moments and forces produced by the engines, jets, and the aero-surfaces and separate them in two parts: (a) the steady-state part that is produced when the effector is at its zero position or nominal thrust, and (b) an adjustable part due to the deflection which is used for controlling and trimming the vehicle.

Moments and Forces Produced by a Control Surface Deflection

The control surfaces generate additional moments and forces that are used to balance the moments and forces on the base vehicle along the trajectory. The aerodynamic moments and forces are functions of the aero-surface increment coefficients, the dynamic pressure, and the reference length, as shown in equation (2.3). The surface coefficients are non-linear functions of four variables: the surface deflection from zero position (δ_{asi}), the angles of attack and sideslip (α and β), and the Mach number (m), for example, $C_m(\alpha, \beta, m, \delta_{asi})$.

$$
\begin{aligned}
L_{XSi} &= \bar{Q}S_{ref}\, b\left\{C_l\left(\alpha,\beta,m,\delta\right)\right\} & F_{XSi} &= -\bar{Q}S_{ref}\left\{C_A\left(\alpha,\beta,m,\delta\right)\right\} \\
M_{YSi} &= \bar{Q}S_{ref}\, \bar{c}\left\{C_m\left(\alpha,\beta,m,\delta\right)\right\} & F_{YSi} &= \bar{Q}S_{ref}\left\{C_Y\left(\alpha,\beta,m,\delta\right)\right\} \\
N_{ZSi} &= \bar{Q}S_{ref}\, b\left\{C_n\left(\alpha,\beta,m,\delta\right)\right\} & F_{ZSi} &= \bar{Q}S_{ref}\left\{C_Z\left(\alpha,\beta,m,\delta\right)\right\}
\end{aligned}
\qquad (2.3)
$$

In order to trim we must solve equation (2.3) for the surface deflections (Δ_{asi}) which are needed to balance the base moments and forces. It is not easy, however, to solve directly for the surface deflections (Δ_{asi}) because the equations are non-linear and not explicitly available, but usually only wind-tunnel data is available. The equations are solved numerically by linearizing them at fixed (α, β, Δ_{asi}, m) for each control surface, and using the control surface derivatives ($C_{m\delta asi}$) etc, to propagate the solution towards a deflection that will balance the vehicle moments and forces. The derivatives are calculated at each iteration and they are also functions of (α, β, Δ_{asi}, m).

The moment/ force increment equation (2.3) is solved for each control surface by separating it in two parts, as shown in equation (2.4), (a) The steady-state part M_{SOi} representing the moments and forces at a nominal surface deflection (Δ_{S0i}), and (b) a linear smaller term representing additional moments/ forces due to small deflection δ_{asi} relative to the nominal deflection Δ_{S0i}. These terms are in addition to the base aero forces described in equation (2.2).

We must also normalize the second term in the moment/ force equation by dividing the small deflection δ_{asi} with the max surface deflection (δ_{asiMax}). We must also multiply the coefficients in the second term by multiplying them with the max deflection (δ_{asiMax}). The input ($\delta_{asi}/\delta_{asiMax}$) in the normalized second term in equation (2.4) becomes non-dimensional, and it can only vary between zero and ±1.

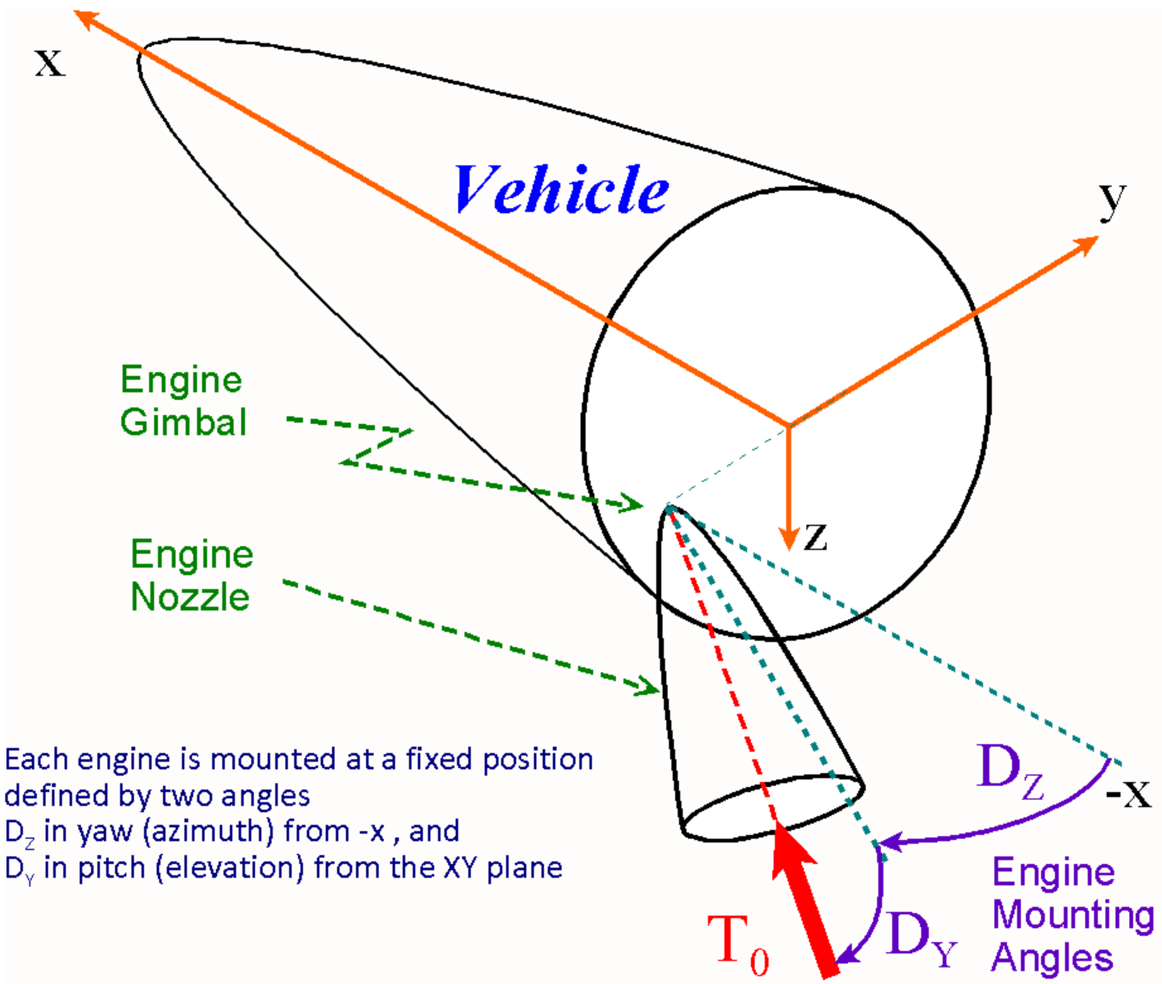
$$
M_{Si} = M_{S0i} + \left[DM_{si}\right]\left(\delta_{asi}/\delta_{asi_{MAX}}\right)
$$

$$
M_{Si} = \bar{Q}S_{ref}\begin{bmatrix} bC_l\left(m,\alpha,\beta,\Delta_{SOi}\right) \\ \bar{c}C_m\left(m,\alpha,\beta,\Delta_{SOi}\right) \\ bC_n\left(m,\alpha,\beta,\Delta_{SOi}\right) \\ -C_A\left(m,\alpha,\beta,\Delta_{SOi}\right) \\ C_Y\left(m,\alpha,\beta,\Delta_{SOi}\right) \\ C_Z\left(m,\alpha,\beta,\Delta_{SOi}\right) \end{bmatrix}_{(i)} + \bar{Q}S_{ref}\,\delta_{asi_{MAX}}\begin{bmatrix} bC_{l\delta as} \\ \bar{c}C_{m\delta as} \\ bC_{n\delta as} \\ -C_{A\delta as} \\ C_{Y\delta as} \\ C_{Z\delta as} \end{bmatrix}_{(i)}\left(\delta_{asi}/\delta_{asi_{MAX}}\right) \qquad (2.4)
$$

Moments and Forces due to an Engine Gimbaling in Pitch and Yaw

The moments and forces on the vehicle generated by a single engine (i) are also non-linear functions of the pitch and yaw deflection angles and they also depend on its thrust variation. We will linearize the force equation produced by an engine (i) and separate it in three parts: (a) the nominal moments and forces generated due to its nominal thrust T_{0i} and at fixed deflections (D_{Zi} and D_{Yi}) that correspond to the engine mounting positions, (b) the moment and force increments generated due to small engine deflections in pitch and yaw (δ_{yei}, δ_{zei}) relative to the engine mounting positions, and (c) the additional moment and force increments due to the variation $D_{Thr(i)}$ in engine thrust from its nominal value. Each term in equation (2.5) is a 6-dimensional vector consisting of 3 moments and 3 forces. The pitch and yaw engine deflections (δ_{Yei}, δ_{Zei}) in equation (2.5) are normalized by dividing with the maximum engine deflections. Similarly, the thrust variation inputs ($D_{Thr(i)}$) are normalized by dividing with the maximum thrust variation of each engine $D_{ThrMax(i)}$.

$$M_{E(i)} = M_{EO(i)} + \left[DM_{E(i)}\right]\begin{Bmatrix} \delta_{Y(i)} / \delta_{YMAX(i)} \\ \delta_{Z(i)} / \delta_{ZMAX(i)} \end{Bmatrix} + \left[DM_{T(i)}\right]\left\{\frac{D_{Thr(i)}}{D_{ThrMax(i)}}\right\} \tag{2.5}$$



The forces at the engine gimbal along the vehicle body axes generated by a single engine are shown in equation (2.6), where (D_Y and D_Z) are the engine trim angles.

$$F_{XO(i)} = T_O(i)\cos(D_Y)\cos(D_Z)$$
$$F_{YO(i)} = T_O(i)\cos(D_Y)\sin(D_Z)$$
$$F_{ZO(i)} = -T_O(i)\sin(D_Y) \tag{2.6}$$

The moment arms distances between an engine (i) and the vehicle CG is:

$$l_{Xei} = X_{ei} - X_{CG}$$
$$l_{Yei} = Y_{ei} - Y_{CG} \tag{2.7}$$
$$l_{Zei} = Z_{ei} - Z_{CG}$$

Equation (2.8) calculates the nominal moments and forces without variations generated by a single engine (i) at its nominal trim deflection angles (D_Y and D_Z), which is the first term $M_{E0(i)}$ in equation (2.5). This term assumes that the engine thrust is at its nominal value $T_{0(i),}$ and it does not include the small angle gimbaling terms.

$$M_{EO(i)} = \begin{Bmatrix} \begin{bmatrix} 0 & -l_{zei} & l_{yei} \\ l_{zei} & 0 & -l_{xei} \\ -l_{yei} & l_{xei} & 0 \end{bmatrix} \begin{bmatrix} F_{XO} \\ F_{YO} \\ F_{ZO} \end{bmatrix}_{(i)} \\ \begin{bmatrix} F_{XO} \\ F_{YO} \\ F_{ZO} \end{bmatrix}_{(i)} \end{Bmatrix} \tag{2.8}$$

The additional moments and forces on the vehicle generated by the small pitch and yaw angle deflections δy(i) and δz(i) of an engine (i) from its trim positions (D_Y and D_Z), are shown in equation (2.9). The deflection inputs to the equation are normalized by dividing with the maximum pitch and yaw engine deflection capability ($\delta_{YMAX(i)}$ and $\delta_{ZMAX(i)}$). This normalization makes the inputs to equation (2.9) vary between zero and ±1. The elements inside the matrix are also scaled accordingly. Equation (2.9) represents the second term in equation (2.5).

$$\left[DM_{E(i)}\right]\begin{Bmatrix} \delta_{Y(i)}/\delta_{YMAX(i)} \\ \delta_{Z(i)}/\delta_{ZMAX(i)} \end{Bmatrix} =$$

$$T_O(i)\begin{Bmatrix} \begin{bmatrix} 0 & -l_{zei} & l_{yei} \\ l_{zei} & 0 & -l_{xei} \\ -l_{yei} & l_{xei} & 0 \end{bmatrix} \begin{bmatrix} -\cos(D_Z)\sin(D_Y)\delta_{YMAX} & -\cos(D_Y)\sin(D_Z)\delta_{ZMAX} \\ -\sin(D_Z)\sin(D_Y)\delta_{YMAX} & \cos(D_Y)\cos(D_Z)\delta_{ZMAX} \\ -\cos(D_Y)\delta_{YMAX} & 0 \end{bmatrix}_{(i)} \\ \begin{bmatrix} -\cos(D_Z)\sin(D_Y)\delta_{YMAX} & -\cos(D_Y)\sin(D_Z)\delta_{ZMAX} \\ -\sin(D_Z)\sin(D_Y)\delta_{YMAX} & \cos(D_Y)\cos(D_Z)\delta_{ZMAX} \\ -\cos(D_Y)\delta_{YMAX} & 0 \end{bmatrix}_{(i)} \end{Bmatrix} \begin{Bmatrix} \delta_{Y(i)}/\delta_{YMAX(i)} \\ \delta_{Z(i)}/\delta_{ZMAX(i)} \end{Bmatrix} \tag{2.9}$$

Similarly, equation 2.10 calculates the moments and forces on the vehicle generated by thrust variations $D_{thr(i)}$ of an engine (i). The throttle control input $D_{thr(i)}$ has no units, and it can be made to vary between zero and ±1 maximum. The actual engine thrust is defined as: $T(i)= T_0(i)\{1+D_{thr(i)}\}$ where $T_0(i)$ is the nominal engine thrust. It means that the engine thrust can be varied between zero and $2T_0(i)$. However, the throttle capability parameter $D_{thrMax(i)}$ of a throttling engine is typically defined to be smaller than |1|. Let's say a typical value of a throttling parameter D_{thrMax}=0.3, which means that the engine thrust can only vary ±30% from nominal. We must normalize the throttle control input the same way we normalized the deflection inputs of the aero-surfaces and the gimbaling engines. We normalize it by dividing it with the throttle parameter $D_{thrMax(i)}$, which represents the maximum throttle variation. So when the normalized throttle input varies between zero and ±1 (same as in equation 2.9) and it represents a thrust variation ±30%. Equation (2.10) calculates the moment and force variations due to an engine throttling, which is the third term in equation (2.5).

$$\left[DM_{T(i)}\right]\left\{\frac{D_{Thr(i)}}{D_{ThrMax}}\right\} = T_O(i)D_{ThrMax}\left\{\begin{matrix}\begin{bmatrix} 0 & -l_{zei} & l_{yei} \\ l_{zei} & 0 & -l_{xei} \\ -l_{yei} & l_{xei} & 0 \end{bmatrix}\begin{bmatrix}\cos(D_Y)\cos(D_Z) \\ \cos(D_Y)\sin(D_Z) \\ -\sin(D_Y)\end{bmatrix}_{(i)} \\ \begin{bmatrix}\cos(D_Y)\cos(D_Z) \\ \cos(D_Y)\sin(D_Z) \\ -\sin(D_Y)\end{bmatrix}_{(i)}\end{matrix}\right\}\left\{\frac{D_{Thr(i)}}{D_{ThrMax}}\right\} \tag{2.10}$$

By normalizing the control inputs it helps to allocate the controls evenly among the effectors when solving the trimming equation numerically. For example, on a three engines vehicle the combined moments and forces due to engine gimbaling and throttling is given by equation (2.11).

$$M_{3TVC} = \sum_{i=1}^{Neng=3} M_{EO(i)} + \left[DM_{E1} \quad DM_{E2} \quad DM_{E3}\right]\begin{bmatrix}\delta_{y1}/\delta_{y1Max} \\ \delta_{z1}/\delta_{z1Max} \\ \delta_{y2}/\delta_{y2Max} \\ \delta_{z2}/\delta_{z2Max} \\ \delta_{y3}/\delta_{y3Max} \\ \delta_{z3}/\delta_{z3Max}\end{bmatrix} + \left[DM_{T1} \quad DM_{T2} \quad DM_{T3}\right]\begin{bmatrix}D_{Thr1}/D_{Thr1Max} \\ D_{Thr2}/D_{Thr2Max} \\ D_{Thr3}/D_{Thr3Max}\end{bmatrix} \tag{2.11}$$

Numerical Solution for Calculating the Effector Trim Deflections and Throttles

Now that we have derived the equations for calculating the moments and forces on the vehicle generated by a each effector separately, that is, aero-surfaces, TVC, and engine throttling, we will combine all the effectors together in a single moment/ force balance equation as follows.

$$M(basic) + \sum_{i=1}^{Neng} M_{EOi} + \sum_{i=1}^{Nsurf} M_{SOi} + M(disturb) - M_V * acceleration = -M(residual) =$$
$$-\delta M(aero\ surface) - \delta M(gimbaling) - \delta M(throttling) \tag{2.12}$$

The terms on the left hand side of equation (2.12) represent the moments and forces at each trajectory point. They are due the base vehicle aerodynamics, the control surfaces at their nominal trim angles, and also due to the engines at their nominal engine thrusts and trim positions. There is also a term included for injecting known external disturbances when trimming and analyzing vehicle controllability against them. If the vehicle was able to trim perfectly without requiring any effector assistance the forces and moments on the left side of the equation (2.12) should perfectly match the (M*acceleration) term which is also on the left side of the equation without requiring any additional assistance from the (δM) control terms which are on the right hand side of equation (2.12). But this is rarely the case. The (δM) terms are due to the three types of normalized control effectors already described. That is, (a) control surface deflections (δ_{asi}) from their nominal trim positions (Δ_{SOi}), (b) pitch and yaw TVC engine deflections (δ_{yei}, δ_{zei}) from their nominal mounting angles (D_{Yei}, D_{Zei}), and (c) additional moments and forces due to engine thrust variations (D_{thri}) from their nominal thrust T_{0i}. If the moments and forces do not balance with the M*acceleration terms on the LHS of the equation, we must solve for the (δM) terms to calculate how much additional forces and moments (due to effector deflections or thrust variations) are needed in order to balance equation (2.12). The equation (2.12) is non-linear and it is solved numerically at each point along the trajectory. The normalized unknown effector increments on the RHS of equation (2.12) are stacked together in a single vector, as shown in the two aero-surface/ two engine example in equation (2.13) below. This shapes the equation (2.12) to a matrix equation form, shown in (2.13).

$$M(resid) = \begin{bmatrix} DM_{S1} & DM_{S2} & | & DM_{E1} & DM_{E2} & | & DM_{T1} & DM_{T2} \end{bmatrix} \begin{bmatrix} \delta_{as1}/\delta_{as1Max} \\ \delta_{as2}/\delta_{as2Max} \\ ---- \\ \delta_{y1}/\delta_{y1Max} \\ \delta_{z1}/\delta_{z1Max} \\ \delta_{y2}/\delta_{y2Max} \\ \delta_{z2}/\delta_{z2Max} \\ ---- \\ D_{Thr1}/D_{Thr1Max} \\ D_{Thr2}/D_{Thr2Max} \end{bmatrix}$$

$$In\_Matrix\_Form: \quad M(resid) = \left[DM\right]\left(\frac{\delta_T}{\delta_{MAX}}\right) \tag{2.13}$$

The matrix equation (2.13) is solved numerically for the effector trim positions vector (δ_T) which is needed to balance the residual forces and moments M(residual), assuming of course that the matrix [DM] is pseudo-invertible. This happens when the rank of [DM] is greater than or equal to the number of vehicle direction dofs that must be trimmed. Notice, the normalized deflections obtained by the pseudo-inversion are multiplied by the max deflections because the inputs of equation (2.13) are already divided by the max deflections and the matrix DM was properly scaled. This scaling allows the effectors that have greater control authority in certain directions to be used more than others which are less capable in those directions.

$$\delta_T = diag(\delta_{MAX})*Pseudo\ Inverse\left[DM\right] \times M(resid) \tag{2.14}$$

The equation (2.14) is solved numerically at each trajectory point as following:

1. Starting with the first trajectory time point at time=T(0). The control surface positions are initialized at zero or bias angles (δ_{S0i}), the engines are initialized at the pitch and yaw mounting angles (D_{Yeoi}, D_{Zeoi}), and the thrusts at nominal To(i). From the Mach number, the angles of attack and sideslip, and the engine positions we calculate the initial matrix $[DM]^0$ of equation (2.13), and the residual moment/ force vector M^0(residual) from the LHS of equation (2.12).

2. Solve the pseudo-inverse equation (2.14) for the trim angle increments (first iteration).
$$\delta_T^1 = diag(\delta_{MAX}) * Pseudo\ \ Inverse\left[DM\right]^0 \times M^0(residual)$$

3. Calculate new values for the control surface deflections, engine gimbal deflections, and thrusts by adding the effector increments and throttle values (obtained from the first iteration step-2) to their corresponding previous values, as shown.
$$D^1_{Y(i)} = D^0_{Y(i)} + \delta^1_{y(i)} \qquad D^1_{Z(i)} = D^0_{Z(i)} + \delta^1_{z(i)}$$
$$\Delta^1_{S(i)} = \Delta^0_{S(i)} + \delta^1_{s(i)} \qquad T^1_{(i)} = T^0_{(i)}\left(1 + D^1_{Thr(i)}\right)$$

4. Obtain new values for matrices $[DM]^1$ and M^1(residual) from equations (2.13) and (2.12), repeat step-2 for the same trajectory time point and solve for the new trim variables using equation (2.14), and repeat the iterations in steps 2 and 3, still for the same trajectory point until the trim angles converge to steady-state values.
$$\delta_T^2 = diag(\delta_{MAX}) * Pseudo\_Inverse\left[DM\right]^1 \times M^1(residual)$$

5. Select the next trajectory point at time=T(1) and repeat the same iterative process described in steps (1 - 4). Initialize using the trim angles from the previous trajectory time point and solve for the trim angles and throttle values at this point. Continue this process with the remaining trajectory points, all the way to T=T(n), and obtain a time history of the effector trim angles and throttle values as a function of trajectory time.

3. Static Performance Analysis Along a Trajectory

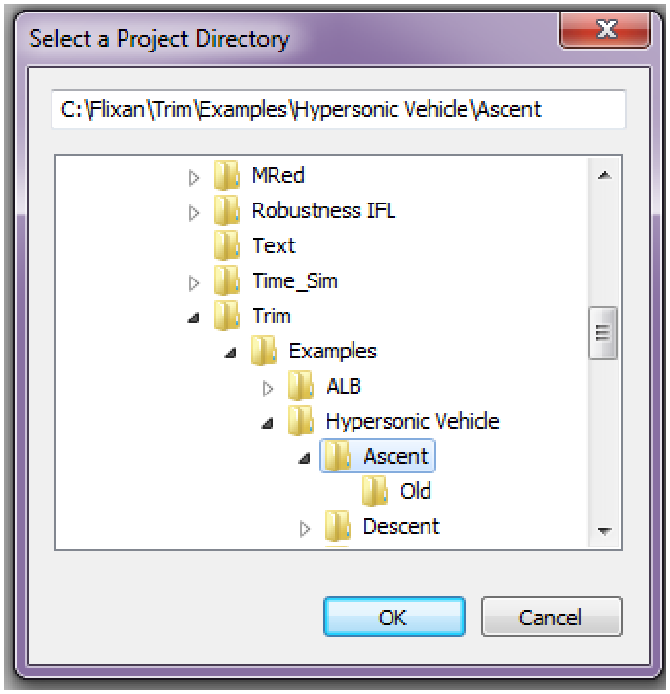
Before analyzing the dynamic characteristics of a flight vehicle the designer must first evaluate if the airframe meets the expected performance along a specified trajectory. Low airspeed and gusts place the greatest demands on control authority. In addition, agile maneuvers accomplished by frequent excursions into high angle-of-attack regimes and high roll performance can result in critical control power conditions, including adverse coupling effects. To achieve a successful design, it is important to assess the control power of a proposed design concept against the performance requirements early in the conceptual stage. The static and dynamic performance of the flight vehicle is captured in its data and its flying performance must be evaluated prior to any control analysis and simulations. The trajectory captures most of the mission requirements and the vehicle control authority, stability and maneuverability characteristics depend on the environment, the vehicle, and the effector data.

The purpose in this section is to define some important parameters that help the analyst to evaluate in a static sense the overall performance quality of a generic flight vehicle by processing the flight vehicle data along the trajectory as a function of time. This evaluation is not just for aircraft but it includes all types of flight vehicles controlled by aero-surfaces, TVC, throttling engines, and RCS jets. The performance parameters are calculated at each trajectory point as a function of the trajectory data, mass properties, aerodynamic coefficients for the vehicle and for the control surfaces, hinge moment coefficients, engine data, reaction control jets (RCS), vehicle geometry, and the control effector combination logic. The effector mixing logic defines the control allocation. That is, how the flight control system demands are converted to effector deflection commands and, therefore, it plays an important role in evaluating the performance. The aero-surface and engine deflections are also used in the performance calculations. For this reason the effectors must be trimmed prior to evaluating the vehicle performance. The performance parameters calculated are as follows: static stability (percent), center of pressure, Aerodynamic Center (along the x-axis), time-to-double amplitude in (sec), short period and Dutch-roll resonances in (rad/sec), $Cn\beta$-dynamic, the control authority of the effectors as a system to maneuver the vehicle against wind disturbances, the lateral control departure parameter (LCDP) which affects roll controllability, inertial coupling effects between axes due to fast maneuvering, hinge moments at the control surfaces which are needed for sizing the actuators, the bank angle due to cross-wind near landing and the sideslip angle β, and also the maximum control accelerations along the control axes. The Trim program provides the capability for the analyst to temporarily modify some of the input data, such as: mass properties, trajectory parameters, CG location, aero coefficients, etc, without exiting the program or editing the actual files. This facilitates the performance re-evaluation and in analyzing the vehicle sensitivity with respect to changes in the environment or vehicle parameters.

6. Generating State-Space Models For Linear Control Analysis

One of the most important functions of the Trim program is its capability of generating input data for the "Flight Vehicle Modeling Program". The FVMP is a Flixan program that calculates linear dynamic models for flight control analysis, simulation and design. After trimming the effectors the analyst can select some critical flight condition points along the trajectory and create dynamic models for those flight cases. The state-space modeling option can either be selected from the Trim main menu or from the menu bar on the top of a trajectory plot. The user is prompted to select a flight time along the trajectory and the program collects the vehicle data that correspond to this flight time from various file sources. The vehicle data are saved as a data-set in standard Flixan input file (.Inp) format and the FVMP reads them, processes them, creates the required dynamic systems in state-space form, and it saves them in a Flixan systems file (.Qdr). To run this program from a trajectory plot, go to the top menu bar of the trajectory window, click on "*Graphic Options*" and then "*Select Time to Create a State-Space System*". So let's take a look at the following example.

From Flixan you must select the project folder that contains the analysis files, such as: trajectory, mass, engine, and control surface aero data. The files must be in the proper format for the program to be able to read them. From the Flixan main menu select "*Analysis Tools*", then "*Flight Vehicle/ Spacecraft Modeling Tools*", and "*Trim/ Static Performance Analysis*".



7. Contour Plots of Performance Parameters Versus (Mach and Alpha)

In Section 3 we demonstrated how to calculate and plot as a function of the trajectory time some critical parameters relating to vehicle stability and performance, such as: static stability, time to double amplitude, lateral departure, control authority, etc. These parameters are functions of the trajectory parameters and also the trim angles at each point along the trajectory. Now let us suppose that some of the performance parameters do not meet our design requirements and we would like to modify the trajectory in order to improve vehicle stability or maneuverability, etc. This would be difficult to do because we wouldn't know how to modify the trajectory for better performance. The vehicle may also deviate from its expected trajectory and we would like to make sure that the planned trajectory is not close to regions of unacceptable performance. Plus the requirements in most of aircraft are not defined along a trajectory but they are defined over a range of speeds (Mach numbers) and angles of attack. So it seems that there is a need for a broader analysis and a capability that would expand our performance perspective in a wider range of Machs and alphas, rather than restricting the analysis in the vicinity of the trajectory. The trajectory variables that mostly affect vehicle performance are the Mach number and the angle of attack because the aero data strongly depend on them. So the Mach versus alpha combination is a good choice of variables along which to analyze performance because the vehicle parameters are heavily dependent on them.

Contour plots are 3-dimensional surface plots that provide a broader picture of how performance varies over the entire Mach versus Alpha range. The Mach number is plotted in the x-axis, the angle of attack (deg) in the y-axis, and the performance parameter under evaluation is plotted in the vertical z-axis, except that instead of using a 3-dimensional display the value of the performance parameter in the z-axis is color coded. The trajectory is shown as a black line traveling across the Mach versus alpha field. This wide array presentation of the performance parameters help the analysts to identify any potentially unacceptable locations and it points to a new trajectory path as necessary for maintaining an acceptable performance by avoiding the bad spots in the Mach versus alpha array. This is not always easy because there are other factors involved in the shaping of a trajectory, such as: payload maximization, structural loading, aero-heating, etc.

7.1 Running the Flixan Contour Plots Program

The easiest way to demonstrate the "Contour Plots" option is to pick an example and run it. In this example we will evaluate some of the performance of a hypersonic glider vehicle from Mach 4.5 all the way to landing using contour plots. Start the Flixan program and select the project folder that includes the analysis files, such as: trajectory, mass, aero data, aero uncertainties, engine

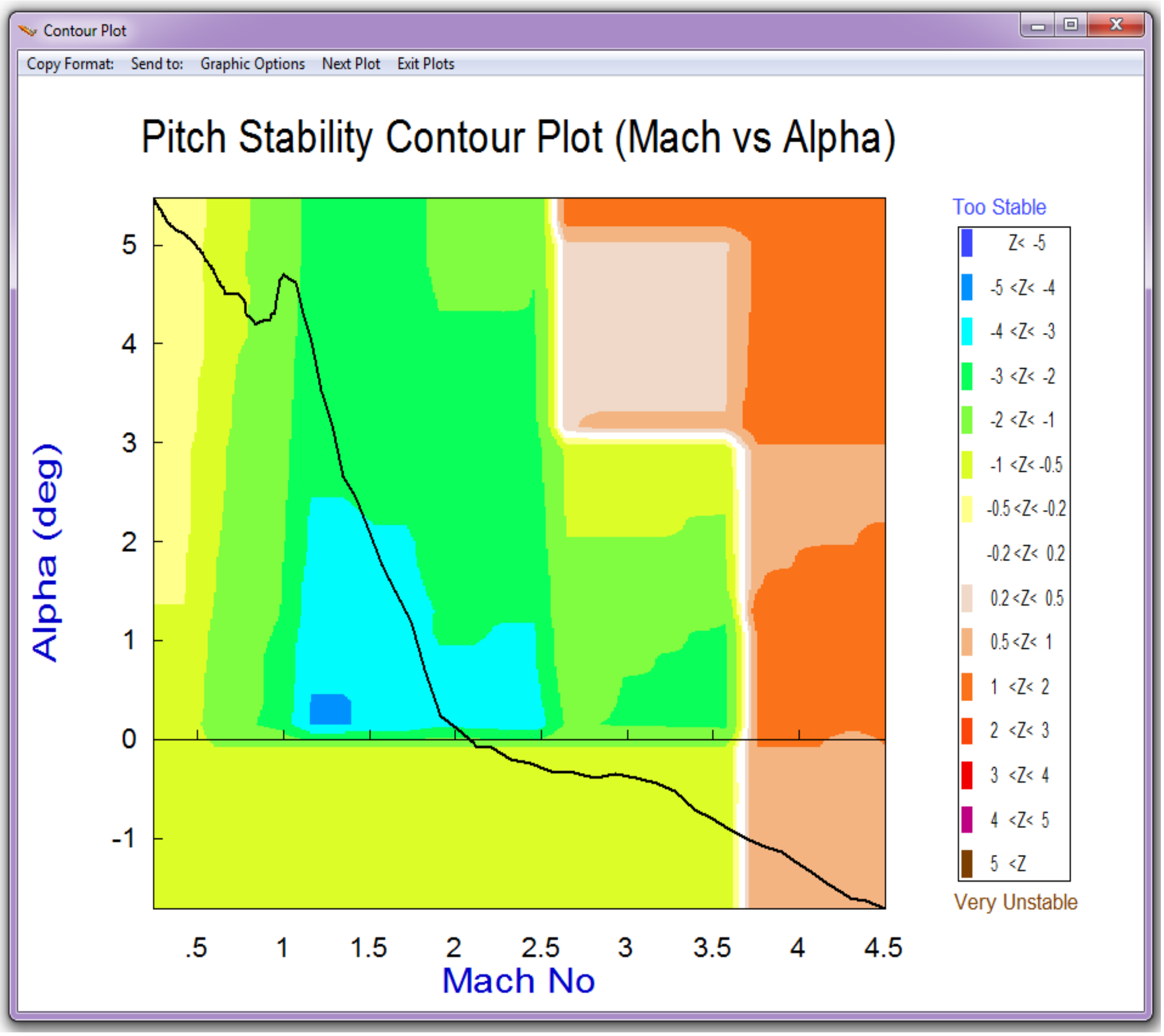
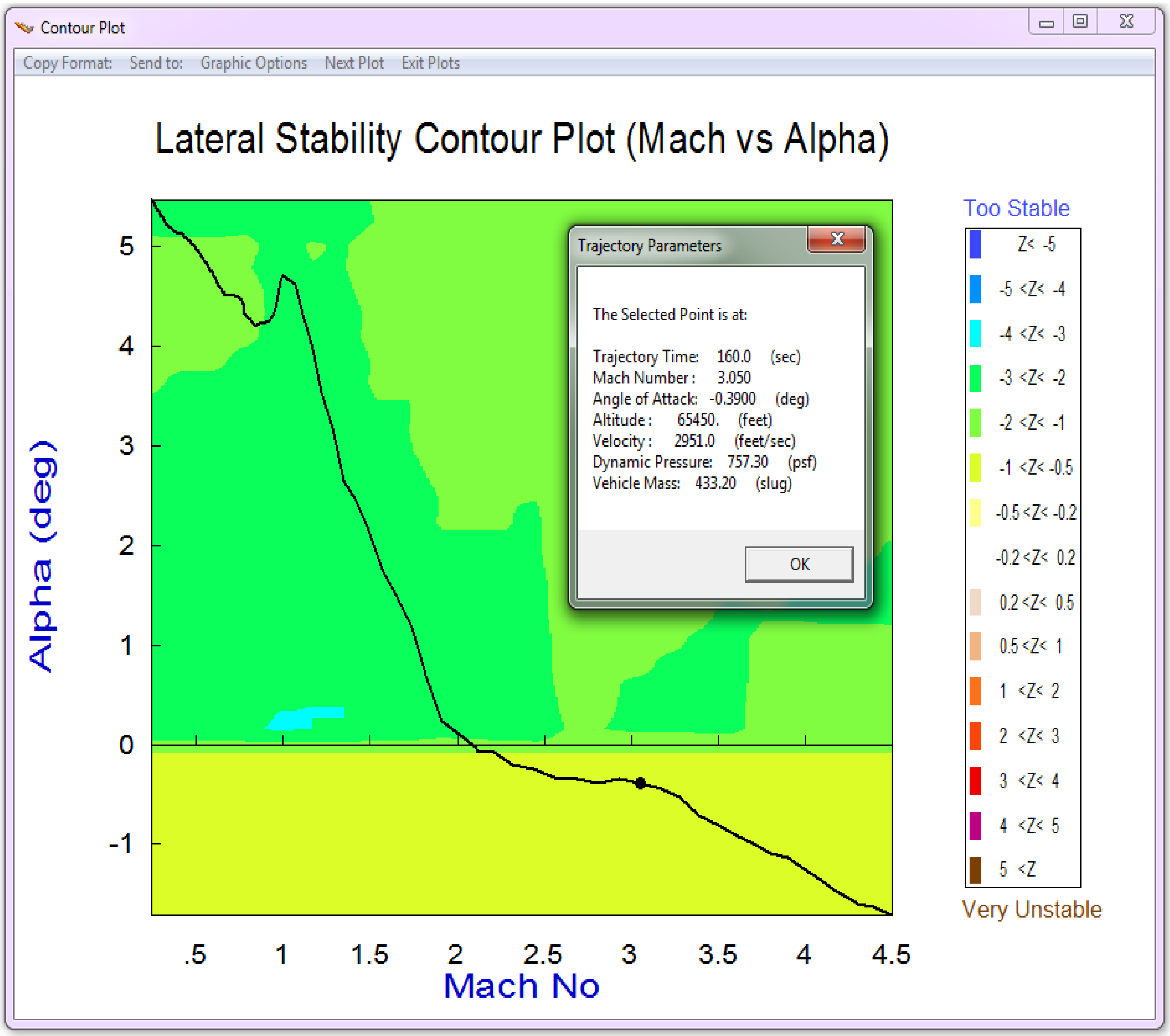


Figure 7.1 Contour Plot showing the vehicle Pitch Stability parameter along the trajectory as a function of Mach and Alpha. The trajectory initially passes through a region of instability and becomes stable below Mach 3.6

From the contour plots menu select the second option to plot the lateral stability parameter as it was defined by Equation (3.18). The color coding is the same as in pitch stability. The vehicle is more stable in the lateral direction without exceeding the acceptable stability range. The peak Dutch-roll resonance is 3.2 (rad/sec). The user may click at a point on the black trajectory curve and a dialog pops-up showing some important trajectory parameters corresponding to the selected point. Click on "Exit Plots" to return to the Contour Plots menu.



From the contour plots menu we may now select the "*Lateral Departure Ratio (LCDP)"* parameter which is shown in Figure (7.2). The LCDP ratio was described in Section (3.3). The color coding for the LCDP ratio is different from the coding used in the stability plots. The unacceptable (red) region is in the middle. Figure (3.3) shows the acceptable and the bad regions in the LCDP ratio as it was described in Section 3. Figure (7.3) defines the color coding used for the mapping of the various LCDP regions in the Mach versus Alpha contour plane. White is an ideal region because the roll/ yaw coordination is perfect. The red is a region to be avoided because this is where the aileron to roll-rate transfer function (3.29) becomes weak and the aileron is not reliable for roll control. It is likely to change sign because of the aero uncertainties. In Figure (7.3) the dark colored regions below the horizontal x axis is where the roll control in the transfer function is reversed. The vehicle may perform great in the grey, dark-yellow, and dark-cyan regions by reversing the roll control. The regions in the left side of the vertical axis are unstable. Lateral instability is usually avoided but in some cases it may be acceptable if it is not too close to the vertical axis and the actuators are fast. The purple/ magenta areas near the vertical axis (both dark and light) should be avoided because the beta transients are too big in those regions. In our hypersonic vehicle example the LCDP ratio contour plot, in figure (7.2), is almost ideal. There are no bad regions, no reversals, across the field and it is close to perfect coordination along the entire trajectory.

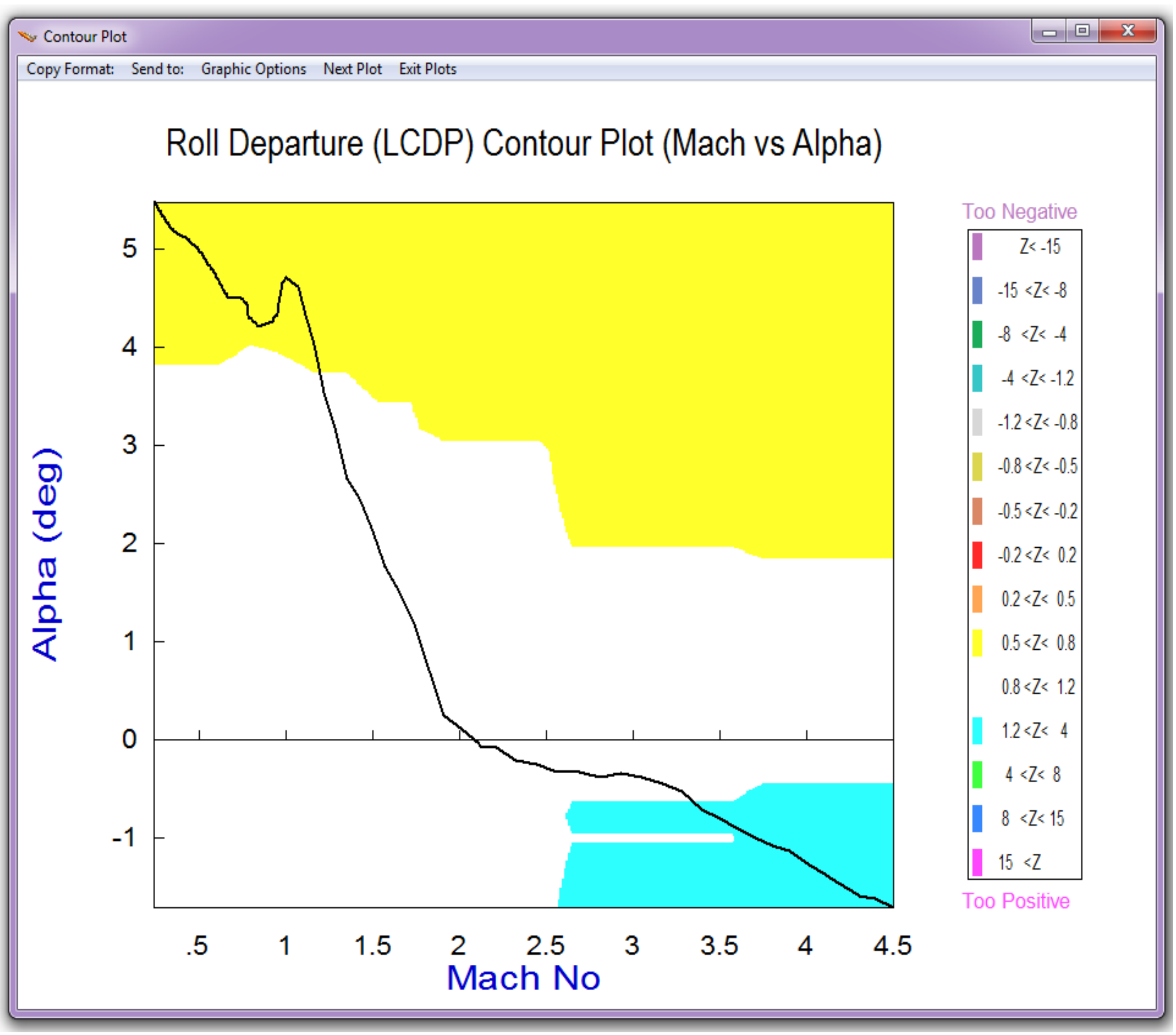


Figure 7.2 The LCDP ratio shows excellent roll performance across the Mach versus Alpha region

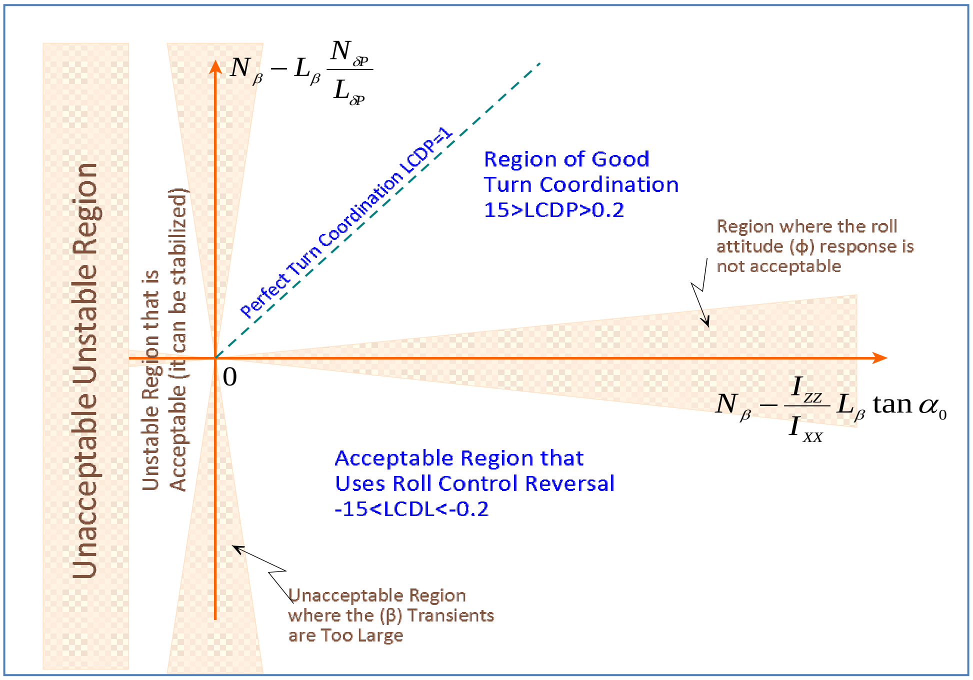


Figure (3.3)

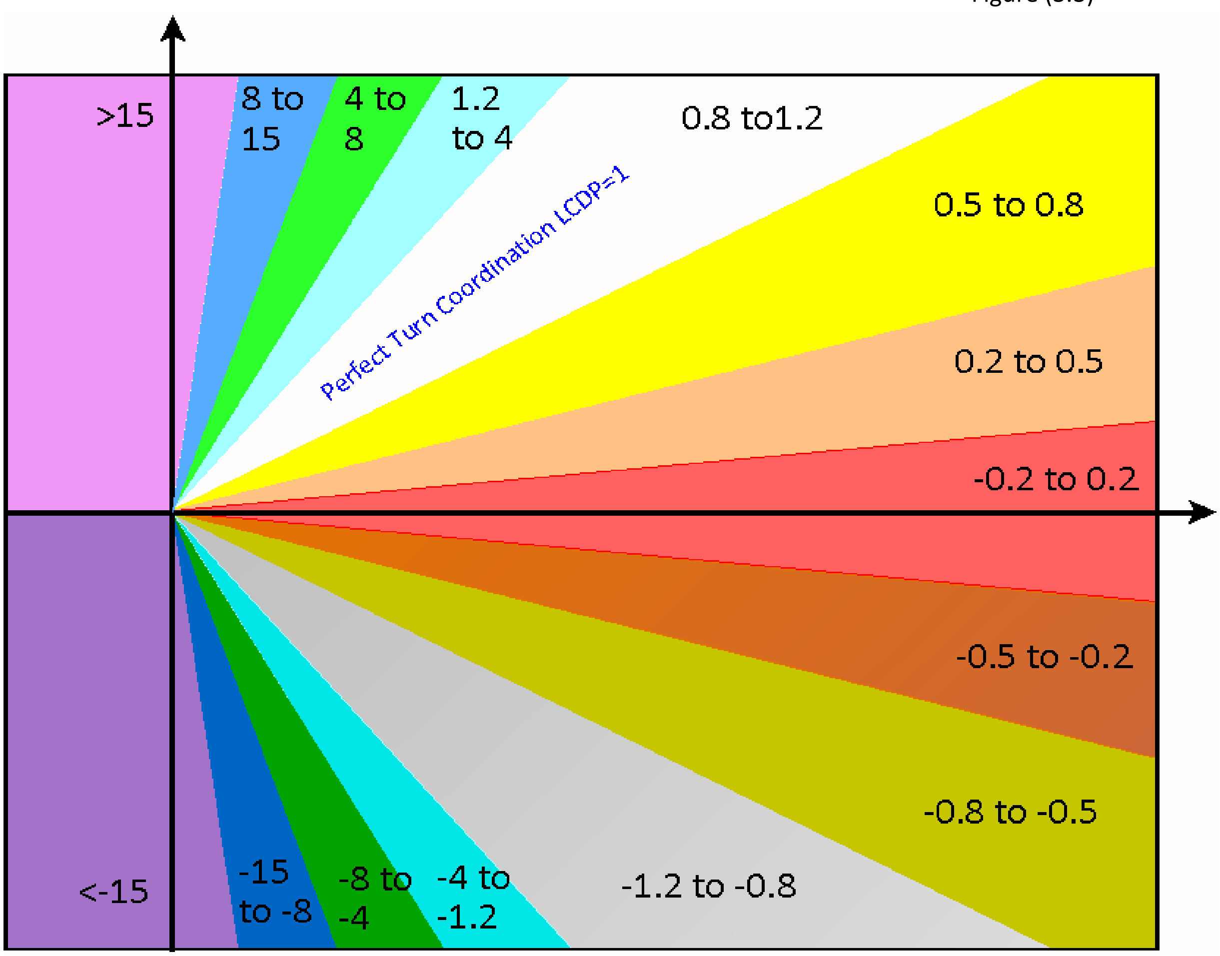
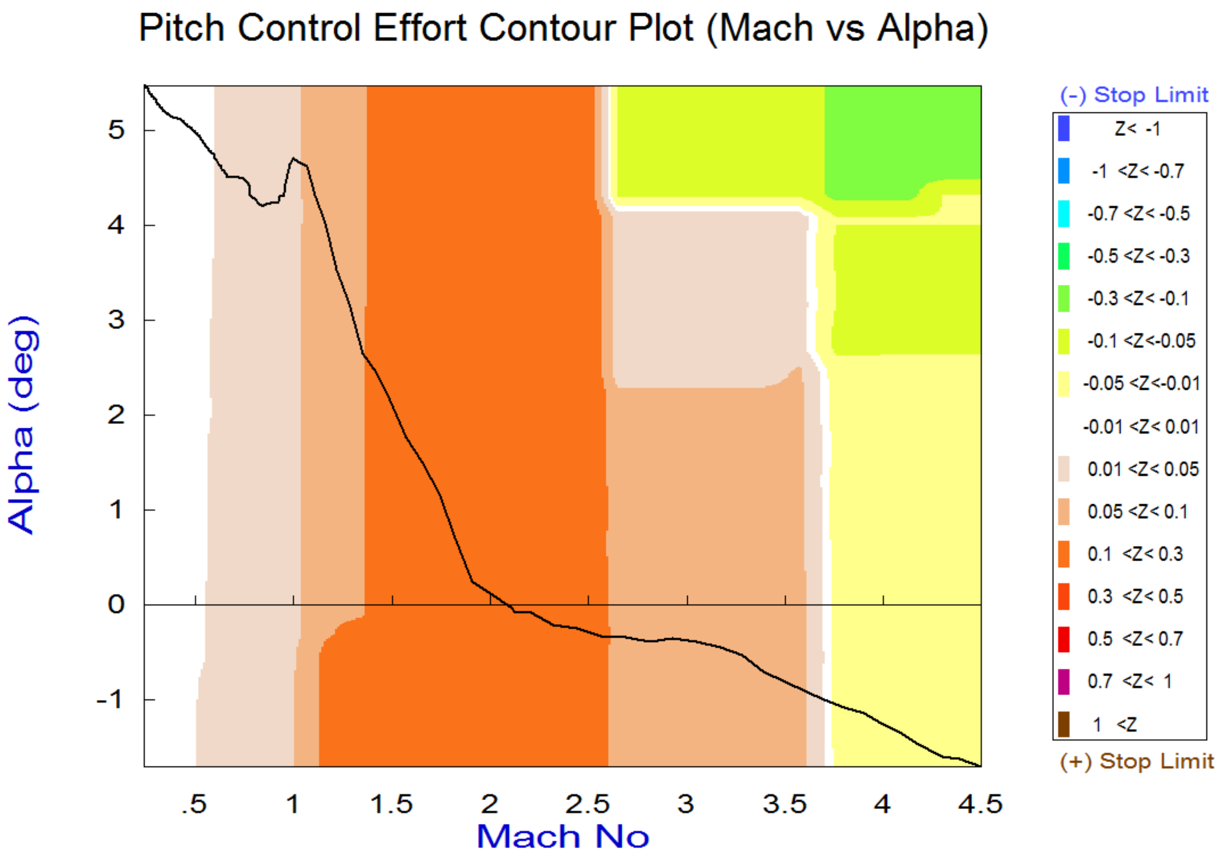


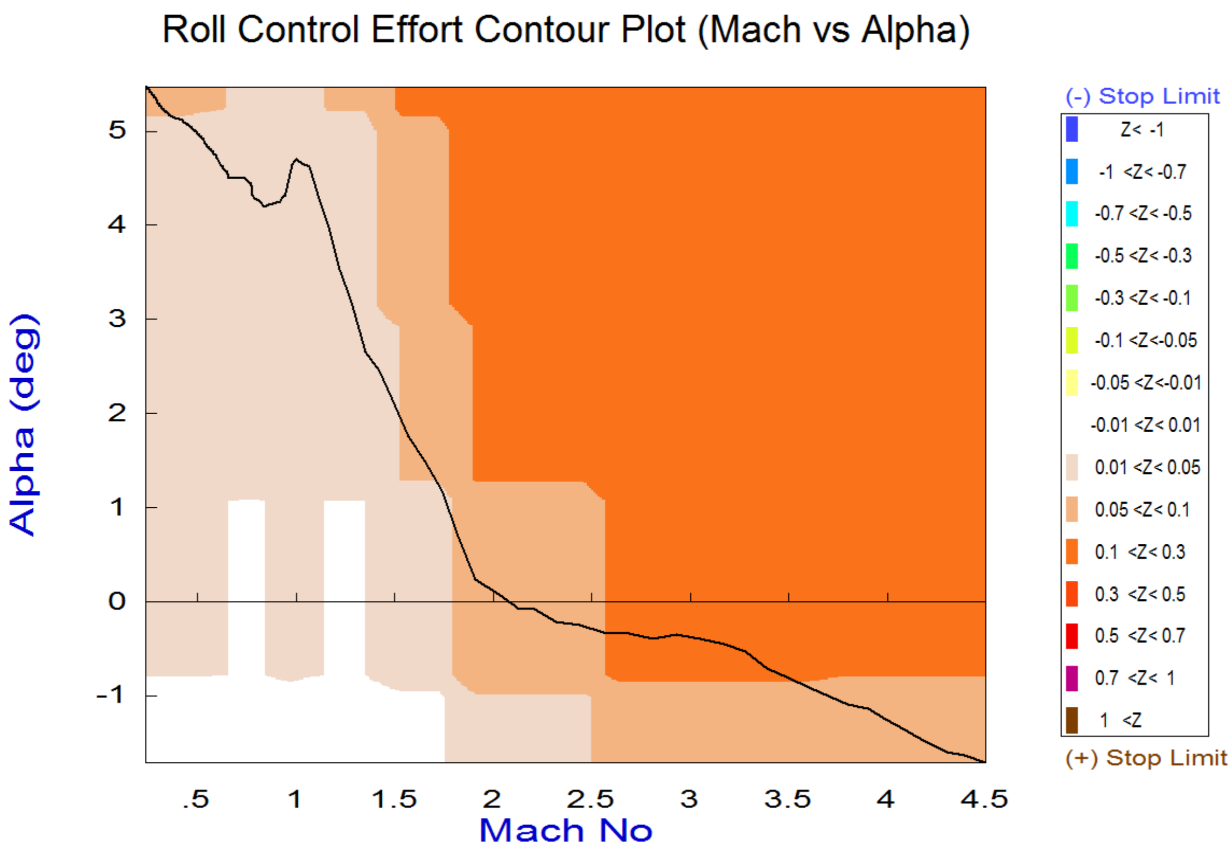
Figure 7.2 Color Coding of the LCDP Contour Plots showing the Good and Unacceptable Regions of the LCDP ratio. The dark color regions below the horizontal axis require Roll-Reversal.

We now return to the contour plots menu and select the last option which analyzes the effectors authority to control the vehicle against aero disturbances as it was described in equation (3.24). The effectors are evaluated collectively as a system combined together with the mixing-logic matrix. The control authority, therefore, strongly depends on the selection of (Kmix). Remember, in the beginning of the contour plots analysis we entered the magnitudes of (α_{max} and β_{max}) dispersions from trim that the vehicle may experience due to winds, maneuvering, etc. The control authority (or effort) in a certain direction is measured by the ratio of the control used against the aero disturbance divided by the maximum control authority in that direction. Its magnitude should obviously be less than 1, or even better, less than 0.5, to allow some control space for other functions, such as gusts, commands etc. The color coding used here in the control effectiveness contour plots is different from the previous color codes. There are obviously two control saturation limits, a positive limit of +1, and a negative limit of -1, corresponding to brown and dark blue colors respectively. White corresponds to regions where the control effort is very small and this happens when the vehicle is close to neutral stability. The next two contour plots show the pitch and roll control efforts. When the vehicle is stable an increase in alpha due to a strong wind shear will have a tendency to deflect the controls in the negative direction towards the yellow, greenish, blue colors and the pitch control will saturate when it reaches the -1 (dark blue) limit. When the vehicle is statically unstable the controls will rotate in the opposite direction towards the orange, red, magenta numbers and it will saturate when it reaches the +1 (brown) limit.

The ideal value of the control effort parameter should be in the white region which indicates that very little control power is needed to counteract the disturbance moments caused by an alpha/ beta wind-shear disturbance. The pitch effort plot shows that our trajectory, being initially stable and close to neutrally stable (in the high Mach and low alpha region), requires very little negative effort (light yellow) to trim the (α_{max}) disturbance. Also near the end before landing (in the low Mach and high alpha region) the control effort is very low there (white). In the intermediate Mach range between Mach# (1 to 3) where the vehicle is unstable the control effort against the (α_{max}) disturbance is in the positive direction and the pitch control effort reaching 0.3 which is very good.

The roll control effort contour plot is not that different. It shows the amount of effort required to counteract against the (β_{max}) disturbance. The user must click on the "Next Plot" option at the top menu bar to show the next plot, or click on "Exit Plots" to return to the previous menu. The yaw effort is not shown here. The user may also click on the trajectory curve (black line). A pop-up display shows some of the important trajectory parameters corresponding to the selected point.


Pitch Control Effort Contour Plot (Mach vs Alpha)
Alpha (deg)
Mach No
5
4
3
2
1
0
-1
.5
1
1.5
2
2.5
3
3.5
4
4.5
(-) Stop Limit
Z< -1
-1 <Z< -0.7
-0.7 <Z< -0.5
-0.5 <Z< -0.3
-0.3 <Z< -0.1
-0.1 <Z<-0.05
-0.05 <Z<-0.01
-0.01 <Z< 0.01
0.01 <Z< 0.05
0.05 <Z< 0.1
0.1 <Z< 0.3
0.3 <Z< 0.5
0.5 <Z< 0.7
0.7 <Z< 1
1 <Z
(+) Stop Limit


Roll Control Effort Contour Plot (Mach vs Alpha)
Alpha (deg)
Mach No
5
4
3
2
1
0
-1
.5
1
1.5
2
2.5
3
3.5
4
4.5
(-) Stop Limit
Z< -1
-1 <Z< -0.7
-0.7 <Z< -0.5
-0.5 <Z< -0.3
-0.3 <Z< -0.1
-0.1 <Z<-0.05
-0.05 <Z<-0.01
-0.01 <Z< 0.01
0.01 <Z< 0.05
0.05 <Z< 0.1
0.1 <Z< 0.3
0.3 <Z< 0.5
0.5 <Z< 0.7
0.7 <Z< 1
1 <Z
(+) Stop Limit

8. Vector Diagram Analysis

Vector diagrams are used for analyzing vehicle controllability in two directions at a fixed flight condition, and compare them against disturbance moments and forces in the same directions. They compare the control moments and forces generated by the effectors system, for example, roll and yaw, or pitch and normal acceleration, against those produced by a steady wind disturbance to assess if the vehicle has sufficient control authority to counteract the disturbance. The disturbance on the vehicle is defined by the maximum alpha and beta dispersion angles from trim (α_{max} and β_{max}) generated by a wind-shear relative to the trajectory's alpha and beta. The dispersion angles can also be due to maneuvering. A vector diagram not only compares magnitudes but it also allows us to examine the directions of the controls versus the disturbance directions. It helps us evaluate the orthogonality of the control system, compare the moments, forces, or accelerations due to the controls and wind disturbance, and to determine if the controls are powerful enough and in the proper directions for counteracting the disturbances along the controlled directions. They also display the effects of aerodynamic uncertainties around the vector tips and help the analyst in making decisions on whether to request more accurate data or to demand airframe modifications. Since our vector plots are limited to 2 directions we typically need several plots to analyze the control authority in multiple directions.

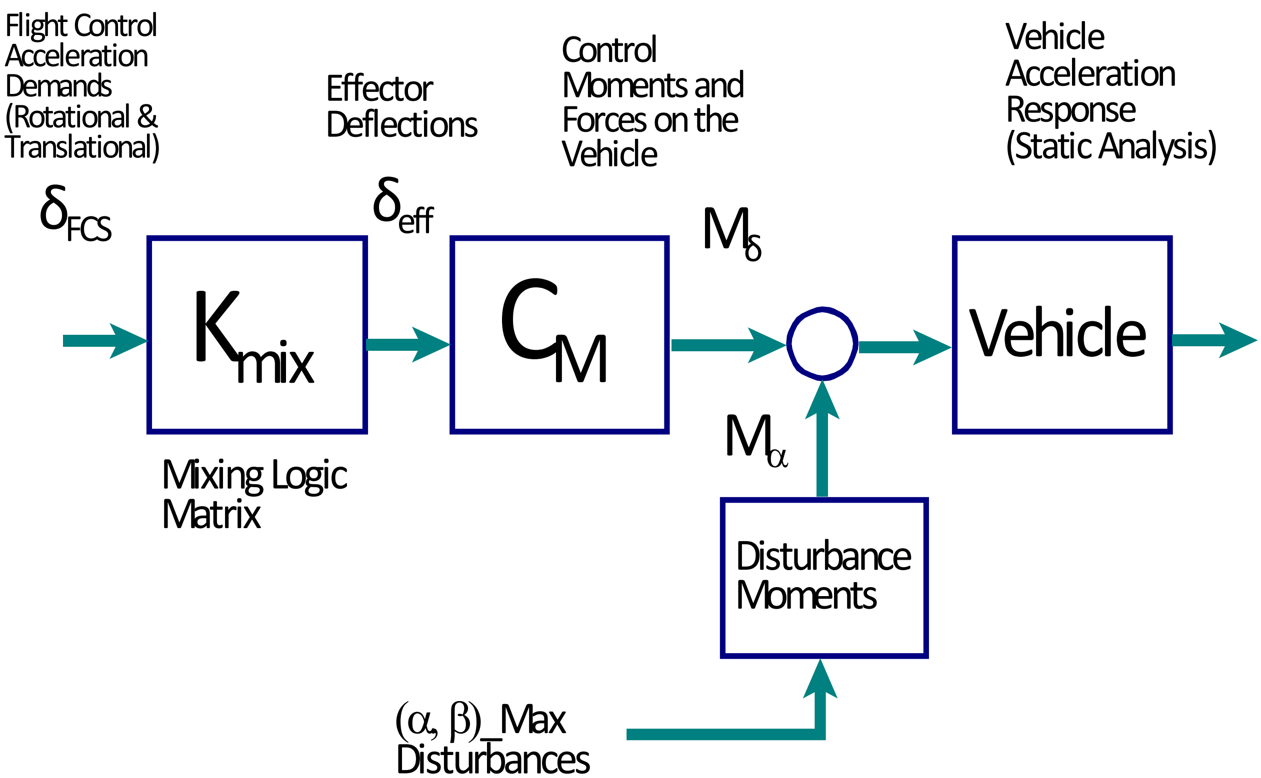


Figure (8.1) Elements used for Evaluating Vehicle Controllability by means of Vector Diagrams

The contributing elements in vector diagram analysis are shown in Figure (8.1). It is obviously an open-loop static analysis of the airframe alone at specific flight conditions and it is not related to closed-loop flight control analysis. On the left hand side we have the control demands coming from the flight control system which are demanding vehicle acceleration along some directions, mainly in roll, pitch, and yaw, and possibly along some translational directions. A mixing logic matrix translates the acceleration demands to effector displacements relative to their trim positions, which are converted to moment and force variation M_δ. The moment and force variation vectors generate the vehicle accelerations, hopefully, in the directions demanded by the FCS. At the bottom of figure (8.1) the analyst specifies the max expected (α_{max} and β_{max}) variations from trim that the vehicle should be able to tolerate in this flight condition. They generate disturbance moments and forces vector M_α and also disturbance accelerations. The Trim program uses four types of vector diagrams:

8.1 (a) Maximum Moment and Force Vector Diagrams.

The maximum moment/ force vector diagrams show the maximum control moments or forces generated by the effectors system in two directions when the FCS demands are maximized in the positive and in the negative directions. The FCS demands are maxed just before saturating the effectors. They are obtained from equation (3.10) when the FCS input demand is maximized in the positive ($\underline{\delta}_{+FCS\ Max}$) and in the negative directions ($\underline{\delta}_{-FCS\ Max}$).

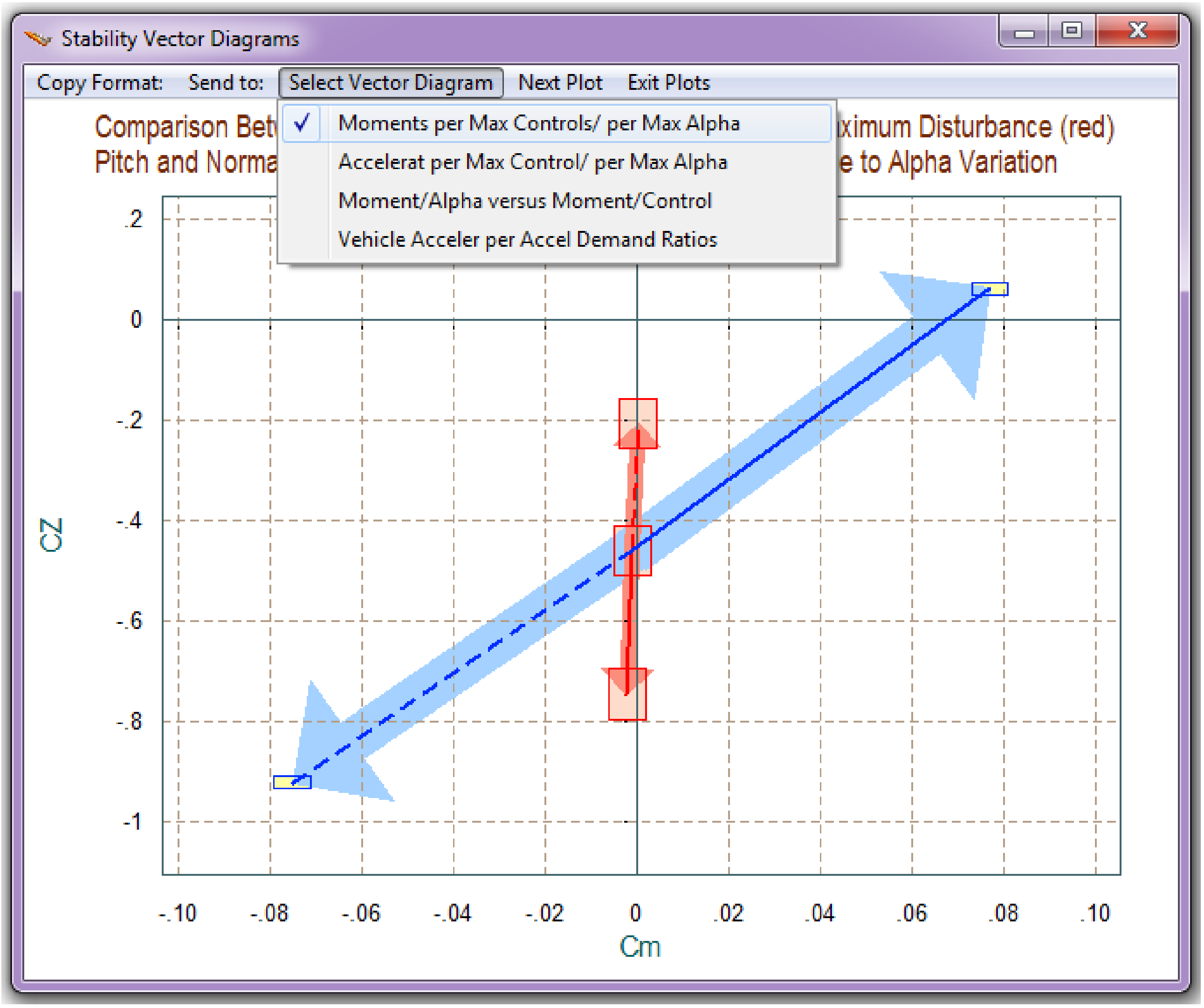
$$M_\delta = C_M\, K_{mix} \delta_{FCS}$$

$$\begin{bmatrix} L_X \\ M_Y \\ N_Z \\ F_X \\ F_Y \\ F_Z \end{bmatrix}_{+Max} = \begin{bmatrix} L_{\delta P} & 0 & L_{\delta R} & 0 & L_{\delta Y} & 0 \\ 0 & M_{\delta Q} & 0 & M_{\delta X} & 0 & M_{\delta Z} \\ N_{\delta P} & 0 & N_{\delta R} & 0 & N_{\delta Y} & 0 \\ 0 & F_{X\delta Q} & 0 & F_{X\delta X} & 0 & F_{X\delta Z} \\ F_{Y\delta P} & 0 & F_{Y\delta R} & 0 & F_{Y\delta Y} & 0 \\ 0 & F_{Z\delta Q} & 0 & F_{Z\delta X} & 0 & F_{Z\delta Z} \end{bmatrix} \begin{bmatrix} \delta_P \\ \delta_Q \\ \delta_R \\ \delta_X \\ \delta_Y \\ \delta_Z \end{bmatrix}_{+FCS\ Max} \quad \textit{for}\ \max\ \textit{positive demands}$$

$$\begin{bmatrix} L_X \\ M_Y \\ N_Z \\ F_X \\ F_Y \\ F_Z \end{bmatrix}_{-Max} = \begin{bmatrix} L_{\delta P} & 0 & L_{\delta R} & 0 & L_{\delta Y} & 0 \\ 0 & M_{\delta Q} & 0 & M_{\delta X} & 0 & M_{\delta Z} \\ N_{\delta P} & 0 & N_{\delta R} & 0 & N_{\delta Y} & 0 \\ 0 & F_{X\delta Q} & 0 & F_{X\delta X} & 0 & F_{X\delta Z} \\ F_{Y\delta P} & 0 & F_{Y\delta R} & 0 & F_{Y\delta Y} & 0 \\ 0 & F_{Z\delta Q} & 0 & F_{Z\delta X} & 0 & F_{Z\delta Z} \end{bmatrix} \begin{bmatrix} \delta_P \\ \delta_Q \\ \delta_R \\ \delta_X \\ \delta_Y \\ \delta_Z \end{bmatrix}_{-FCS\ Max} \quad \textit{for}\ \max\ \textit{negative demands}$$

The max moment/ force vector diagrams also show the disturbance moments and force vectors produced by the ($\pm\alpha_{max}$ and $\pm\beta_{max}$) dispersions from trim. The control vectors are then compared against the disturbance vectors and this helps the analyst to decide on how well the vehicle can be maneuvered and also capable to handle wind disturbances.

This type of diagrams are selected from the horizontal menu bar located on the top of the vector diagrams window. Click on "*Select Vector Diagrams*" and then from the vertical pop-up menu select "*Moments per Max Controls, and per Max Alpha*", as shown in the CZ versus Cm figure below.



The next figure shows a roll/ yaw vector diagram (Cl versus Cn). The solid blue vector corresponds to max positive yaw FCS demand (δR_{+FCS_Max}) and the dashed blue vector corresponds to max negative yaw demand (δR_{-FCS_Max}). Similarly, the green vectors correspond to the ±max roll FCS demands (δP_{+FCS_Max} and δP_{-FCS_Max}). The two red vectors represent the roll and yaw moments generated by the wind disturbance defined in terms of angles of attack and sideslip ($\pm\alpha_{max}$ and $\pm\beta_{max}$) variations from their trim positions. The dispersion angles typically range between 2° to 5° depending on the flight condition. The solid red arrow in figure (8.2) shows an increase in yaw moment when the angles of attack and sideslip are increased from (α_{trim}) to ($\alpha_{trim}+\alpha_{max}$) and from ($\beta_{trim}$) to ($\beta_{trim}+\beta_{max}$). The effect is mainly due to the sideslip and demonstrates that the vehicle is statically stable in yaw because it turns towards the wind. The dashed red vector shows the moments when the angles of attack and sideslip are reduced from (α_{trim}) to ($\alpha_{trim}-\alpha_{max}$) and from

(β_{trim}) to (β_{trim}-β_{max}), β_{trim}=0 in this case. The moments are non-dimensional. Notice that the yaw control vector does not couple into roll but the roll control has some component in yaw. This is not accidental but it is caused by the mixing logic which provides lateral decoupling to compensate against the vehicle Ixz cross-product of inertia. In the vector diagrams we like to see the two control vectors perpendicular to each other and their magnitudes bigger than the red disturbance vectors, as in this case.

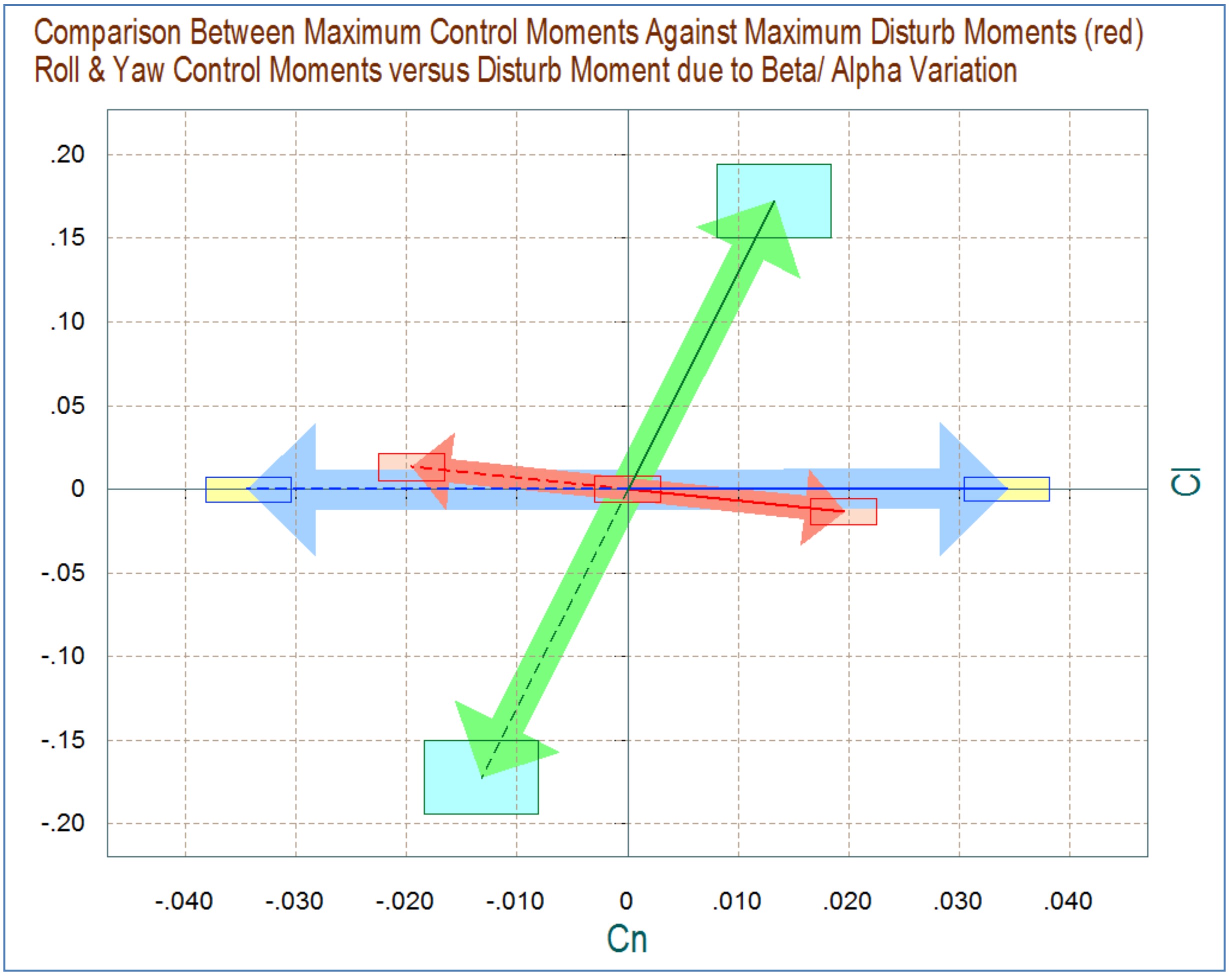
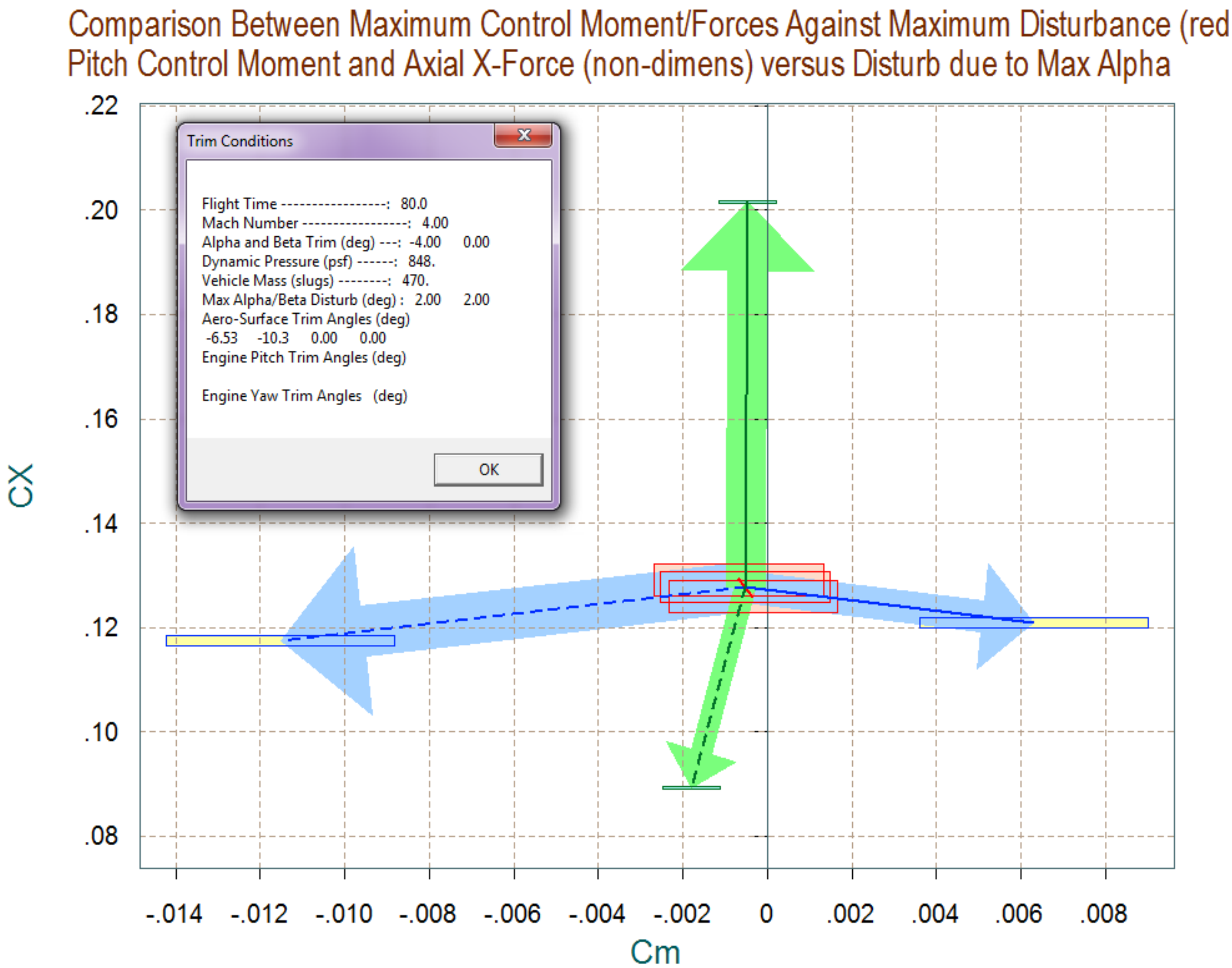


Figure (8.2) Maximum Moments Vector Diagram in the Roll and Yaw Plane

The square rectangles at the tip of the vectors represent the effect of the aero uncertainties, the bigger the uncertainties the larger the rectangles. The uncertainties are read from the uncertainties file that has an extension (.Unce). It is not necessary to provide an uncertainties file. If the file is not available only the vectors will be shown without the rectangles at their tips. The reddish uncertainty rectangles at the tips of the disturbance (red) vectors represent the possible variations of that vector due to the uncertainty in aero coefficients (C_l and C_n). The yellow rectangles at the tips of the yaw control vector (blue) represents the possible variation of this vector due to the uncertainties in the control surface derivatives, which are mainly the rudder ($C_n\delta_{rudd}$), but also the aileron ($C_n\delta_{ailr}$) is contributing because they combine. Similarly, the cyan blocks at the tips of the roll control vector (green) represent its possible variation due to the derivative uncertainties, mainly the aileron ($C_l\delta_{ailr}$), but the rudder ($C_l\delta_{rudd}$) is also contributing in roll. The uncertainty rectangles should be small enough to preclude the possibility that the disturbance forces may overcome the controls in neither direction.



The above vector diagram shows the vehicle controllability in pitch moment and x-force directions. This time the vectors are not symmetrical as it was in the lateral directions. It is twice easier to torque and to rotate the vehicle in the negative pitch direction when you apply a negative pitch control (δQ_{-FCS_Max}) than it is to rotate it in the positive direction with positive control (δQ_{+FCS_Max}). This is because the vehicle is statically unstable in this flight condition and it is flying with a negative α= -4°, and therefore, its natural tendency would be to rotate in the negative direction and to produce more negative pitch acceleration. Notice also that δQ_{-FCS_Max} is not necessarily equal to $-\delta Q_{+FCS\_Max}$ because of the biases in the effectors. The vehicle has a throttling engine that provides control in the x direction. At nominal thrust the axial force is C_X=0.128 which means that it is accelerating. There is also unsymmetry in the x-force direction due to the effector mixing (throttling engines and surfaces). The force increase is greater when applying a maximum positive throttle (δX_{+FCS_Max}), than it is when slowing down by applying a negative throttle (δX_{-FCS_Max}).

10. Design Examples

The following examples are included in this document for the purpose of demonstrating the capability of the Trim program to analyze a variety of flight vehicle types and to create dynamic models for control analysis and design.

1. **Hypersonic Rocket-Plane Analysis during Ascent and Descent phases.**
2. **F-16 Fighter Aircraft Analysis, Control Design, and 6-dof Simulation**
3. **Air-Launched Vehicle with Wings and Tails Design in Multiple Phases**
4. **Re-Usable Launch Vehicle with Multiple-Engines During Ascent and Descent**
5. **Lifting-Body Aircraft Descent from Space, Vertical Take-Off, and 6-dof Simulation**
6. **Re-Entry Vehicle Design and Analysis Using Aero-Surfaces and RCS Jets**

The examples presented, in addition to trimming and performance analysis, they demonstrate also the following methodologies and capabilities of the Flixan/ Trim program.

- Capability of the Trim algorithm to automatically allocate trim control authority based on the individual effector capabilities in specific directions.
- How to graphically modify the trimming conditions and to trade off activity of some effector against others in situations where multiple effectors are available.
- How to analyze the effects of parameter dispersions on vehicle performance.
- Generating dynamic models at selected flight conditions along the trajectory.
- Using the Flixan program to create various types of design models for synthesizing control systems, time-domain simulations, models for stability analysis using Matlab.
- Using Flixan to generate models for Robustness Analysis to Structured Uncertainties, and how to use these models to perform μ-Analysis.
- Using Flixan derived models to synthesize flight control laws. Evaluate control designs using analysis and simulations. The dynamic models, vehicle data files, design software, detailed methodologies, Flixan & Matlab files, etc, are included in the example packages.
- Developing 6-dof Non-Linear Simulations using Flixan derived control laws, interpolated between design points. Using the 6-dof simulations to generate trajectories.
- Demonstrating interactive graphics using vector diagrams, contour plots, overlays, user interactive trajectory modifications, interactive features (menus, dialogs, etc).
- How to efficiently combine effectors in vehicles that use multiple types of effectors by creating effector combination matrices that produce the demanded accelerations, and reduce the dynamic coupling between the control axes.
- Huge amount of flight vehicle design information is available, including data files, Matlab scripts, Simulink files, and 6-dof non-linear simulations.

The user/ analysts are encouraged to study, repeat, and run some of these examples in their own computers in order to familiarize themselves with the process and tool. They may begin with the examples which are similar to their flight vehicle application and gradually modify them by introducing data from their own applications.