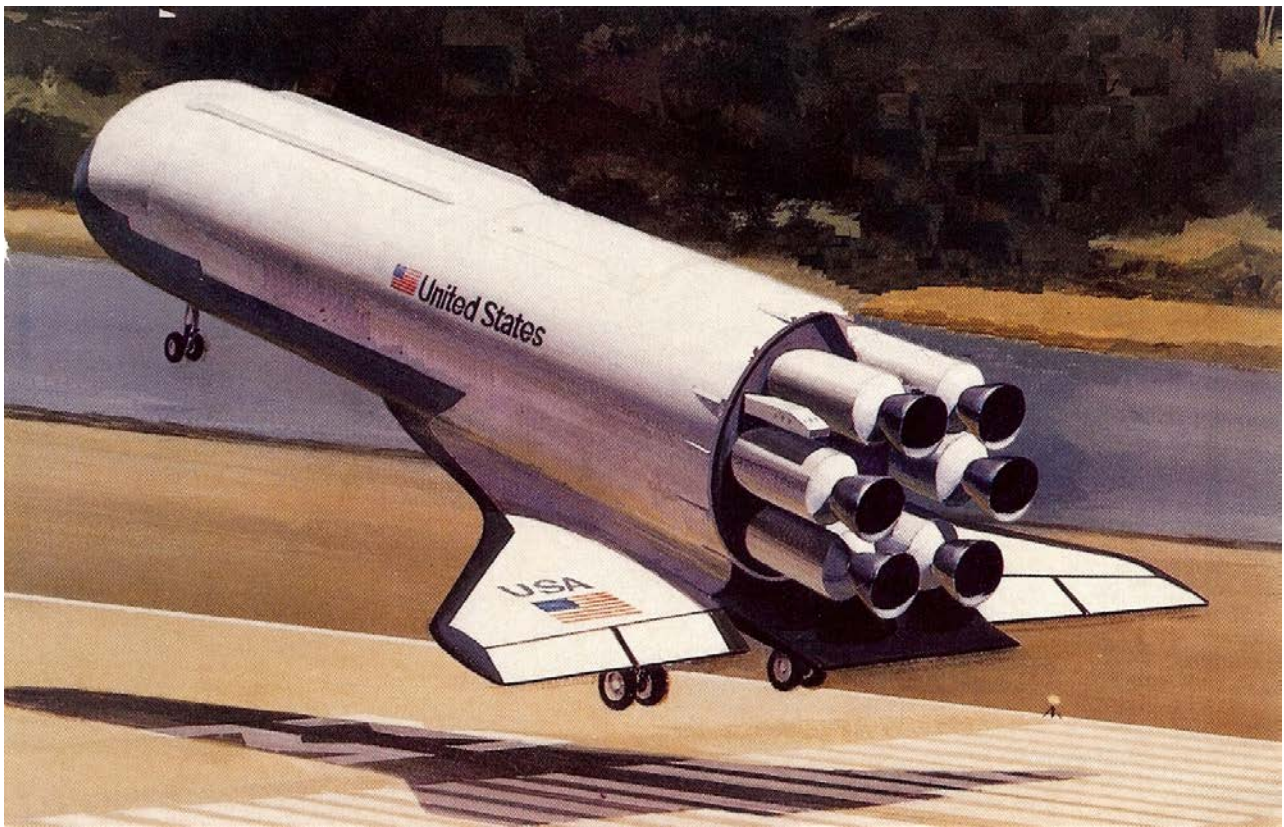


# Reusable Launch Vehicle



In this example we have a reusable rocket-plane that looks like a launch vehicle with wings, similar to the one shown in the picture. It takes off vertically like a typical launch vehicle, reaches up to low earth orbits and it returns to land like a glider without engine power. During ascent it uses 6 engines that provide 350,000 (lb) of total thrust. The engines in this example are different types with different thrusts for the purpose of demonstrating the Trim program capability to combine different engines. The engines configuration are shown in Figure (1). The first two engines are gimbaling  $14^\circ$  in pitch and  $12^\circ$  yaw and they have constant thrusts of 70,000 (lb) each. Engines #3 and #4 are also gimbaling  $5^\circ$  and they can also vary their thrusts from 9,200 to 100,000 (lb) each. Engines #5 and #6 can also vary their thrusts from 9,200 to 100,000 (lb) each, but they are fixed and they do not gimbal. The space-plane also uses two elevon/ flaps which are located at the wings. During ascent the vehicle uses the two flaps to control roll because the gimbaling engines are too close together and they do not provide sufficient rolling moment for control. Notice, that the space-plane does not have a vertical tail and rudder. During descent the engines are not active but the vehicle uses the two flaps for pitch and roll control and two lateral thruster jets for yaw control which are located near the front. Two different trajectories were created for analyzing the vehicle performance during ascent and descent.

In the following sections we will use the Trim program to analyze the vehicle performance during ascent and descent in separate sections and different folders. Evaluate its performance and

stability parameters, its trim capability against aero disturbances, effects due to lack of lateral symmetry caused by the YCG offset, engine and actuator failures, create linear models, design flight controls, and analyze its dynamic performance.

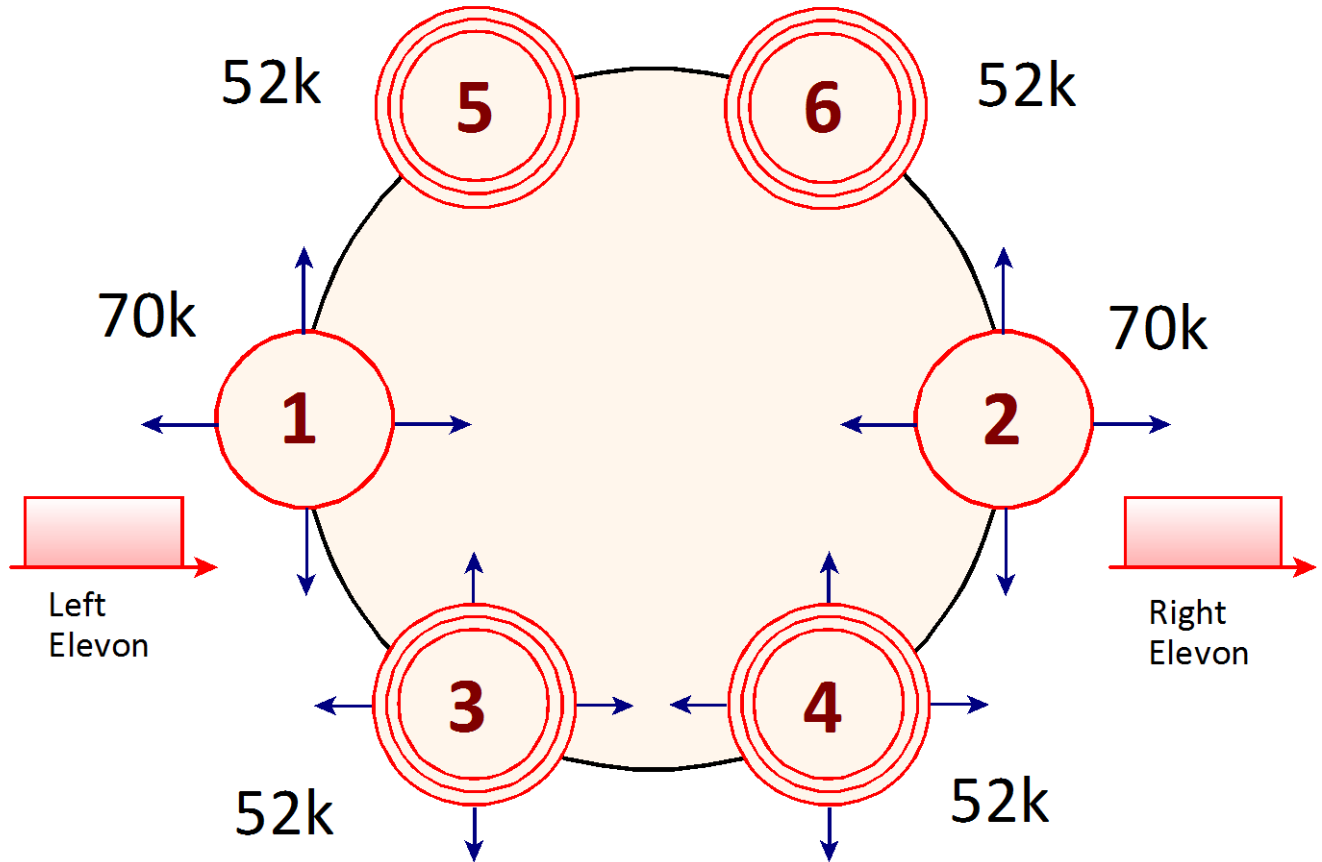


Figure (1) During Ascent the Vehicle Uses Six Main Engines and Two Elevon/ Flaps

## 1.0 Ascent Analysis Files

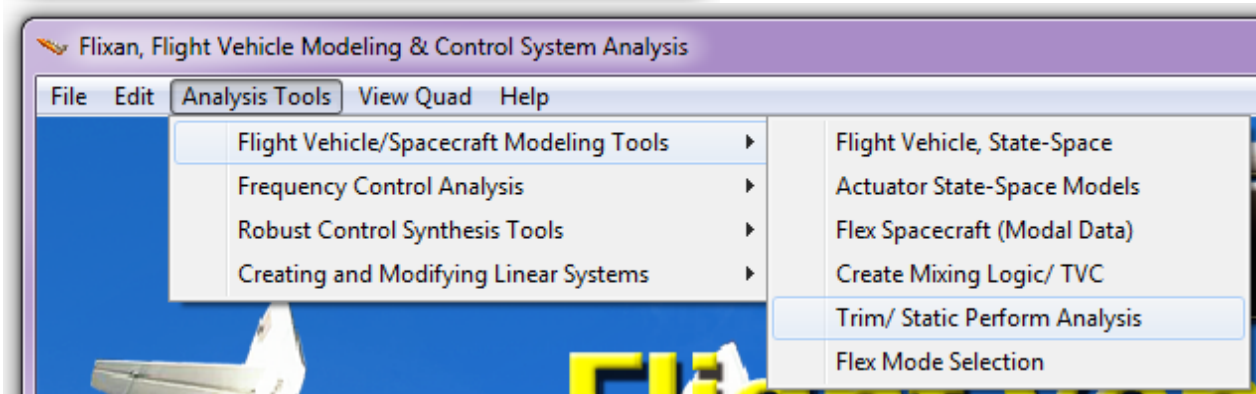
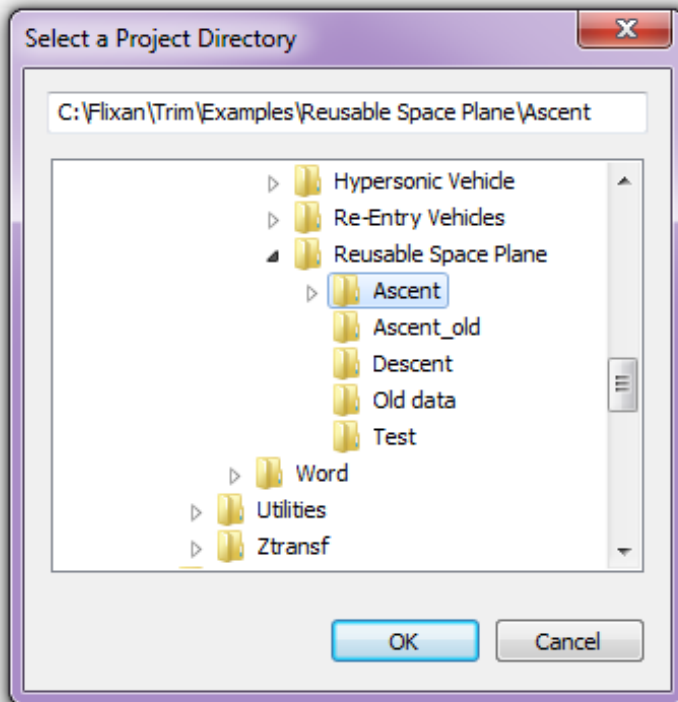
The input data files for the ascent analysis have already been formatted for the Trim program. They are located in folder "*C:\Flixan\Trim\Examples\Reusable Space Plane\Ascent*", that contains the following files:

- **Trajectory Files:** There are two trajectory files containing only the boost phase of the trajectories when the engines are firing. In both cases the vehicle takes off vertically and reaches approximately the same altitude of about 260,000 (ft), but the maneuvers are very different. The first trajectory is in file "*ReSP\_Ascent\_A.Traj*" and performs a 360 pitch maneuver at high altitude, and the second one is in file "*ReSP\_Ascent-B.Traj*".
- **Mass Properties:** The vehicle mass properties are located in file "*ReSP.Mass*". It contains the inertias and CG location as a function of vehicle mass (slugs) in descending magnitude.

- **TVC/Engine Data:** The data for the 6 main engines, such as: thrusts, positions, mounting angles, max deflections, throttle parameters, etc, are located in file "*ReSP6.Engn*". All 6 engines are mounted in the back with a  $-2^\circ$  tilt in pitch ( $D_y=2^\circ$ ,  $D_z=0^\circ$ ). The first two engines have a constant thrust of 70,000 (lb) and they do not throttle but are gimbaling  $14^\circ$  in pitch and  $12^\circ$  yaw. The next two engines have a nominal thrust of 52,000 (lb) and they can vary their thrust  $\pm 95\%$  from nominal. They can also gimbal  $5^\circ$  in pitch and yaw. The last two engines are not gimbaling but only throttling. They have a nominal thrust of 52,000 (lb) each (at zero throttle command) and they can vary their thrust  $\pm 95\%$  from nominal.
- **Basic Aero Data:** The aero data base in this example is kind of poor. We only have force and moment coefficients at two Mach numbers, 3 betas and 12 alpha increments. They are located in file "*ReSP.Aero*".
- **Aero-Surface Data:** This vehicle has two aero-surfaces, the left and right flaps. The aero-coefficient increments of both surfaces are in file "*ReSP.Delt*". They correspond to the same numbers of Mach, alpha, and beta as the basic aero data, and they also include 4 increments for each aero-surface.
- **Damping Derivatives:** In this example we have a damping derivatives file "*ReSP.Damp*" that corresponds to the same Mach numbers and alphas as the basic aero data.
- **Hinge Moment Coefficients:** We also have a hinge moment coefficients file "*ReSP.HMco*" that corresponds to the same Mach numbers, alphas, betas, and surface increments as in the aero-surface data.
- **Propellant Sloshing Data:** This launch vehicle contains two propellant tanks: a liquid oxygen (LOX) and a liquid hydrogen (LH2) tank. Although the Trim program itself does not analyze sloshing, the slosh data, however, are transferred to the flight vehicle modeling program and used for creating slosh dynamic models. The slosh data file "*ReSP.Slsh*" contains data for the two sloshing tanks as a function of the vehicle mass in (slugs). The data consist of: slosh masses, slosh frequencies (at 1g acceleration), damping, and the location of the slosh masses in vehicle coordinates. The slosh frequencies are converted by the vehicle modeling program from 1g to the frequencies that correspond to the instantaneous vehicle acceleration.

## 1.1 Trajectory (a) Analysis

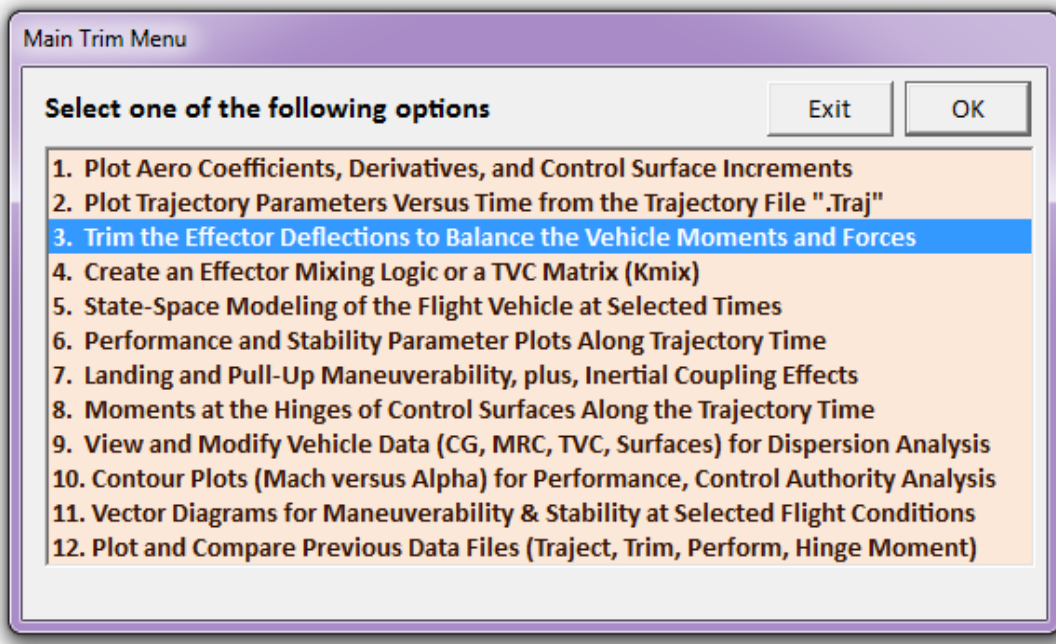
It is a good practice to familiarize ourselves with the trajectory by taking a good look at it before analyzing the vehicle performance. After starting the Flixan program the user must select the project folder that contains the analysis files, then from the Flixan main menu select "Analysis Tools", "Flight Vehicle/ Spacecraft Modeling Tools", and then "Trim/ Static Performance Analysis", as shown below.



## 1.2 Trim Analysis Using Trajectory (A)

During ascent the space-plane uses the two flaps and 6 engines. The flaps are used mainly for roll control because the TVC does not have sufficient control authority in roll. The 6 engines are in file "*ReSP6.Engn*" and they are different, as already described. The Trim program calculates the throttle commands, the TVC deflections, and the aero-surface deflections required to balance the moments and the axial force along the trajectory. The throttle parameters of the 4 throttling engines is 0.95 which defines the maximum thrust variation from nominal. During trimming the throttle commands define the actual thrust variation from nominal as a function of time, required to balance the moments and acceleration, and they can vary between zero and  $\pm 1$ .

Return to the Trim main menu and choose option (3) to trim the effectors. There is a total of 14 effectors in this configuration. From the next two menu/dialogs, do not select an initialization file, and in the menu that selects the directions to be balanced choose the third option to trim along 3 moments, plus axial acceleration since the vehicle has a variable thrust capability. The program calculates the engine and surface deflections and the thrust variations required to balance the 3 moments and the axial acceleration along the trajectory. The deflections are based on the individual effector control capability.

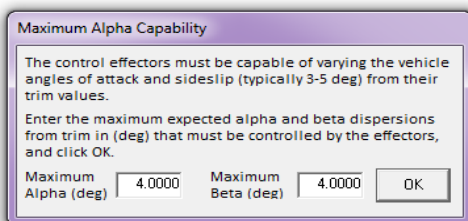
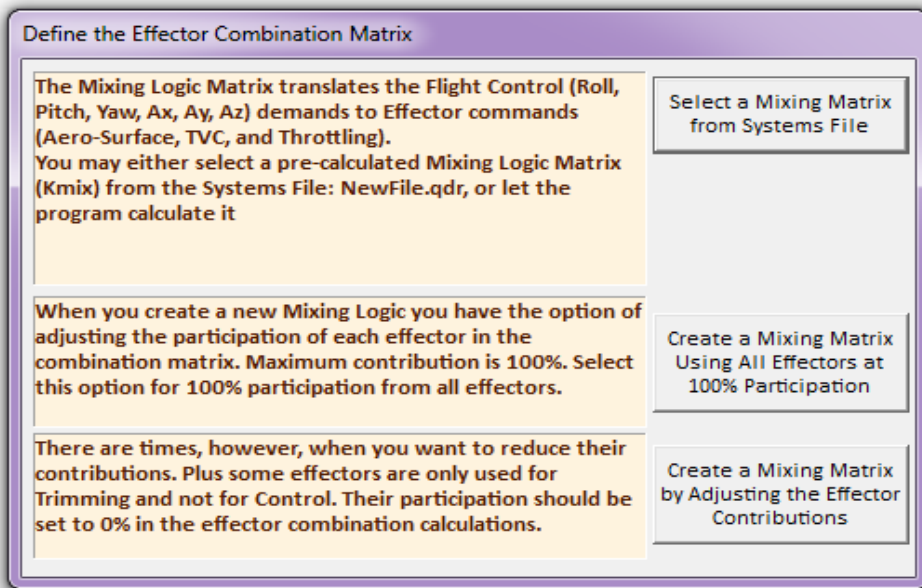
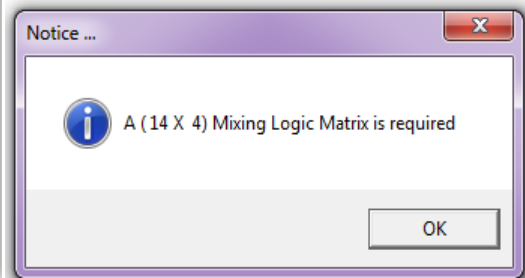
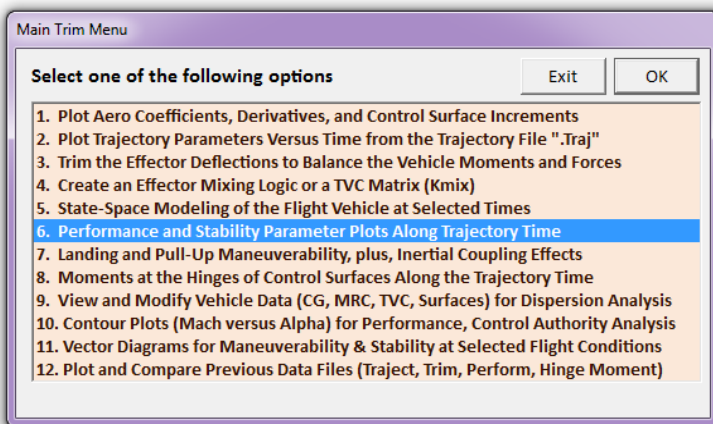


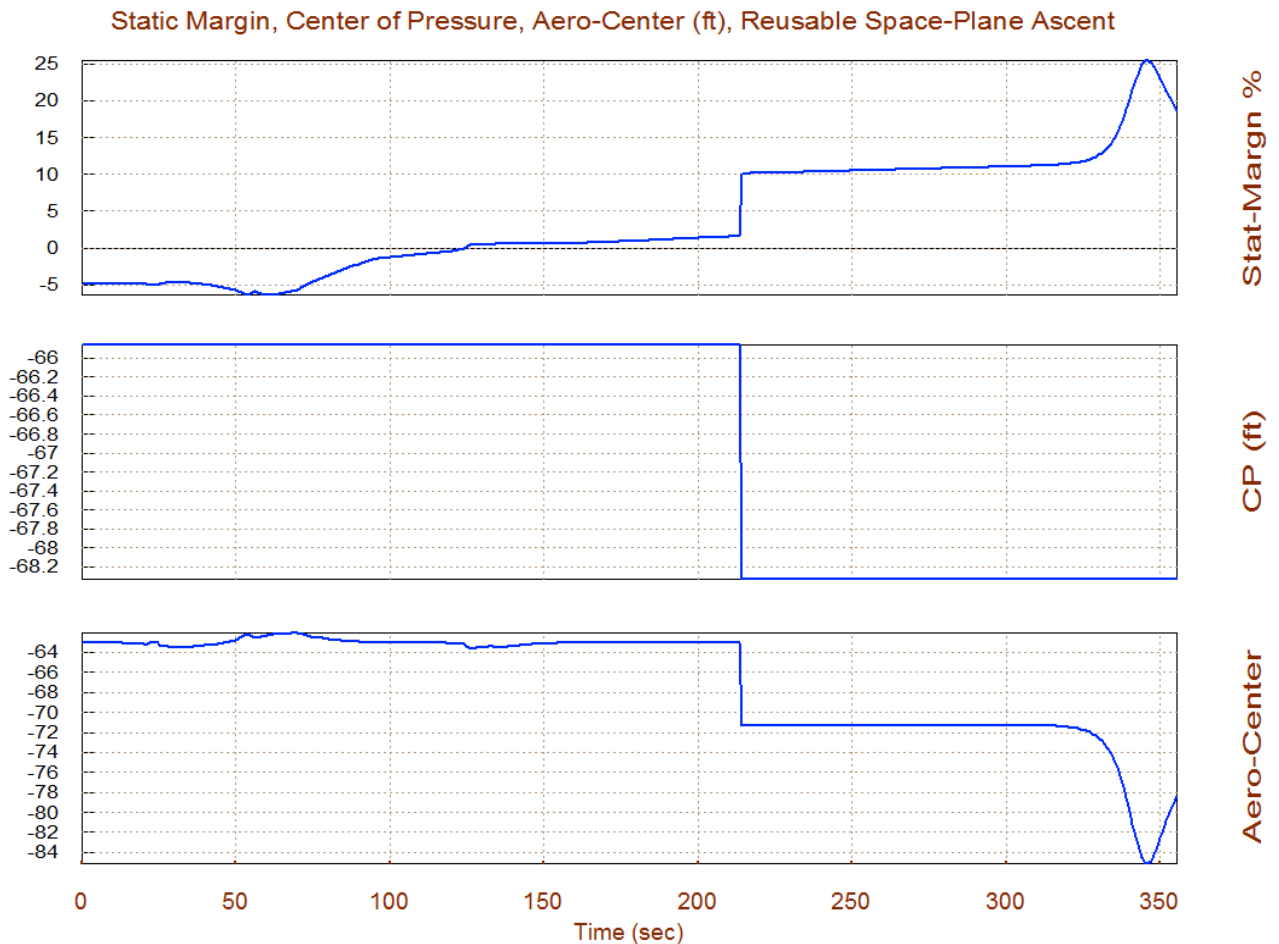
The +YCG offset causes the vehicle to yaw slightly creating a  $0.25^\circ$  beta sideslip. The four TVC engines are, therefore, balancing the yaw moment due to beta by rotating approx  $0.3^\circ$  in yaw ( $\delta_z$ ). The pitch deflections of the four engines ( $\delta_y$ ) are not the same because they are balancing the rolling moment. The thrusts of the throttling engines vary significantly when trimming in order to match the acceleration defined in the trajectory. There is also a small difference in the deflections of the two flaps which help in balancing the rolling moments generated due to beta and the  $Y_{CG}$  offset.



### 1.3 Performance Analysis of Trajectory (a)

Let us now analyze vehicle static performance parameters along the first trajectory, as described in Section 3. Return to the main menu and select option (6) to check the performance and stability parameters along the trajectory time. However, before evaluating the vehicle performance the program needs to know how the effectors combine together to trim along the four directions. The flight control system will also independently control these four directions. A mixing logic matrix defines the effector allocation along the four control directions. The program can either read a matrix from the systems file or calculate it from the vehicle parameters. In this case we allow the program calculate a Kmix along the trajectory by selecting the 2<sup>nd</sup> option "Calculate a Mixing Matrix Using All Effectors at 100% Participation".

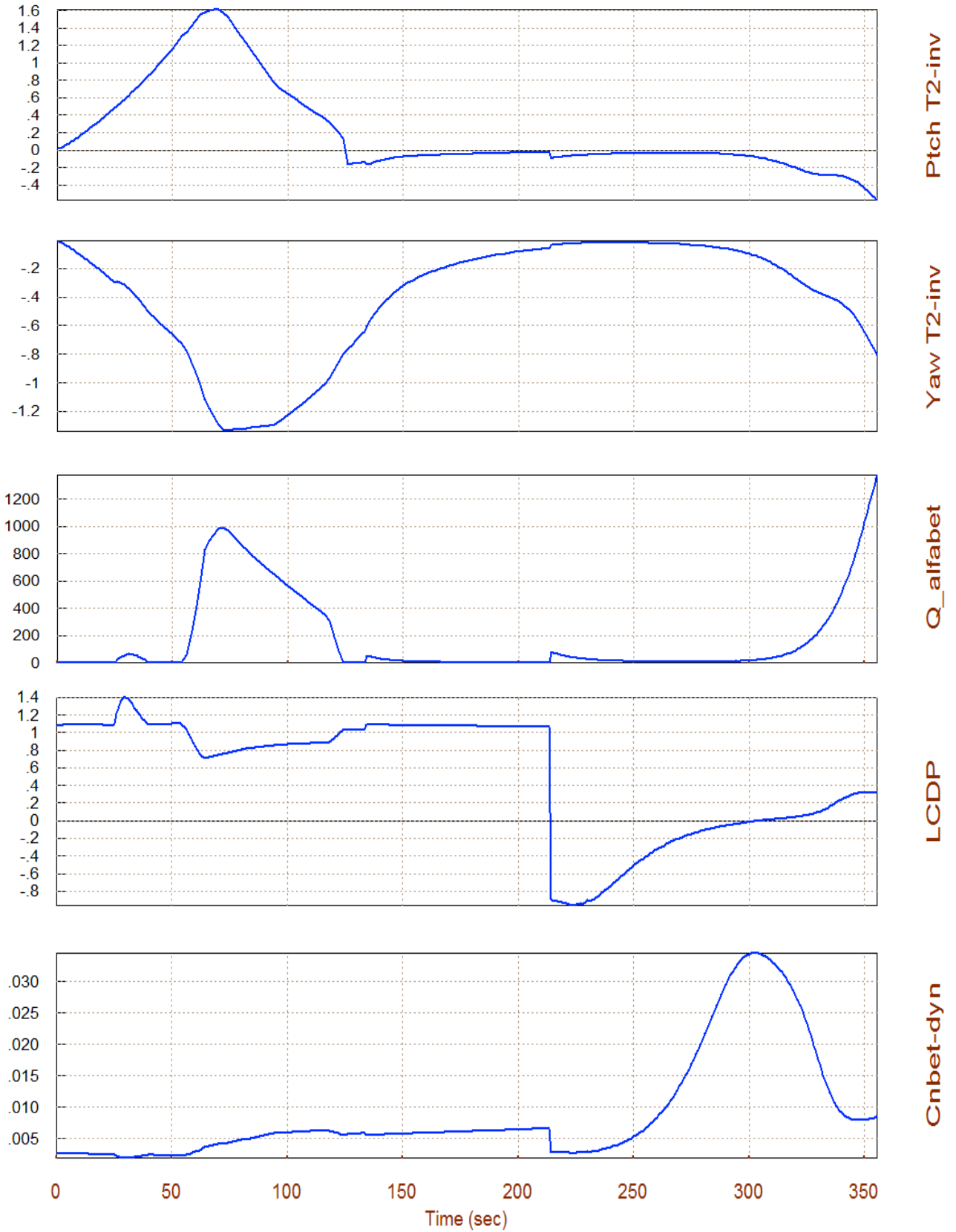




The effector combination matrix will have 4 inputs for the 4 control directions (3 rotations plus x-acceleration), and 14 outputs (8 TVC, 4 throttling, and 2 aero-surfaces). In the following dialog enter the maximum amount of angle of attack and sideslip angles ( $\alpha=4^\circ$ ,  $\beta=4^\circ$ ) expected during flight. These are aerodynamic disturbances generated due to wind-shear or due to maneuvering, producing moments and forces that the vehicle should have the control power to counteract. The effector system control authority is measured by its capability to counteract them.

The static margin is telling us that this launch vehicle is unstable in pitch during the first 130 seconds of the trajectory and later on it becomes stable. From the pitch T2-inverse plot we see that the max instability occurs at 70 seconds and the time to double amplitude  $T_2=0.62$  (sec). In the lateral direction, however, is always stable with a positive  $C_n\beta$ -dynamic. The yaw T2-inverse curve is negative (stable), and the Dutch-Roll resonance peaks to 1.3 (rad/sec) at 75 sec. The combined (Q-alpha and Q-beta) loading at ( $\alpha=4^\circ$  &  $\beta=4^\circ$ ) is acceptable. The LCDP parameter looks great up to 210 sec and then it suddenly changes sign a couple of times because of the large pitch maneuver. This sign reversal in the LCDP implies that the control gains must also change sign after the  $180^\circ$  maneuver. This is not a surprise since the entire control law must be changed during this low dynamic pressure large angle maneuver using a combination of RCS and TVC.

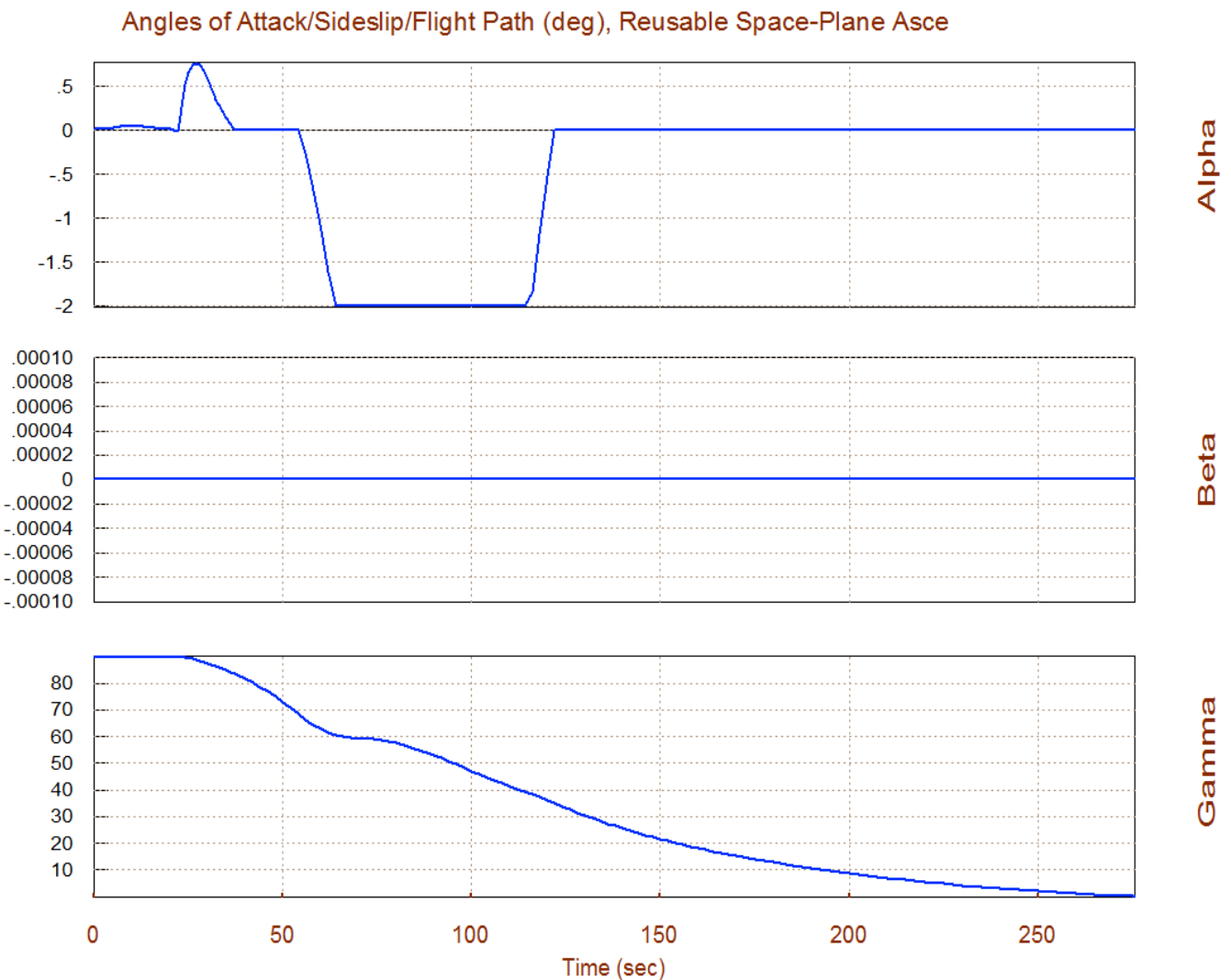
Short-Period (w)/ Time-to-Double-Ampl-Inverse (/sec), Q\_alpha\_beta (deg-lb/ft^2)



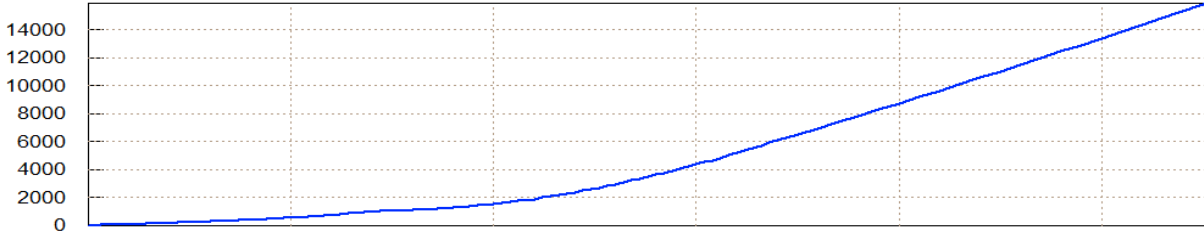


## 1.4 Trajectory (B) Analysis

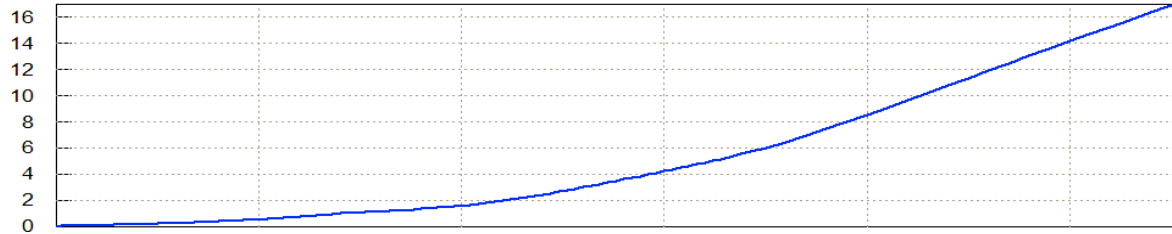
Let us now repeat the Trim analysis to check the vehicle performance along the second trajectory. We must first re-run Flixan, select the same project folder but this time in the filename selection menus we must select the second trajectory file "*ReSP\_Ascent\_B.Traj*". Before beginning our analysis let us first take a look at the trajectory. From the Trim main menu select option (2) for plotting the trajectory parameters. This trajectory is different from the previous one because it does not perform the large 180° pitch maneuver and the thrust does not have the big drop from 360,000 to 180,000 (lb), as it did in the first trajectory, but it is gradually reduced to zero. Also, the angle of attack is small, unlike the wild alpha maneuver that we saw in the first trajectory.



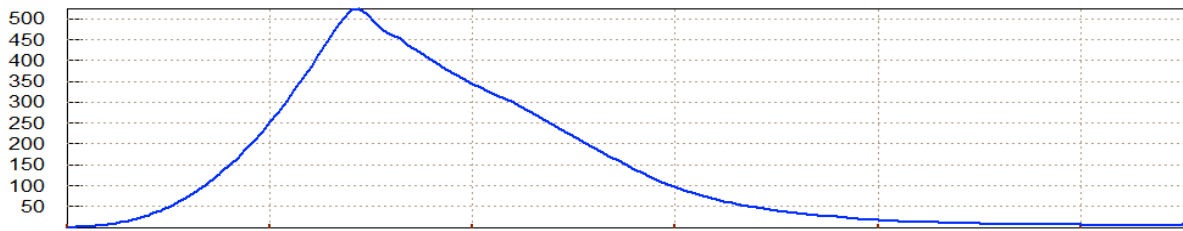
Velocity, Dynamic Pressure, Reusable Space-Plane Ascent Trajectory-2



Veloc (ft/s)

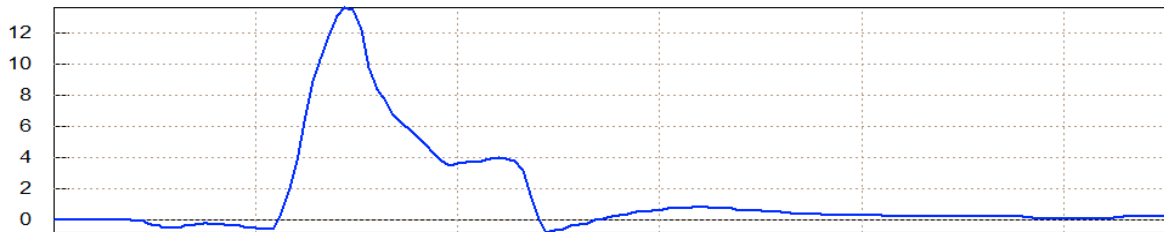


Mach Number

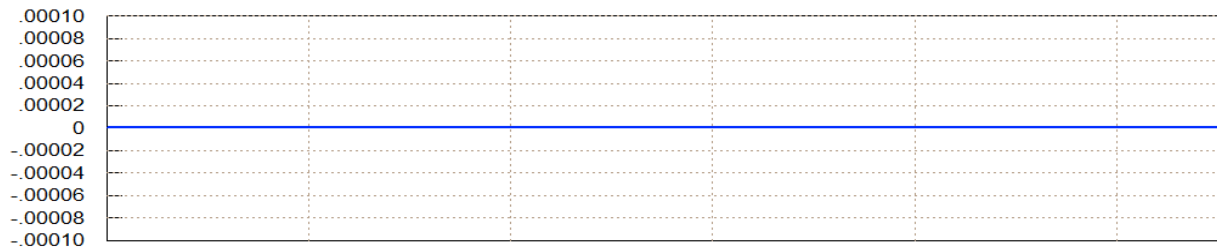


Q-bar (PSF)

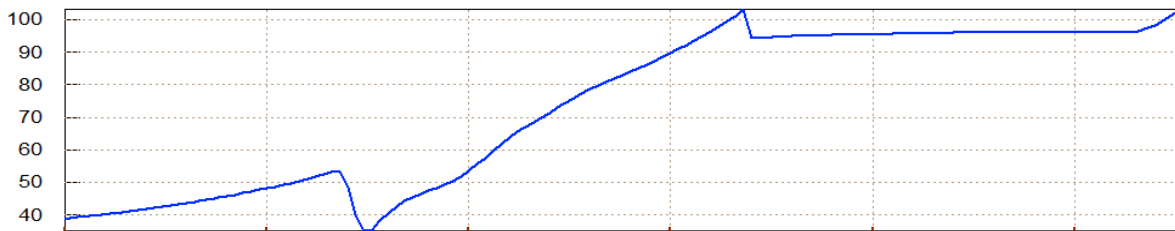
Sensed Acceleration in (ft/sec^2), Reusable Space-Plane Ascent Trajectory-2



Accel-Z



Accel-Y

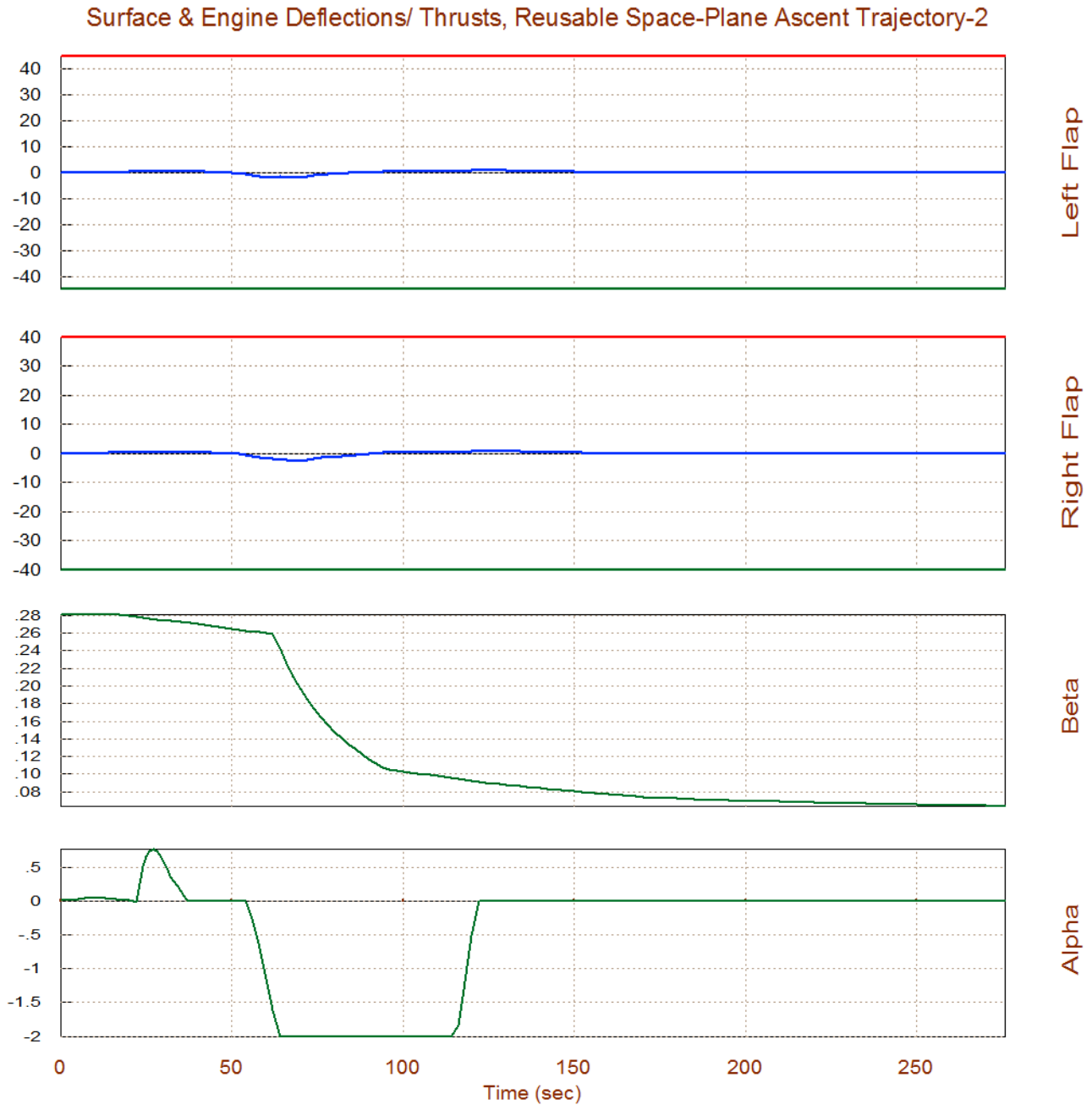


Accel-X

Time (sec)

### 1.5 Trim Analysis Using Trajectory (B)

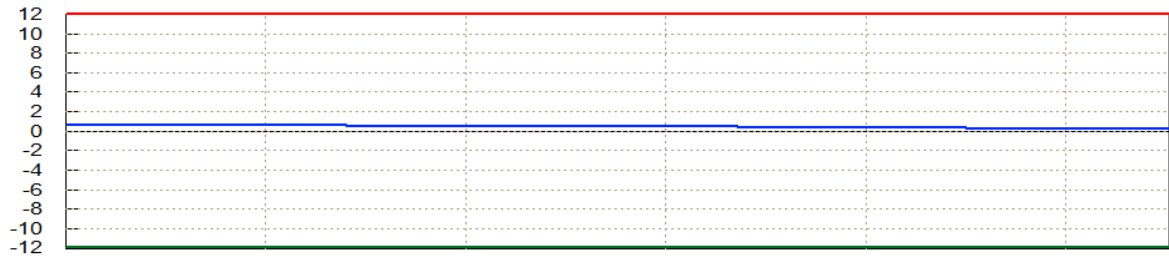
Now let us trim the effectors using the second trajectory. The program still uses the same effector files, only the trajectory has changed and, obviously, the trim history of the 2 flaps and the 6 engines thrusts and deflections will be different in this trajectory. Return to the Trim main menu and choose option (3) to trim the effectors as in Section 1.2. From the two menu/dialogs, do not select an initialization file (because it's the first time), and choose to trim along 3 rotations plus the axial acceleration, like before. The program will calculate the new effector deflections and thrust variations required to trim the moments and forces along trajectory (b), as shown below. Also, similar to the previous trajectory the  $+Y_{CG}$  offset causes the vehicle to yaw, creating a sideslip ( $\beta$ ).



Surface & Engine Deflections/ Thrusts, Reusable Space-Plane Ascent Trajectory-2



Dy\_Engine 1



Dz\_Engine 1

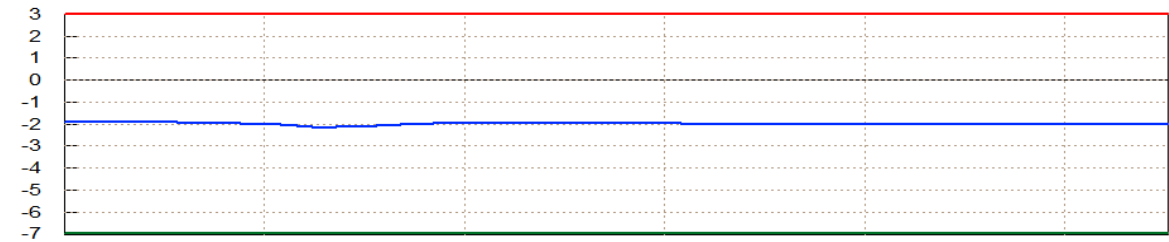


Dy\_Engine 2

Surface & Engine Deflections/ Thrusts, Reusable Space-Plane Ascent Trajectory-2



Dz\_Engine 2



Dy\_Engine 3



Dz\_Engine 3

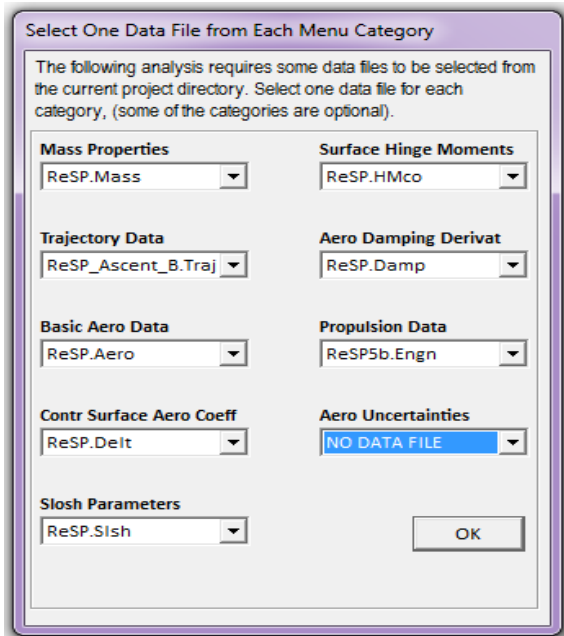
0 50 100 150 200 250  
Time (sec)

## 1.6 Analyzing Engine Failures

Engine failures are not uncommon in launch vehicles with multiple engines and in most cases they should be able to complete their missions in the event of at least one engine failure. Failures may be due to a loss of thrust or a TVC actuator failure. In the analysis that follows we will demonstrate that we are able to trim and to control the vehicle in the event of loss of thrust from one of the 6 engines or in the event of an actuator failing to gimbal. We will fail one of the throttling engines, the last one #6, and see if the remaining 5 engines with the control surfaces are able to compensate and to continue trimming the vehicle along the same trajectory. It is easy to fail an engine in Trim, either by removing one of the engines from the engines data file, or by setting its effectiveness to zero. The second method is more convenient because it makes it easier when co-plotting and comparing the data (before and after failure). We are still keeping engine #6 in file, but its thrust and throttling capability are set to a very small number, which essentially is eliminating its authority. Instead of modifying the original engines data file "*ReSP6.Engn*", it makes more sense to copy it under a different name "*ReSP5b.Engn*" and modify that file instead. So let us restart the Trim program and select the failed engine file instead of the original.

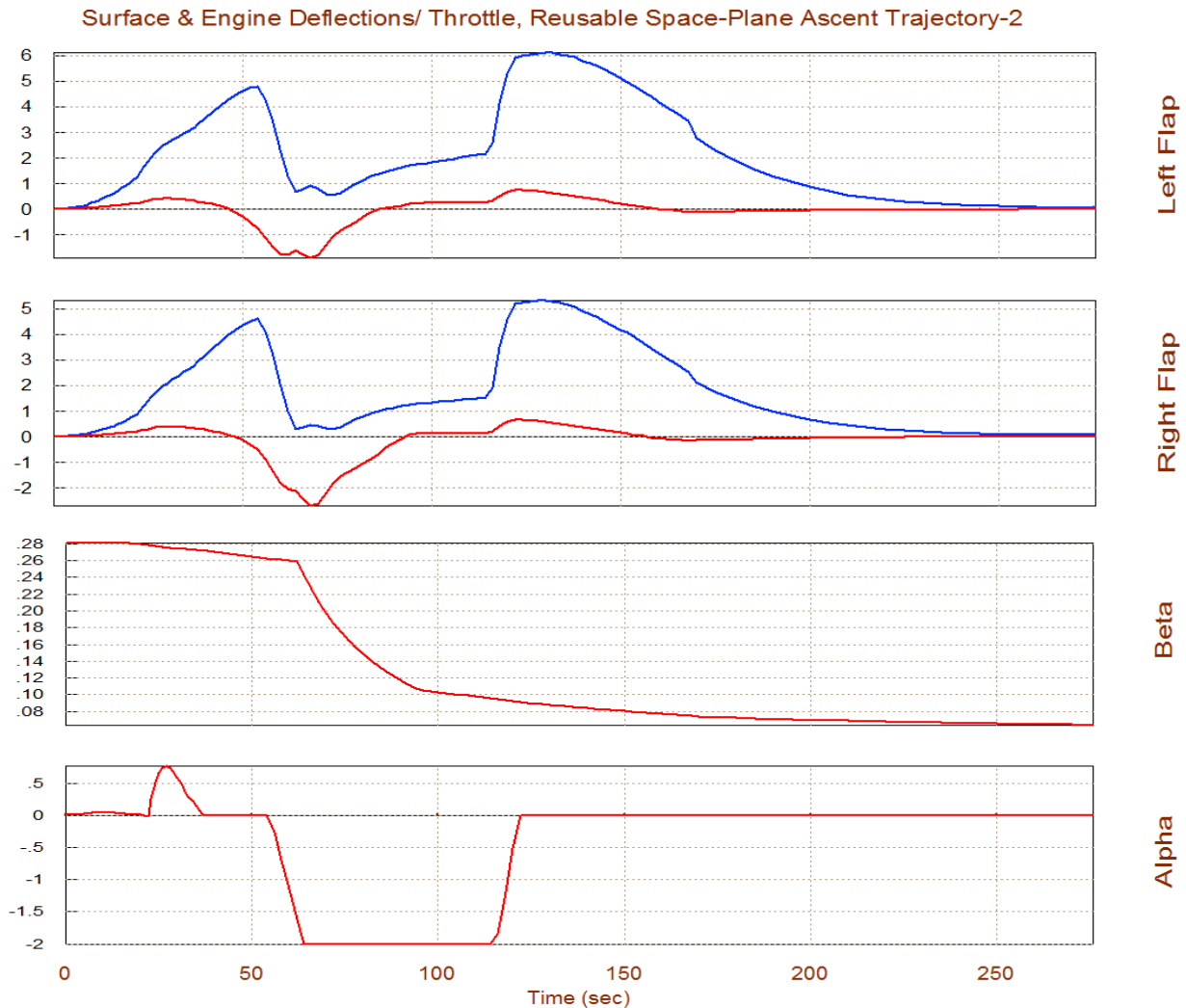
Reusable Launch Vehicle data for the 6 Engines  
 Engine #6 Has Lost its Thrust. Normally it was Throttling but Not Gimbaling

Engine Description,	Thrust (lb)	Mass (slug)	Ieng (slug-ft <sup>2</sup> )	Mom Arm (ft)	Location (x,y,z) (feet)			Mounting Angles (Dy, Dz) Elevat, Azimuth (degr)		Max Deflection Dym,Dzm (deg)		Max Throttl (0-1)
Main Engine#1	70000.0	100.0	1500.0	2.8	-89.0	-4.8	0.0	-2.0	0.0	14.0	12.0	0.0
Main Engine#2	70000.0	100.0	1500.0	2.8	-89.0	+4.8	0.0	-2.0	0.0	14.0	12.0	0.0
Main Engine#3	52000.0	100.0	1500.0	2.8	-89.0	-2.25	+3.9	-2.0	0.0	5.0	5.0	0.95
Main Engine#4	52000.0	100.0	1500.0	2.8	-89.0	+2.25	+3.9	-2.0	0.0	5.0	5.0	0.95
Main Engine#5	52000.0	100.0	1500.0	2.8	-89.0	-2.25	-3.9	-2.0	0.0	0.0	0.0	0.95
Main Engine#6	0.01	100.0	1500.0	2.8	-89.0	+2.25	-3.9	-2.0	0.0	0.0	0.0	0.01



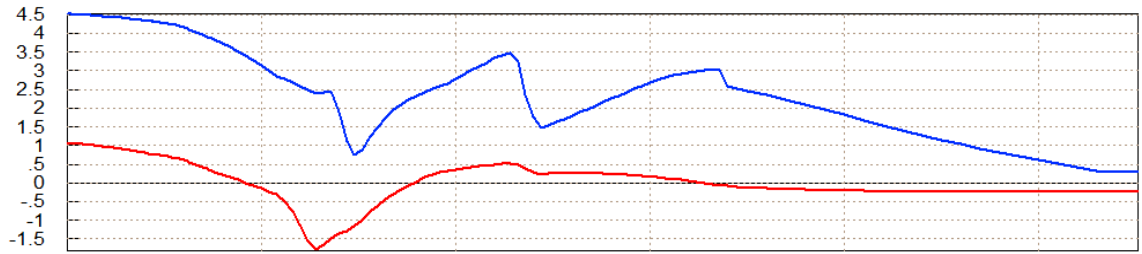
We must trim again along 3 rotations plus the axial acceleration using again the second trajectory and the 5 engines file this time, without selecting an initialization file, as before. When the trim is complete return to the main menu and chose option (12) to co-plot and compare the two trim data files: the original trim file "*ReSP\_Ascent\_B1.Trim*" with the 6 engines, against the latest one "*ReSP\_Ascent\_B.Trim*" that has one disabled engine.

There is a sideslip beta due the +YCG offset like before. Only the throttle values instead of actual thrusts are shown in the comparison plots (the actual thrusts are only shown in Option-3). The red curves are the effector positions from the original 6-engines file (saved by the program in *ReSP\_Ascent\_B1.Trim*), and the blue curves are from the recently created file with the failed engine #6 (*ReSP\_Ascent\_B.Trim*). Both trim files were obtained from trajectory (b). The throttle value and also the thrust of Engine #6 (Throttle 4) are zero according to the way it was defined in the modified engine file. The throttle values of the remaining 3 throttling engines (Throttle 1 to 3) have been increased in comparison with the 6 engines case, as expected, in order to make up for the failed engine. The yaw deflections ( $\delta_{z1}$ ,  $\delta_{z2}$ ,  $\delta_{z3}$ ,  $\delta_{z4}$ ) of the four gimbaling engines have been increased in order to provide a negative yawing moment and to make up against the positive yawing moment produced by the absence of engine #6 thrust. The pitch deflections ( $\delta_{y1}$ ,  $\delta_{y2}$ ,  $\delta_{y3}$ ,  $\delta_{y4}$ ) in the four TVC engines is also increased in order to make up for the lack of symmetry. The flap deflections are increased and also the difference between left and right flaps is also increased in order to counteract the rolling moment caused due to the lack of lateral symmetry.

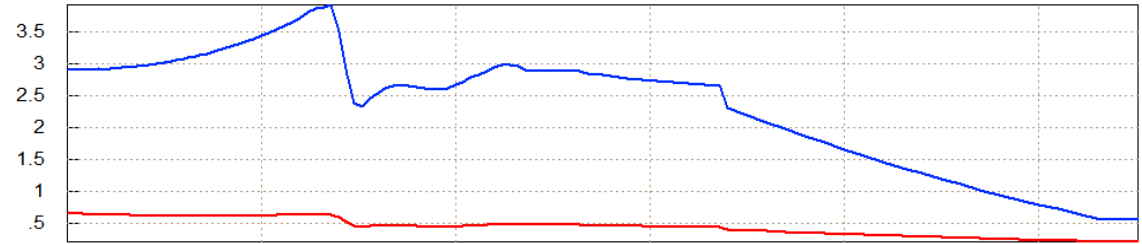




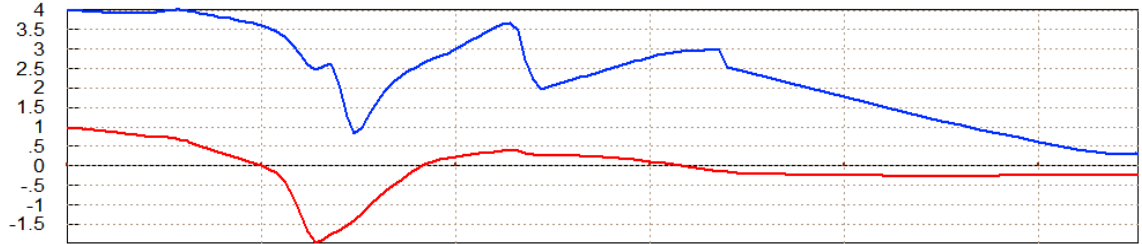
Surface & Engine Deflections/ Throttle, Reusable Space-Plane Ascent Trajectory-2



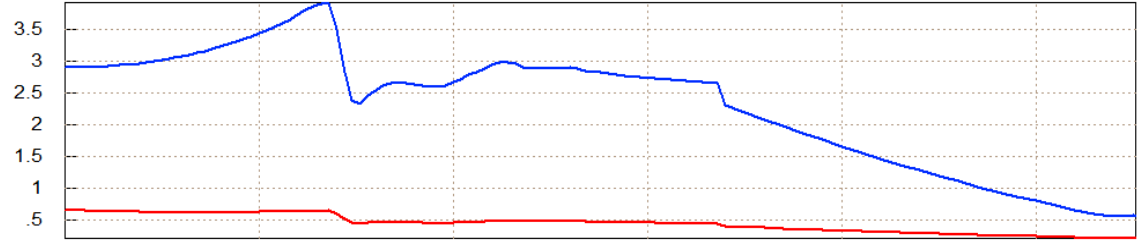
Dy\_Engine 1



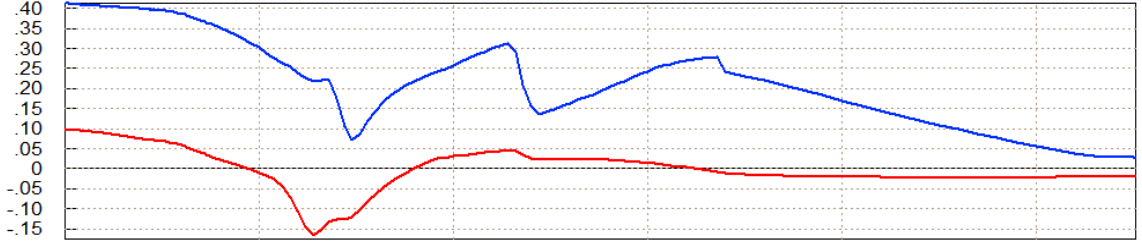
Dz\_Engine 1



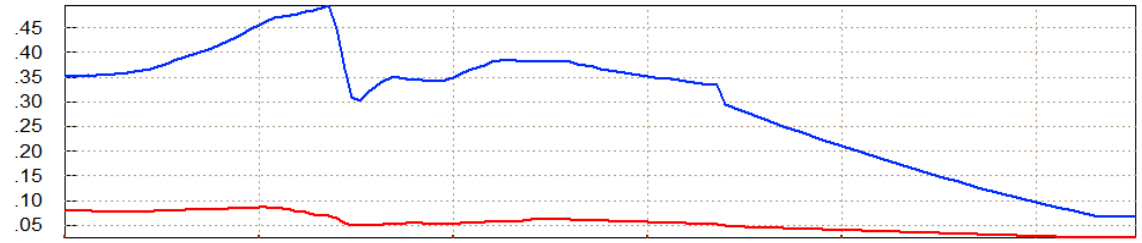
Dy\_Engine 2



Dz\_Engine 2



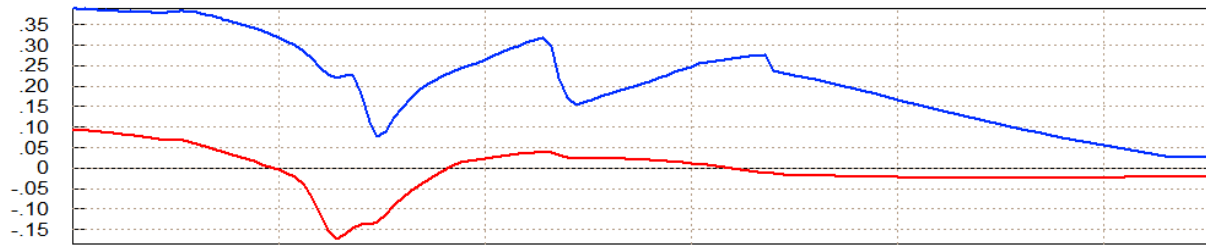
Dy\_Engine 3



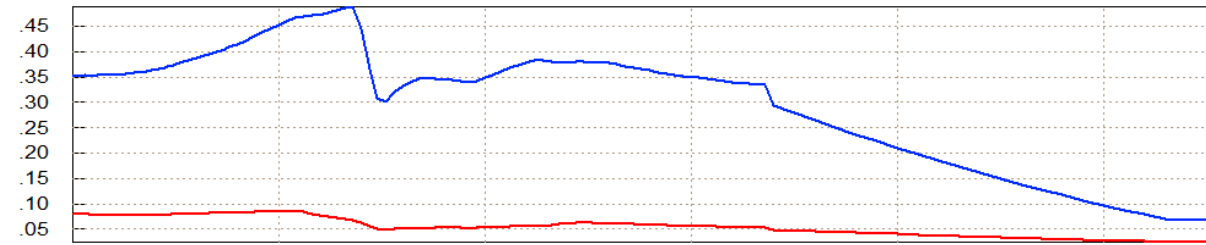
Dz\_Engine 3

0 50 100 150 200 250  
Time (sec)

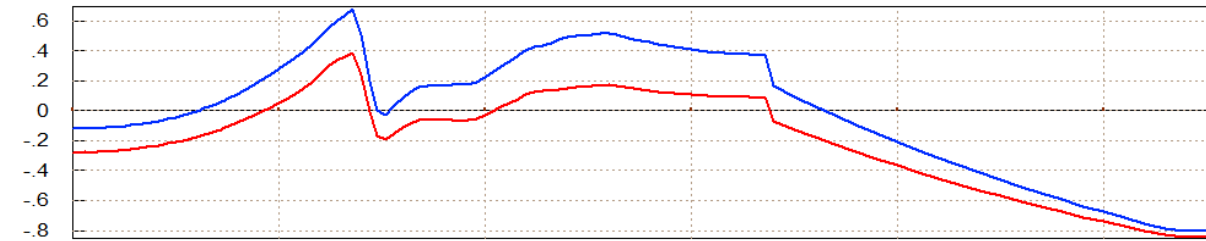
### Surface & Engine Deflections/ Throttle, Reusable Space-Plane Ascent Trajectory-2



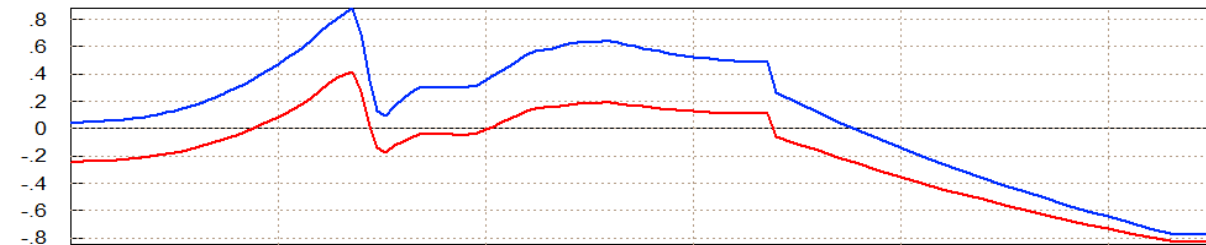
Dy\_Engine 4



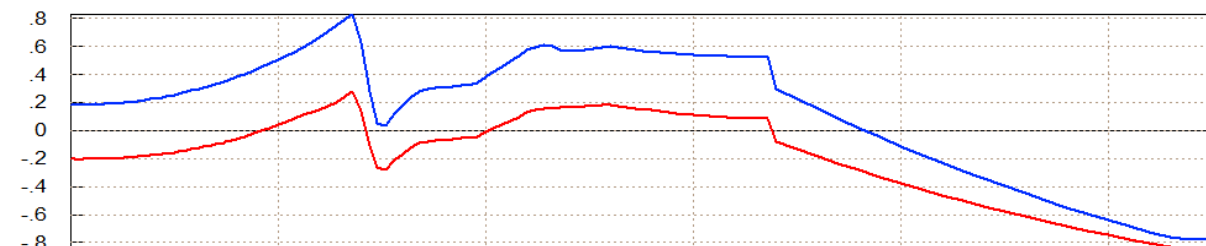
Dz\_Engine 4



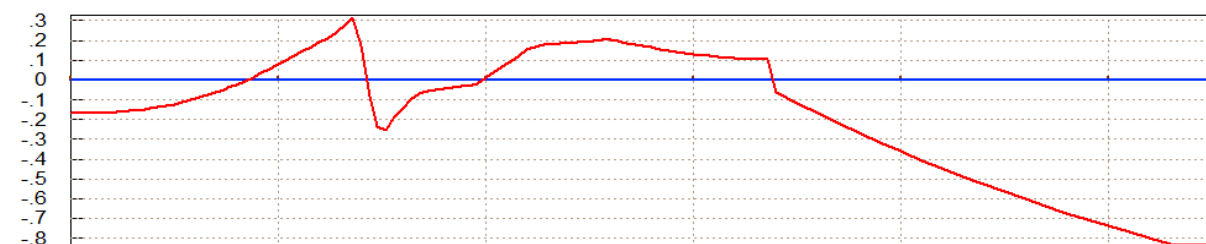
Throttle 1



Throttle 2



Throttle 3



Throttle 4

0 50 100 150 200 250  
Time (sec)

## Engine Actuator Failure

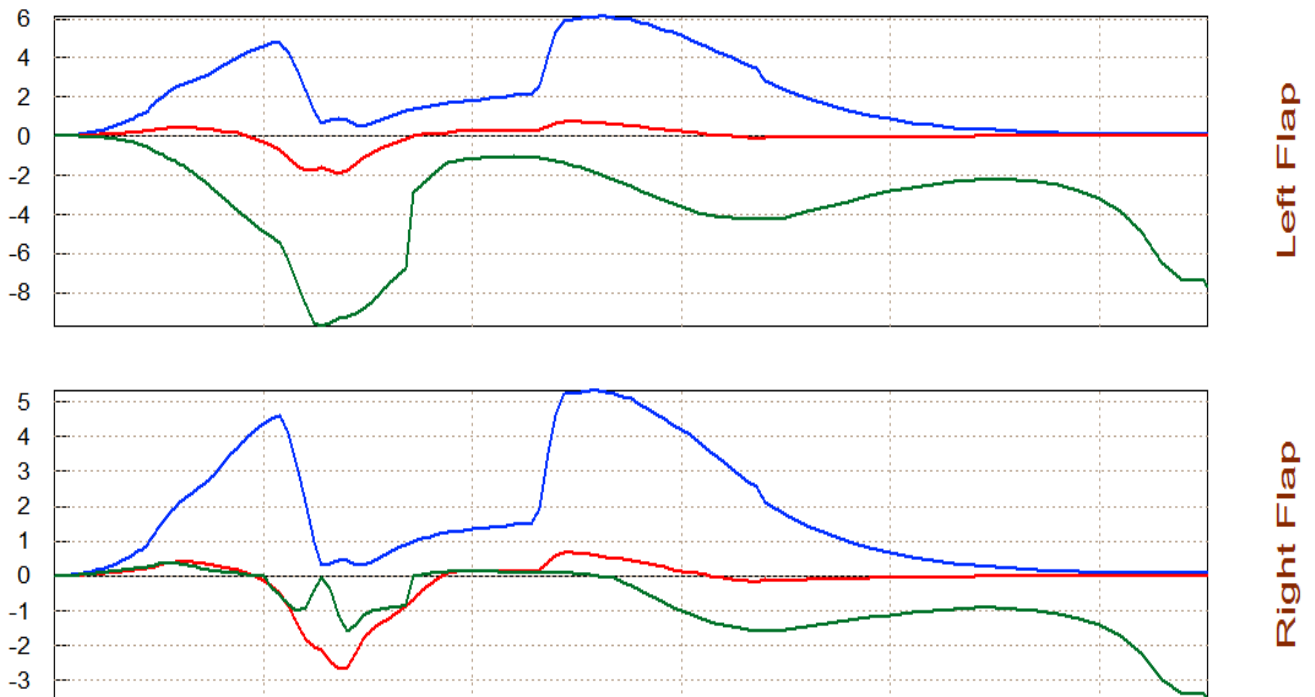
Now let's take a look at a case where the actuators of a gimbaling engine fail to gimbal and the engine is stuck with a constant thrust and at fixed bias position  $\delta y = -0.5^\circ$  and  $\delta z = -0.5^\circ$ . This situation is defined in a separate engines file "*ReSP5c.Engn*", shown below.

Reusable Space-Plane Data for the 6 Engines  
 Engine #1 Actuator failure (No Gimbaling, No Throttling, Fixed Thrust, Fixed Gimbal Position)

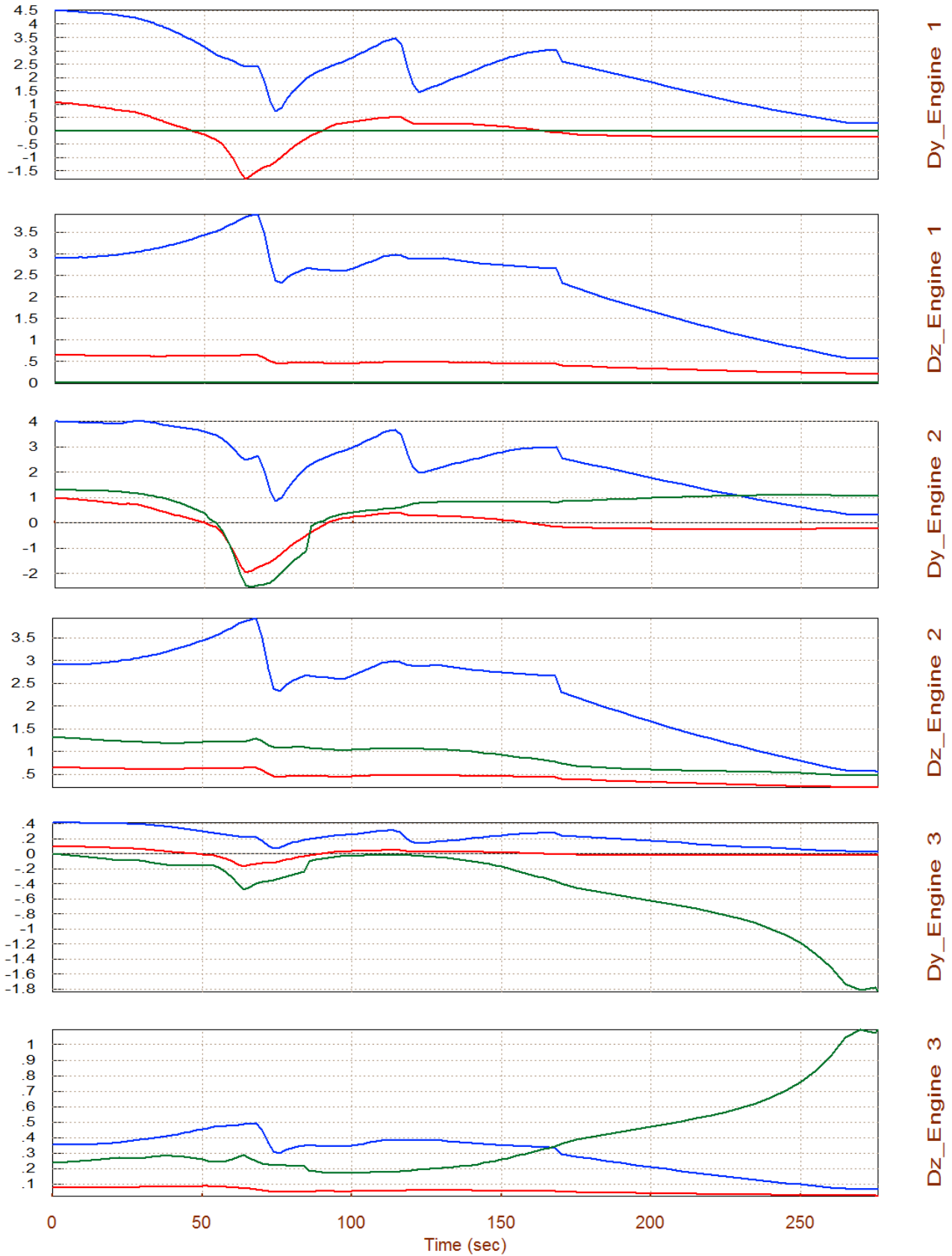
Engine Description, Thrust (lb)	Mass (slug)	Ieng (slug-ft <sup>2</sup> )	Mom Arm (Ft)	Location (x,y,z) (feet)	Mounting Angles (Dy, Dz) Elevat, Azimuth (degr)	Max Deflection Dym,Dzm (deg)	Max Thrott1 (0-1)
Main Engine#1	70000.0	100.0	1500.0	2.8 -89.0 -4.8 0.0	-0.5 -0.5	0.001 0.001	0.0
Main Engine#2	70000.0	100.0	1500.0	2.8 -89.0 +4.8 0.0	-2.0 0.0	14.0 12.0	0.0
Main Engine#3	52000.0	100.0	1500.0	2.8 -89.0 -2.25 +3.9	-2.0 0.0	5.0 5.0	0.95
Main Engine#4	52000.0	100.0	1500.0	2.8 -89.0 +2.25 +3.9	-2.0 0.0	5.0 5.0	0.95
Main Engine#5	52000.0	100.0	1500.0	2.8 -89.0 -2.25 -3.9	-2.0 0.0	0.0 0.0	0.95
Main Engine#6	52000.0	100.0	1500.0	2.8 -89.0 +2.25 -3.9	-2.0 0.0	0.0 0.0	0.95

The trimming process is repeated using the failed actuator file and the results are shown by the green curves in the plots below using option (12). The red and blue curves are the same as before. The red is from the original 6 engines file "*ReSP6.Engn*", the blue is from the failed thrust file "*ReSP5b.Engn*", and green is from the failed actuator file "*ReSP5c.Engn*". Note that the deflections in option (12) are measured from the engine bias positions, so the zero deflections of TVC engine #1 (green curves) corresponds to the failed positions ( $\delta y = -0.5^\circ$ ,  $\delta z = -0.5^\circ$ ). The green curves show that the remaining TVC engines and aero-surfaces are deflecting further to make up for the disability of the Engine #1 to gimbal.

Surface & Engine Deflections/ Throttle, Reusable Space-Plane Ascent Trajectory-2



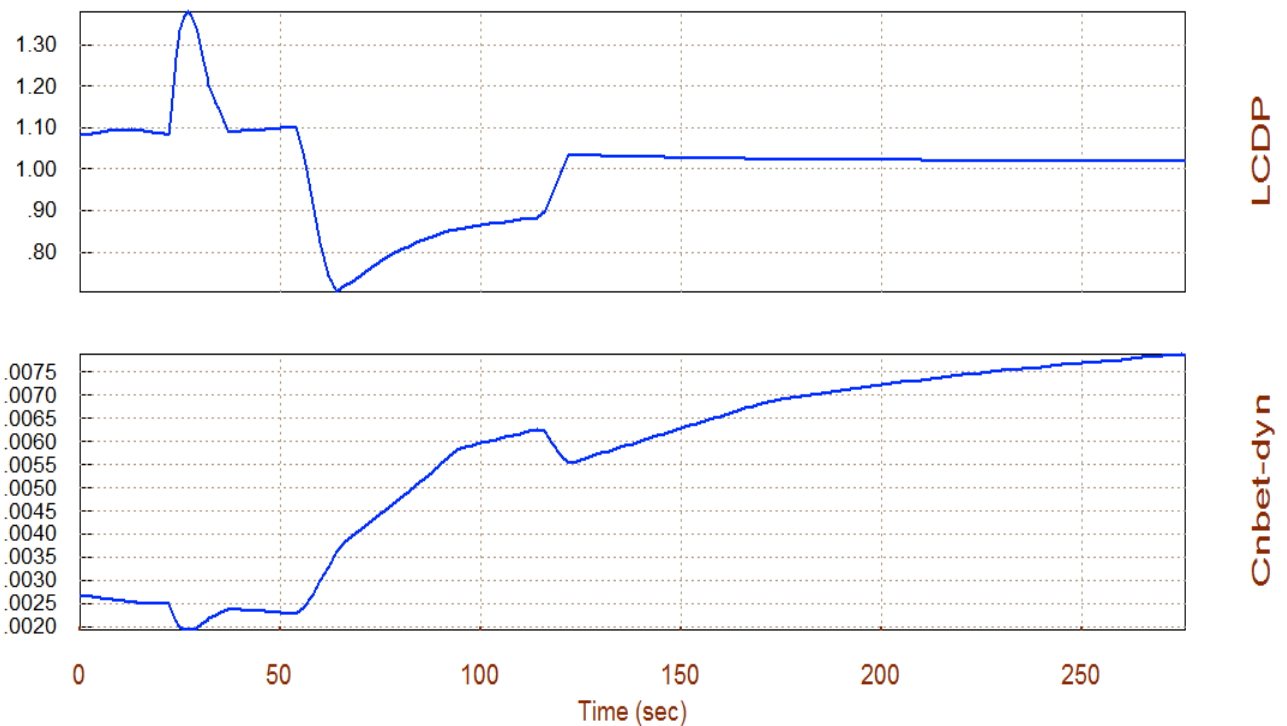
Surface & Engine Deflections/ Throttle, Reusable Space-Plane Ascent Trajectory-2



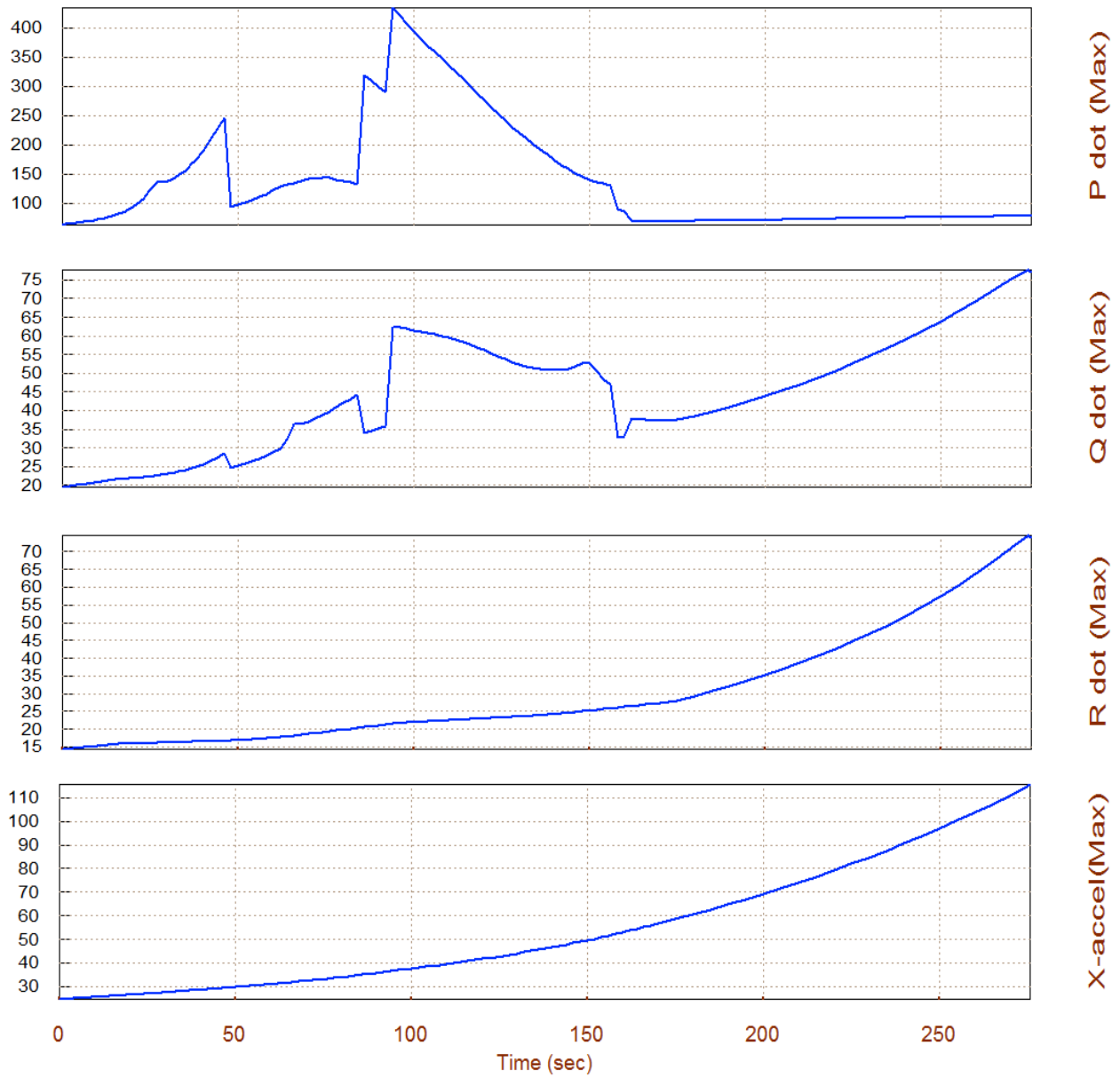
## 1.7 Performance Analysis Using Trajectory (B)

Let us now re-evaluate the performance parameters (described in section 3) along the second trajectory. The second trajectory is easier to evaluate because it does not have the large pitch maneuver. Return to the main menu and select option (6) that plots the performance and stability parameters along the trajectory. Allow the program to calculate its own TVC matrix, like before. The effector combination matrix will have 4 inputs for the 4 control directions (3 rotations plus x-acceleration), and 14 outputs (8 TVC, 4 throttling, and 2 aero-surfaces). In the disturbance dialog enter the max angles of attack and sideslip expected due to wind-shear ( $\alpha_{\max}=4^\circ$ ,  $\beta_{\max}=4^\circ$ ).

The static stability results of the vehicle along the second trajectory are similar to the results obtained from the first trajectory. This launch vehicle is unstable in pitch during the first 130 seconds and later it becomes stable. When unstable the shortest time to double amplitude  $T_2=0.62$  (sec) and occurs at 70 sec. In the lateral direction the vehicle is always stable with a positive  $C_n\beta$ -dynamic. The yaw  $T_2$ -inverse parameter is negative (stable), and the Dutch-Roll resonance peaks to 1.4 (rad/sec) at 80 sec. The (Q-alpha , Q-beta) loading at ( $\alpha_{\max}=4^\circ$ ,  $\beta_{\max}=4^\circ$ ) is also acceptable. The control effort in all four directions (roll, pitch, yaw, plus x-acceleration) is also good. It requires less than 50% of full control in all 4 directions to counteract disturbances due to the ( $\alpha_{\max}=4^\circ$ ,  $\beta_{\max}=4^\circ$ ) excursions from trim. This allows sufficient control authority for maneuvering and against wind-gusts. It means that changes in the accelerations due to wind-shear disturbances can easily be counteracted by the controls.



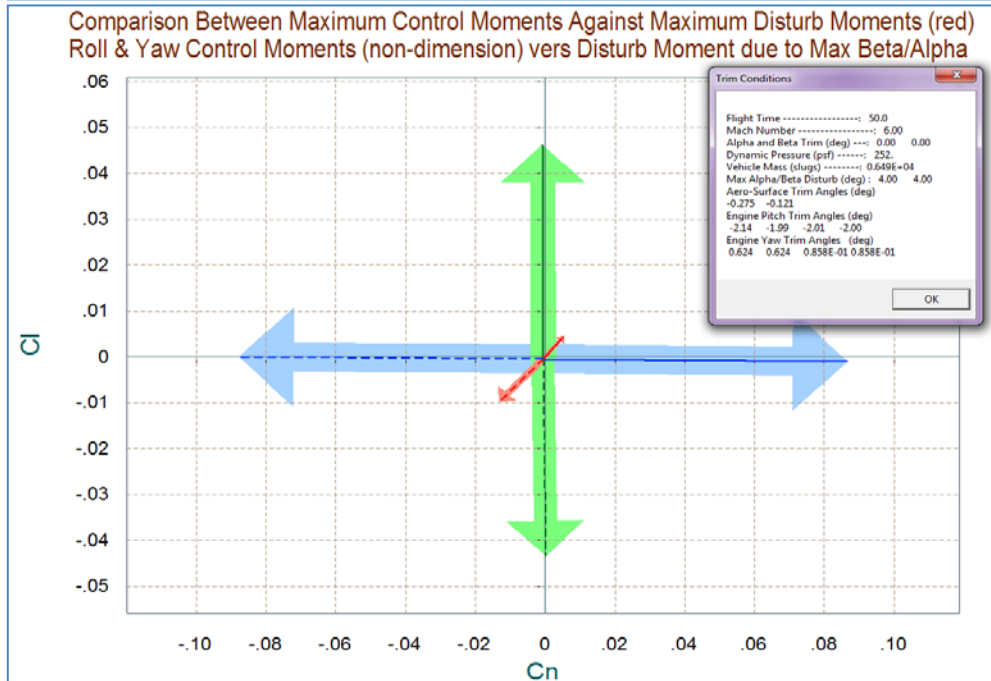
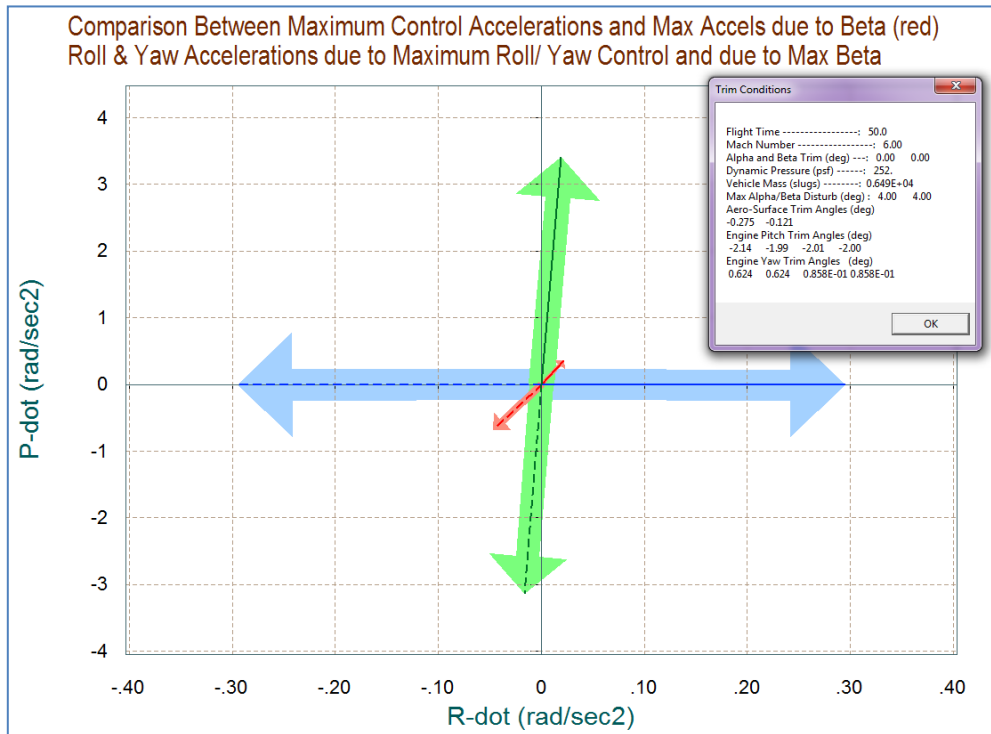
Max Angular Accelerat (deg/sec<sup>2</sup>), at Max Control Demand Reusable Space-Plane As



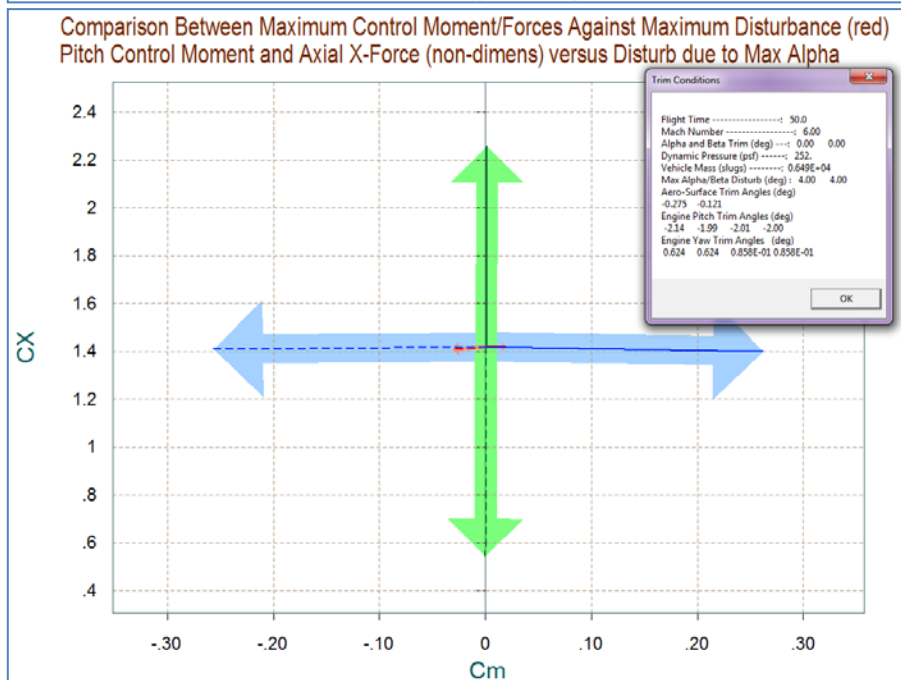
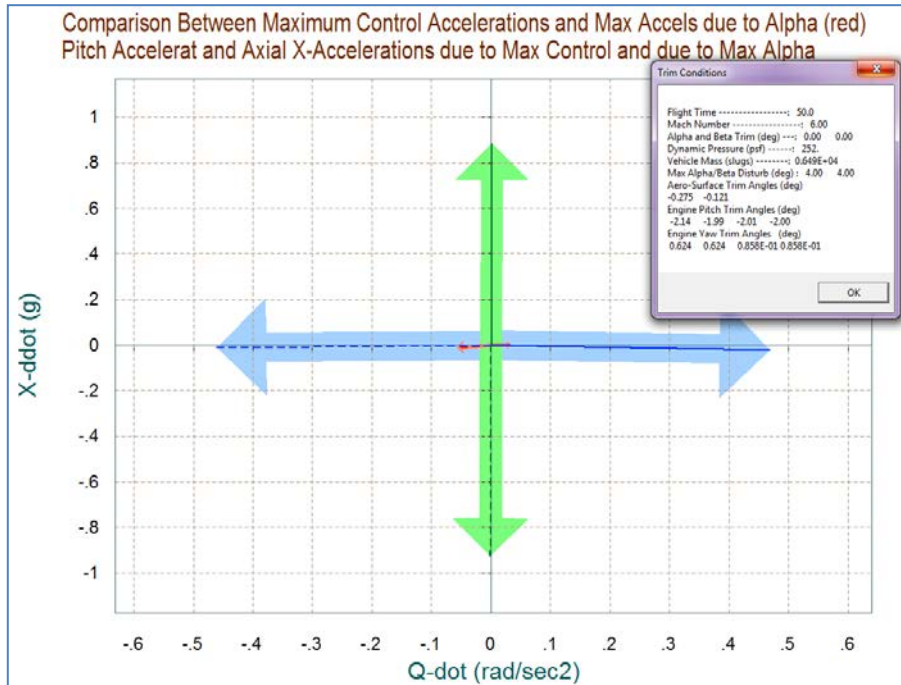
This trajectory has a definite performance advantage in the LCDP in comparison with the previous trajectory. In the previous performance analysis the LCDP had changed signs a couple of times because of the large angle pitch maneuver. Small magnitudes and sign reversals in the LCDP is undesirable because it also implies reversals in the control law gains. In this trajectory, however, the LCDP remains positive and very close to one (which implies good turn coordination) through the entire ascent trajectory. It means that we can rely on the flaps and the TVC to control roll along trajectory (b) without any assistance from RCS jets.



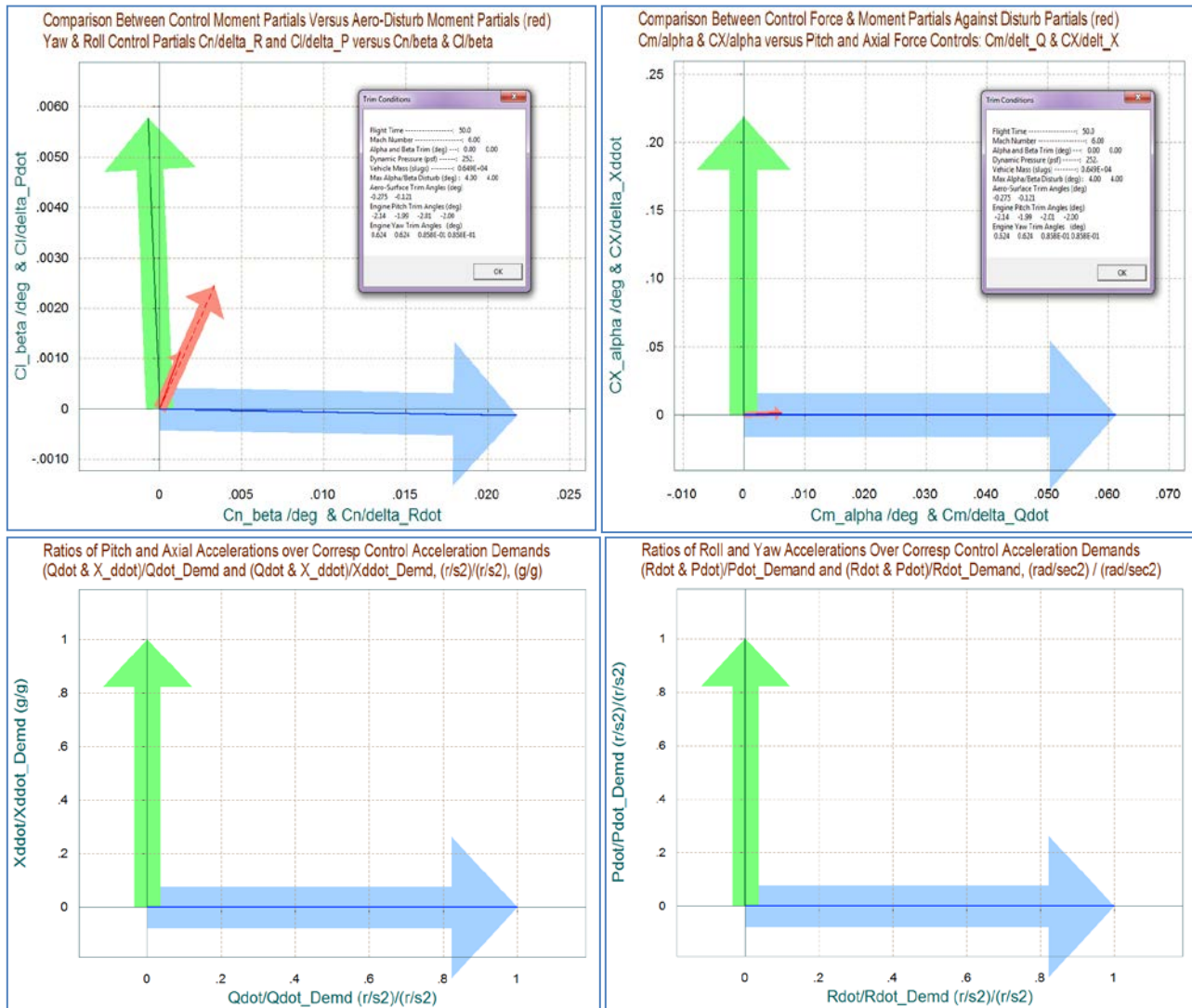
$\pm$ maxed. The green vectors show the moments or accelerations when the roll control is  $\pm$ maxed. The solid line vectors show the moments or accelerations due to positive roll and yaw demands and the dashed vectors are due to negative demands. The red vectors represent the roll/yaw moments or accelerations generated by the  $(\alpha_{max}, \beta_{max})$  dispersions from trim. The plots show that all directions are controllable, decoupled from each other and that the controls are more powerful than the disturbance moments and forces. Right-clicking the mouse button displays information about the trimming conditions.



The next two plots show the change in pitch acceleration ( $\text{rad/sec}^2$ ) versus change axial acceleration in (g) and the pitch moment versus axial force (non-dimensional). The pitch axis is controlled by gimbaling the first 4 TVC engines in pitch. The axial acceleration is controlled by throttling the last 4 engines. The axial force coefficient  $C_x$  is positively biased, centered at 1.4, because the vehicle is accelerating at that time. The horizontal blue vectors show the  $\pm$ max range in control moment and acceleration in the pitch direction due to maximizing the pitch demand ( $\pm\delta Q_{FCS\_Max}$ ). The vertical green vectors show the  $\pm$ max range of the control force and acceleration in the axial direction due to maximizing the axial acceleration demand ( $\pm\delta X_{FCS\_Max}$ ). The red vectors show the effects due to ( $\pm\alpha_{max}$  and  $\pm\beta_{max}$ ). The disturbance affects mainly the pitch direction  $C_m$ , not  $C_x$ . A negative  $\alpha_{gust}$  creates a negative pitching moment because the vehicle is unstable.



The next diagram compares the partials of the aero moments per beta ( $C_l\beta$ ,  $C_n\beta$ ) (red vectors) against the roll and yaw moments partials per roll and yaw flight control acceleration demands. The green vector is  $(C_l\delta P_{FCS}, C_n\delta P_{FCS})$ , and the blue vector is  $(C_l\delta R_{FCS}, C_n\delta R_{FCS})$ . The blue and green control vectors are scaled appropriately to make them comparable to the disturbance red vectors, as described in equations (8.1 through 8.4). The disturbance vectors (red) due to  $\beta_{max}$  are small in comparison to the control partials, so there is no controllability problem in roll and yaw directions. The results are similar in the longitudinal directions shown in the second diagram which compares the axial force and pitch moment partials. The green vector is the x-force and moment per axial acceleration demand  $(C_x\delta X_{FCS}, C_m\delta X_{FCS})$ . The blue vector is the x-force and moment per pitch acceleration demand  $(C_x\delta Q_{FCS}, C_m\delta Q_{FCS})$ . The red vectors are the x-force and pitch moment partials due to alpha  $(C_x\alpha, C_m\alpha)$ . The control vector partials are almost orthogonal to each other and they are pointing towards the demanded directions, the roll partial in the roll direction, the yaw partial in yaw, etc. The decoupling between axes is accomplished by the  $K_{mix}$  matrix, and it is more evident in the acceleration per acceleration demand partials below.

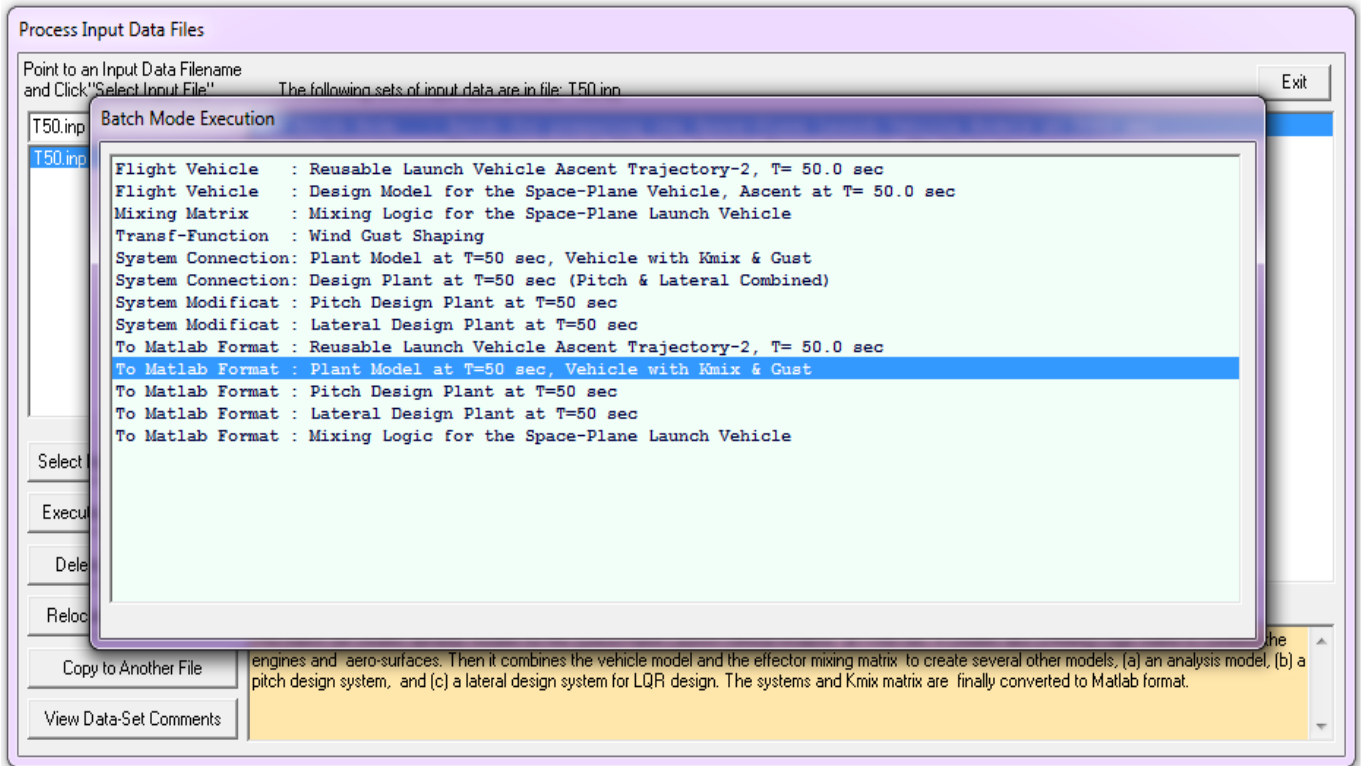
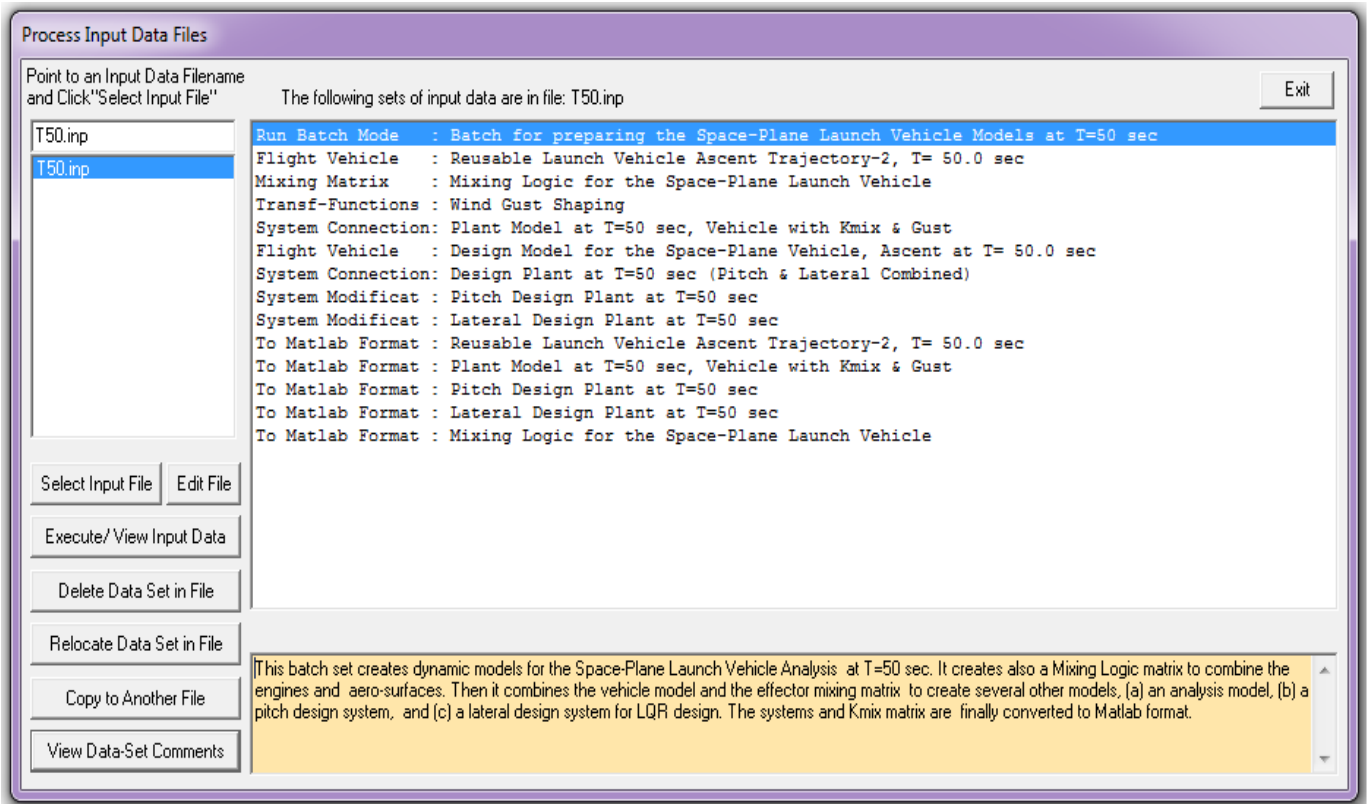


## 1.9 Creating an Ascent Dynamic Model

We now come to the interesting part of the analysis and use the Flixan and Trim programs to create a dynamic model for our launch vehicle at a fixed flight condition,  $t=50$  sec after lift-off, and in the next section we will design a flight control system and analyze it. This vehicle is a good example for demonstrating the capability of the program to create dynamic models of vehicles that use multiple types of effectors and the methodology for creating a mixing logic that optimally combines the effectors together for the purpose of maximizing control authority in the controllable directions. The dynamic model will be created in file "T50.Qdr". It has a total of 14 control inputs: 8 gimbal deflection inputs for the four TVC engines (4 pitch and 4 yaw gimbals) in (radian), 4 throttling inputs for the four engines that have a variable thrust, and 2 aero-surface inputs (deg). There are 6 engines in the vehicle data, two engines are only gimbaling, two are only throttling, and two engines are both gimbaling and throttling. The engine nozzles are not mounted parallel to the vehicle x axis but they are tilted  $-2^\circ$  in pitch, also their trim positions in this flight condition are close to  $-2^\circ$ . The max deflections of the first two engines are  $\pm 14^\circ$  in pitch and  $\pm 12^\circ$  yaw. Engines #3 and #4 can gimbal  $\pm 5^\circ$  in pitch and yaw. The four throttling engines have a nominal thrust of 55,300 (lb) and they can vary their thrusts  $\pm 95\%$ . That is, from 9,200 (lb) when the throttle input is -1 to 101,400 (lb) when the throttle input is +1. Zero throttle input corresponds to the nominal thrust. Inputs 13 and 14 correspond to the two flap deflections in (rad). The 15<sup>th</sup> input to the dynamic model is a wind-gust velocity disturbance in (ft/sec). The gust direction relative to the vehicle is defined in the input data file "T50.inp".

This vehicle has a multitude of effectors and the coordination between the effectors is taken care by the mixing logic matrix which is calculated by Flixan using the same vehicle model and corresponds in this flight condition. The flight control system is expected to control the same four directions as in trimming, that is, roll, pitch, yaw, and axial velocity. The mixing logic matrix translates the four flight control system acceleration demands (roll, pitch, yaw, axial) to 14 effector commands. There is one additional feature that makes this vehicle configuration even more interesting. It includes two tanks containing sloshing propellants and Flixan will create dynamic models for the two slosh resonances. There is a liquid oxygen (LOX) tank located near the rear of the vehicle and it is much heavier than the liquid hydrogen (LH2) tank which is located closer to the front of the vehicle. The slosh parameters are automatically selected from the slosh file "ReSP.Slsh" where they are listed as a function of the vehicle mass. The program selects a set of parameters that correspond to the vehicle mass at the flight condition being analyzed, which is at  $t=50$  sec. The modeling procedure and analysis is identical for any other flight time.

So let us continue the analysis using the Trim program. Select the same files as before, and enter an input filename "T50.inp" in the filenames selection menu. This file will contain the vehicle input data plus other Flixan related data-sets. Enter also an output filename "T50.Qdr" to save the vehicle systems and matrices. Re-trim the effectors using a mass properties file that has the YCG set at zero. It is better to use an unbiased model for the control design so that the gains do not



## 1.10 Ascent Control Design, Analysis and Simulation

We will now use the dynamic models developed in section 1.9 to design control laws for our space-plane, analyze stability, and develop a simulation model in Matlab/Simulink. This vehicle is particularly interesting because it has fuel sloshing and it blends multiple types of effectors, such as: aero-surfaces, gimbaling TVC engines, and thrust varying engines. The analysis is performed in directory: "C:\Flixan\Trim\Examples\Reusable Space Plane\Ascent\Analysis". We will begin by designing the state-feedback control laws, performing stability analysis in the frequency domain, developing a simulation model, and simulating the system's response to flight control commands and also to wind-gusts.

### Flight Control Design

The flight control gains are calculated by the Matlab script file "*design.m*", shown below. It uses the pitch and lateral design systems in files "*vehi\_pitch.m*" and "*vehi\_later.m*", created in section 1.9, to generate state-feedback matrices *Kq.mat* and *Kpr.mat* for the pitch and lateral systems. The slosh resonances are not included in the design models. The pitch state-vector consists of:  $[\theta, q, \alpha, \theta\text{-integral}]$ , and the lateral state-vector consists of states:  $[\phi, p, \psi, r, \beta]$ .

```
% Pitch LQR Design for Space-Plane
[Ap,Bp,Cp,Dp]= vehi_pitch;           % Load the Pitch Design Model
[Api,Bpi,Cpi,Dpi]= linmod('Pitch_Design'); % Augment Pitch Simulink model
sys1=SS(Api,Bpi,Cpi,Dpi);
% Weights[thetas, q, alpha, theta_int]
Q= diag( [5.5, 3.5, 0.01, 0.1]); % Weights(thetas,q,alpha,thetas_int)
R=1; % Control Weights R=2
[Kq,S,E]= LQR(sys1,Q,R);
save Kq.mat Kq -ascii % Save the LQR gains in Kq.mat

% LQR Lateral Design for Space-Plane
[A1,B1,C1,D1]= vehi_later; % Load the Lateral Design Model
sys2=SS(A1,B1,C1,D1);
% states: [phi, p, psi,r, beta]
Q=diag([ 0.5, 0.2, 0.5, 0.2, 0.001]);
R=diag([1 1]*1); % Control Weights
[Kpr,S,E]= LQR(sys2,Q,R);
save Kpr.mat Kpr -ascii % Save the LQR gains in Kpr.mat
```

### Simulation Model

The closed-loop simulation model is in file "*Sim\_Ascent.mdl*", shown in Figure (1.10.1). It is initialized by the Matlab file "*runsim.m*", shown below, which loads the system with title: "*Reusable Launch Vehicle Ascent Trajectory-2, T= 50.0 sec*", from file "*vehicle.m*", that was created in section 1.9. It also loads the mixing logic matrix and the state-feedback gain matrices.

```
% Initialize Simulation
r2d=180/pi; d2r=pi/180;
design % Calculate Gains using LQR Design
[Av,Bv,Cv,Dv]= vehicle; % Load the Vehicle Analysis Model
load Kmix_50 -ascii % Load Mixing Logic Matrix
load Kq -ascii % Load LQR Gains
load Kpr -ascii
```



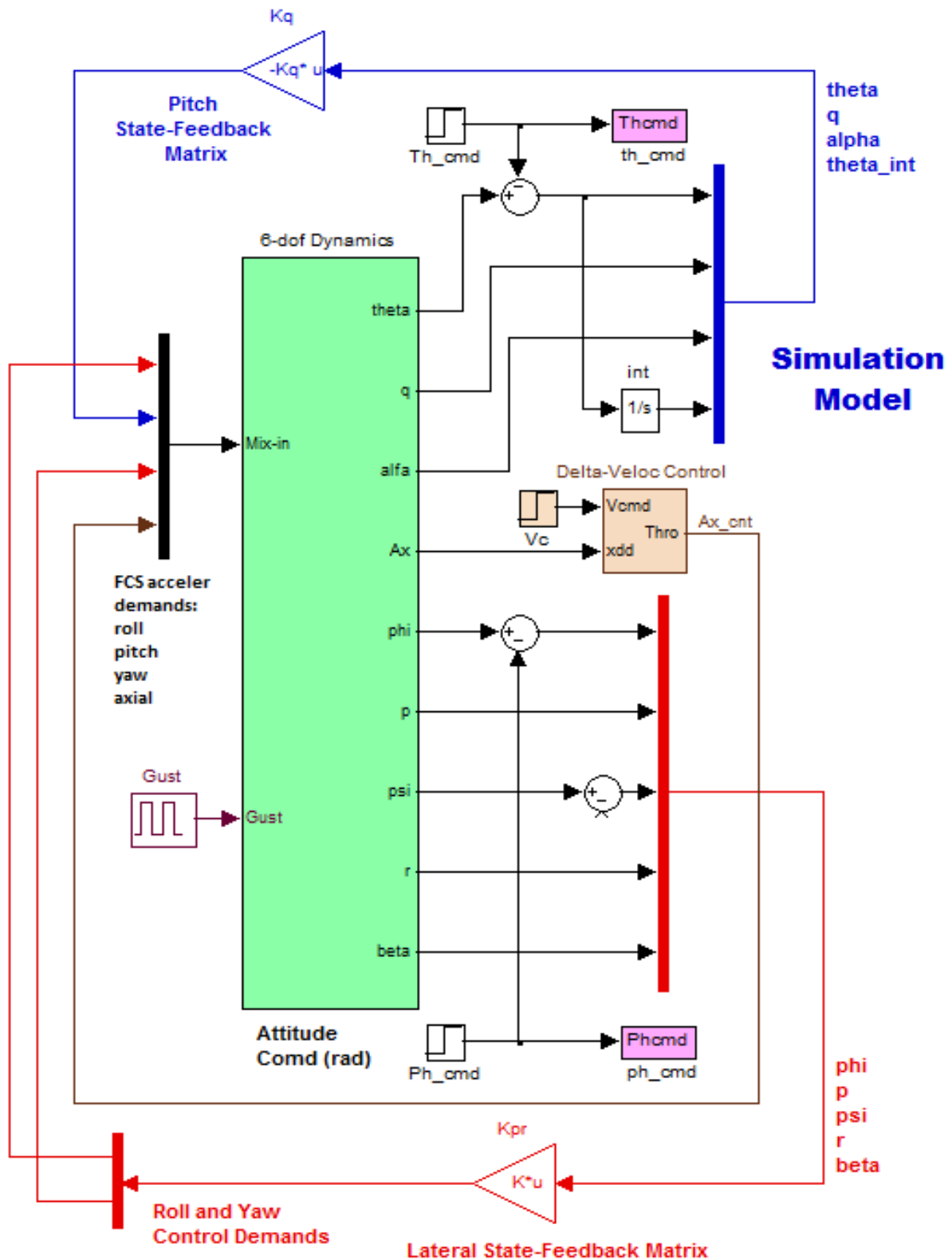
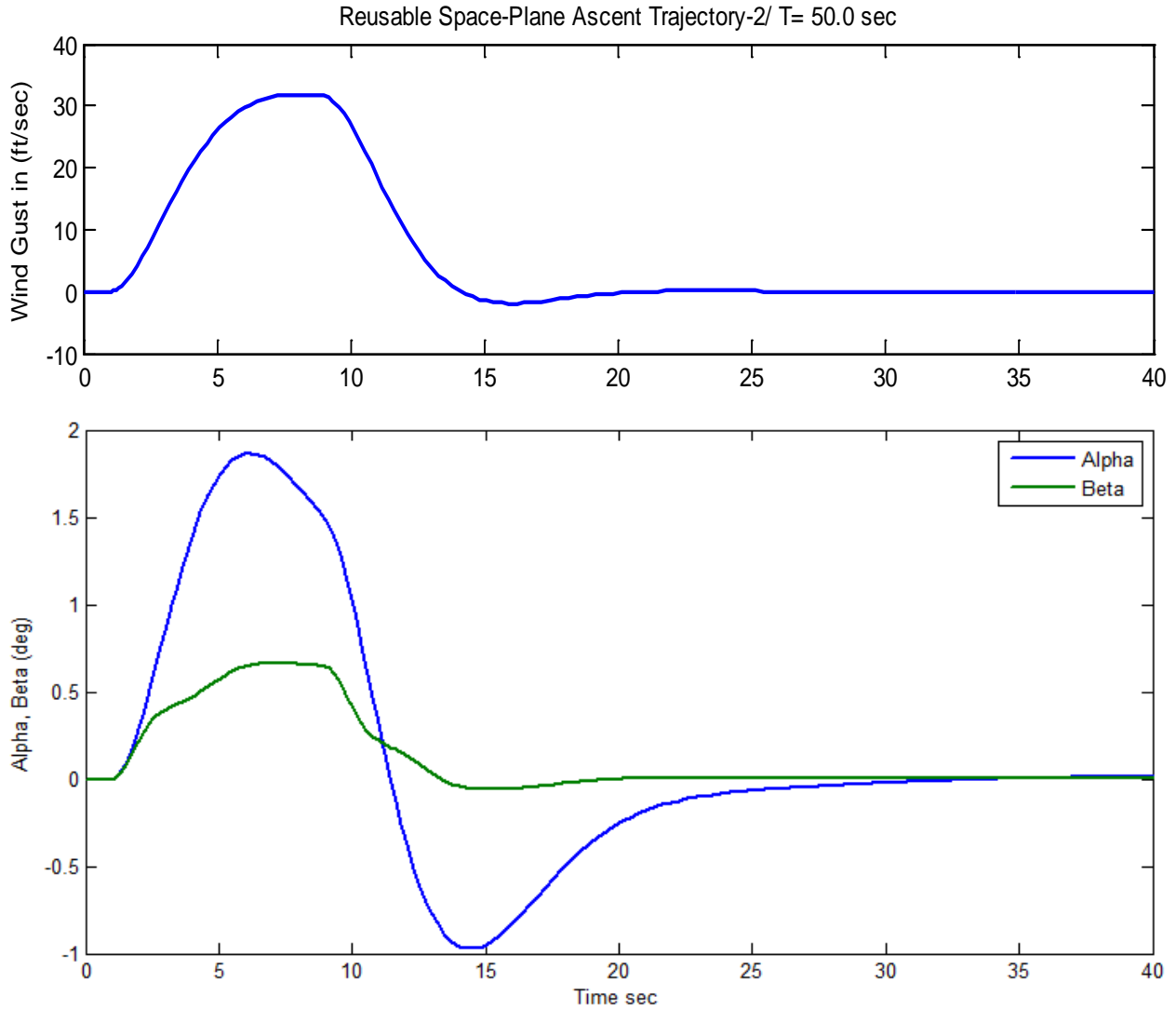
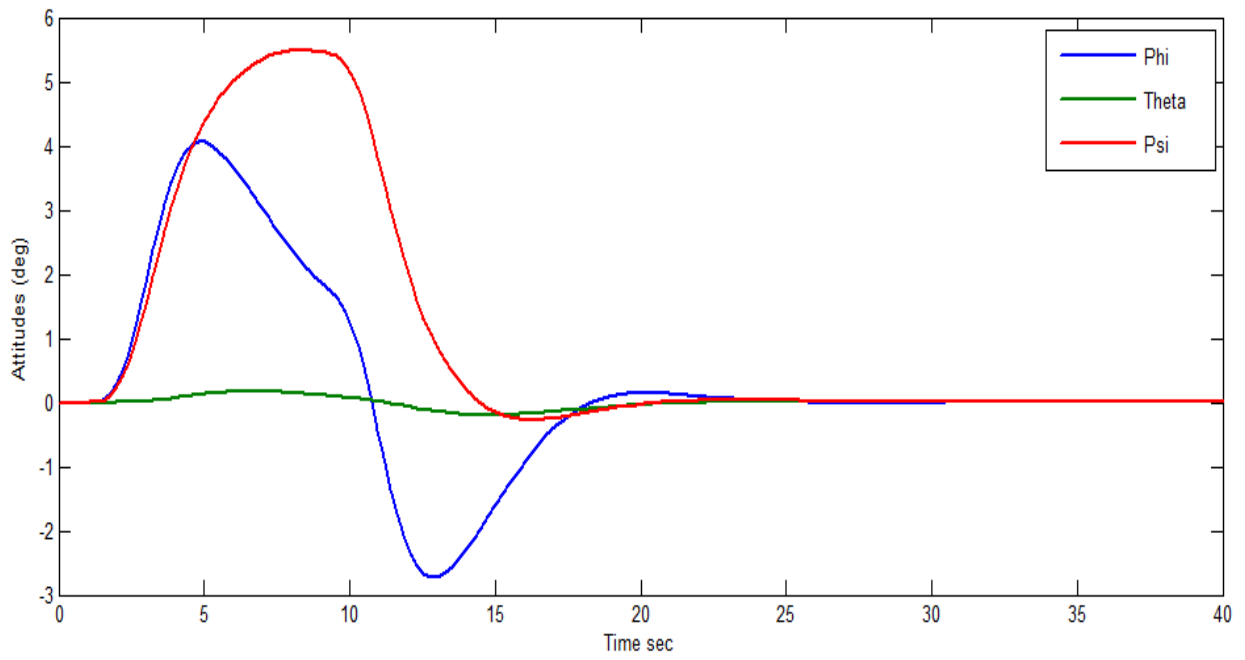
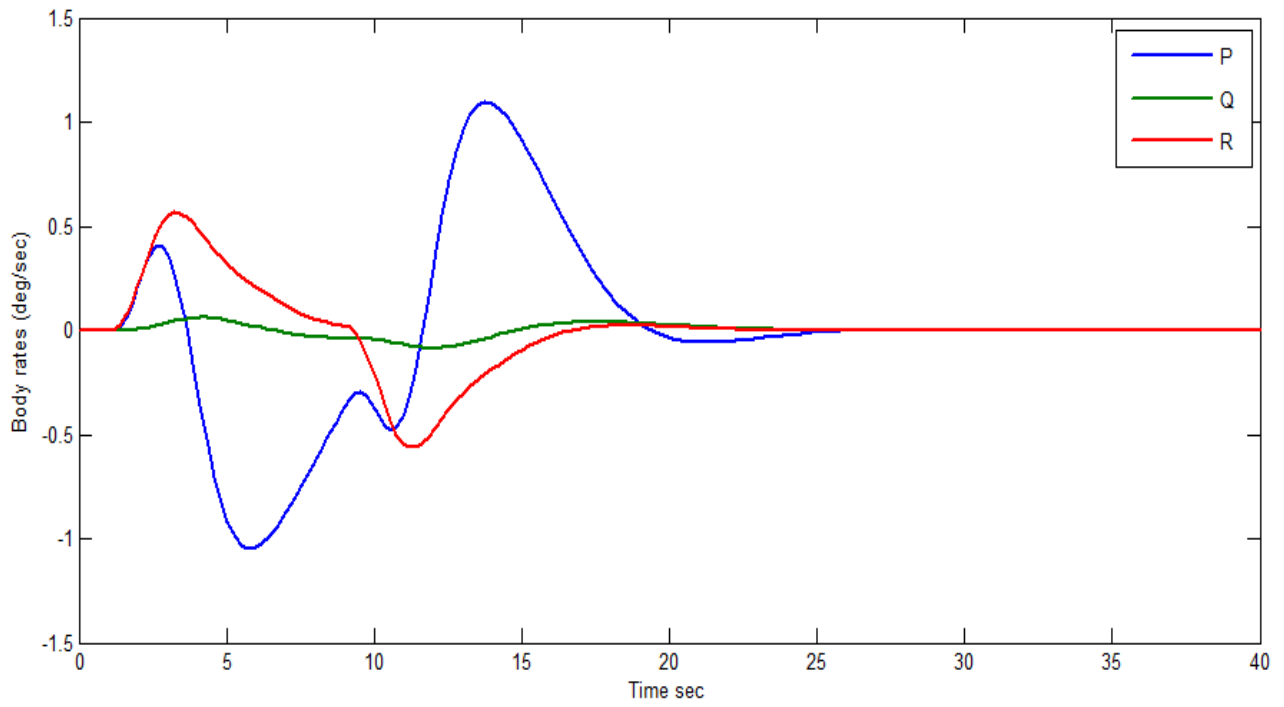


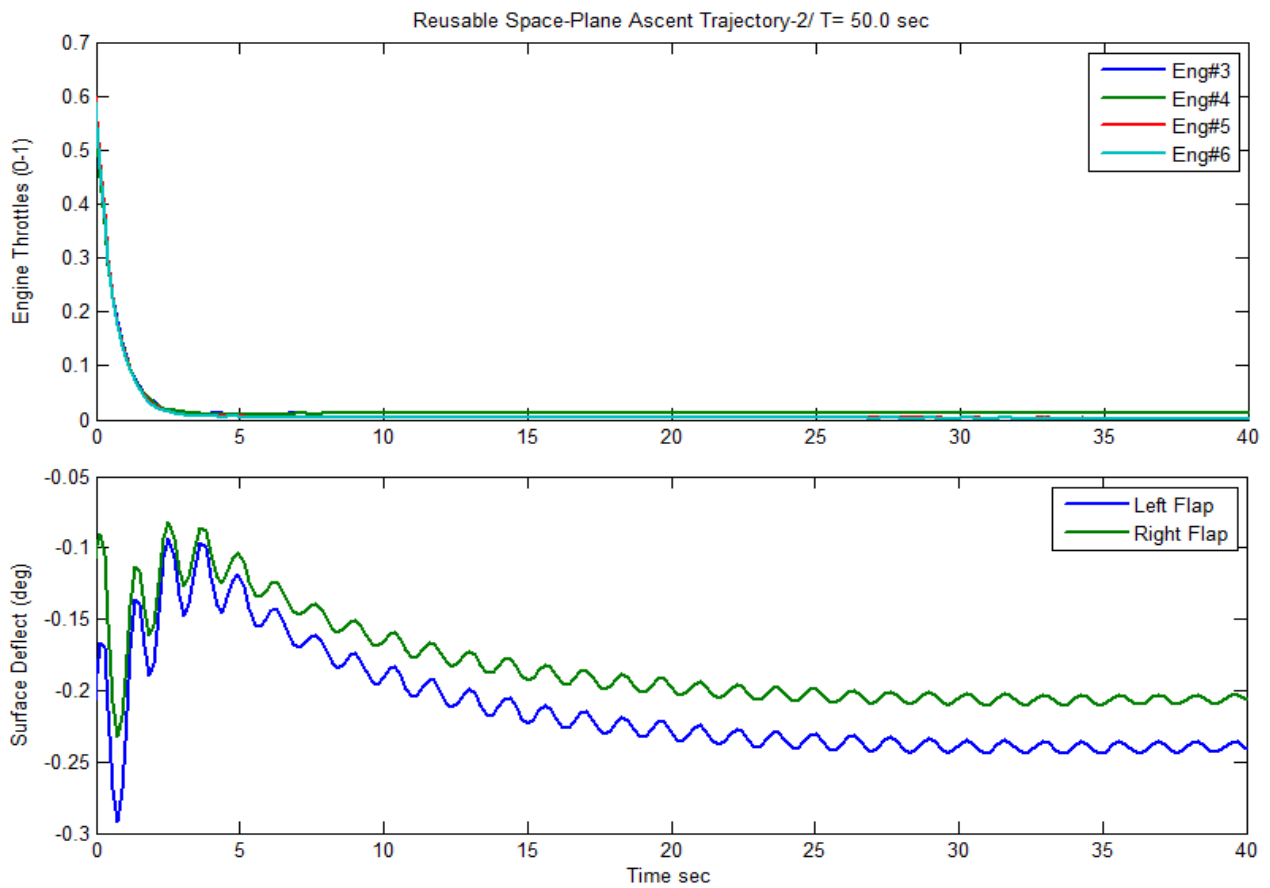
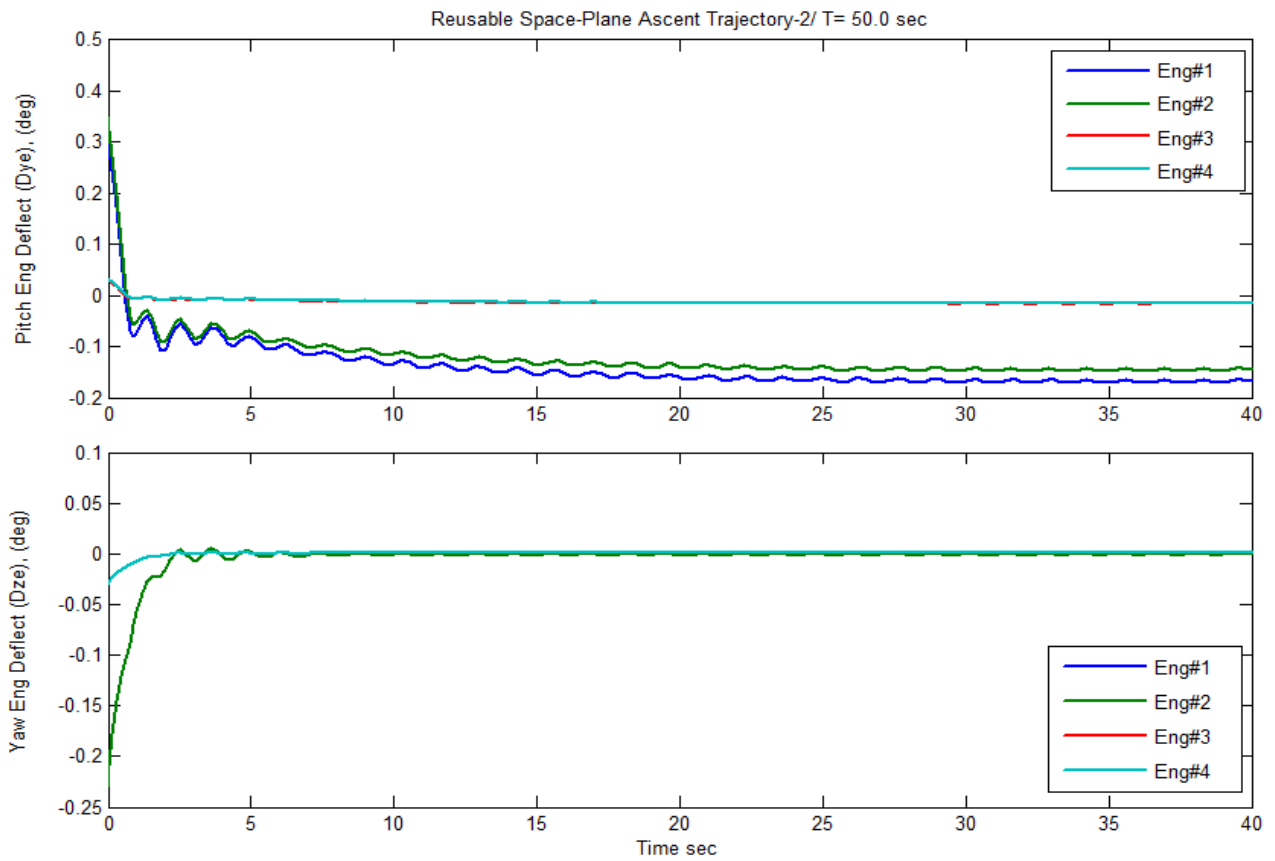
Figure 1.10.1a Space-Plane Simulation Model in File "Sim\_Ascnt.mdl".

## Wind-Gust Simulation

The wind-gust is applied as a square pulse of 30 (ft/sec) velocity for a period of 4 sec. It is smoothed out by the low-pass filter that makes it behave like a real gust, shown below. The gust is exciting both pitch and yaw because its direction is set at 45° between the +Y and the +Z directions and perpendicular to the X-axis. It is creating positive alpha and beta transients. Notice how both, the TVC engines and the aero-surfaces asymmetrically react against the gust disturbance as they are stabilizing the vehicle.

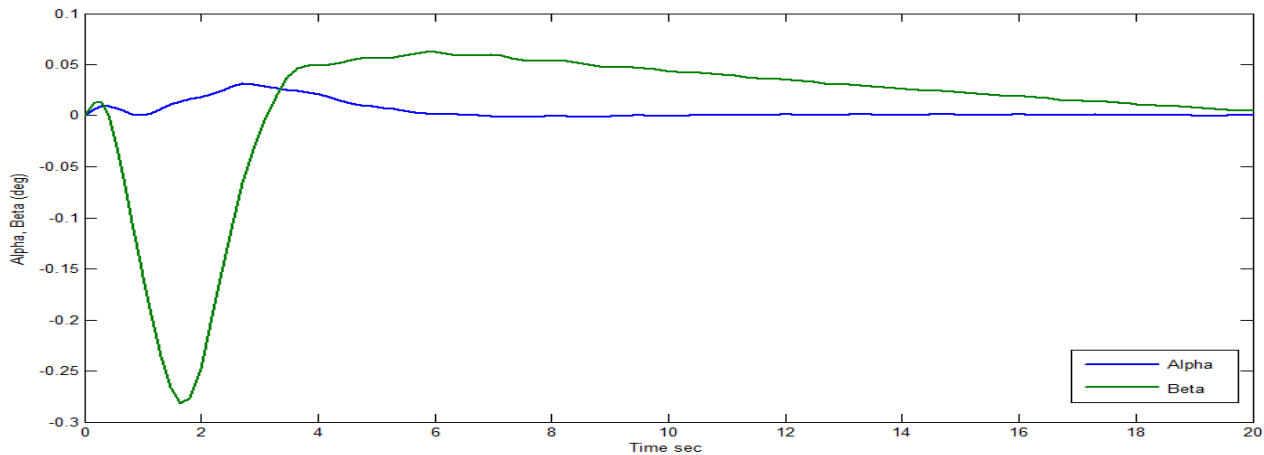
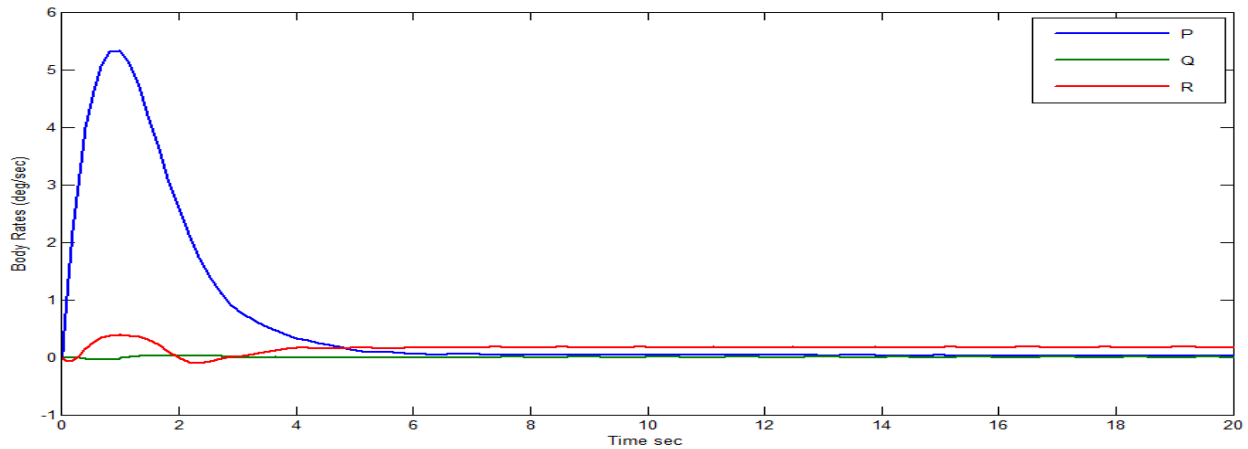
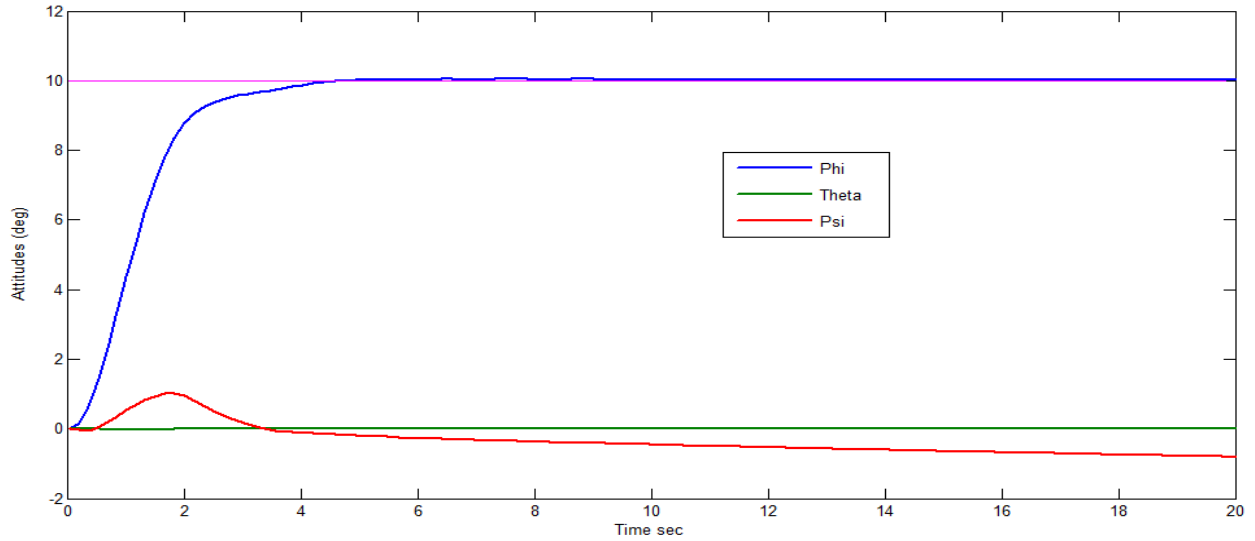






## Roll Command Simulation

Set the roll attitude command to a step input of  $\phi_{cmd} = 10^\circ$ , and all other commands set to zero. The commanded roll attitude is achieved by deflecting the two aero-surfaces differentially and also two of the TVC engines (eng#1 & eng#2) differentially in pitch. The differential deflections are not symmetric because of the YCG offset. The yaw TVC deflections are in the same direction.



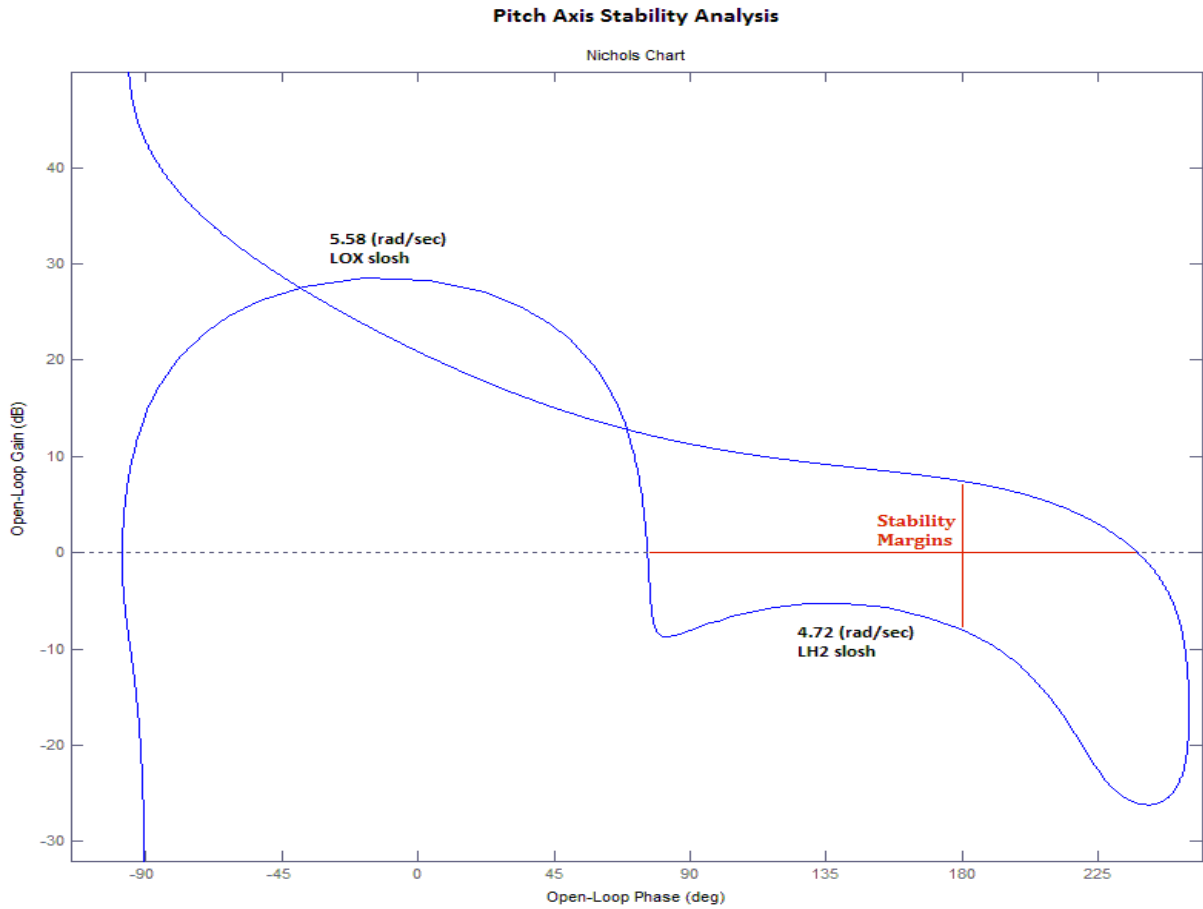
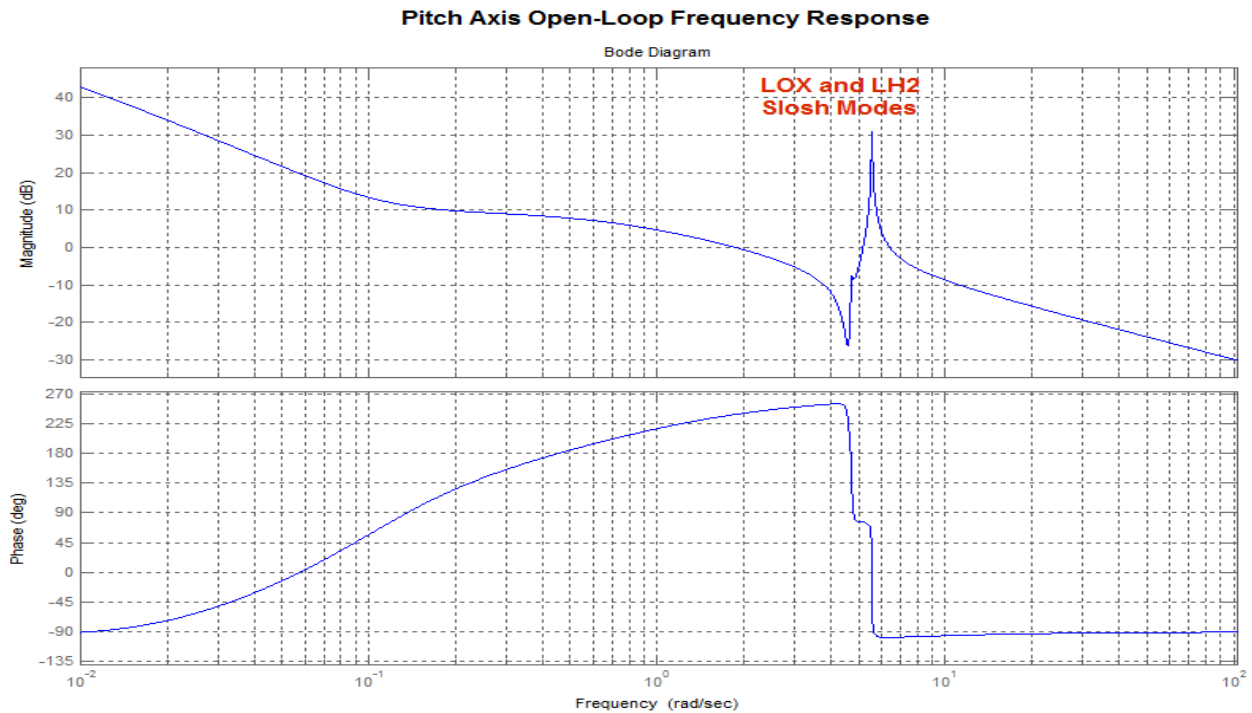


Figure 1.10.4 Pitch Axis Bode and Nichols Plots show stability margins and the two slosh resonances. Notice that the LH2 slosh mode has a tendency towards instability. It was stabilized by increasing the damping coefficient which implies that baffles are needed in the LH2 tank. The LOX mode is much stronger but it is phase stable and it does not require baffles.

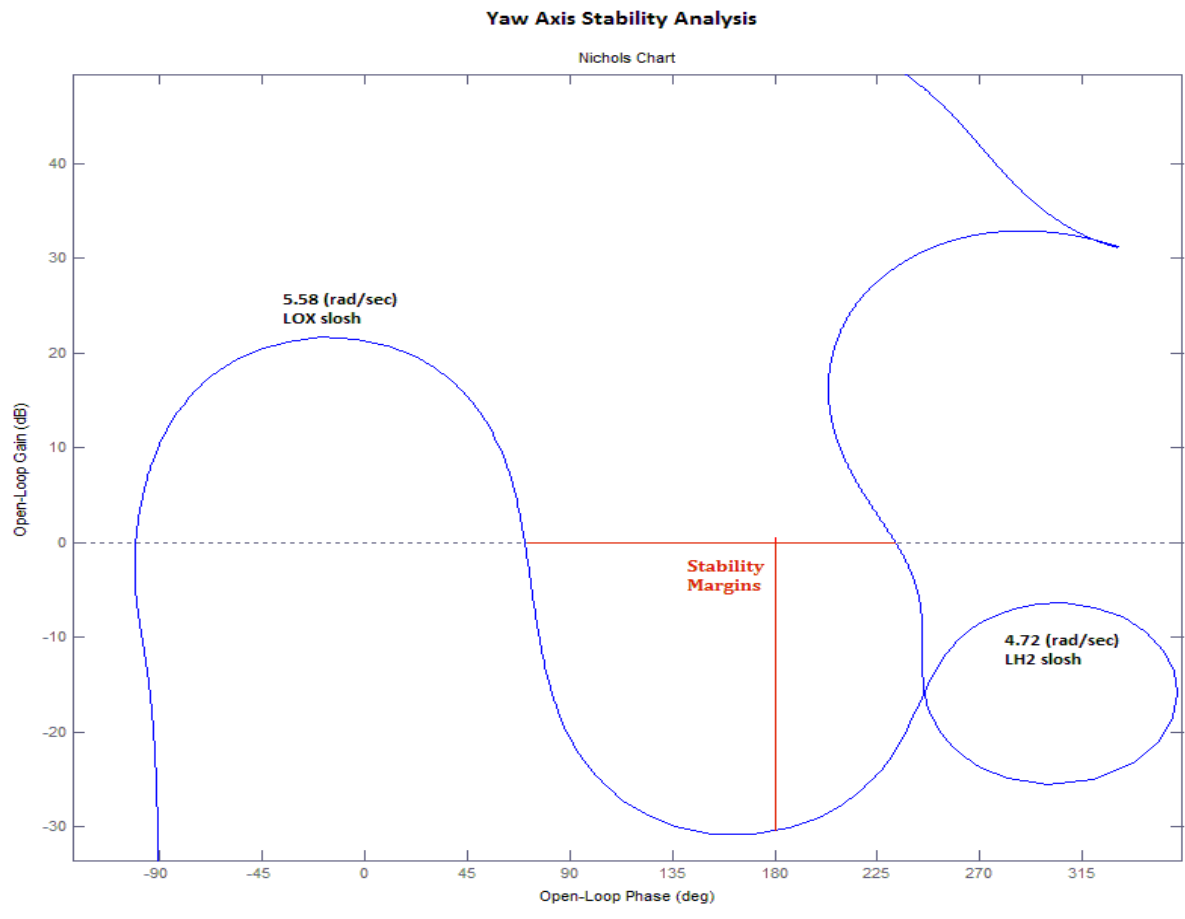
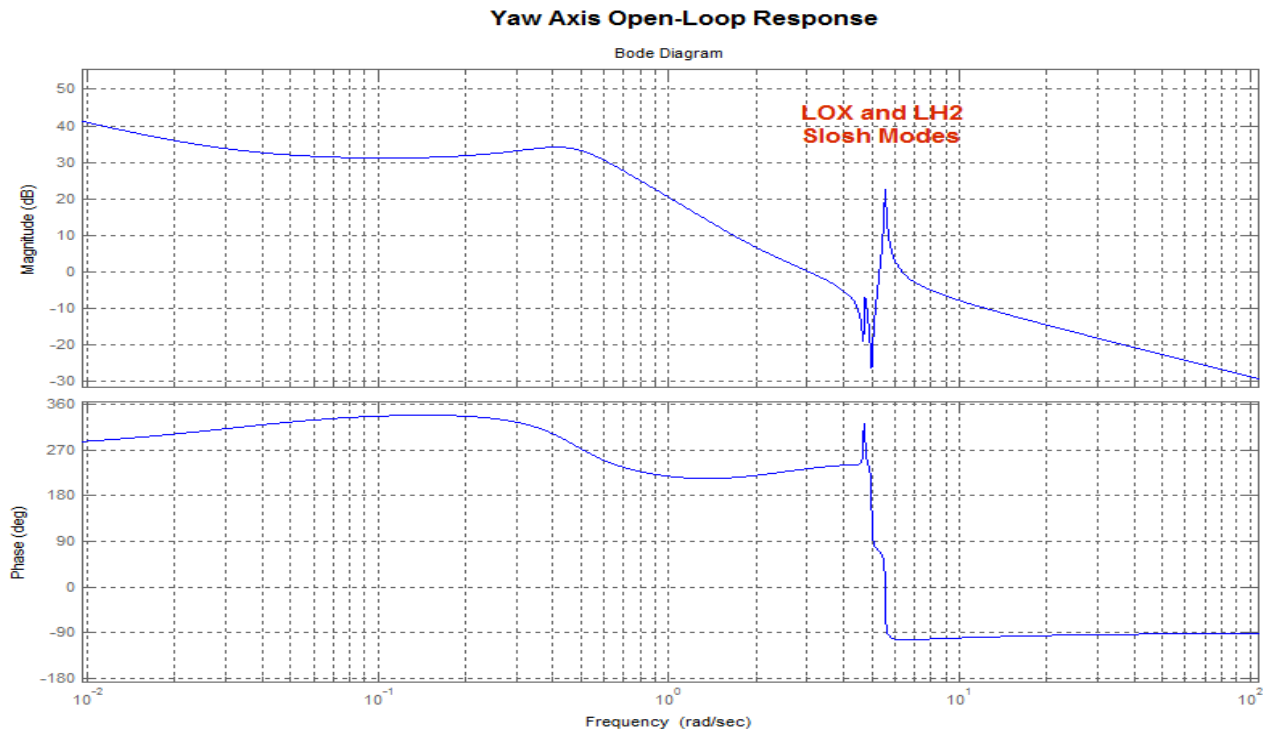


Figure 1.10.5 Yaw axis Bode and Nichols plots show stability margins and the two slosh resonances. Both slosh resonances are phase-stable in yaw.



## 2.0 Descent Phase

During descent the space-plane uses the two flaps for pitch and roll control and two side RCS jets for yaw control because it does not have a rudder. The jets provide 2,500 (lb) of thrust along the  $\pm y$  direction, and they are located near the front of the vehicle for larger yawing moment. The analysis files during descent are in directory "*C:\Flixan\Trim\Examples\Reusable Space Plane\Descent*", and it includes the following files:

- The descent sections of the two trajectories are in files "*RSP\_Entry-A.Traj*" and "*RSP\_Entry-B.Traj*", where the thrust is now zero.
- The propulsion file "*RSP\_Entry.Engn*" that contains only the information about the RCS jet. A pair of back-to-back RCS firing jets in the  $\pm y$  direction are implemented in Trim as a single thruster that generates either positive, negative or zero thrust in the  $y$  direction. The throttle parameter for the jet is set to one, which means, that the actual jet thrust can vary between zero and  $\pm 2,500$  (lb). The thrust is a continuous signal (not a bang-bang). During trimming it is controlled by the throttle control input which varies between zero and  $\pm 1$ . The jet is rotated  $Dz=90^\circ$  in yaw from the  $-x$  direction and a positive throttle input generates a  $+y$  force and a positive yaw. The engine mass, inertia, and moment arm are irrelevant in this case because the thruster is fixed and it does not gimbal.
- There are two aero-surface coefficient files "*ReSP0.Delt*" and "*ReSP1.Delt*". The second one is used in this analysis because the flaps are more effective than the first one.
- The mass properties, the basic aero, the damping derivatives, and the hinge moment coefficient files are the same as during ascent.

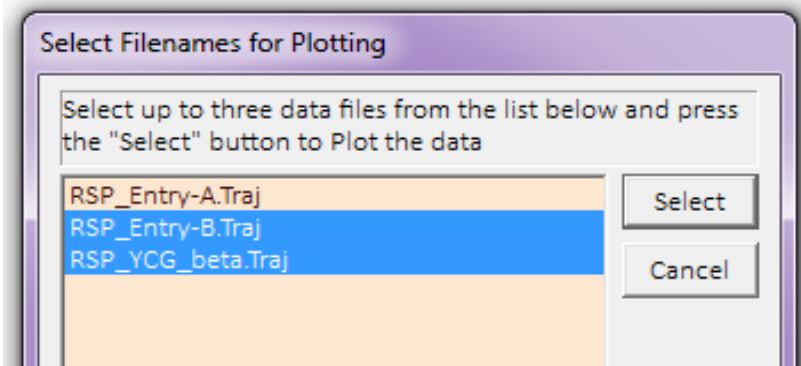
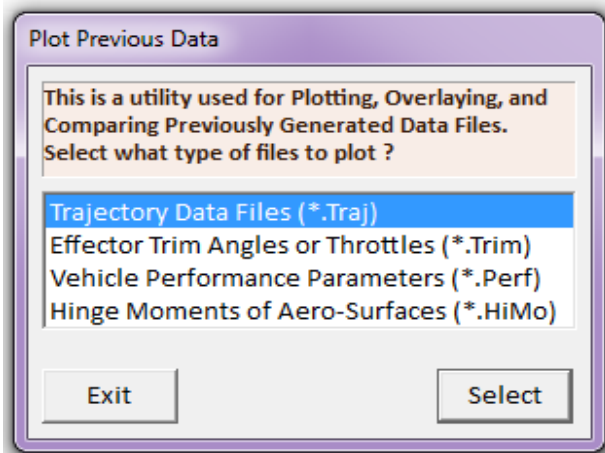
## 2.0 Trajectory (a) analysis

Let us take a look at the first trajectory. You must first select the filename "*RSP\_Entry-A.Traj*" from the filename selection menu, and from the Trim menu we select option (2) for plotting it, as shown below. This represents the descent phase of the first trajectory whose ascent part was analyzed in the ascent analysis. The CG is now constant because there is no significant fuel depletion. The angle of attack is initially high and it is gradually reduced to  $5^\circ$ . The flight path ( $\gamma$ ) is mostly negative as the vehicle is descending. The peak velocity is in the beginning of the descent flight,  $V=4,500$  (ft/sec) at Mach 4. The max dynamic pressure is about 200 (psf) towards the lower end of the flight.

## 2.2 Trajectory (b) analysis with Dispersions

The descent phase of the second trajectory is in file "*RSP\_Entry-B.Traj*". The ascent segment of this trajectory was analyzed in the ascent section. This trajectory was modified slightly from its original version by introducing a few dispersions for analysis, a variation in beta and a CG offset. A trajectory is modified by going to "*Graphic Options*", then "*Modify a Trajectory Plot*", and selecting a variable to modify from the menu. However, we are not showing the details because this process was demonstrated in previous examples. The modified trajectory was saved and then renamed in a separate file "*RSP\_YCG\_beta.Traj*".

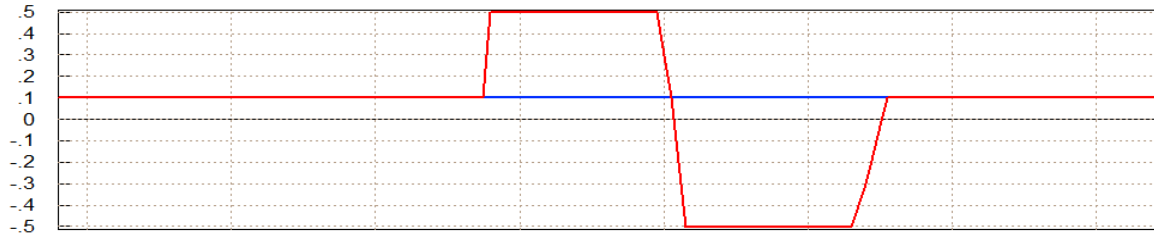
Before analyzing them let us plot the two trajectories together and show their differences. From the Trim menu select option (12), and from the menu that selects the file type select the trajectory type (.Traj). Then from the trajectory filenames menu select the two trajectory files, as shown. The blue curves represent the original and the red curves are the modified trajectory. The Ycg was moved from its original position +0.1 to 0.5 (ft) for a period of 120 sec, and then to -0.5 (ft) for another 120 sec, and then back to its original position. The Xcg and the Zcg were also modified. An angle of sideslip dispersion doublet was also introduced at t=400 sec. It was changed from  $\beta=0^\circ$  to  $\beta=2^\circ$ , then to  $\beta=-2^\circ$ , and then back to  $\beta=0^\circ$ . The  $\beta$ -disturbance and the CG variation occur at different times in order to analyze the effects separately.



Vehicle CG in (feet), User Modified Trajectory ...



Xcg



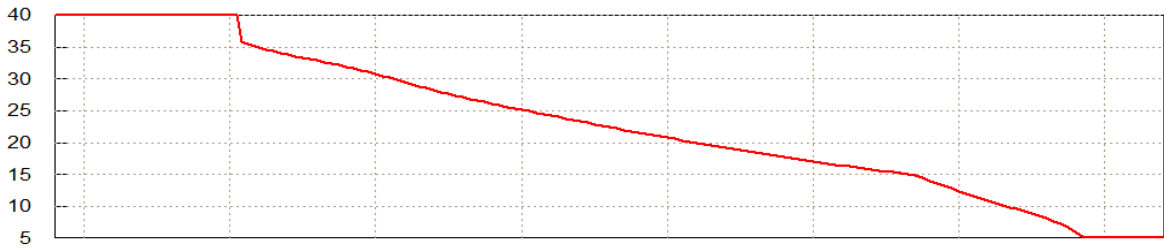
Ycg



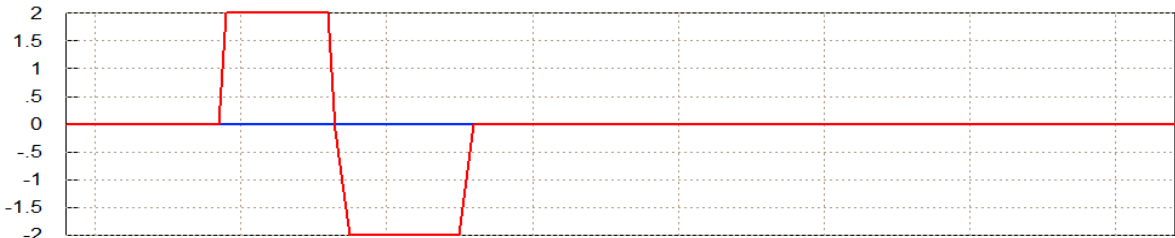
Zcg

Time (sec)

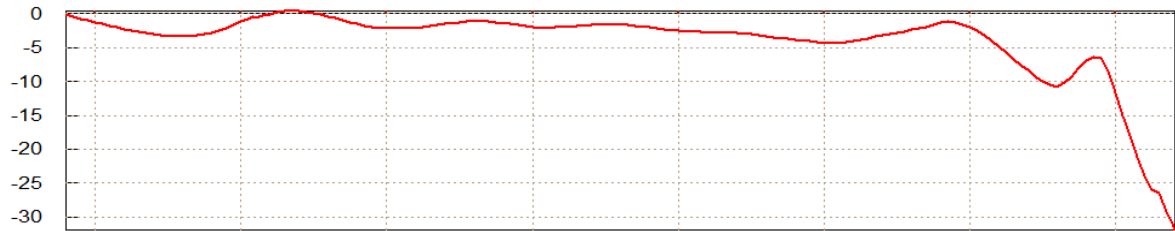
Angles of Attack/Sideslip/Flight Path (deg), User Modified Trajectory



Alpha



Beta

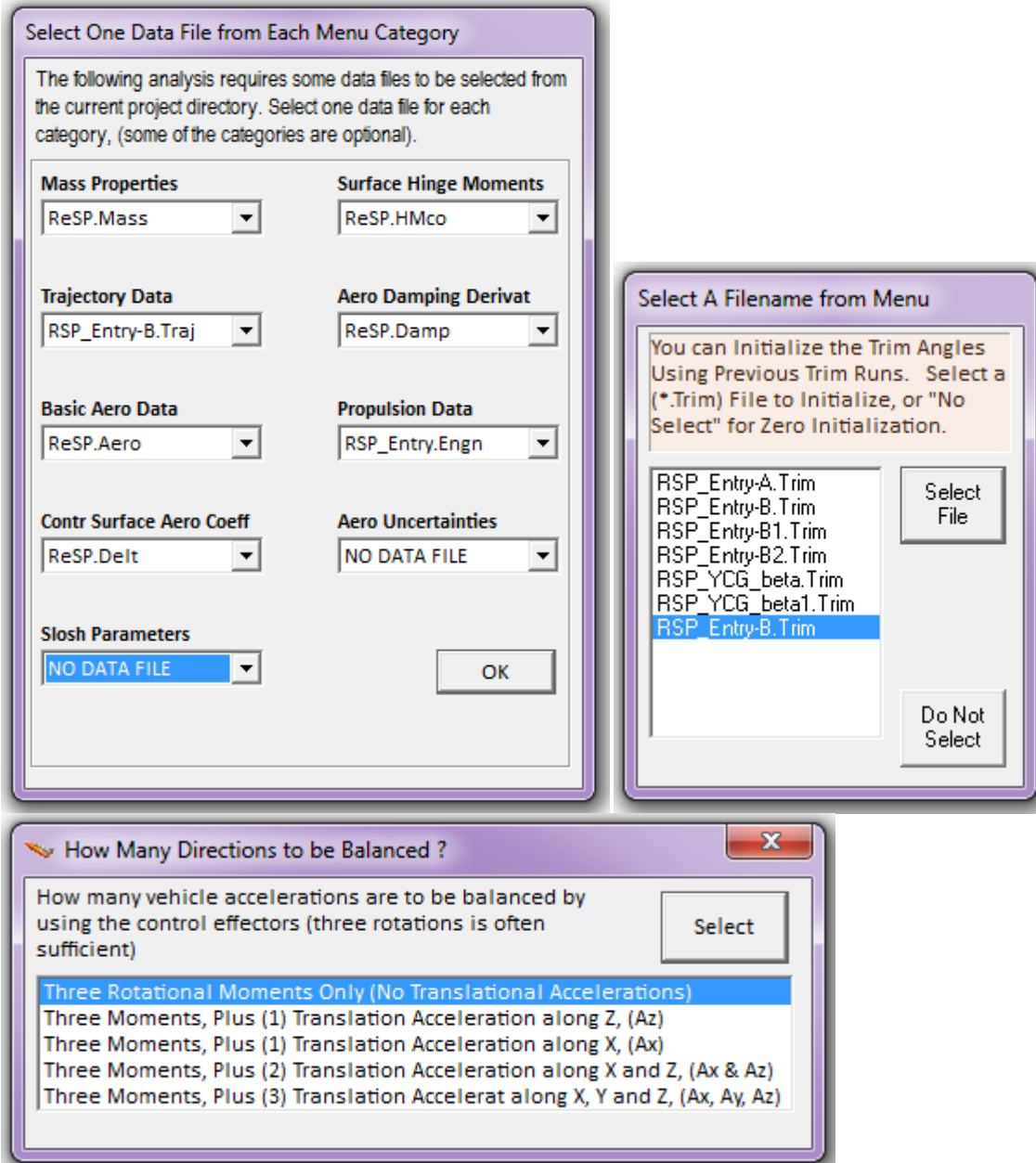


Gamma

Time (sec)

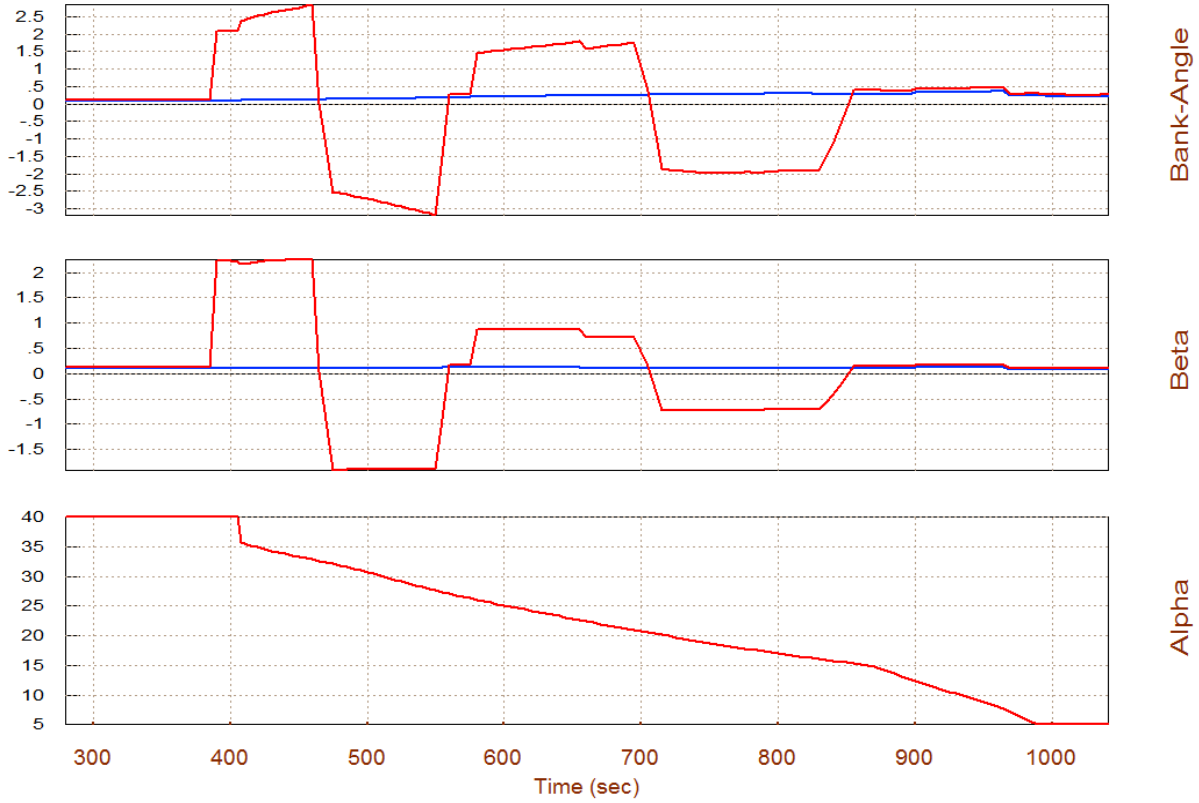
## 2.3 Trimming with Trajectory (b) and the Dispersion Trajectory

Now we return to the Trim main menu and select option (3) to trim the vehicle, first along the two trajectories separately, the original trajectory (b) and then using the modified trajectory. Since we do not have the effector capability to trim along any translations we must trim only along the 3 moments.

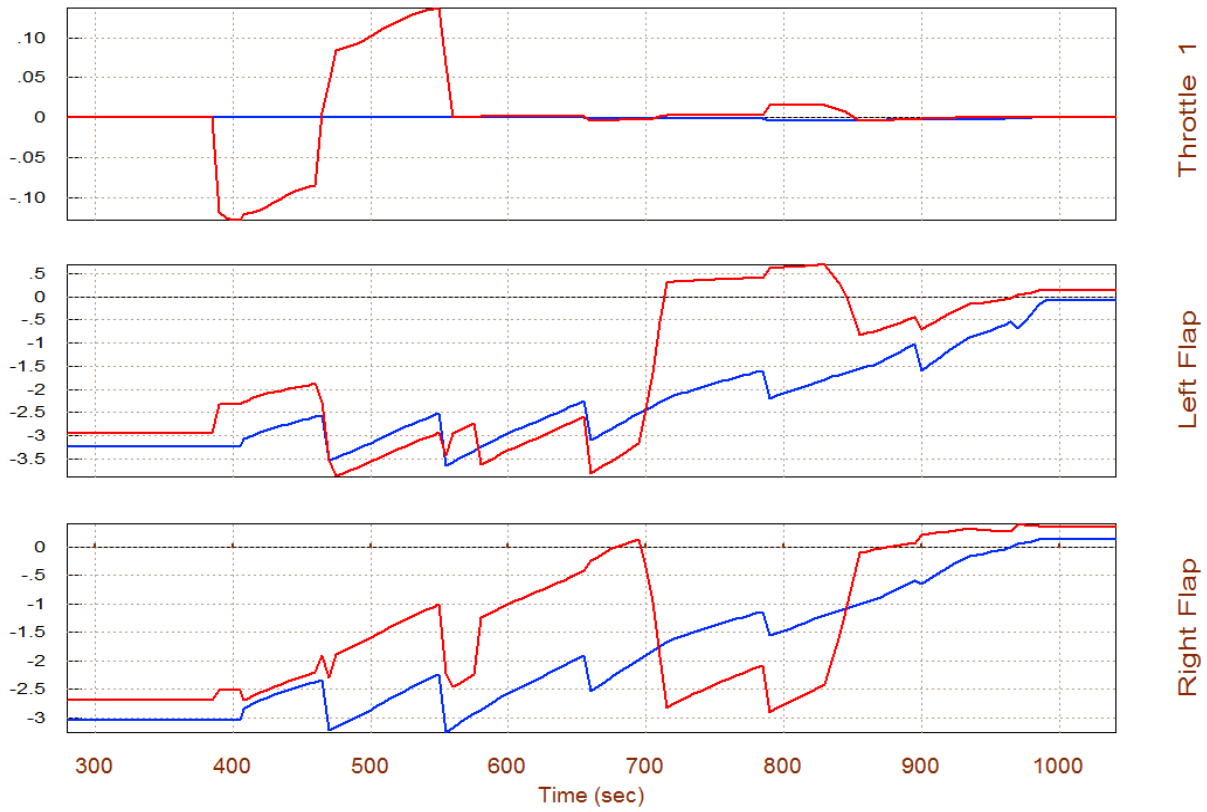


The generated trim files for the original and modified trajectories are: "*RSP\_Entry-B.Trim*" and "*RSP\_YCG\_beta.Trim*" respectively. After trimming, return to the Trim menu and select option (12) to plot these two files together and compare the trim results. The trim variables from the original trajectory are shown in blue and those from the modified trajectory are in red.

Alpha, Beta, and Bank Angle (phi), Assuming Effectors are Trimmed (deg)



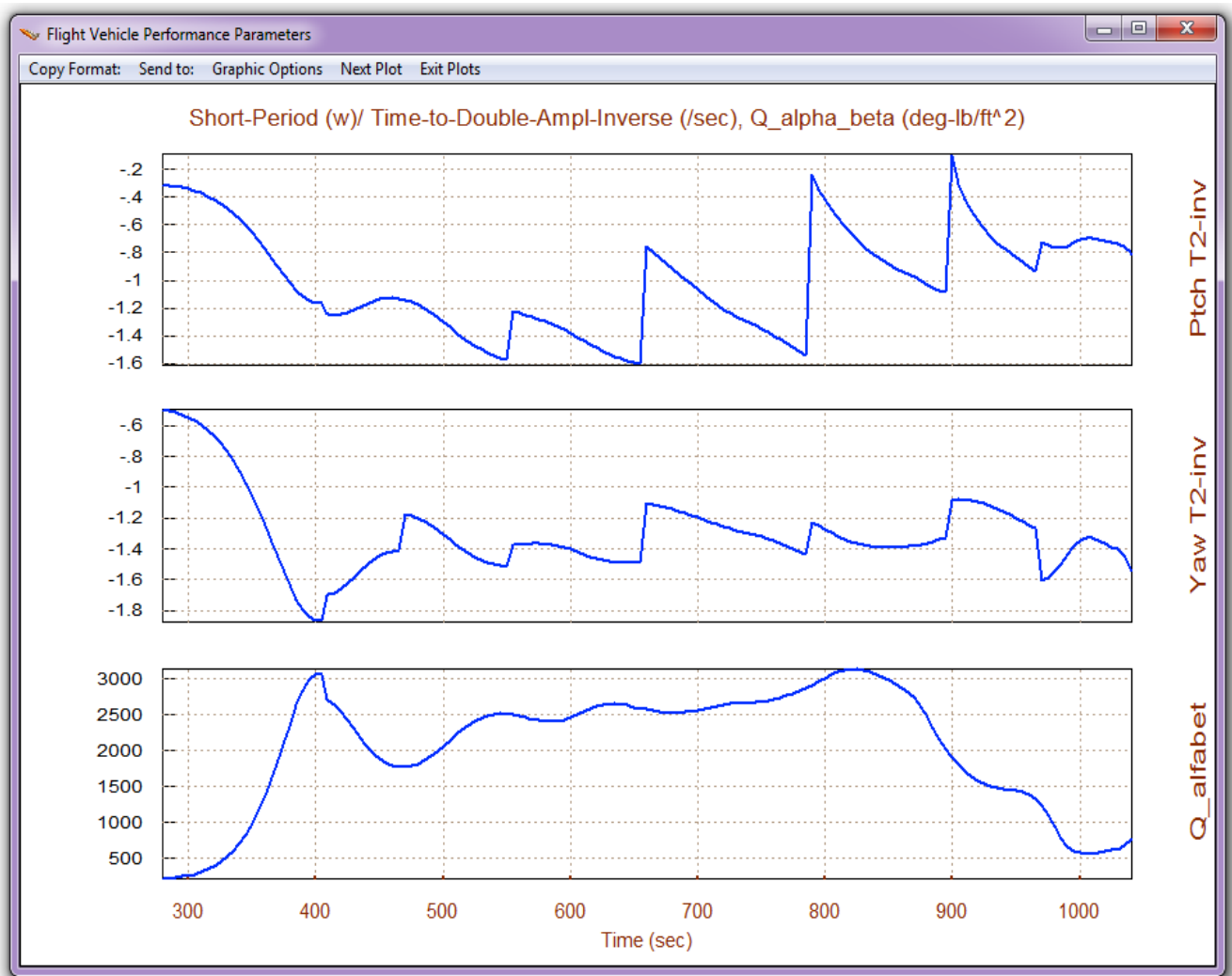
Surface & Engine Deflections/ Throttle, User Modified Trajectory ...

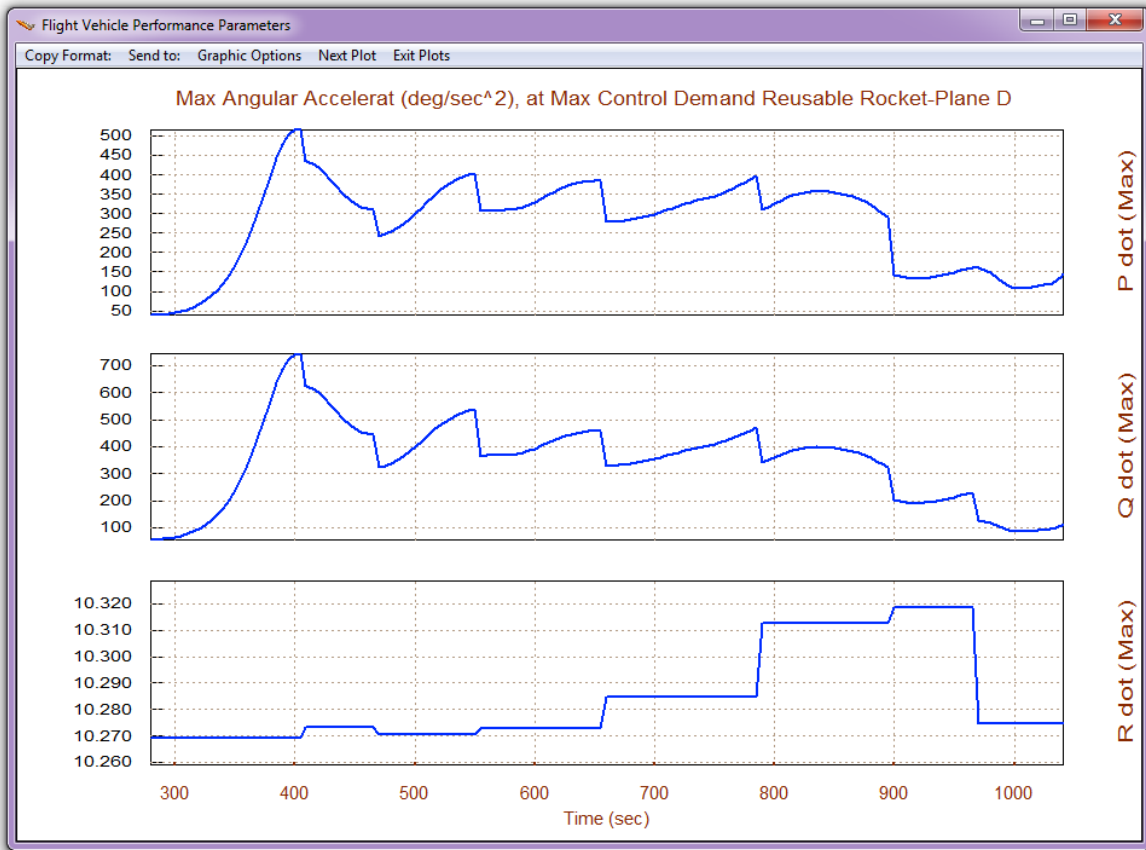
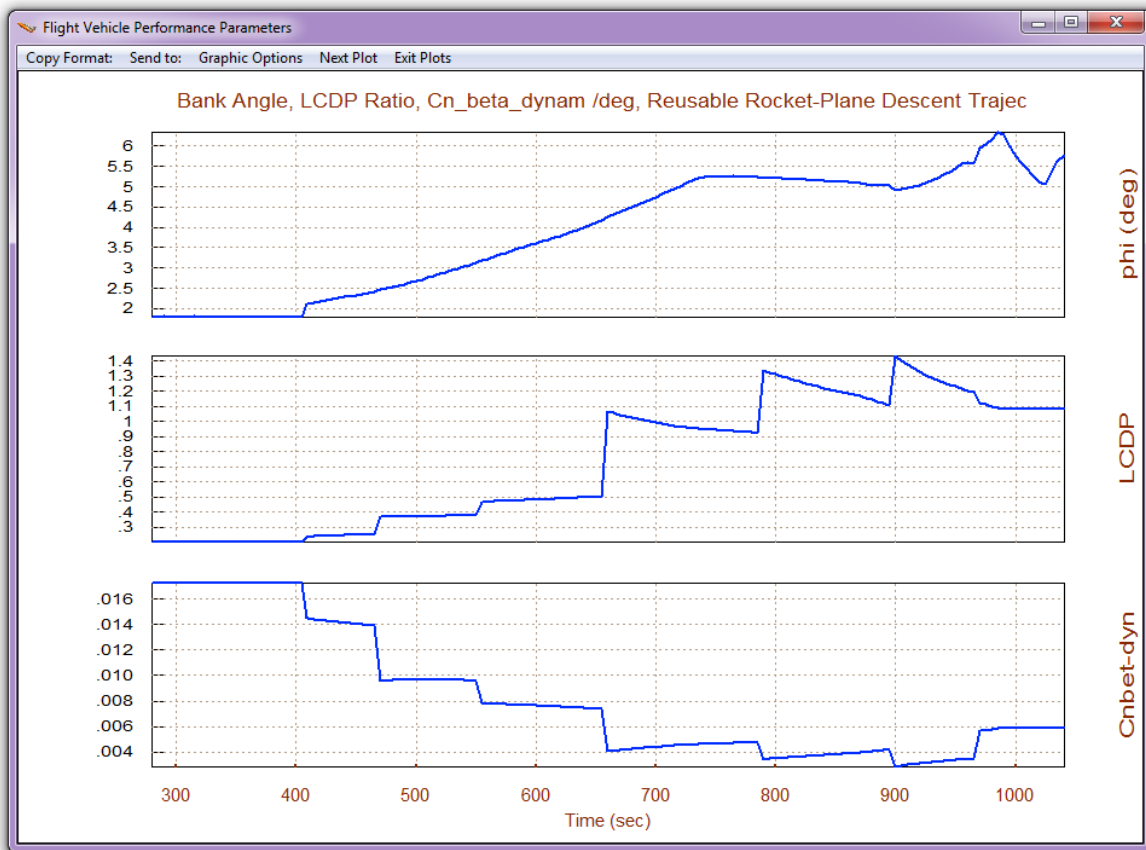


## 2.4 Performance Analysis Along Trajectory (b)

Let us return to the Trim main menu and select option (6) to analyze the vehicle performance parameters along trajectory (b). We will assume a max disturbance  $\alpha_{\max}=2^\circ$  and  $\beta_{\max}=2^\circ$ , and allow the program calculate the (3x3) mixing-logic matrix that converts the roll, pitch and yaw demands to flap deflections and Y-jet throttle command.

From the static margin and the pitch T2-inverse parameter we conclude that the vehicle is stable in pitch, throughout descent, with a short period resonance of 1.6 (rad/sec). The yaw axis is also stable because  $C_n\beta$ -dynamic  $> 0$ , and the yaw T2-inverse parameter is negative showing a Dutch-Roll resonance that peaks to 1.82 (rad/sec). The  $C_n\beta$ -dynamic becomes very small at  $t=900$  sec (barely acceptable), which causes the LCDP to spike at that time. The LCDP ratio is also small during the first 100 seconds (barely greater than 0.2). The control effort is acceptable in all 3 control directions because it is less than 0.5. The bank angle ( $\phi$ ) is generated by the cross-wind due to  $\beta_{\max}=2^\circ$ , and it is used to evaluate the landing conditions. The last chart shows the max accelerations that can be produced in the 3 direction before saturating the effectors.

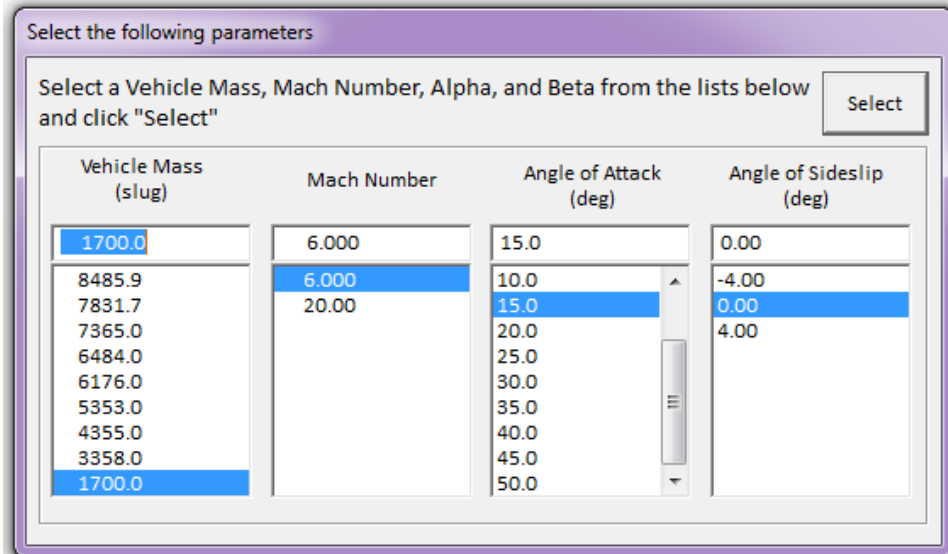
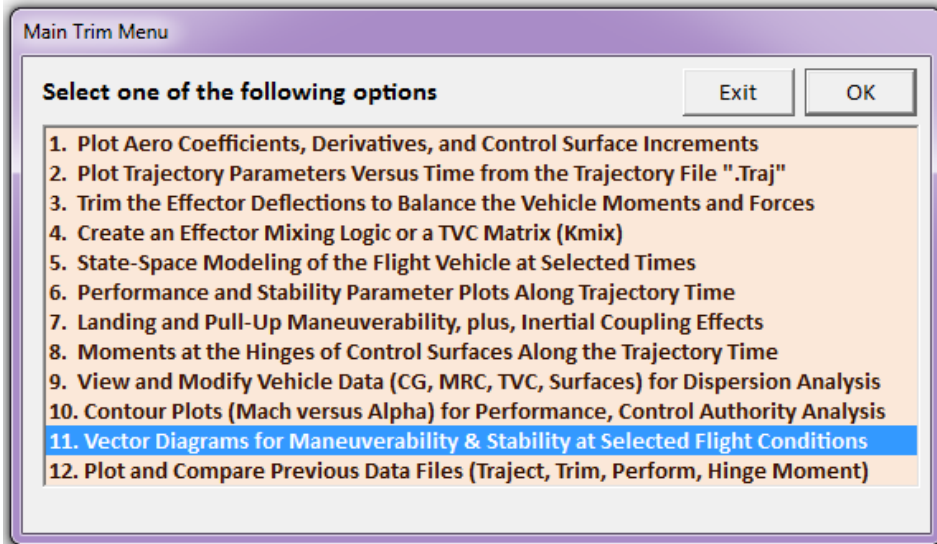




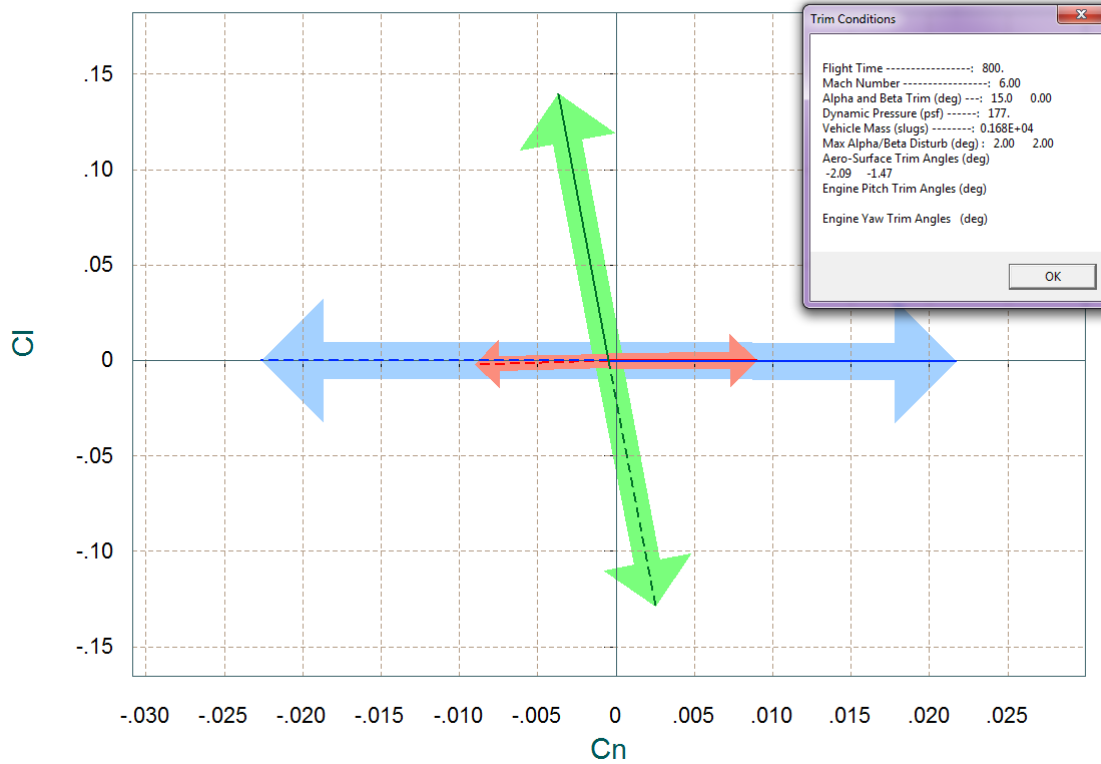


## 2.5 Vector Diagram Analysis

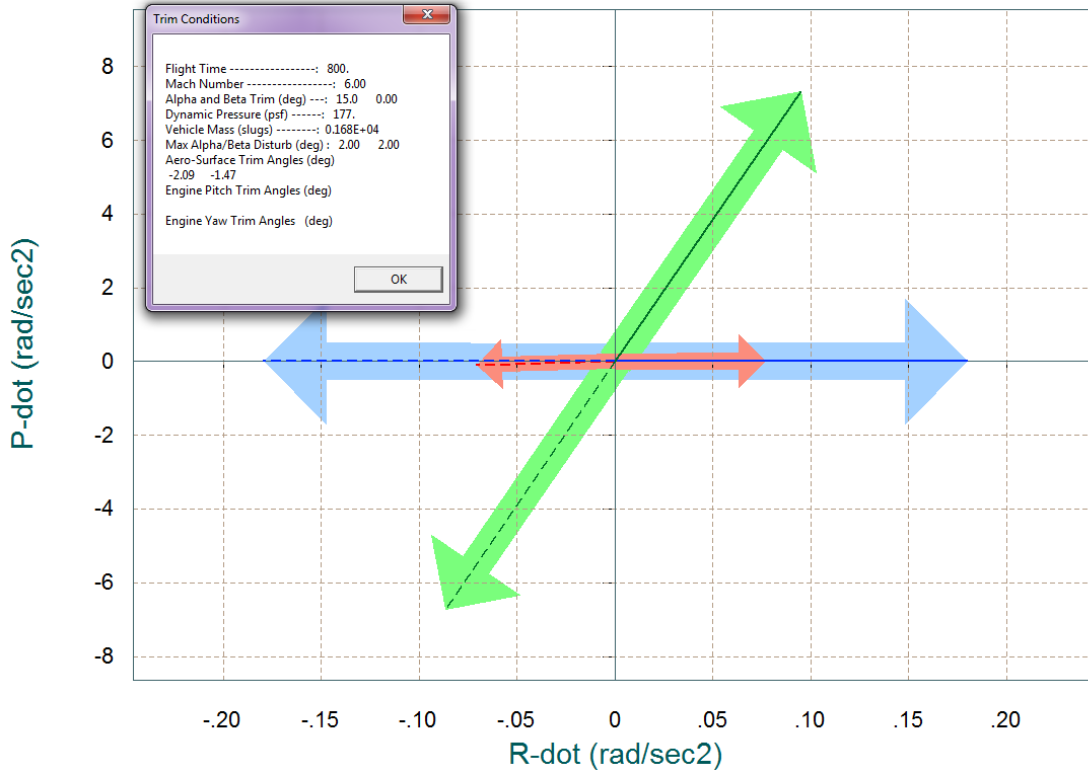
Vector diagrams are 2-dimensional diagrams used for analyzing the vehicle controllability at a fixed flight condition. We compare the control capability of the aero-surfaces in two directions against the effects on the vehicle of wind-shear disturbance that is expressed by the disturbances that it causes in the angles of attack and sideslip, and we determine if the vehicle provides sufficient control authority to counteract against the disturbance moments and forces. It also allows us to examine the directions of the controls versus the disturbance directions. It helps us evaluate the orthogonality of the control system, compare the acceleration magnitudes due to the controls and wind, and to determine if the controls are powerful enough and their directions are capable of counteracting the disturbance moments along roll, pitch, and yaw. From the Trim menu select option (11), and then a flight condition at  $t=800$  sec. The following dialog consists of menus used for selecting the vehicle mass, Mach number, alpha, and beta. Select the default values which correspond to this flight condition and click "Select".



Comparison Between Maximum Control Moments Against Maximum Disturb Moments (red)  
 Roll & Yaw Control Moments (non-dimension) vers Disturb Moment due to Max Beta/Alpha

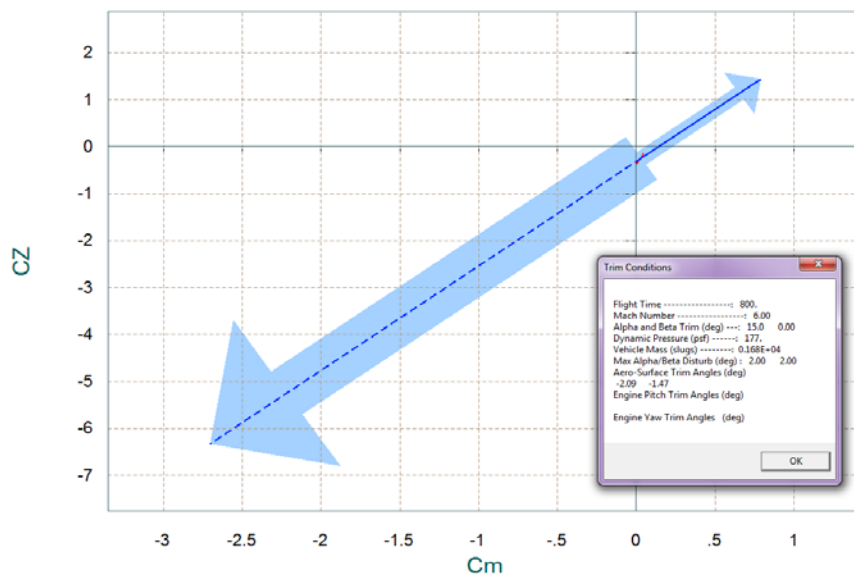


Comparison Between Maximum Control Accelerations and Max Accels due to Beta (red)  
 Roll & Yaw Accelerations due to Maximum Roll/ Yaw Control and due to Max Beta

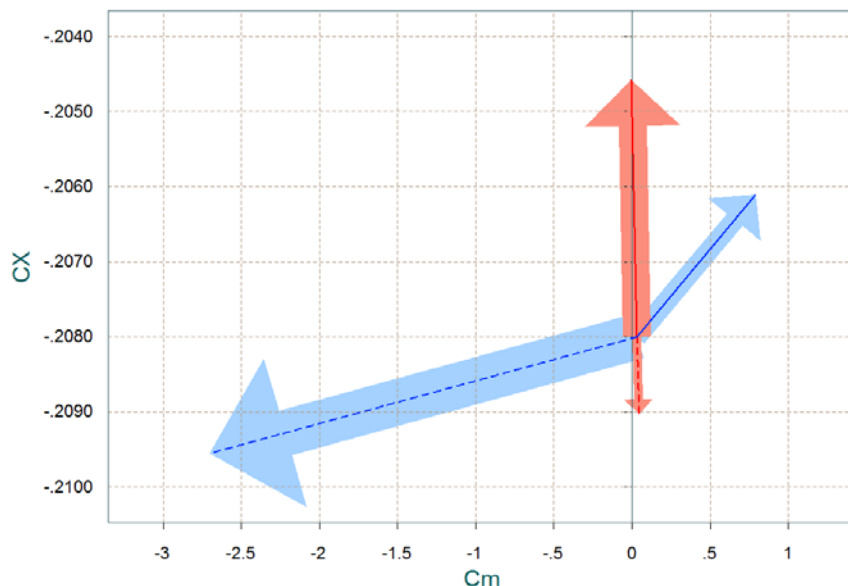


The next plot shows the effects in the longitudinal moments forces ( $C_m$ ,  $C_x$ , and  $C_z$ ) when the pitch control is maximized, fully positive and fully negative. When the pitch control is max negative (blue dashed vectors) it produces a negative pitching moment and negative z-force which is bigger in magnitude than when the pitch control is maxed positive (blue solid vector). This asymmetry is due to the fact that the vehicle is statically stable and it is flying with a positive  $\alpha=15^\circ$ , and therefore, it is a lot easier to pitch down than it is to pitch up. Notice that when the pitch control is zero the vehicle experiences negative aero forces in both x and z directions ( $C_x=0.208$ , and  $C_z=0.3$ ). The wind disturbance due to  $\alpha_{max}$  affects mainly the axial force asymmetrically. The drag force ( $C_A=-C_x$ ) is reduced (or acceleration along x is increased), shown by the red solid vector pointing upwards, when  $\alpha_{max}=2^\circ$ .

Comparison Between Maximum Control Moment/Forces Against Maximum Disturbance (red) Pitch and Normal-Z Control Moments and Forces versus Disturb due to Alpha Variation

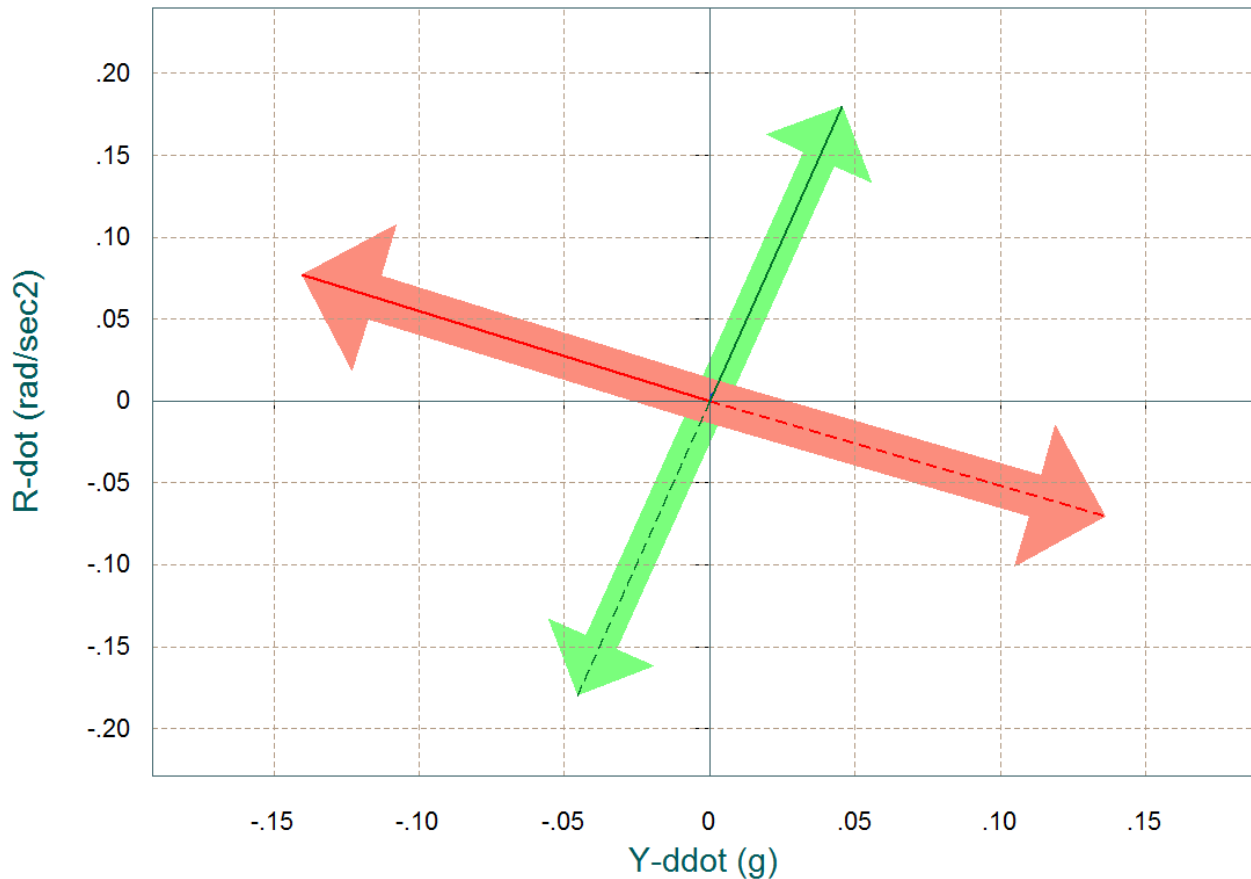


Comparison Between Maximum Control Moment/Forces Against Maximum Disturbance (red) Pitch Control Moment and Axial X-Force (non-dims) versus Disturb due to Max Alpha

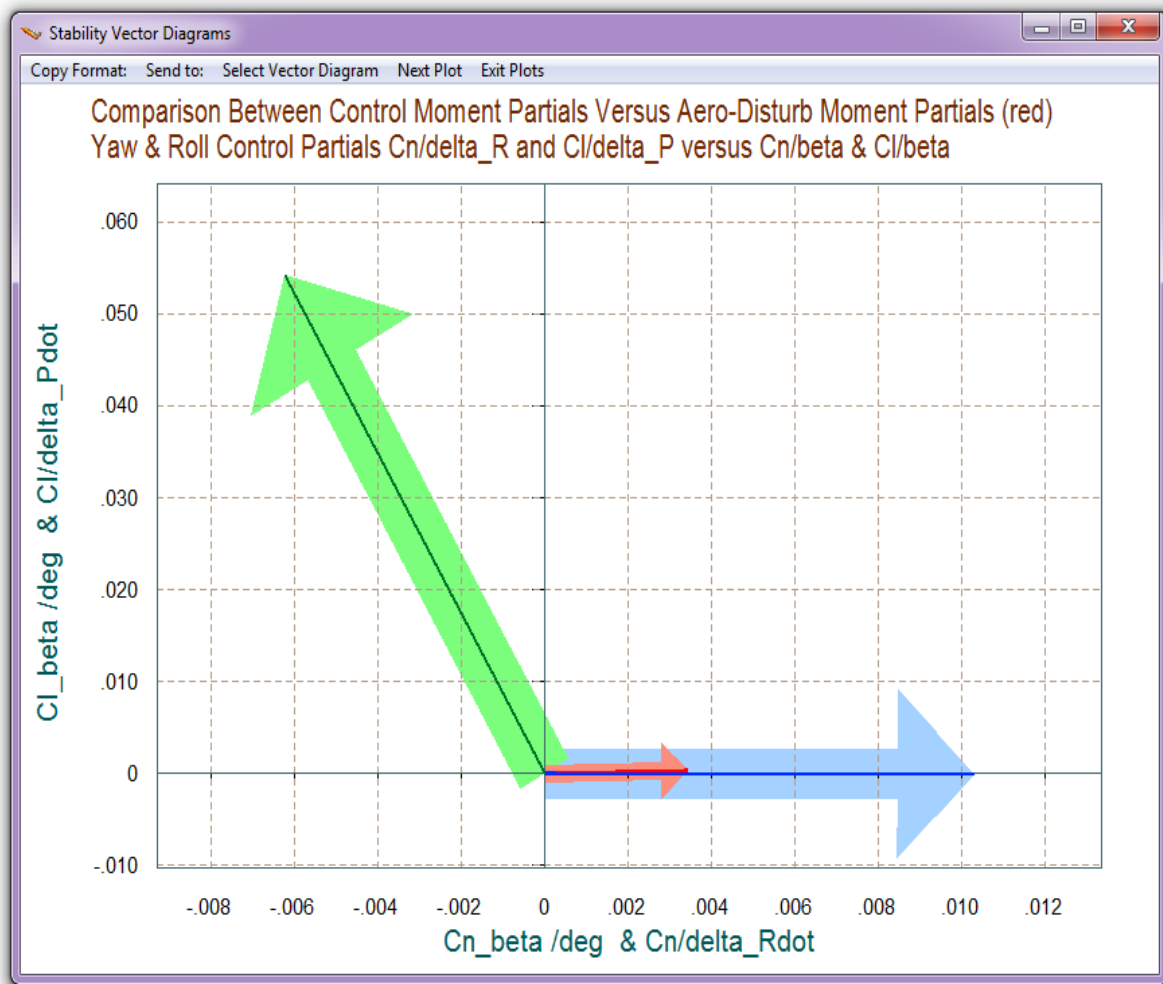


The next plot shows a pair of green vectors representing the variation in yaw ( $\dot{r}$ ) and side ( $\dot{y}$ ) accelerations as a result from maximizing the yaw control. A positive yaw demand due to the RCS jet firing causes positive accelerations in both yaw ( $\dot{r}$ ) and in ( $\dot{y}$ ), as shown by the solid green vector. The red vectors show the accelerations due to  $\beta_{\max}=\pm 2^\circ$ . A positive beta produces a negative side acceleration and a positive yaw acceleration (solid red line). The yaw control acceleration dominates the disturbance in yaw, but not in the side acceleration (which is not controllable).

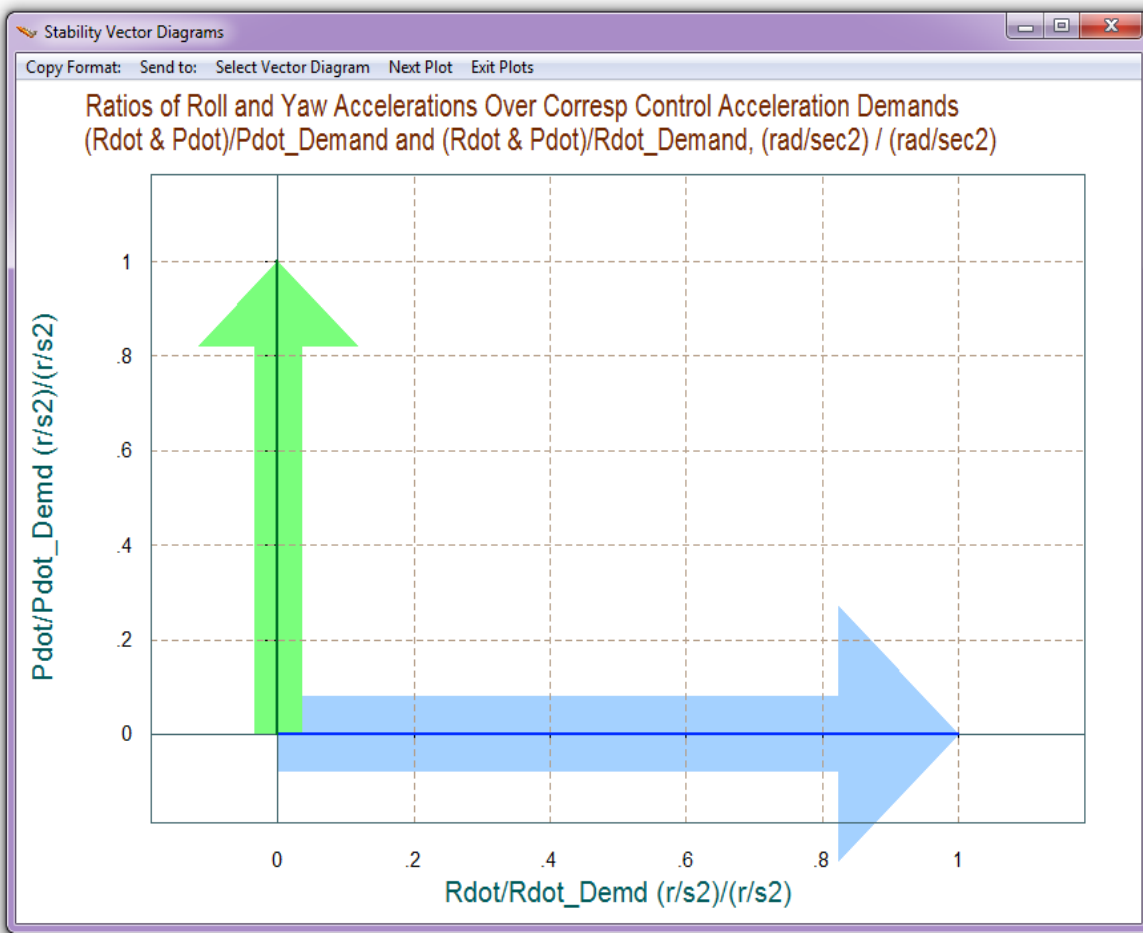
Comparison Between Maximum Control Accelerations and Max Accels due to Beta (red)  
 Yaw and Side-Force Accelerations due to Max Control and Accelerat due to Max Beta



The next figure is a moments partials vector diagram showing the variation in roll and yaw moments per acceleration demands in roll and yaw in (rad/sec<sup>2</sup>). The blue vector is the moments partial  $\{C_n\delta R_{FCS}, Cl\delta R_{FCS}\}$  due to yaw control demand which is entirely in the yaw direction. The green vector is the moments partial  $\{C_n\delta P_{FCS}, Cl\delta P_{FCS}\}$  due to roll control demand and it is mostly in roll but it also couples into yaw. They are close to being orthogonal to each other. The red vector is mostly due to the  $C_n\beta$  partial. The  $Cl\beta$  partial is small. It shows that the control authority is good because the control vectors dominate over the partials due to beta.



The last plot below shows the partials of roll and yaw accelerations per acceleration demand. The green vector is  $\{\dot{P}/\delta P_{FCS}, \dot{R}/\delta P_{FCS}\}$ , and the blue vector is  $\{\dot{P}/\delta R_{FCS}, \dot{R}/\delta R_{FCS}\}$ . The axis units are in  $(rad/sec^2)/(rad/sec^2)$ . They are unity vectors and completely decoupled from each other, showing that the mixing-logic matrix is perfectly diagonalizing the plant model.



## 2.6 Creating a Dynamic Model for Descent

We will now create a dynamic model for our space-plane at a fixed flight condition during descent and in the next section we will design a flight control system and analyze the system. During descent the space-plane is using the two flaps and a pair of yaw jets. The two jets are represented by a single thruster that produces either positive or negative forces in the y direction. Its thrust is limited to  $\pm 2,500$  (lb).

Start the Trim program, select the same files as before and make "T800.inp" to be the Flixan input data file. This file contains the vehicle data plus other Flixan related model generation and data reduction data sets that will be discussed later. Re-trim the effectors to make sure that you are using a current trim condition. Do not select a trim initialization file, and trim only along the 3 moments (no translational trimming). From the Trim main menu select option (5) to create a state-space dynamic model. A dialog reminds the user how to select a flight time for the dynamic model, click "OK". From one of the trajectory plots go the top menu bar, and choose "Graphic Options", and then from the vertical pop-up menu click on "Select Time to Create State-Space System". Then using the mouse click at time  $t=800$  sec, along the x axis. Confirm that you have selected the correct time by clicking "OK", as shown below. Otherwise, click "Cancel" and try again.

**Flight Vehicle Parameters**

**Vehicle System Title**  
Reusable Rocket-Plane Descent Trajectory-2/ T= 800.0 sec

**Number of Vehicle Effectors**

Gimbaling Engines or Jets. Include Tail-Wags-Dog?	1	WITH TWD	WITHOUT TWD
Rotating Control Surfaces. Include Tail-Wags-Dog?	2	WITH TWD	WITHOUT TWD
Reaction Wheels?	0		

**Momentum Control Devices**

Single Gimbal CMGs?	0	Include a 3-axis Stabilized Double Gimbal CMG System?	Yes	No
---------------------	---	---	-----	----

**Number of Sensors**

Gyros	0
Acceleromet	3
Aero Vanes	0
External Torques	0

**Modeling Options (Flags)**

Output Rates in	Turn Coordination ?
Body Axes	Include Turn Coordin
Stability Axes	Without Turn Coordin
Aero-Elasticity Options	Attitude Angles
Include GAFD, H-param	Euler Angles
Flex Coupl. data only	Integrals of Rates
Neither Gafd nor Hpar	LVLH Attitude

**Number of Modes**

Structure Bending	0
Fuel Sloshing	0

Reaction Wheels | Single Gimbal CMGs | Double Gimbal CMG System | Slewing Appendages | Gyros | Accelerometer | Aero Sensors | Fuel Slosh | Flex Modes | User Notes  
Mass Properties | Trajectory Data | Gust/ Aero Paramet. | Aero Force Coeffs | Aero Moment Coeffs | Control Surfaces | Gimbal Engines/ RCS | External Torques

**Vehicle Mass Properties**

Moments/ Products of Inertia (slg-ft <sup>2</sup> )				Location of Center of Gravity (ft)		Vehicle Mass in (slugs)	
Ixx	127176.6	Ixy	0.000000	Xcg	-59.81394	Vehicle Mass in (slugs)	1684.200
Iyy	997833.6	Ixz	13527.07	Ycg	0.1000000	Accelerat. due to Gravity (ft/sec <sup>2</sup> )	32.17400
Izz	819847.1	Iyz	0.000000	Zcg	1.139163	Earth Radius (Re) in (feet)	0.2089600E+08

The program is now ready to create a dynamic model at the selected flight time. The above dialog shows the flight vehicle parameters prepared by Trim which are extracted from the data files. There are multiple tabs with data categories. The user can modify some of the data or titles using this dialog before saving it. Click on "Update Data" after any modifications. Do not run this file yet because there is more work to be done and more data to be included in T800.inp. Click on "Save in File" and the vehicle data will be saved in file "T800.inp", under the title "Reusable Space-Plane Descent Trajectory-2/ T= 800.0 sec". The file "T800.inp" will be processed by Flixan to generate the



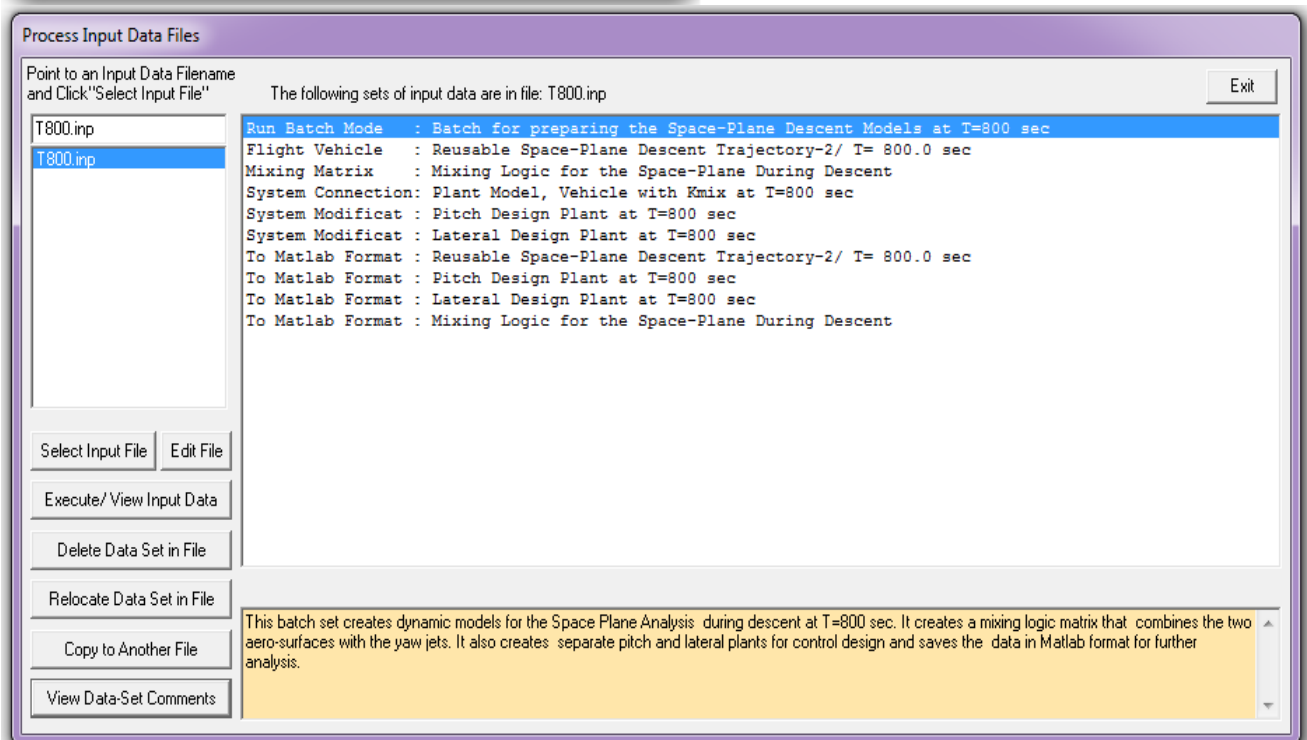
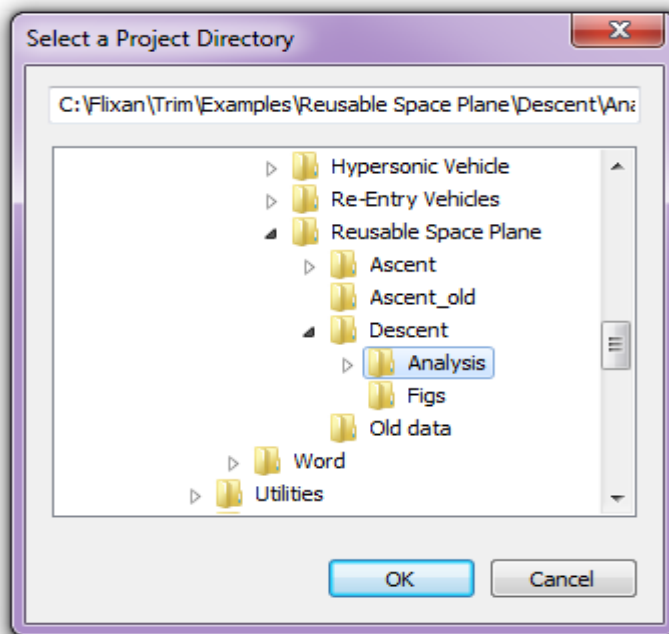
systems for control design and analysis using Matlab/ Simulink. In addition to the vehicle data the input file contains also system interconnection and modification data related to this analysis that will be processed by Flixan. The systems and matrices generated by Flixan will be saved in file "T800.Qdr".

Let us now take a look and see what is in file "T800.inp" before processing it in Flixan. The file contains several sets of data and each set corresponds to and it is processed by a Flixan utility.

- At the top of the file there is a batch set that can process all other data-sets in batch mode. This is faster because it processes them all together instead of interactively. Its title is "*Batch for preparing the Space-Plane Descent Models at T=800 sec*".
- Below the batch we see the flight vehicle data for the flight condition that we have just created. The first 3 inputs of the vehicle state-space model correspond to the 3 control inputs. That is, the yaw jet throttle input that varies between -1 to +1 representing a thrust variation between -2500 to +2500 (lb), and the two flap aero-surfaces. The 4<sup>th</sup> input is a wind-gust velocity disturbance in (ft/sec). The gust direction relative to the vehicle is defined in file T50.inp.
- The next data-set generates the (3x3) mixing logic matrix that converts the 3 (roll, pitch, yaw) rotational acceleration demands to flap deflections and yaw jet throttling commands. The data-set title is "*Mixing Logic for the Space-Plane During Descent*" and the matrix title is Kmix\_800.
- It is a good practice to include the mixing logic matrix in series with the vehicle model, at its input. This reduces the number of inputs, it decouples the plant, and makes it easier to analyze and to design flight control laws. The next set of data is a system interconnection set that combines the vehicle model with the mixing matrix. Its title is: "*Plant Model, Vehicle with Kmix at T=800 sec*". It will be split into pitch and lateral and used for LQR design.
- Next, we have two system modification data-sets for separating the previous plant in two separate sub-systems: (a) a pitch design plant "*Pitch Design Plant at T=800 sec*", and (b) a lateral design plant "*Lateral Design Plant at T=800 sec*", by extracting the corresponding inputs, states, and outputs.
- Finally, the Flixan input file also includes data-sets for converting the systems to Matlab m-functions for further analysis using Matlab. The original vehicle model is saved in file "*vehicle.m*". The pitch and lateral design plants are saved in files "*vehi\_pitch.m*" and "*vehi\_later.m*" respectively. The effector mixing matrix is saved in file "*Kmix\_800.mat*". These files will be loaded and analyzed in Matlab, as it will be described in the next section.

Let us now process this input file using Flixan. The preparation of the batch set and the system manipulation sets is not documented here because it is beyond the scope of this example and not difficult to follow. Plus these processes are documented in detail in Flixan. Start the Flixan program and select the same project folder. Then go to "Edit", "Manage Input Files (.Inp)", and "Process/

*Edit Input Data*", as shown below. The following dialog is for viewing and processing Flixan input files. It has two menus. The one on the left lists the input files in the project directory. There is only one in this case, select "T800.Inp" and click on "Select Input File". The menu on the right side shows the data-sets which are in "T800.Inp". They can be processed individually, but select the batch set on the top to process all of them together, and click on "Execute". Flixan will now process the input file and create the systems in file "T800.Qdr". It will also create the function files that will be loaded into Matlab. These files are transferred in subdirectory "Descent\Analysis" where we will perform the control design and analysis.



## 2.7 Descent Control Design and Analysis

We will now use the dynamic models developed in section 2.6 to design control laws for our space-plane during descent, analyze stability, and develop simulation models in Matlab/Simulink. The analysis is performed in directory: "C:\Flixan\Trim\Examples\ Reusable Space Plane\Descent\Analysis". We will begin by designing control laws, then perform stability analysis in the frequency domain, and eventually develop a simulation model.

### Flight Control Design

The flight control gains are calculated by the Matlab script file "*design.m*", shown below. It uses the pitch and lateral design systems in files "*vehi\_pitch.m*" and "*vehi\_later.m*", created in section 2.6, to generate state-feedback matrices *Kq.mat* and *Kpr.mat* for the pitch and lateral systems. The pitch state-vector consists of  $[\theta, q, \alpha, \alpha\text{-integral}]$ . The lateral state-vector consists of  $[\phi, p, \psi, r, \beta]$ .

```
% Pitch LQR Design for Space-Plane
[Ap,Bp,Cp,Dp]= vehi_pitch;                               % Load the Pitch Design Model
[Api,Bpi,Cpi,Dpi]= linmod('Pitch_Design');              % Augment Pitch Simulink model
sys1=SS(Api,Bpi,Cpi,Dpi);
% Weights[thet, q, alpha, alfa_int]
Q= diag( [0.01, 0.1, 5.0, 1.5]);                          %
Weights(thet,q,alpha,alfa_int)
R=1;                                                       % Control Weights R=2
[Kq,S,E]= LQR(sys1,Q,R);
save Kq.mat Kq -ascii                                     % Save the LQR gains in Kq.mat

% LQR Lateral Design for Space-Plane
[Al,Bl,Cl,Dl]= vehi_later;                               % Load the Lateral Design Model
sys2=SS(Al,Bl,Cl,Dl);
% states: [phi, p, psi, r, beta]
Q=diag([ 3.5, 0.2, 3.5, 0.2, 0.00001]);
R=diag([1 1]*1);                                         % Control Weights
[Kpr,S,E]= LQR(sys2,Q,R);
save Kpr.mat Kpr -ascii                                  % Save the LQR gains in Kpr.mat
load Kmix_800 -ascii                                     % Load Mixing Logic Matrix
```

## Stability Analysis

The script file "*freq\_anal.m*" below performs stability analysis in the frequency domain. It uses the Simulink model "*Open\_Loop.mdl*", shown in figure (2.7.1) for yaw analysis.

```
[Av,Bv,Cv,Dv]= vehicle; % Load the Vehicle Analysis Model
[Ao,Bo,Co,Do]= linmod('Open_Loop'); % Augment Pitch Simulink model
w=logspace(-3, 3, 10000); % Define Frequency Range
sys= SS(Ao,Bo,Co,Do); % Create SS System
figure(1); Nyquist(sys,w) % Plot Nichol's Chart
figure(2); Nichols(sys,w) % Plot Nichol's Chart
figure(3); Bode(sys,w); grid on % Plot Bode Plot
```

Figure (2.7.2) shows the open-loop frequency response Bode plots of the pitch and yaw axes. The short period and Dutch-Roll resonances are close to those predicted by the performance analysis parameters in Section 2.4. The Nyquist diagrams in Figure (2.7.3) show stable margins in both pitch and yaw.

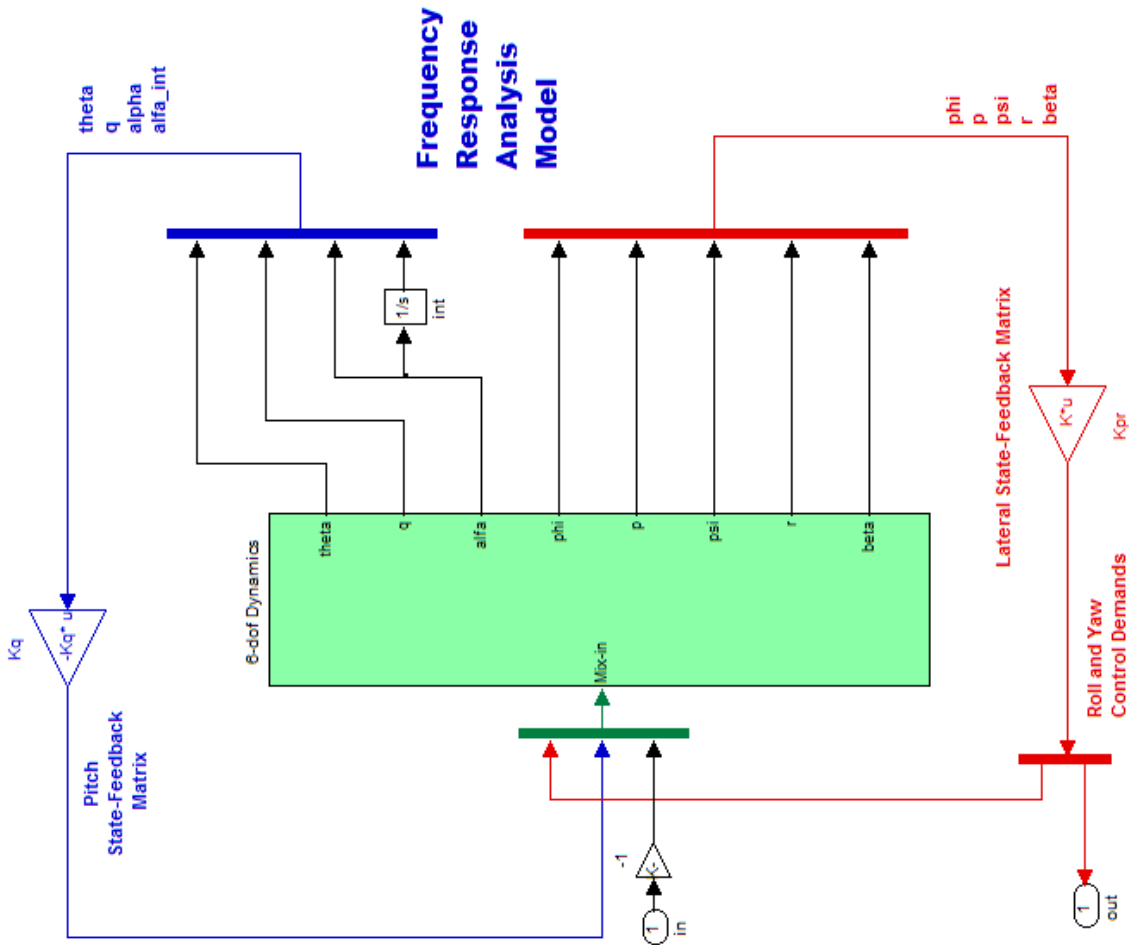
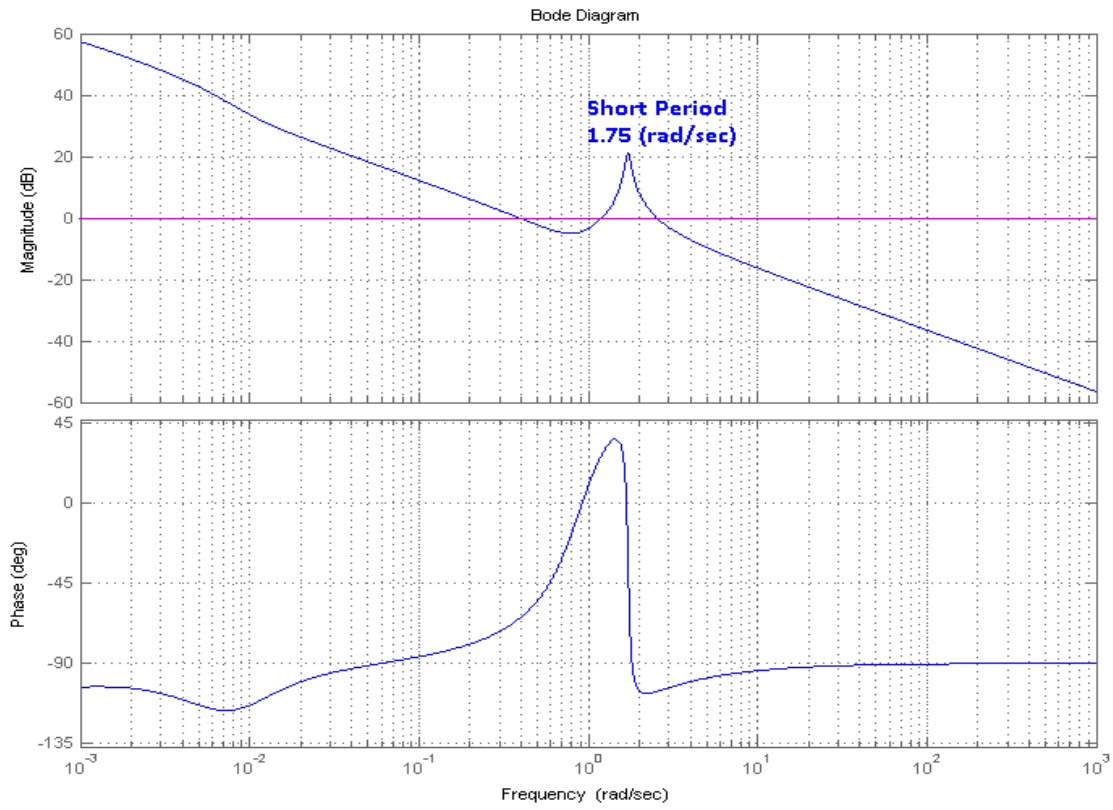


Figure 2.7.1 Open-Loop Frequency Response Analysis Simulink Model "*Open\_Loop.Mdl*".



**Yaw Axis Open-Loop Frequency Response**

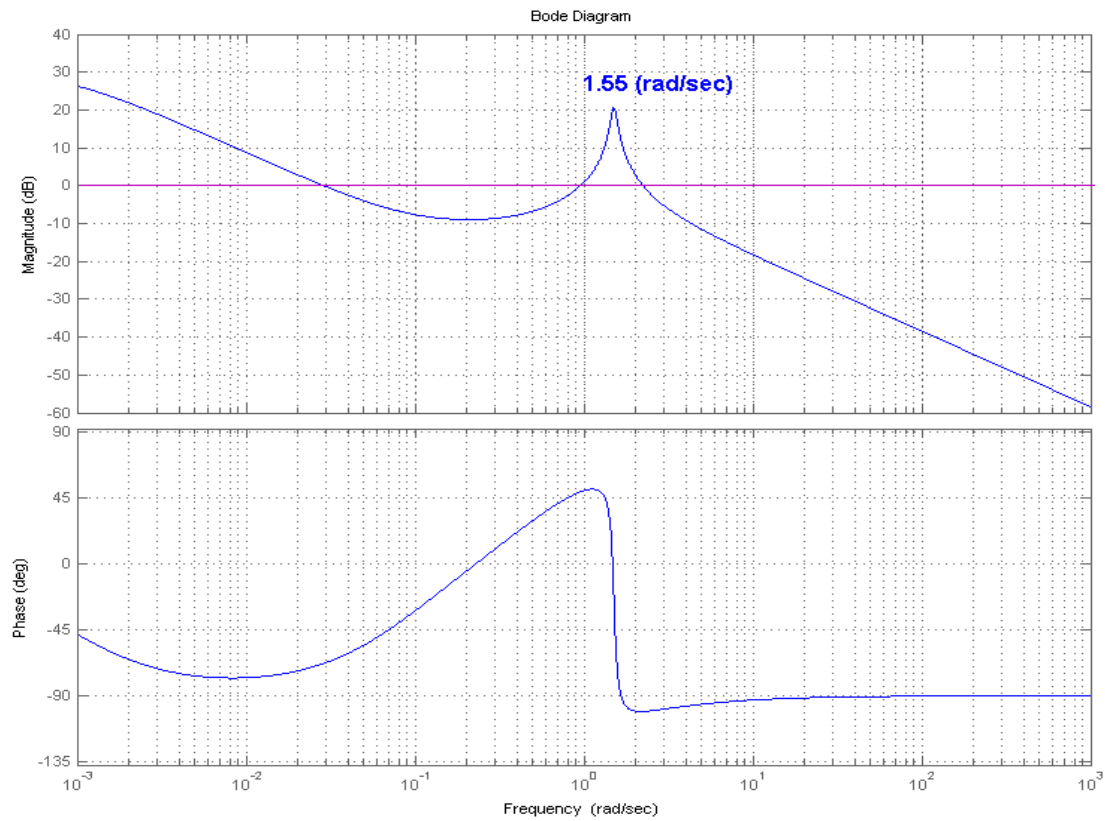


Figure 2.7.2 Open-Loop Bode Plots for the Pitch and Yaw Axes.

## Simulation

The closed-loop simulation model is in file "Sim\_6dof.mdl", shown in Figure (2.7.4 a&b). It is initialized by the Matlab file "runsim.m", shown below, and uses the system in file "vehicle.m", with title: "Reusable Space-Plane Descent Trajectory-2/ T= 800.0 sec" that was created in section 2.6. The file "runsim.m" also loads the mixing logic matrix and the two gain matrices.

```

% Initialize Simulation
r2d=180/pi; d2r=pi/180;
design
[Ay,Bv,Cv,Dv]= vehicle;
load Kmix_800 -ascii
load Kq -ascii
load Kpr -ascii
% Calculate Gains using LQR Design
% Load the Vehicle Analysis Model
% Load Mixing Logic Matrix
% Load LQR Gains

```

The simulation model has three flight control loops: roll, pitch, and yaw. The pitch state-feedback control law via matrix  $K_q$  is seen in the upper section (blue), and the roll/yaw state-feedback control law via matrix  $K_{pr}$  is seen in the lower section (red). The model receives 3 FCS commands from guidance. In this case they are represented with step inputs. There is an alpha command ( $\alpha_{cmd}$ ), a roll command ( $\phi_{cmd}$ ), and a yaw command (not shown). There is also a wind-gust input pulse that is shaped into a more realistic gust by a low-pass filter. The wind-gust direction is defined in the input file and it is set at  $45^\circ$  to excite both: the pitch and yaw axes. When the simulation is complete the file "pl.m" plots the data. We are showing two cases that were made using this simulation model: (a) a roll attitude step command, and (b) an alpha command in combination with a wind-gust disturbance.

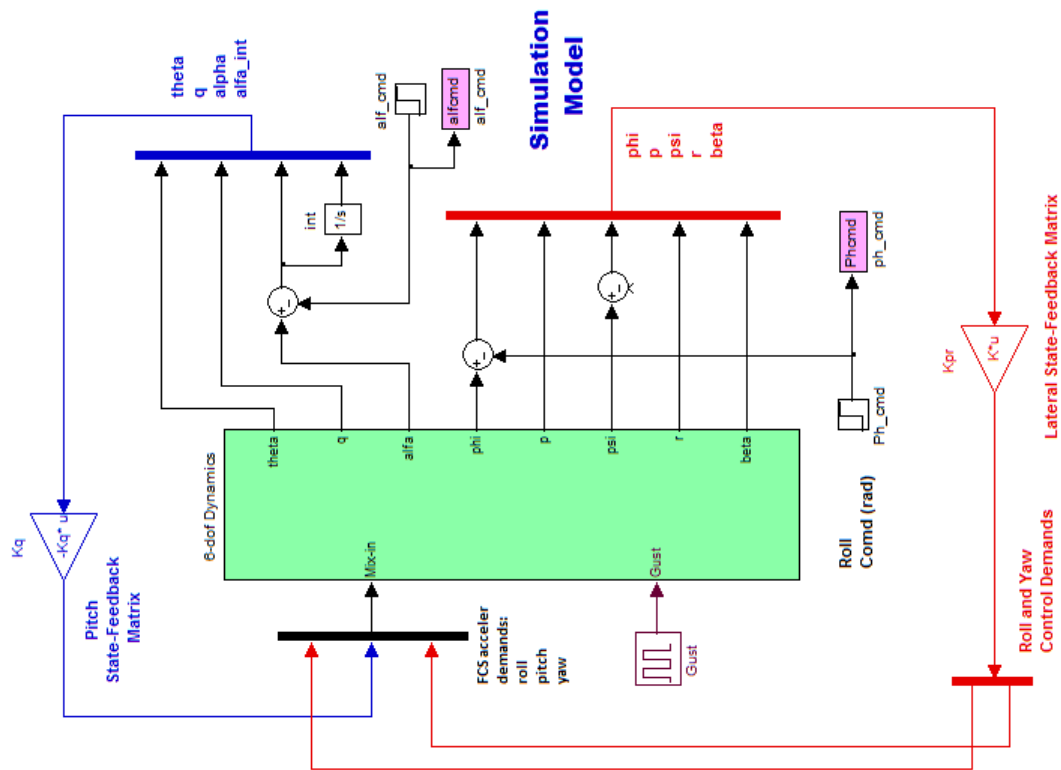


Figure 2.7.4a Descent Simulation Model "Sim\_6dof.mdl".

### Simulation with a Phi-Command

In this case the Phi-command is set to 1 (deg). The alpha-command and the gust inputs are set to zero. The plot below shows that the roll angle approaches 1° with a small steady-state error. This error can be eliminated with a phi-integrator, however, this is taken care by the closed guidance loop. The Phi-command causes a small steady-state beta. The maneuver is achieved by pivoting the left and right flaps differentially and firing the yaw thruster with a steady -100 (lb) thrust. The steady thrust is needed to counteract the positive moment due to the steady beta.

