Re-Entry Glider Vehicle



In this example we will use the Flixan/ Trim program to analyze the performance of a re-entry, Shuttle type of unpowered vehicle in different phases, from the point where it begins to enter the atmosphere at 360,000 (ft) of altitude, all the way to landing. Descent begins after the vehicle fires its orbital maneuvering engines to slow-down, performs a large pitch attitude maneuver and positions itself at an angle of attack of approximately 41°, and gravity begins to take effect reducing its flight-path angle to γ =-1°. Initially the dynamic pressure is very low and the vehicle uses mainly reaction control jets in combination with aero-surfaces for trimming and control. As the dynamic pressure increases the surfaces become more effective in controlling the vehicle and the RCS usage is reduced.

The vehicle uses six aero-surfaces for trimming and control: 2 flaps, 2 rudders, a body-flap, and a speed-brake, see Figure (4.1). The arrows show the hinge vector directions for positive rotations about the hinges. The speed-brake is only used near landing for speed control. The body-flap is only used for trimming and not flight control. Initially it is positioned at -20° which causes the vehicle to enter the atmosphere with a stable trim of approximately α =41°. The RCS jets configuration consisting of 6 continuous (analog) jets is preliminary and intended mainly for sizing the thrusters and not for detail simulations. A flight control system consisting of RCS and aero-surfaces helps maintain a shallow flight-path angle γ =1° and α =41°, which prevent the vehicle from overheating. Although alpha is constant during the early part of re-entry, the bank angle is not always zero. As soon as the dynamic pressure increases to 5 (psf) the vehicle rolls about its velocity vector, and banks approximately 60° which

prevents it from bouncing back off into space. Further down it performs several bank reversal maneuvers while the angle of attack slowly decreases. The bank reversals at high Mach numbers are used for managing its excess energy. When the speed drops below Mach 1 the flight control mode switches to normal acceleration, Nz-control, for controlling altitude, and the flight path angle comes down steeper at -20° to maintain high speed for the approach and landing maneuver. During this period the vehicle also uses its speed-brake for velocity control.



Figure 4.1 Six Aero-Surfaces and the Directions of the Hinge Vectors

The analysis that follows is separated in three sections.

- An early phase where the dynamic pressure is low, the angle of attack is 41°, and the flight path angle is -1°. The pitch control system is initially controlling alpha at 41° and later it switches to γ-control. In the lateral direction it controls roll about the velocity vector V₀ and performs roll maneuvers.
- 2. A mid-phase analysis during which the vehicle begins by controlling the angle of attack, initially at 41° and then reducing it to lower values. Further down the flight-path angle is controlled at a steady shallow value of -1°. Although the dynamic pressure is sufficient for aero-surfaces control the RCS is also partially active in order to augment controllability. During periods of reversed or uncertain LCDP the RCS controls the roll and yaw axes and the control surfaces control the longitudinal axes.
- 3. The approach and landing phase during which the RCS jets are turned off and the speed-brake and body-flap become active in longitudinal control. The vehicle altitude and speed are controlled independently. The altitude is controlled by a combination of pitching using the V-tails and Nz-control using the flaps, and the speed using a combination of speed-brake and body-flap from their partially deployed positions. In lateral the heading direction is controlled by banking about the velocity vector V₀.

This is a great example for demonstrating a vehicle that uses a combination of aero-surfaces and RCS jets for trimming and control. We will analyze each of these phases separately by trimming the effectors, analyzing static performance along the trajectory, using contour plots and vector diagrams for analyzing performance and maneuverability, generating dynamic models, designing flight control laws, and analyzing dynamic stability at selected flight conditions.

1.0 Early Re-Entry Phase

In this early phase of the trajectory the dynamic pressure is very low and the vehicle uses mainly the reaction control jets in combination with the aero-surfaces for flight control. As the dynamic pressure increases the surfaces become more effective and the RCS is gradually phased out. During this phase the vehicle uses only 5 of its aero-surfaces. The 2 flaps, 2 rudders, and the body-flap. The speed-brake is reserved for near landing. The aero-surfaces and mainly the body-flap are trimmed to provide static stability and a steady angle of attack at approximately 41°. The integrated RCS and aero-surfaces flight control system helps maintain a shallow flight-path angle of approximately -1.2° to prevent the vehicle from burning due to friction. Although alpha is constant the bank angle is changing. As soon as the dynamic pressure increases to 5 (psf) the vehicle rolls about its velocity vector and banks approximately 70° to the right to prevent it from bouncing off back into space. It continues performing several bank reversal maneuvers while it is slowly decreases its angle of attack. The bank reversals are also used to manage its excess energy since the speed-brake is not used in hypersonic speeds.

In this early re-entry phase we are not interested to control or trim along the normal and axial accelerations. In the pitch direction we want to accurately control the angle of attack and the flight path angle because this regulates heating. In the lateral direction we should be able to perform roll maneuvers about the velocity vector V₀ because this minimizes sideslip and lateral loading. The control system, therefore, consists of roll, pitch, and yaw control by means of a combination of RCS and aero-surfaces that helps maintain a predefined alpha and flight path angle trajectory and to perform roll maneuvers about the velocity vector. The simplified RCS system used in this analysis is only for sizing purposes and it does not represent a typical RCS consisting of multiple thrusters in various directions. Although both aero-surfaces and RCS operate in parallel during this phase, the RCS jets dominate during the first 350 sec and the surfaces gradually take over when the dynamic pressure increases.

The files for this early re-entry phase are in directory "C:\Flixan\ Trim\ Examples\Re-Entry Glider\Early Phase". The trajectory file "X_Early.Traj" contains only the early part of the descent trajectory. The definitions of the RCS jets are in file "X_6RCS_Jets.Engn", which consist of 6 throttling jets that provide moments in all 3 axes. Each thruster is defined with a throttling parameter of (1) which represents a pair of back-to-back firing jets with zero nominal thrust, that can generate either positive or negative continuous thrust. In contrast with a throttling engine that has a non-zero nominal thrust Te, a throttling parameter which is less than one, and can vary its thrust about Te. Trimming is used to determine the thrust requirement for each jet. The exact modeling of the RCS locations, thrusting directions, and bang-bang control laws is typically implemented later in the simulation models which are created by the Flixan vehicle modeling program (FVMP).



After finishing the trim-history modification of one effector, click on "*Continue Adjusting the Next Trimming Effector*" to select the next one from the effectors menu. When all 6 thruster modifications are complete, click on "*Re-Trim*" and the Trim program will re-trim the effectors by taking into consideration the user modifications which constrain thruster usage after 400 sec. The program searches for other effectors which are capable of balancing moments under the new constrain in throttling and it converges to a new solution because the aero-surfaces are able to contribute more. Otherwise, if it cannot find other effectors it will violate the position limits.





In the following menu/ dialog select the default options for vehicle mass, Mach number, Alpha, and Beta which correspond to t= 200 sec. In the effector mixing matrix selection dialog we must define the source of the mixing-logic matrix, which is needed for the calculation of the control vectors. We can either select a pre-calculated matrix or let the program calculate a matrix. In this case we choose the third option to calculate the mixing-logic matrix but with an adjustable effector participation. This is because we want to eliminate the Body-Flap from participating in the mixing-logic matrix. The menu on the right lists the 11 effectors and initially their participation is set to 100%. So we reduce the participation of the Body-Flap to 0% and click on the button in the upper right hand corner. So in this flight condition we will allow all effectors, except for the body-flap, to participate using their full capacities.



100

0

The following vector diagram compares the maximum roll and yaw accelerations (shown in green and blue colors respectively) against the accelerations produced by the aero disturbance. The disturbance accelerations in this flight condition are small because the dynamic pressure is small. The cross-coupling between the accelerations is small. The diagram below shows the moment partials. The blue vector is the moment partials {Cn δR_{FCS} , Cl δR_{FCS} } due to yaw control demand and the green vector is the moment partials {Cn δP_{FCS} , Cl δP_{FCS} } due to roll demand. The small red vectors are the (Cl β and Cn β) partials.



Comparison Between Control Moment Partials Versus Aero-Disturb Moment Partials (red) Yaw & Roll Control Partials Cn/delta_R and Cl/delta_P versus Cn/beta & Cl/beta



The following vector diagram shows the effect of max pitch control (blue vector) in two directions: pitch acceleration in (rad/sec^2) and normal z-acceleration in (g). The red vector shows the effect (in the same two directions) of varying alpha and beta $\pm 4^\circ$ about $\alpha_0=42^\circ$. It obviously affects mainly the z acceleration and not pitch. The pitch control is much stronger in the pitch direction in comparison to the disturbance vector. The partials chart below shows the same outcome. The pitch acceleration due to pitch FCS demand (blue vector) is stronger than Cm α . Cz α dominates in the z-acceleration.



Comparison Between Control Force & Moment Partials Against Disturb Partials (red) Cm/alpha & CZ/alpha versus Pitch and Norml Z-Force Controls: Cm/delt_Q & CZ/delt_Z



1.5 Control Design and Analysis during Early Re-Entry

There are several control approaches to consider in combining the aero-surfaces and RCS jets together. One approach might be to allow the aero-surfaces control system to be operating in the inner loop and the RCS to provide an outer loop control and acting as a back-up system. In this example, however, we will use two separate control systems operating in parallel, which are also capable controlling the vehicle independently from each other. One system is using the 4 aero-surfaces and the other is using the 6 RCS jets. The vehicle begins its descent trajectory from orbit by firing its orbital maneuvering engines to slow it down and placing it to a shallow re-entry flight path angle γ =-1°. During this early phase the flight control system attempts to control the angle of attack, initially at α =41° and gradually decreasing it to smaller alphas. This angle is optimal for heat protection.

The design models already include the mixing matrices so the input to the pitch plant model is pitch acceleration demand. The state-vector was modified from its original definition in files pdes rcs.m and pdes surf.m. The velocity and altitude states (δV and δh) were not included because during this phase we are not interested to control these variables which affect mainly the phugoid mode. The integral of the angle of attack was included in the state-vector, so the control state vector consists of: (θ , q, α , α integral). In the lateral direction the vehicle is expected to perform roll maneuvers about its velocity vector V₀. The design models are in the stability axes, that is, roll and yaw are defined relative to the velocity vector V₀ which is at α =41°. Now why do we want our design models to be in stability axis rather than in the body axis? It would make more sense to design the gains relative to the body axis since the rate and acceleration measurements are coming in the body axis. The reason is that in the lateral axes the vehicle is commanded to roll about the velocity vector V₀. This minimizes the sideslip angle β transient. So it makes sense to optimize our LQR gains based on the stability model and command a roll directly in the stability axis. A roll command about the velocity vector corresponds to a simultaneous roll and yaw command in the body axis. This reduces the beta transients. The lateral design model state vector is augmented and it consists of (p_stab, r_stab, β , p_stab-integral, and β integral). The inputs are roll and yaw demands.

LQR Design

The following Matlab script "Init.m" loads the four LQR design models, the simulation model, and the two mixing-logic matrices. It performs the four LQR designs for aero-surfaces and RCS, and generates four state-feedback matrices. Two lateral (KJpr and KSpr), and two pitch (KJq, and KSq).

```
d2r=pi/180; r2d=180/pi;
[Apj, Bpj, Cpj, Dpj] = pdes_rcs;
[Aps, Bps, Cps, Dps] = pdes_surf;
[Alj, Blj, Clj, Dlj] = ldes_rcs;
[Als, Bls, Cls, Dls] = ldes_surf;
[Ave, Bve, Cve, Dve] = vehi_sim;
load KSmix.mat -ascii;
load KJmix.mat -ascii;
```

```
% Load Pitch RCS Jet Design Model
% Load Pitch aero-surf Design Model
% Load Lateral RCS Jet Design Model
% Load Lateral aero-surf Design Model
% Simulation Model 6-dof
% Load Surfaces Mix Logic (4 x 3)
% Load RCS Jets Mix Logic (6 x 3)
```

```
alfa0=41.5; V0=24124.0; Thet0=40.3; ge=32.174; % Additional Vehicle Parameters
calfa=cos(alfa0*d2r); salfa=sin(alfa0*d2r); % for Body to Stability Transform
% Convert Lateral State Vector from Body to Stability Axes, Outputs=States
Al2= Clj*Alj*inv(Clj); Bl2= Clj*Blj;
                                               % for the Jets
Cl2= Clj*inv(Clj);
                   Dl2= Dlj;
% Lateral LQR Design Using Only the RCS Jets
[Al5,Bl5,Cl5,Dl5]= linmod('LRdes5x');
                                               % 5-state model {p,r,bet,pint,betint}
R=[1,1]*20; R=diag(R);
Q=[1 1 0.1 0.5 0.005]*3; Q=diag(Q);
R=[1,1]*20; R=diag(R);
                                               % CS LQR Weights R=[1,1]*20
                                               % CS LQR Weights Q=[1 1 0.1 0.5 0.005]*3
[KJpr,s,e]=lqr(Al5,Bl5,Q,R);
                                               % Perform LQR design on Jets
% Convert Lateral State Vector from Body to Stability Axes, Outputs=States
Al2= Cls*Als*inv(Cls); Bl2= Cls*Bls;
                                               % Transform from Body to Stability
Cl2= Cls*inv(Cls); Dl2= Dls;
                                                % for the Surfaces
% Lateral LQR Design Using Only the Aero-Surfaces
[Al5,Bl5,Cl5,Dl5]= linmod('LRdes5x');
                                              % 5-state model {p,r,bet,pint,betint}
R=[1,1]*20; R=diag(R);
                                              % CS LQR Weights R=[1,10]*2
Q=[1 1 0.1 0.2 0.002]*2; Q=diag(Q);
                                              % CS LQR Weights Q=[1 1 1 0.04 0.04]*0.4
[KSpr,s,e] = lqr(Al5,Bl5,Q,R);
                                               % Perform LQR design on Jets
% Pitch LQR Design Using Only the RCS Jets
Apd=Apj; Bpd=Bpj; Cpd=Cpj; Dpd=Dpj;
[Ap4, Bp4, Cp4, Dp4] = linmod('Pdes4x');
                                               % 4-state des model {theta,q,alfa,alf_int}
Ap5= Cp4*Ap4*inv(Cp4); Bp5= Cp4*Bp4;
                                               % Convert to
Output=State={theta,q,alfa,alf_int}
Cp5= Cp4*inv(Cp4); Dp5= Dp4;
R=4; Q=[0.05 0.5 1 1]; Q=diag(Q);
                                               % RCS LQR Weights {theta,q,alfa,alf_int)
[KJq,s,e] = lqr(Ap5,Bp5,Q,R);
                                               % Perform LQR design on Surf
% Pitch LQR Design Using Only the 4 Aero-Surfaces
Apd=Aps; Bpd=Bps; Cpd=Cps; Dpd=Dps;
                                             % 4-state des model {theta,q,alfa,alf_int}
% Convert to
[Ap4, Bp4, Cp4, Dp4] = linmod('Pdes4x');
Ap5= Cp4*Ap4*inv(Cp4); Bp5= Cp4*Bp4;
Output=State={theta,q,alfa,alf_int}
Cp5= Cp4*inv(Cp4);
                    Dp5= Dp4;
R=2; Q=[0.05 0.5 1 1]; Q=diag(Q);
                                         % AS LQR Weights {theta,q,alfa,alf_int}
[KSq,s,e] = lqr(Ap5,Bp5,Q,R);
                                                % Perform LQR design on Surf
```

The two simple simulation models in files "*Sim_Later_Simple.mdl*" and "*Sim_Pitch_Simple.*mdl", shown below, are used to evaluate the pitch and lateral LQR designs and to adjust the LQR weights. They use the design models and they include both the RCS and the aero-surface state-feedback loops operating in parallel. Each loop is contributing half of the amount that it should be contributing if the other loop was absent. Since the mixing-logic matrices are included in the vehicle systems their inputs are acceleration demands (δP , δQ , δR) coming from the state-feedback matrices. The mixing-logic matrices in the models are used to convert the control demands to effector deflections which must be taken into consideration while adjusting the LQR weights. We would typically like to keep the surface deflections below ±30° during a maneuver and the throttles below ±1.

Case B, Phi-Command Simulation

In this case the vehicle is initially commanded to perform a 30° roll maneuver about its velocity vector V_0 . The vehicle performs the roll maneuver in 7 sec, using a combination of aero-surfaces and RCS. The peak transient in beta is about 1.6°. The roll and yaw body rates are both responding to the ϕ -stability command. Both the RCS and the aero-surface control systems are equally contributing towards the maneuver, the RCS by differential throttling and the surfaces by differential deflections (mainly the 2 rudders). The steady bank angle also causes a steady cross-range velocity.



6-45

2.0 Mid-Phase

In the mid-phase period of the trajectory the dynamic pressure is large enough to control the vehicle by using the aero-surfaces alone without any assistance from the RCS. There is an exception, however, near the LCDP transitioning. Only the two flaps, the two rudders, and the Body-Flap are used for trimming. The Speed-Brake is not used because energy is dissipated by the roll maneuvers about the velocity vector. So the vehicle arrives in the approach and landing section aligned with the landing strip and having the proper altitude and speed. In actual flight the surfaces are scheduled by open-loop commands derived from the trim analysis. The flight control commands are added to the scheduled positions. The body-flap, however, is not used for flight-control but only scheduled open-loop. Initially the angle of attack is controlled at 41° as in the previous phase. It is gradually reduced it to approximately 10°. The descent flight path angle is controlled steadily at -1.2°, but towards the end of the mid-phase it starts coming down steeper at -16° to maintain speed as it approaches Mach 2 and the dynamic pressure increases to 200 psf.

During most of the flight and mainly in the high Mach and high alpha regions the LCDP ratio is negative and it does not vary much which requires reversal in the roll control. As the vehicle transitions to smaller angles of attack, the LCDP ratio changes sign a few times and Cn β -dynamic becomes negative. This changing in the LCDP ratio indicates that we cannot rely on the ailerons for roll control during this period and that we must also use RCS. Negative values of Cn β -dynamic indicate lack of directional stability and the RCS is also required for yaw control during this period. This is because the terms (ω_{ϕ}) and (ω_D) in the transfer function that relates roll acceleration per roll control become uncertain and in the steady-state it may become unstable or change sign.

$$\frac{\dot{p}(s)}{\delta_{P_{FCS}}(s)} = \frac{L_{\delta p}}{I_{XX}} \left\{ \frac{s^2 + \omega_{\phi}^2}{s^2 + \omega_D^2} \right\} \quad \text{where}$$
$$\omega_{\phi}^2 = \frac{\cos \alpha_o}{I_{ZZ} L_{\delta p}} \left[N_{\beta} L_{\delta p} - L_{\beta} N_{\delta p} \right] \quad \text{and} \quad \omega_D^2 = \left[\frac{N_{\beta} \cos \alpha_o}{I_{ZZ}} - \frac{L_{\beta} \sin \alpha_o}{I_{XX}} \right]$$

A solution to the problem during this uncertain transitioning period is to include RCS control as an outer loop around the aileron and rudder loops. The data files for this mid-phase analysis are in directory "*C*:*Flixan**Trim**Examples**Re-Entry Glider**Middle Phase*". The following figures show the trajectory variables versus time during the mid-phase period.





2.1 Mid Phase Trimming

The purpose for trimming the aero-surfaces is to obtain open-loop scheduling commands as a function of time. In actual flight the flight control feedback will provide additional deflections for vehicle stabilization, guidance, and for counteracting wind disturbances. Start the Trim program and select the mid-phase folder. From the filenames selection menu select the mid-phase trajectory file "*X*-*Middle.Traj*". The file "*X*_*4RCS_Jets.Engn*" contains four jets that will be assisting the surfaces in roll/yaw control. Although the surfaces alone are capable of trimming the moments the jets are included to improve lateral controllability. Plus the jets also need to be included in the dynamic models. In the aero-data files only the first 5 surfaces are included instead of 6. The speed-brake is not read. The Body-Flap is biased at -10° in the aero-coefficients file.

In the second filename selection menu make sure that you include the systems file "Mixing-Matrix.Qdr" which includes the aero-surface/RCS combination logic. It will be used in the following sections. Do not select an initialization file for trimming, and in the menu that selects the directions to be balanced select the first option (3 moments only). Since the speed-brake is disabled and alpha is high, we do not have any axial or Nz control. Therefore, we must only trim the moments and not attempt to trim in the axial and Nz directions.





The stability parameter (T2-inverse) that measures static-stability or time-to-double amplitude is initially negative in both: pitch and lateral directions. It means that both directions are stable with short-period and Dutch-roll resonances in the order of 1.5 (rad/sec). Later on they become positive and change signs a few times. The times-to-double amplitude when unstable do not get much shorter than 0.5 second. The (Q-alpha, Q-beta) loading parameter peaks at about 3,500 (psf-deg). The control authority is very good. The effort is less than 0.5 in all 3 directions. It means that the wind-shear dispersions due to (α_{max} and b_{max})=4° cannot saturate the controls, plus have additional 50% control to spare.

The LCDP ratio is better seen in the second (expanded) plot. It begins at -0.2 and it remains negative for a long period. Then towards the end of the mid-trajectory it changes sign a few times. The Cn-beta-dynamic parameter also becomes negative towards the end which shows directional instability. The bank angle (phi) in the presence of a wind-shear due to (β_{max}) is not applicable here. The following plot shows the max accelerations achieved in roll, pitch, and yaw without saturating the controls.



Max Angular Accelerat (deg/sec^2), at Max Control Demand Re-Entry Vehicle with 6

2.3. Contour Plots Analysis

Now let us take a look at some key performance parameters using contour plots which allow us to have a wider perspective of vehicle performance over the entire Mach versus Alpha range. In general, the mixing logic matrix generated by the Flixan algorithm optimizes the control authority of the effector system, but it does not necessarily optimize all vehicle performance parameters. Sometimes we may have to trade-off between performance parameters, as we shall see in this case. Let us first analyze the contour plots performance by letting the program calculate its own mixing-logic matrix and then use a pre-calculated matrix from file "*Mixing_Matrix.qdr*" and repeat the analysis. Make sure that this matrix file is selected when you start Flixan. From the Trim menu select option (10) and from the dialog that selects an effector combination matrix select the third option to allow the program create a mixing matrix but with adjustable contributions for each effector. From the menu on the right zero-out the participation of all 4 thrusters and also the body-flap, since the body-flap is only used for trimming and not for closed-loop control. Click on the button in the upper right hand corner to continue. We removed the jets because we want to check the vehicle performance using aero-surfaces alone.



The first contour plot shows the pitch stability parameter (T2-inverse) versus Mach and alpha. The trajectory is shown by the black line that travels across the Mach versus alpha field. It begins in the upper right-hand side where both alpha and Mach are high and it ends near the lower left-hand corner at Mach 10. The trajectory crosses through a mostly stable region, although is passes through a region of mild instability. The second plot shows the lateral stability parameter (T2-inverse) with similar results. The third contour plot shows the LCDP ratio which does not look very good. It is red through most of the high Mach, high alpha region, which indicates a "sluggish" and uncertain roll controllability. The next three contour plots show the control authority in roll, pitch, and yaw, and they look very good in all 3 directions.



Pitch Stability Contour Plot (Mach vs Alpha)



Lateral Stability Contour Plot (Mach vs Alpha)



Pitch Control Effort Contour Plot (Mach vs Alpha)

Yaw Control Effort Contour Plot (Mach vs Alpha)



ſ	Select a Gain I	Matrix
	Select one o	f the following Matrices from the Systems File View Matrix Cancel Select Matrix
	KMIX1400	: Mixing Logic for Re-Entry Vehicle with 6 Aero-Surfaces at Time: 1400.0
	KMIX1401	: Mixing Logic for Re-Entry Vehicle with 6 Aero-Surfaces at Time: 1401.0

The pitch and lateral stability did not change very much with the new mixing matrix. The LCDP ratio, however, has changed considerably. It has become more negative (dark colors) in the high Mach and high alpha region. It is no longer in the red "sluggish" and uncertain roll control region (±0.2) but it has shifted further negative (brown, dark yellow, grey regions). It requires, however, a reverse roll control which is okay until we get below Mach #3, where we have to rely on the RCS for roll control because the LCDP becomes unpredictable for a period. When it gets below Mach #1, the LCDP becomes positive and we can use the aero-surfaces again for lateral control, as we shall see in the next phase.



Roll Departure (LCDP) Contour Plot (Mach vs Alpha)



Pitch Control Effort Contour Plot (Mach vs Alpha)



Roll Control Effort Contour Plot (Mach vs Alpha)

This modification in the effector combination matrix, however, and the improvement in LCDP did not come without a cost. Although the pitch control authority did not change, and the roll authority improved, the control authority in the yaw directions is slightly deteriorated with the pre-calculated mixing matrix, but it is still within the acceptable region.

2.3. Vector Diagram Analysis

Vector diagrams are 2-dimensional diagrams used for analyzing the vehicle controllability at a specified flight condition. We compare the control capability of the effectors as a system in two directions against the effects on the vehicle of a wind-shear disturbance that is expressed by the dispersions it causes in the angles of attack and sideslip, and we determine if the vehicle provides sufficient control authority to counteract against the disturbance moments. It is not just a magnitude comparison but it also allows us to examine the directions of the controls versus the disturbance directions. This helps us evaluate the orthogonality of the control system, compare the acceleration magnitudes due to the controls and wind, and to determine if the controls are powerful enough and their directions are capable of counteracting the disturbance moments along the roll, pitch and yaw directions. From the Trim menu select option (11), and then an arbitrary flight condition at t=1320 sec, near the middle of the mid-phase trajectory, corresponding to Mach 8. Define the disturbance angles (α_{max} and β_{max})=4°, and select the same (9x3) effector combination matrix "Kmix1401" from file "Mixing_Matrix.Qdr", as before.







Comparison Between Control Moment Partials Versus Aero-Disturb Moment Partials (red) Yaw & Roll Control Partials Cn/delta_R and Cl/delta_P versus Cn/beta & Cl/beta

The first vector diagram shows the roll and yaw moments, non-dimensional ($C_l \& C_n$), produced when the roll and yaw FCS demands are maximized (before saturating the effectors). The solid blue vector corresponds to max positive yaw FCS demand ($+\delta R_{FCS_Max}$) and the dashed blue vector to max negative yaw demand ($-\delta R_{FCS_Max}$). Similarly, the green vectors correspond to the roll FCS demands ($\pm \delta P_{FCS_Max}$). There is a significant amount of control cross-coupling in both directions but fortunately the controls are nearly orthogonal to each other. The two red vectors show the roll and yaw moments generated by the variations in the angles of attack and sideslip ($\pm \alpha_{max}$ and $\pm \beta_{max}$) from their trim positions. The rectangles at the tips of the control vectors represent the uncertainties in the control moments. The uncertainties are obtained from file "*Aero_Uncert5.Unce*".

The above figure is a moments partials vector diagram showing the variation in roll and yaw moments per acceleration demands in roll and yaw in (rad/sec^2) . The blue vector is $\{Cn\delta R, Cl\delta R\}$ due to yaw demand (δR) , and the green vector is $\{Cn\delta P, Cl\delta P\}$ due to roll demand (δP) . Both vectors couple into both directions. The red vectors pointing downward are the $\{Cn\beta, Cl\beta\}$ partials. They are two because they are calculated at the two extreme values of $(\pm \alpha_{max} \text{ and } \pm \beta_{max})$. Notice that $Cl\beta$ is negative due to the dihedral caused by the V-tail and it is bigger in magnitude than $Cn\beta$. The red rectangle centered at the tip of the $\{Cn\beta, Cl\beta\}$ vector is due to the uncertainties in the two partials. Similarly the yellow

2.4. Flight Control Design and Analysis

We will now select a flight condition to analyze further, create dynamic models, design control laws, and analyze stability. The flight condition chosen was at t=1550 sec, because this is the time period when the LCDP ratio changes sign, which implies the possibility of sluggish roll control using aero-surfaces alone. The Cn β -dynamic parameter also becomes very small and negative, which implies directional divergence. At earlier times the Mach number and the angle of attack are high, and the LCDP ratio is consistently negative. Plus the Cn β -dynamic parameter is positive and it does not change much. Reversing the roll control gain in this earlier time period takes care of the roll and yaw axis stabilization in the aero-surfaces control law. In the next section we will see that during the approach and landing phase those two parameters are also consistent and they do not change and, therefore, there is no problem in controlling the lateral directions using aero-surfaces alone during approach and landing. In this transitioning period, however (between t=1500 to 1600), we cannot fully rely on the aero-surfaces to control the lateral directions and it is safer to use the RCS jets as a primary system. In the longitudinal direction we do not have this problem and we can still use the aero-surfaces for pitch control.

We will use this vehicle model generated by Flixan to perform preliminary LQR control designs for the pitch and lateral axes. The control analysis is performed in directory "*C*:*Flixan**Trim**Examples**Re-Entry Glider**Middle Phase**Matan_T1550*". The file "*T1550.Inp*" includes the vehicle input data plus the mixing matrices and system reduction data. Let's take a look at the input data file and describe what is in it.

- 1. On the top of the file there is a batch set "*Batch for analyzing a Re-Entry Vehicle in the Mid-Phase at t=1550 sec*" which processes the remaining data-sets. It generates three types of vehicle models, design plants, mixing-logic matrices, and converts the data to Matlab.
- 2. The first vehicle system is a simulation model with rates defined in the body-axes. Its title is "Re-Entry Vehicle with Aero-Surfaces and RCS Jets, T=1550 sec (Simulation Model)". The body axes definition flag in the flight vehicle data is located below the title and comments. It includes the first 4 aero-surfaces plus the four RCS jets (defined as throttling). The four jets provide a maximum of ±100 (lb) each. The first two generate roll torques and the other two yaw torques. Pitch thrusters are not included because only the aero-surfaces are used for pitch control. This simulation system is saved in Matlab file "Vehicle_Sim.m".
- 3. The second system is a design model that includes only the 4 aero-surfaces. The jets are not included. Its output rates are defined in the stability axis where roll is measured about the velocity vector. Its title is "Re-Entry Vehicle with Aero-Surfaces Only, T=1550 sec (Stab Axis)". It is used for control design using aero-surfaces, mainly in the pitch axis but also for the lateral control backup.
- 4. The next system is also a design model for the lateral axis that includes only the four jets, no surfaces. Its output rates are also defined in the stability axis. Its title is *"Re-Entry Vehicle with"*

3.0 Approach and Landing Phase

The approach and landing phase begins below Mach 1 at trajectory times between 1650 and 1890 sec. The angle of attack is positive and the flight-path angle gets steeper to -21°. The max dynamic pressure is 200 psf. The vehicle banks 40° to perform its final maneuver and to align itself with the runway. During this final period the vehicle no longer dissipates its extra energy by roll-maneuvering but it uses the speed-brake. The flight control system now controls 5 variables. in addition to roll, pitch, and yaw, it controls also the vehicle altitude by means of vertical acceleration (Nz) control, and also its speed. It uses a combination of: flaps and body-flap for Nz control, and the speed-brake for speed control.



In the final segment of the trajectory we must, therefore, trim the vehicle along the 5 controlled directions (roll, pitch, yaw, Ax, Az) and to determine the trim deflections of the 6 aero-surfaces. This will tell us if we have the available control authority to fly along this trajectory, which is in file "*X_Land.Traj*". The basic aero coefficients file is the same as before, but the aero-surface coefficients file "*X_6Surf.Delt*" this time contains all 6 aero-surfaces, including the speed-brake. The surface bias positions are different than the previous phases. Notice that the speed-brake has only positive deflections. The analysis files for the landing phase are in folder "*C:\Flixan\Trim\ Examples\Re-Entry Glider\Landing Phase*". The mass properties file is the same "*XRV.Mass*". We have introduced some lateral unsymmetry by shifting the YCG towards the right by a small amount of 0.1 (ft), because we want to make sure that we have sufficient control authority to land with a small CG offset. So let us begin the Flixan program, choose the project folder for landing, and select the data files for this final phase. In the input/system filenames selection menu choose the default "NewFile" names, as shown

below. Let us first take a look at the trajectory. From the Trim main menu select option (2) which plots the trajectory variables, as shown below.





The 5th column of this aero-surface mixing-logic matrix that controls Az acceleration is set to zero. The matrix controls only roll, pitch, yaw, plus axial acceleration. Even though the 6 aero-surfaces are capable to provide control in all 5 directions, however, there is no need to independently control pitch and Az. Plus it reduces control authority if you spread the control in many directions.



Static Margin, Center of Pressure, Aero-Center (ft), Re-Entry Vehicle with 6 Aer

The static-margin and the pitch T2-inverse parameters are telling us that overall the pitch axis is statically stable with a short period resonance of about 1.5 to 2.5 (rad/sec). Initially, for the first 15 seconds it is neutrally stable to unstable with 1 sec time to double amplitude. The lateral axis is also statically stable with an average Dutch-Roll resonance of 2 (rad/sec). The (Q-alpha, Q-beta) load for a 2° (α_{max} and β_{max}) wind disturbance is below 1800 (psf-deg), which is also acceptable.

The Cn β -dynamic parameter is positive indicating that the vehicle is directionally stable. The LCDP ratio is also consistently positive which indicates that roll controllability is a good region and it does not have any undesirable roll-reversals. It momentarily drops below 0.25 during the bank maneuver. The phi angle with a β_{max} =2° cross-wind is 7°. This parameter is a little higher than expected. We like to see the roll angle less than 5° during landing.

The control authority (rotational and x-translational) plots are telling us that the vehicle has the control authority to trim against the (α_{max} and β_{max}) disturbances not only about the three moments (roll, pitch, and yaw) but also along the x direction, with plenty of margins left to respond against gusts, guidance commands, etc. The control efforts are less than 0.5 in all 4 directions.



Pitch Stability Contour Plot (Mach vs Alpha)



Roll Departure (LCDP) Contour Plot (Mach vs Alpha)



Pitch Control Effort Contour Plot (Mach vs Alpha)





Control Yaw Moment & Side-Force against Moment and Force due to Max Beta Variation



Figure 3.4.1 Lateral Moments and Sideforce Produced by Maximizing the Roll and Yaw Control Demands and by the Wind Disturbance $(\pm \beta_{max})$



The figure above shows the longitudinal partials of accelerations per acceleration demands in pitch and axial directions. The blue vector is $\{\dot{Q}/\delta Q_{FCS}, \ddot{X}/\delta Q_{FCS}\}$, and the green vector is $\{\dot{Q}/\delta X_{FCS}, \ddot{X}/\delta X_{FCS}\}$. It shows that both vectors are not far from unit vectors pointing mainly in their demanded directions, indicating good longitudinal controllability.

3.5 Creating a Dynamic Model Near Landing

We will now use the Flixan and Trim programs to create dynamic models of our re-entry vehicle at t=1860 (sec), which is near landing and in the next section we will create a surface mixing matrix, design a flight control system, and perform stability analysis and simulation. The vehicle at this point uses all 6 aero-surfaces and no RCS jets. The dynamic model has 6 control surface inputs and a wind-gust velocity disturbance input. The gust direction relative to the vehicle is defined in file T1860.inp. The flight control system in this flight condition controls not only the three rotations but speed and the rate of descent. Although the descent rate and speed are controlled independently by guidance commands, the commands, however, in absence of thrust are coordinated by guidance to provide a proper descent rate and alpha and to avoid stalling. The coordination between the 6 aero-surfaces is taken care by the mixing logic matrix which is calculated by Flixan (although a modified version was used). It converts the 5 flight control system demands (roll, pitch, yaw, plus axial and normal accelerations) to surface commands for the 6 aero-surfaces.

So let us resume the analysis using the Trim program. Select the same files as before, and make "T1860.inp" to be the Flixan input data file that will contain the vehicle data, plus other Flixan related model building data. Re-trim the surfaces, if you have not done it already to make sure that you are using a current trim condition. Do not select a trim initialization file and trim along the 3 moments plus axial and normal (A_x and A_z) accelerations. From the Trim main menu select option (5) to create a state-space dynamic model. A dialog reminds the user how to select a flight time for the dynamic model, click "OK". From one of the trajectory plots go the top menu bar, and choose "*Graphic Options*", and then from the vertical pop-up menu click on "*Select Time to Create State-Space System*". Then using the mouse click at time t=1860 sec, along the x axis, and confirm that you have selected the correct time by clicking "OK". Otherwise, click "Cancel" and try again.

Main Trim Menu					
Select one of the following options	Exit OK				
 Plot Aero Coefficients, Derivatives, and Control Surface Increments Plot Trajectory Parameters Versus Time from the Trajectory File ".Traj" Trim the Effector Deflections to Balance the Vehicle Moments and Forces Create an Effector Mixing Logic or a TVC Matrix (Kmix) 					
5. State-Space Modeling of the Flight Vehicle at Selected Times 6. Performance and Stability Parameter Plots Along Trajectory Time					
 Performance and Stability Parameter Piots Along Trajectory Time Landing and Pull-Up Maneuverability, plus, Inertial Coupling Effects Moments at the Hinges of Control Surfaces Along the Trajectory Time 					
 9. View and Modify Vehicle Data (CG, MRC, TVC, Surfaces) for Dispersion Analysis 10. Contour Plots (Mach versus Alpha) for Performance, Control Authority Analysis 11. Vector Diagrams for Maneuverability & Stability at Selected Flight Conditions 					
12. Plot and Compare Previous Data Files (Traject, Trim, Perf	orm, Hinge Moment)				

The program is now ready to create a dynamic model at the selected time. The above dialog shows the flight vehicle parameters prepared by Trim that were extracted from the data files. The user can modify some of the data or titles in this dialog before saving it. Click on "*Update Data*" after any modifications. Do not run it yet because there is more work to be done and more data to be included in T1860.inp. Click on "*Save in File*" and the vehicle data will be saved in file "*T1860.inp*", under the title "*Re-Entry Vehicle with 6 Aero-Surfaces, Landing Phase, T= 1860 sec*". The file "*T1860.inp*" will later be processed by Flixan to generate the systems for control design and analysis using Matlab/ Simulink. In addition to the vehicle data this input file contains also system interconnection and modification data related to this analysis that will be processed by Flixan. The systems and matrices generated by Flixan will be saved in file "*T1860.Qdr*".

Processing the Input Data File

Let us now take a look and see what is inside the already existing file "*T1860.inp*" before processing it in Flixan. This file contains several sets of data and each set corresponds to and is processed by a Flixan utility. Flixan will create systems and matrices that will be used for control analysis in the next section using Matlab.

- The first set of data in this file is a batch for processing the remaining data-sets in batch mode. This is faster because it processes them all together, instead of processing each set interactively. Its title is "Batch for analyzing the Re-Entry Vehicle during Approach and Landing, at t=1860 sec".
- Below the batch there is a flight vehicle data set that generates a vehicle model for the flight condition that was selected above. Its title is "*Re-Entry Vehicle with 6 Aero-Surfaces, Landing Phase, T= 1860 sec*". Its output rates are body rates and it will be used in simulations.
- 3. The next set is also flight vehicle data that generates a vehicle model for the same flight condition as above. Its output rates, however, are stability rates (with respect to the velocity vector) and it also includes the turn-coordination terms with the vehicle dynamics. Its title is "*Re-Entry Vehicle Landing Phase (Stability Axis)*" and it will be used for control design.
- 4. The next set of data generates the (6x5) mixing logic matrix that converts the FCS acceleration demands (roll, pitch, yaw, Ax, Az) to surface deflection commands. The data-set title is "*Mixing Logic for Re-Entry Vehicle Landing Phase*" and the matrix name is Kmix_T1860. This matrix, however, was not used in the analysis but a different one "KSmix" was used instead.
- 5. The next set has a title "*Re-Entry Vehicle with (dV), Landing Phase (Simulation Model)*". It is very similar to the original vehicle model created in step-2 with a small modification. It includes one additional output in its outputs vector, the change in velocity (δ V). This output is extracted from the state vector (state #10) and it will be used in the flight control feedback. This model will be used in the Matlab simulation.
- 6. The next two sets are system modification sets which create the design plants for the LQR algorithm that is implemented in Matlab. Their titles are "*Re-Entry Vehicle Landing Phase Pitch Design Model*" and "*Re-Entry Vehicle Landing Phase Lateral Design Model*". They basically

The following dialog is used for editing and processing Flixan input files. The left menu shows the only input file in the project directory, which is "*T1860.Inp*". Select it and click on "*Select Input File*". The menu on the right side shows the data-sets which are included in "*T1860.Inp*". They can be processed individually, but select the batch set on the top to process them all together, and click on "*Execute/View*". Flixan will process the input file and create the systems in file "*T1860.Qdr*". It will also create the Matlab system files. The procedure described can be used to create models and analyze other flight condition along the trajectory.

Process Input Data Files					
Point to an Input Data Filename and Click''Select Input File''	The following sets of input data are in file: T1860.inp	Exit			
T1860.inp T1860.inp Select Input File Edit File	Run Batch Mode : Batch for analyzing the Re-Entry Vehicle during Approach and Landing, at t=1860 set Flight Vehicle : Re-Entry Vehicle with 6 Aero-Surfaces, Landing Phase, T= 1860 set Flight Vehicle : Re-Entry Vehicle Landing Phase (Stability Axis) Mixing Matrix : Mixing Logic for Re-Entry Vehicle Landing Phase System Modificat : Re-Entry Vehicle with (dV), Landing Phase (Simulation Model) System Modificat : Re-Entry Vehicle Landing Phase Pitch Design Model System Modificat : Re-Entry Vehicle Landing Phase (Simulation Model) System Modificat : Re-Entry Vehicle Landing Phase Eateral Design Model To Matlab Format : Re-Entry Vehicle Landing Phase Pitch Design Model To Matlab Format : Re-Entry Vehicle Landing Phase Pitch Design Model To Matlab Format : Re-Entry Vehicle Landing Phase Design Model To Matlab Format : Re-Entry Vehicle Landing Phase Lateral Design Model To Matlab Format : Mixing Logic for Re-Entry Vehicle Landing Phase	ec.			
Execute/View Input Data Delete Data Set in File					
Relocate Data Set in File Copy to Another File View Data-Set Comments	This batch set creates dynamic models for control design and simulations for the re-entry vehicle during the approach and landing phase. During this phase the altitude is controlled by a combination of pitching and flaps, and the velocity by the speed-brake. The speed-brake is partially opened. The batch creates a model in the body axis for simulation purposes. It creates also two additional models (pitch and lateral) for control design using the LQ method. The change in altitude and velocity (phugoid) states are also included in the pitch design. The surfaces mixing matrix uses the 4th column for drag-modulation control, and the 5th column for Nz control using the flaps. All six aero-surfaces are used to control the vehicle.	e R yr			

3.6 Control Design and Analysis

The control analysis is performed using Matlab in the subdirectory "*C*:*Flixan**Trim**Examples**Re*-Entry *Glider**Landing Phase**Matan_1860*". The following Matlab script file "init.m" is used to load the models and matrices generated in the previous section and to design simple state-feedback flight control laws for the longitudinal and the lateral directions using the LQR method. The longitudinal and lateral design plants are loaded from files: "*pitch_des.m*" and "*later_des.m*", and also the surface mixing matrix "KSmix.mat". The design plants are augmented to include the corresponding columns of matrix KSmix. This is done in Simulink files "Ldes4x.Mdl" and "Pdes6x.Mdl", see Figure (3.6.1), and the 5 control demands are separated to lateral and longitudinal. The roll and yaw demands which correspond to KSmix columns (1 and 3) are included in the lateral design plant "Ldes4x.Mdl". The pitch, Nz, and axial acceleration demands which correspond to KSmix columns (2, 5, and 4) go to the longitudinal design plant "Pdes5x.Mdl". Notice that both pitch and Nz control demands are used for altitude control. It provides a little better performance than using only pitch for altitude control. The lateral state-vector consists of (ps, rs, β, \oplis). The longitudinal state-vector includes both, the short-period and the phugoid modes and its state-vector consists of (θ, q, α, δh, δv). All variables are variations from their nominal variables, which are: θ_0 =-16°, α_0 =6°, h_0 =5600 (ft), v_0 =465 (ft/sec).

```
% LQR Design & Param Initialization file init.m
d2r=pi/180; r2d=180/pi;
[Aps, Bps, Cps, Dps] = pitch_des;
[Als, Bls, Cls, Dls] = later_des;
                                               % Load Pitch aero-surf Design Model
                                               % Load Lateral aero-surf Design Model
[As, Bs, Cs, Ds] = vehicle_sim;
                                                % Simulation Model 6-dof
load KSmix.mat -ascii; Kmix=KSmix;
                                                % Load Surfaces Mix Logic (6 x 5)
alfaO=6.0; VO=465.37; ThetO=-16.0; ge=32.174; % Additional Vehicle Parameters
calfa=cos(alfaO*d2r); salfa=sin(alfaO*d2r);
                                               % for Body to Stability Transform
% Lateral LQR Design Using States: (ps, rs, beta, phi-stab)
% Convert Lateral State Vector from Body to Stability Axes, Outputs=States
[A14,B14,C14,D14] = linmod('Ldes4x');
                                               % 5-state model {p,r,beta,pint}
A15= C14*A14*inv(C14); B15= C14*B14;
                                                % Stability axis System
C15= C14*inv(C14);
                      D15= D14;
R=[1,1]*1; R=diag(R);
                                                % LQR Weights R=[1,1] *2
Q=[10 2 1.5 20]; Q=diag(Q);
                                                % LQR Weights Q=[10 2 0.5 20]
[Kpr,s,e]=lqr(A15,B15,Q,R);
                                               % Perform LQR design on Jets
save Kpr_T1860.mat Kpr -ascii
                                                💲 Lateral State-Feedback Gain
% Pitch LQR Design Using the 6 Aero-Surfaces, States: {theta,q,alfa,dH,dV}
[Ap4,Bp4,Cp4,Dp4] = linmod('Pdes6x'); % Include Kmix in design model
R=[3,1,1]; R=diag(R);
                                                % LQR Control Weights (Pitch, Nz, Ax)
Q=[0.0001 0.1 0.01 14 4]; Q=diag(Q);
[Kq,s,e]=lqr(Ap4,Bp4,Q,R);
                                               % LQR State Weights {theta,q,alfa,dH,dV}
                                                % Perform LQR design on Surf
save Kq T1860.mat Kq -ascii
                                                % Longitudinal State-Feedback Gain
```

Flight Vehicle Dynamics Re-Entry Vehicle Near Landing, Simulation Model



Figure 3.7.2 Re-Entry Vehicle Dynamic Model

The vehicle dynamics block is shown in detail in figure (3.7.2). The state-space system's title is "*Re-Entry Vehicle with (dV), Landing Phase (Simulation Model)*" that was created in file "*vehicle_sim.m*". On the left side it receives the lateral (red) and longitudinal (blue) flight control demands which are converted to surface deflection commands by the surface mixing matrix KSmix. The deflection commands are filtered by a set of first order actuators and become surface deflections that drive the first 6 inputs of the vehicle model. Input number 7 is a wind-gust velocity disturbance input. The direction of the disturbance is defined in the vehicle data. This input is excited by a noise generator that creates random gusts with amplitudes varying ± 10 (ft/sec). The system's outputs are listed above, and some of the outputs are used for flight control feedback. The output data are saved for plotting using file "pl.m".

Control System Architecture

During the final approach and before landing the vehicle aligns its direction with the runway and controls its rate of descent and speed. The lateral control system is shown in figure (3.7.3). It receives a heading direction command (ψ_{cmd}) from guidance and aligns its heading with the runway. The heading error becomes a roll command that temporarily rolls the vehicle to change its heading direction.



Figure 3.7.3 Heading Direction Control System

The state-feedback control gain (Kpr) was calculated by the LQR method. However, instead of beta in the state vector the lateral accelerometer measurement (Ny) is used. The scaled Ny is a good approximation for beta in this case.



Figure 3.7.4 Longitudinal Flight Control System Controls Altitude and Speed

The longitudinal flight control system used in the simulation model is shown in figure (3.7.4). In the longitudinal direction the control system receives coordinated altitude and speed commands. Guidance calculates the changes in altitude (δh_{cmd}) and speed (δV_{cmd}) commands which control its rate of descent and speed. It uses the longitudinal state-feedback control gain (Kq) that was calculated by the LQR method. The inputs to the state-feedback matrix are: (θ , q, scaled Nz instead of α , altitude error δh , and speed error δV). Filtered accelerometer measurements are used instead of α and β because they are directly measured, otherwise α and β must be estimated.

Simulation

We will now use the simulation model described in figure (3.7.1) to perform a descent and landing simulation of the re-entry vehicle starting from an altitude of 5,560 (ft), α =6°, and a speed of 460 (ft/sec), which corresponds to the linearized model at t=1860 (sec), all the way to landing. This linear simulation is of course an approximation because the vehicle dynamic parameters change with time. For more accurate analysis a non-linear 6-dof simulation is required to accurately model the approach and landing phase (similar to the simulations presented in the F16 or Lifting-Body aircraft examples). But even this linear simulation is very useful in evaluating the flight control performance, plus it is much easier to implement in comparison with a non-linear 6-dof simulation.







In the longitudinal axis the control system receives from guidance coordinated altitude and speed commands versus time. The above plot shows the altitude versus time response of the vehicle. The altitude follows the altitude descent command through a steep gamma glide-slope the flare (nose up) and landing (where γ =0). The effects of the noisy wind-gusts have a effect in the velocity versus time plot.

The next plot shows the deflections of the control surfaces. Note that these deflections are not measured from zero, but they are increments relative to their trim values. That is, the trim deflections at t=1860 sec, which are shown in the input data file "T1860.Inp". Notice how the speed-brake and body-flap deflections increase prior to the flare to dissipate the extra energy and to control the angle of attack as needed in order to slow down the rate of descent.

Stability Analysis

The next important analysis to perform is to check the gain and phase margins of the LQR control design. The Simulink model used for frequency response analysis is in file "*Stab_Anal.Mdl*", shown in Figure (3.7.5). It is used from the script file "freq.m" that calculates the frequency response across the opened loop. The loop opening is adjustable. Either the heading loop, velocity loop, or the altitude loop is opened at a time, with the other two closed. The stability analysis Bode and Nichols Charts are shown in Figure (3.7.6).

```
% Stability Analysis Script
d2r=pi/180; r2d=180/pi;
init
label='Velocity Control Loop Opened, all other loops are closed'
[Ao,Bo,Co,Do]=linmod('Stab_Anal'); % Frequ analysis system
sys= SS(Ao,Bo,Co,Do);
w=logspace(-3, 3, 30000);
figure(1); Nichols(sys,w); title(label);
figure(2); Bode(sys,w); title(label);
```



Figure 3.7.5 Simulink Model "*Stab_Anal.Mdl*" used for frequency response stability analysis. It is shown in two different configurations: (a) for heading loop stability, and (b) for altitude control loop stability analysis.



