## Air Launched Rocket

In this example we have a booster rocket that is launched to orbit after being dropped from a carrier aircraft. The rocket has two stages and it carries an upper-stage payload. It is released from an altitude of $20,000(\mathrm{ft})$ and it glides like an airplane for a while by means of a system of wings and tails that help it separate from the aircraft, provide lift and prevent it from losing altitude, and pointing it upwards before igniting a $400,000(\mathrm{lb})$ booster engine. The wings and tails remain attached on the rocket for another 20 seconds providing lift and helping it to achieve a desirable flight path angle $(\gamma)$. They separate at $\mathrm{T}=90$ seconds and the vehicle continues to ascent into orbit like a typical booster rocket. At $\mathrm{T}=500$ seconds the first stage separates and the second stage ignites. During the initial unpowered glide phase the vehicle is controlled by two elevons and a rudder located in the tail section. The wing has no controls. During the boost phase the vehicle is controlled by the TVC engine in pitch and yaw and also by the aero-surfaces. After the wings and tails separate a pair of RCS jets is used to control the vehicle in roll.

This operation consists of several phases, but in this example we will analyze only the first three phases during first stage:

1. Unpowered Glide Phase. This phase is unpowered, from $\mathrm{T}=(0$ through 72$)$ seconds. That is, from the carrier aircraft separation to first stage engine ignition. The wing is attached on the vehicle and it uses three aero-surfaces for flight control which are located in the tail section. The aileron for roll control, the elevon for pitch, and the rudder for yaw control.
2. Boost Phase with Wings and Tails: This phase runs between $T=(72$ to 90$)$ seconds, from stage1 ignition to wing and tails separation. The TVC (thrust vector control) system is turned on while the vehicle retains its wings and tails for another 18 sec . In this phase the flight control system, in addition to the elevon and rudder, it also uses the TVC for pitch and yaw control. The aileron is used for roll control in combination with the rudder and TVC.
3. Boost Phase without Wings and Tails: This phase begins at wing and tail separation to first stage burnout. That is, between $\mathrm{T}=(90$ to 150$)$ seconds trajectory time. During this phase the rocket uses the TVC engine for pitch and yaw control and the two RCS jets for roll control.

This is a great example for demonstrating the capability of the Flixan/Trim program to analyze a vehicle that gradually transforms itself from an airplane to a rocket. The analysis data files are in directory "C:\Flixan\Trim\Examples\Air Launched Vehicle" and a detailed analysis will be performed for each phase in separate subdirectories. The analysis consists of trimming the effectors, stability, controllability against aero-disturbances, and an overall performance evaluation will be performed for each of the three phases separately. In addition, dynamic models will be created at specific flight conditions. The models will be used to design control laws, and the dynamic models will be used to perform Matlab simulations for each phase.


Figure 1 Air-Launched Rocket Vehicle

## Unpowered Glide Phase with Wings and Tails

After being dropped from the carrier aircraft at an altitude of $24,000(\mathrm{ft})$, during this phase the vehicle is gliding down using its wing and tails without an engine power for 72 seconds to a lower altitude. However, it is gaining speed and increasing its flight path angle before igniting the TVC engine. It is controlled by the Aileron, Elevon, and Rudder which are located in the tail section. The wings have no controls. The data files for the glide phase are in the subdirectory: "C:|Flixan | Trim \Examples\Air Launched Vehicle\Phase A Glide With Wing". It includes the mass properties in file "Air-Launch.Mass", which contains weights and inertias for all phases. The mass data in the top line correspond to the glide phase and also the trajectory file "AL_Glide.Traj" which contains the trajectory parameters between zero and 72 seconds. The mass is constant ( 10,550 slugs) and the thrust is zero during this unpowered phase. The file "Wing_On_Bas.Aero" contains the basic aero data with the wings and tails attached, which corresponds to 6 Mach numbers, 13 angles of attack, and 3 sideslip angles. The file "AL_AeroSurf.Delt" contains the aero-surface increment coefficients for the Elevon, the Aileron, and the Rudder. There is no thruster data file for this phase. The file "Kmix3.qdr" is just a scaling and aero-surface re-ordering matrix that will be used later in the analysis. The files "Glide_Phase_T20.Inp" and "Glide_Phase_T20.Qdr" contain the vehicle input data and dynamic models that will be used later in the closed-loop dynamic analysis.

## Trajectory Analysis

Let us begin the analysis by taking a look at the trajectory. Start Flixan and select the project folder that contains the analysis files. Then, from the Flixan main menu select "Analysis Tools", "Flight Vehicle/ Spacecraft Modeling Tools", and then "Trim/ Static Performance Analysis". From the Trim filename selection menu select the following files.


## Angles of Attack/Sideslip/Flight Path (deg), Air Launched Vehicle Firs


Sensed Acceleration in（ft／sec＾2），Air Launched Vehicle First Stage Glide T



Aero Lift／Drag Forces，Eng．Thrust in（lb），Air Launched Vehicle First Stage



The above plot shows the roll angle ( $\phi$ ) that would occur when trimming along the trajectory at $4^{\circ}$ of sideslip ( $\beta$ ), due to cross-wind. The LCDP is acceptable for the most part. It is slightly low at high alphas which moderately reduces the roll maneuverability, but it does not change sign, which is good. The Cn-beta-dynamic is positive which indicates lateral stability. Its minimum value is around 0.003.

## Contour Plots

Contour plots are 3-dimensional plots of some vehicle performance parameters versus alpha and Mach. They provide a broader performance perspective in the entire Mach versus alpha range, rather than in the vicinity of the trajectory. Occasionally the vehicle may deviate from its nominal alpha versus Mach trajectory and we would like to make sure that the trajectory is not close to regions of unacceptable performance. They also help reshaping the trajectory towards regions of improved performance. From the main menu select option (10) to create contour plots of some parameters against Mach and Alpha. Similar to our previous performance analysis we must specify the aero disturbance environment which in steady-state is defined by the maximum angles ( $\alpha_{\max }$ and $\beta_{\max }$ ) relative to the wind. Enter $4^{\circ}$ for both $\alpha_{\text {max }}$ and $\beta_{\text {max }}$, as shown below. The program also requires the aero-surface combination matrix (Kmix3) from file Kmix3.Qdr.

Pitch Stability Contour Plot (Mach vs Alpha)


Very Unstable

## Lateral Stability Contour Plot (Mach vs Alpha)



Pitch Control Effort Contour Plot (Mach vs Alpha)


Yaw Control Effort Contour Plot (Mach vs Alpha)


Roll Control Effort Contour Plot (Mach vs Alpha)


(+) Stop Limit

Roll Departure (LCDP) Contour Plot (Mach vs Alpha)


Too Negative


Too Positive

## Controllability Analysis Using Vector Diagrams

Vector diagrams are used for visualizing the vehicle controllability against aerodynamic disturbances which are caused due to alpha and beta dispersions from trim. They are two-dimensional vector diagrams that compare the magnitudes and directions of the vehicle's response to flight control demands against its response to alpha and beta disturbances. The disturbance vectors must be small in comparison with the control vectors. Assuming that the initialization files and trim conditions are the same, from the main menu select option (11) for plotting vector diagrams. Select a flight condition at time=20 seconds, and from the following menu you may either keep the default values of Mass, Mach, $\alpha$, and $\beta$ which correspond to the selected time, or modify them to different values. In this case we select the default values.


We must also select the maximum alpha and beta dispersions like before, and the mixing-logic matrix which in this case is just a surface reordering and scaling matrix.


Select a Gain Matrix


KMIX3
Mixing Logic for Air Launched Vehicle First Stage, Glide Phase

Two vector diagrams are shown below. The first one shows the max roll and yaw moments Cl and Cn (green and blue vectors respectively) due to the aileron and rudder at maximum deflections, in both positive (solid line) and in the negative (dashed line) directions. Both the aileron and rudder surfaces excite both, the roll and yaw directions. Despite the cross-coupling the control vectors are nicely orthogonal to each other, which is very good property. The disturbance moments due to $\pm \alpha_{\text {max }}$ and $\pm \beta_{\max }$ are shown by the red vectors. A positive $\beta_{\max }$ causes negative rolling moment (solid red vector pointing down). Their magnitudes are significantly smaller than the moments due to the aerosurfaces. The second vector diagram shows the roll and yaw moment partials per roll and yaw acceleration demands ( $\left.\delta_{p} \& \delta_{R}\right)_{\text {Fcs }}$, green and blue vectors. The red vectors also show the roll and yaw partials per beta disturbance ( $\mathrm{Cl} \beta, \mathrm{Cn} \beta$ ), calculated at two extreme alphas. The control vectors are scaled as described in equations ( 7.1 through 7.4). As you can see, the control partials dominate significantly over the aero disturbance partials.

Comparison Between Maximum Control Moments Against Maximum Disturb Moments (red) Roll \& Yaw Control Moments (non-dimension) vers Disturb Moment due to Max Beta/Alpha


Comparison Between Control Moment Partials Versus Aero-Disturb Moment Partials (red) Yaw \& Roll Control Partials Cn/delta_R and Cl/delta_P versus Cn/beta \& Cl/beta


## Wind-Gust Excitation

In this case the vehicle model is excited with a 30 ( $\mathrm{ft} / \mathrm{sec}$ ) wind-gust impulse, shown below. The gust excites attitudes and rates in all directions, which are eventually damped by the control system.





## Gamma-Command Simulation

Now the vehicle model is commanded to increase its flight-path angle gamma by one degree from steady-state. All other commands and excitations are set to zero. This causes a steady increase in altitude and a drop in velocity. There is also a positive Elevon deflection transient that decays to zero. This causes a positive alpha and normal acceleration transients which increase gamma as commanded.







# First Stage Boost with Wings and Tails 


#### Abstract

At time $=71.6$ seconds the $400,000(\mathrm{lb})$ first stage engine ignites, and the vehicle accelerates for 18.5 seconds with the wings and tails still attached. During this phase the flight control system uses both, the aero-surfaces and the TVC. The angle of attack remains positive and the wings help increase gamma to $55^{\circ}$ before they are ejected at Mach 0.9 . During pull-up the vehicle experiences a $1.5(\mathrm{~g})$ normal load. The data files for this phase are similar to the glide phase and they are located in the subdirectory: "C:\Flixan\Trim\Examples\Air Launched Vehicle\Phase B Boost With Wing ". This folder also includes the TVC engine data in file "AL_Stg1.Engn". The main engine has a 400,000 (lb) thrust. Its thrust direction is along the $x$ axis. It can gimbal up to $5^{\circ}$ in pitch and yaw, and it can vary its thrust $\pm 60 \%$ from nominal. The file Kmix4.Qdr contains the effector combination mixing logic matrix Kmix4b. This matrix converts the four flight control demands (which are: roll, pitch, yaw, and axial acceleration), to effector deflection commands, (which are: TVC pitch and yaw deflections, engine throttle command, Elevon, Aileron, and Rudder deflections).


## Trajectory Analysis

We can first take a look at the trajectory as we did for the glide phase. Start Flixan, select the project folder "C:\Flixan\Trim\Examples\Air Launched Vehicle\Phase B Boost With Wing", and then from the Flixan main menu select "Analysis Tools", "Flight Vehicle/ Spacecraft Modeling Tools", and "Trim/ Static Performance Analysis". From the Trim filename selection menu select the following files, and from the next filename selection menu select the file Kmix4.Qdr that contains the effector combination matrix. From the Trim main menu select the $2^{\text {nd }}$ option for plotting the trajectory data.

## Main Trim Menu

Select one of the following options
Exit

1. Plot Aero Coefficients, Derivatives, and Control Surface Increments
2. Plot Trajectory Parameters Versus Time from the Trajectory File ".Traj"
3. Trim the Effector Deflections to Balance the Vehicle Moments and Forces
4. Create an Effector Mixing Logic or a TVC Matrix (Kmix)
5. State-Space Modeling of the Flight Vehicle at Selected Times
6. Performance and Stability Parameter Plots Along Trajectory Time
7. Landing and Pull-Up Maneuverability, plus, Inertial Coupling Effects
8. Moments at the Hinges of Control Surfaces Along the Trajectory Time
9. View and Modify Vehicle Data (CG, MRC, TVC, Surfaces) for Dispersion Analysis
10. Contour Plots (Mach versus Alpha) for Performance, Control Authority Analysis
11. Vector Diagrams for Maneuverability \& Stability at Selected Flight Conditions
12. Plot and Compare Previous Data Files (Traject, Trim, Perform, Hinge Moment)


We must also define the max alpha and beta excursions from nominal due to winds and also for maneuvering. This is required for determining the maneuverability and control authority of the vehicle. In this case we pick $3^{\circ}$ for both angles.

## Maximum Alpha Capability

$$
\begin{aligned}
& \text { The control effectors must be capable of varying the vehicle } \\
& \text { angles of attack and sideslip (typically } 3-5 \text { deg) from their } \\
& \text { trim values. } \\
& \text { Enter the maximum expected alpha and beta dispersions } \\
& \text { from trim in (deg) that must be controlled by the effectors, } \\
& \text { and click OK. } \\
& \begin{array}{ll}
\text { Maximum } \\
\text { Alpha (deg) }
\end{array} \begin{array}{ll}
3.0000 & \begin{array}{l}
\text { Maximum } \\
\text { Beta (deg) }
\end{array} \\
\hline
\end{array}
\end{aligned}
$$

The performance plots indicate that this vehicle is statically stable in both pitch and lateral directions. The 3,500 (psf-deg) Q-alpha, Q-beta load factor at $3^{\circ}$ of alpha and beta is reasonable. The control authority parameters in roll pitch yaw and x-acceleration are all acceptable (they are less than one). The roll angle ( $\phi$ ) which would occur along the trajectory at $3^{\circ}$ of sideslip ( $\beta$ ), due to cross-wind, is less than $2.2^{\circ}$. The LCDP in this case is negative. Its magnitude is slightly low but it does not change sign, which is good. The Cn-beta-dynamic is positive which indicates lateral stability, and its minimum value is around 0.011.

Static Margin, Center of Pressure, Aero-Center (ft), Air Launched Vehicle First



Short-Period (w)/ Time-to-Double-Ampl-Inverse (/sec), Q_alpha_beta (deg-lb/ft^2)



$\stackrel{\rightharpoonup}{ }$
$\underset{1}{1}$
$\stackrel{1}{\perp}$
$\underset{\sim}{3}$
$\underset{\sim}{\sim}$


Rotation Control Authority (dQ/dQmax)<1 for 3 (deg) of Alpha \& Beta Variation




Bank Angle, LCDP Ratio, Cn_beta_dynam /deg, Air Launched Vehicle First Stage Boo




Pitch Stability Contour Plot (Mach vs Alpha)


| z<-5 |
| :---: |
| \| - $5 \lll<-4$ |
| -4<2<-3 |
| \| $-3<2<-2$ |
| -2 \lll - 1 |
| -1<z<-0.5 |
| -0.5 \lll - 0.2 |
| -0.2 \lll 0.2 |
| $0.2 \lll 0.5$ |
| - $0.5<2<1$ |
| \| $1<2<2$ |
| - $2<2<3$ |
| - $3<2<4$ |
| - $4<2<5$ |
| - $5<2$ |

Very Unstable



Pitch Control Effort Contour Plot (Mach vs Alpha)

(-) Stop Limit

(+) Stop Limit

## Yaw Control Effort Contour Plot (Mach vs Alpha)


(-) Stop Limit

| $z<-1$ |
| :---: |
| $-1<z<-0.7$ |
| $-0.7<z<-0.5$ |
| $-0.5<z<-0.3$ |
| $-0.3<z<-0.1$ |
| $-0.1<z<-0.05$ |
| $-0.05<Z<-0.01$ |
| $-0.01<z<0.01$ |
| $0.01<z<0.05$ |
| $0.05<z<0.1$ |
| $0.1<z<0.3$ |
| $0.3<z<0.5$ |
| $0.5<z<0.7$ |
| $0.7<z<1$ |
| 1 |

(+) Stop Limit

## Controllability Analysis Using Vector Diagrams

Vector diagrams are used for visualizing the vehicle controllability against aerodynamic disturbances which are caused due to alpha and beta dispersions from trim. They are two-dimensional diagrams of vectors that compare the magnitudes and directions of the vehicle's response to flight control demands against its response to alpha and beta disturbances. The disturbance vectors must be small in comparison with the control vectors. Assuming that the initialization files and trim conditions are the same as before, from the main menu select option (11) for plotting vector diagrams. Select a flight condition at time=90 seconds, and from the following menu you may either keep the default values of Mass, Mach, $\alpha$, and $\beta$ which correspond to the selected time, or modify them to different values. In this case we select the default values.

## Main Trim Menu

| Select one of the following options | Exit | OK |
| :--- | :--- | :--- |

1. Plot Aero Coefficients, Derivatives, and Control Surface Increments
2. Plot Trajectory Parameters Versus Time from the Trajectory File ".Traj"
3. Trim the Effector Deflections to Balance the Vehicle Moments and Forces
4. Create an Effector Mixing Logic or a TVC Matrix (Kmix)
5. State-Space Modeling of the Flight Vehicle at Selected Times
6. Performance and Stability Parameter Plots Along Trajectory Time
7. Landing and Pull-Up Maneuverability, plus, Inertial Coupling Effects
8. Moments at the Hinges of Control Surfaces Along the Trajectory Time
9. View and Modify Vehicle Data (CG, MRC, TVC, Surfaces) for Dispersion Analysis
10. Evaluate Performance, Authority Using Contour Plots (Mach versus Alpha)
11. Vector Diagrams for Maneuverability \& Stability at Selected Flight Conditions
12. Plot and Compare Previous Data Files (Traject, Trim, Perform, Hinge Moment)


Select the following parameters
Select a Vehicle Mass, Mach Number, Alpha, and Beta from the lists below and click "Select"

Select


Comparison Between Maximum Control Accelerations and Max Accels due to Beta (red) Roll \& Yaw Accelerations due to Maximum Roll/ Yaw Control and due to Max Beta


Comparison Between Maximum Control Moments Against Maximum Disturb Moments (red) Roll \& Yaw Control Moments (non-dimension) vers Disturb Moment due to Max Beta/Alpha


Comparison Between Maximum Control Accelerations and Max Accels due to Alpha (red) Pitch Accelerat and Axial X-Accelerations due to Max Control and due to Max Alpha


Comparison Between Maximum Control Moment/Forces Against Maximum Disturbance (red) Pitch Control Moment and Axial X-Force (non-dimens) versus Disturb due to Max Alpha


The next plot shows the roll and yaw moment partials per roll and yaw FCS acceleration demands ( $\delta_{\text {PFCS }} \& \delta_{\text {RFCS }}$ ), green and blue vectors respectively. It also shows the roll and yaw moment partials per beta disturbance ( $\mathrm{Cl} \beta \& \mathrm{Cn} \beta$ ). It is calculated at two extreme alphas. The control vectors are scaled as described in equations ( 7.1 through 7.4) to make them better comparable with ( $\mathrm{Cl} \beta \& \mathrm{Cn} \beta$ ). The control partials dominate significantly over the aero disturbance partials, as expected.

The next vector diagram shows the control versus disturbance partials in the longitudinal directions. It is the partials of pitch and axial force per pitch and x -acceleration FCS demands ( $\delta_{\text {QFCS }} \& \delta_{\text {XFCS }}$ ). That is, $\left(\mathrm{Cm} \delta_{\mathrm{a}}, \mathrm{CX} \delta_{\mathrm{Q}}\right)$ blue vector and ( $\mathrm{Cm} \delta_{\mathrm{x}}, \mathrm{CX} \delta_{\mathrm{x}}$ ) green vector respectively. The red vectors show the pitch moment partial per alpha and the axial force partial per alpha ( $\mathrm{Cm} \alpha, \mathrm{C}_{x} \alpha$ ). There are two of these red vectors calculated at two extreme betas.

The plot below shows the roll and yaw acceleration partials per acceleration demands. The green vector shows the roll and yaw acceleration partials per roll demand ( $\delta_{\text {PFCS }}$ ). The blue vector shows the roll and yaw acceleration partials per yaw demand ( $\delta_{\text {PFCS }}$ ). As you can see, they are not entirely decoupled like in other examples, but this is OK, they don't have to be, as long as they are sufficiently orthogonal.


## Air-Launched Rocket Simulation (Coupled Model)



Figure 2.3a Air-Launched Vehicle Simulation Model in Simulink file "Sim_Air_Launch.mdl"

Flight Vehicle Dynamics, Simulation Model Air Launched Vehicle First Stage Boost Trajectory/ T=89.0 sec

Inputs = 7
1 Engine No 1 Pitch Deflect. (rad), Dymax= 5.0000 deg
2 Engine No 1 Yaw Deflect. (rad), Dzmax $=5.0000$ deg
3 Throttle Input dTh/Th for Engine No $1(-)$
4 Aero Surface No 1 Elevon Deflection (radians)
5 Aero Surface No 2 Aileron Deflection (radians)
6 Aero Surface No 3 Rudder Deflection (radians)
7 Wind Gust Azim, Elev Angles=( 45.0 90.0) (deg)


Outputs = 14
1 Roll Attitude (phi-123) (radians)
2 Roll Rate (p-body) (rad/sec)
3 Pitch Attitude (thet-123) (radians)
4 Pitch Rate (q-body) (rad/sec)
5 Yaw Attitude (psi-123) (radians)
6 Yaw Rate (r-body) (rad/sec)
7 Angle of attack, alfa, (radians)
8 Angle of sideslip, beta, (radian)
9 Change in Altitude, delta-h, (feet)
10 Forward Acceleration (V-dot) (ft/sec)
11 Cross Range Velocity (Vcr) ( $\mathrm{ft} / \mathrm{sec}$ )
12 Accelerom \# 1, (along X), (ft/sec^2) Translat. Acceler
13 Accelerom \# 2, (along Y), (ft/sec^2) Translat. Acceler
14 Accelerom \# 3, (along Z), (ft/sec^2) Translat. Acceler
Figure 2.3b Vehicle Dynamic Model Using the System in File "Vehicle.M"


Figure 2.3c Lateral Flight Control System that uses LQR State-Feedback

## Simulation Results

The simulation model in Figure (2.3) can be excited from guidance signals (shown as step inputs here) of changes in gamma, phi, and velocity commands. It can also be excited by wind gusts and turbulence. The wind-gust direction is perpendicular to the vehicle $x$-axis and skewed at $45^{\circ}$ between the $+y$ and $+z$ axes, as it is defined in the vehicle input data file. We will use this model to calculate the vehicle responses to various excitations.

## Phi-command Simulation

A phi-command $=20^{\circ}$ is applied to the simulation model with all other inputs set to zero. The figures below show the bank angle which responds to the command and the yaw attitude which begins to drift due to the steady turn. The cross-range velocity also increases due to banking. Banking is used by guidance as a means to control the flight direction and the cross-range velocity.







## Gamma-Command Simulation

In this case the simulation model is commanded to increase its flight-path angle gamma by one degree from steady-state. All other commands and excitations are set to zero. This causes a positive alpha, a steady increase in altitude and a drop in velocity that is compensated by throttling up. The elevon and the engine briefly deflect in the negative direction to cause the pitch-up, and they decay to zero (or trim positions).









## First Stage Boost without Wings and Tails

The wings provided the lift required for the vehicle to achieve a desired gamma trajectory without losing much altitude but they are extra weight and drag and at a certain point they are no longer needed. After ejecting the wings and tails at time $=90$ seconds, the vehicle no longer looks like an aircraft, but it is a typical booster rocket, still at first stage. This rocket, however, is not a perfect cylinder but there is some unsymmetry in the payload that causes aerodynamic coupling between the roll and yaw axes. A significant amount of rolling moment is introduced due to beta that cannot be ignored. The vehicle in this configuration has lost its aero-surface controls. The TVC provides plenty of pitch and yaw control. For roll it uses two pairs of back-to-back RCS jets firing in the $\pm z$ direction. Each pair is located on the left and right sides, 3.6 ( ft ) away from the cylinder centerline. The sizing of the jet thrust is based on controllability requirements. Roll disturbances are mainly caused by the $\mathrm{Y}_{\mathrm{CG}}$ offset and also due to sideslip disturbances from cross-winds or maneuvering. The rocket should be controllable with a $\mathrm{Y}_{\mathrm{CG}}$ offset of 0.1 foot. It should also have enough roll control to handle one degree of sideslip angle dispersion. We will use Trim to make sure that those requirements are satisfied with two 500 (lb) thruster pairs.


Figure 3.1 The vehicle has three effectors in this configuration, a main engine that gimbals and throttles, and two RCS jet pairs for roll control.

The data files for this phase are located in the subdirectory: "C:\Flixan\Trim\Examples\Air Launched Vehicle\Phase C Boost No Wing ". This folder includes a basic aero-data coefficients file of the vehicle without wings and tails in "Wing_Off_Stg1.Aero". There is no aero-surfaces file (.Delt). It has a propulsion data file "AL_Stg1.Engn" that includes the TVC engine and the two RCS jet pairs. The main engine is defined as having $300,000(\mathrm{lb})$ of nominal thrust along the $x$ axis. It can gimbal up to $\pm 8^{\circ}$ in pitch and yaw, and it can vary its thrust $\pm 80 \%$ from nominal. The RCS jets are 500 (lb) each and their thrust direction is along the $\pm z$ axis.

## Obtaining a User Modified Trajectory

Our nominal trajectory is not qualified to size the RCS jets because it does not include lateral disturbances. We must, therefore, create another trajectory for trimming that will include a beta disturbance and a $\mathrm{Y}_{\mathrm{CG}}$ offset. From one of the trajectory plot windows go to the top menu bar, click on "Graphic Options" and from the drop-down menu select "Modify a Trajectory Plot Using the Mouse", as shown below. The following trajectory parameters menu lists the trajectory parameters that can be modified using the mouse. Select the sideslip angle (beta) and click on "Select a Variable to Modify".

| \$. Trajectory Parameters |
| :--- |
| Copy Format: Send to: Graphic Options Next Plot Exit Plots |
|  |

The dialog below shows the sideslip angle beta which is zero (green line). We must modify this line to a different profile in the time period between 100 and 110 seconds. It is like introducing a square beta-gust of $1^{\circ}$ in that time period, as shown below. This can be done graphically by modifying the green curve one section at a time, using the mouse. The final beta profile is shown by the yellow line below. Click on "Continue with Another Variable to Modify" and this time select "CG Location Along Y $(f t)$ ". The nominal $\mathrm{Y}_{\mathrm{CG}}$ is very small. Similarly introduce another square pulse of amplitude $0.1(\mathrm{ft})$ in the time period between 120 and 135 seconds. Click on "Save the Modified Trajectory" and it will be saved in file "AL_Stg1_Woff1.Traj" which was renamed to "AL_Stg1_Mod.Traj". The modified parameters are shown in the following trajectory figures.

Angles of Attack/Sideslip/Flight Path (deg), User Modified Trajectory



Vehicle CG in (feet), User Modified Trajectory ...








The plot above shows that $1^{\circ}$ of beta causes the vehicle to trim by banking $7^{\circ}$ towards the wind. The next two plots show the trim results in the presence of the disturbances. The engine TVC deflects in pitch to provide the required pitching moment to trim in pitch. It also varies its thrust in order to match the axial acceleration of the trajectory. The left and right RCS jets, labeled as Throttle $2 \& 3$, vary their thrust differentially in order to trim by counteracting the beta disturbance and the YCG offset which occur at different times. There is also a small amount of yaw gimbaling due to the disturbances. The trimming of the RCS jets did not exceed the 500 (lb) thrust limit set in the propulsion data file. It means that $500(\mathrm{lb})$ thrusters are sufficient for roll control. Notice, that in Trim we always assume that the RCS jets are continuous and not "on-off" devices because its purpose is sizing and not for dynamic modeling. Flixan dynamic models, however, can be used to simulate the effects of "on-off" firing of the RCS jets and to perform non-linear control analysis using Simulink models, as it will be shown later in this example.


## Select a Gain Matrix

| Select one of the following Matrices from the Systems File | View Matrix | Cancel |
| :--- | :--- | :--- | Select Matrix


| KMIX4A | : Mixing Logic for Air Launched Vehicle First Stage Boost Trajectory at Time: 119.52 |
| :--- | :--- | :--- |
| KMIXAC | : Mixing Logic for Air Launched Vehicle First Stage Boost (User Modified) | KMIX4C : Mixing Logic for Air Launched Vehicle First Stage Boost (User Modified)

We must also define the max alpha and beta dispersions from nominal caused by winds and also due to maneuvering. This is required for determining the maneuverability and control authority of the vehicle. In this case we at higher altitude and the wind disturbances are smaller. We pick $1^{\circ}$ for both angles.


This vehicle is statically unstable in both pitch and lateral directions with times to double amplitude equal to 0.5 sec and 1 sec respectively. Rockets are usually unstable, as long as they do not diverge too fast. The Q-alpha, Q-beta load factor (with $1^{\circ}$ of alpha and beta) remains flat at an acceptable level of 3,500 (psf-deg). The control authority parameters in roll pitch yaw and $x$-acceleration are all acceptable (they are less than one in all 4 directions). The worst one is in roll because it is using the jets that have limited power. Some improvement in roll authority was obtained by introducing some roll and yaw coupling in the mixing matrix, at the expense, of course, in yaw control authority. But there is plenty of yaw authority. The X-translational control authority parameter is very small which indicates that the vehicle has plenty of throttle capability to overcome forces in the $x$ direction due to the winds defined.

The roll angle ( $\phi$ ) that would occur along the trajectory at $1^{\circ}$ of sideslip ( $\beta$ ), due to cross-wind, is less than $6^{\circ}$. The LCDP is positive and sizeable showing good maneuverability in roll. The Cn-beta-dynamic is negative because the vehicle is directionally unstable. The static margin is also negative indicative of pitch instability.

Static Margin, Center of Pressure, Aero-Center (ft), Air Launched Vehicle First





Short-Period (w)/ Time-to-Double-Ampl-Inverse (/sec), Q_alpha_beta (deg-lb/ft^2)




Bank Angle, LCDP Ratio, Cn_beta_dynam /deg, Air Launched Vehicle First Stage Boo




Pitch Stability Contour Plot (Mach vs Alpha)


Lateral Stability Contour Plot (Mach vs Alpha)


Too Stable


## Roll Departure (LCDP) Contour Plot (Mach vs Alpha)



The above plot is a perfect example of the advantages offered by the contour plots. It shows the LCDP parameter which is an indicative of roll maneuverability (not the control authority). It indicates the dynamic controllability in the roll axis, which is very good in this case, positive and in a good range of values (light cyan). The white areas indicate perfect roll/yaw coordination. There is a bad region, however, in the upper left-hand corner where the parameter changes sign and crosses in the negative direction, indicating roll-reversal. Negative LCDPs are represented with dark colors (red and above in the chart). Our trajectory, however, is far from crossing that region.

## Pitch Control Effort Contour Plot (Mach vs Alpha)



## Yaw Control Effort Contour Plot (Mach vs Alpha)



Comparison Between Maximum Control Moments Against Maximum Disturb Moments (red) Roll \& Yaw Control Moments (non-dimension) vers Disturb Moment due to Max Beta/Alpha


Comparison Between Maximum Control Accelerations and Max Accels due to Beta (red) Roll \& Yaw Accelerations due to Maximum Roll/ Yaw Control and due to Max Beta


Comparison Between Maximum Control Moment/Forces Against Maximum Disturbance (red) Pitch Control Moment and Axial X-Force (non-dimens) versus Disturb due to Max Alpha


Comparison Between Maximum Control Accelerations and Max Accels due to Alpha (red) Pitch Accelerat and Axial X-Accelerations due to Max Control and due to Max Alpha


