# Hypersonic Rocket Plane

This example analyzes a rocket-plane that can either take off horizontally from the ground or it can be dropped from another aircraft. When it takes off from the ground it climbs up to an altitude of 76,000 feet. Then it turns off its engine and glides back to the ground and lands like an airplane using only aero-surfaces for control. The vehicle is powered by a 63,000 (lb) rocket engine during ascent and uses four aero-surfaces during both ascent and entry: an elevon, a body-flap, an aileron, and a rudder. In this example we will use the "Trim" program to calculate the aero-surface trim angles and evaluate the vehicle performance during the entire flight, from ground take off to landing. The analysis is separated into two parts, the boost phase and the descent to the ground phase.

## 1. Ascent Phase

During ascent the rocket plane uses a variable thrust booster engine to regulate its axial acceleration. The engine thrust is along the vehicle x axis and the engine does not gimbal. The aircraft is controlled by the control surfaces. We are going to use the "Trim" program to perform the following analysis: (a) calculate the trim angles of the control surfaces and the engine thrust as a function of time, (b) evaluate the vehicle performance during the entire boost phase, (c) investigate the effects of XCG variations on the trim angles and performance, (d) analyze stability by means of contour plots, (e) analyze maneuverability against disturbances by using vector diagrams, (f) take advantage of multiple effectors which are available in pitch to perform trimming modifications, and (g) analyze the lateral effects of a beta disturbance and of a YCG shift.

## 1.1 Flight Vehicle Data Files

The data for the hypersonic vehicle example during ascent is in folder "*C:\Flixan\Trim\Examples \Hypersonic Vehicle \Ascent*". This folder contains data files which are inputs to "Trim" and files which are generated by the "Trim" program. We will first describe the input data files. There are two ascent trajectory files in this directory, a high altitude release "*Drop_Boost.Traj*", and a ground take-off trajectory file "*Hyp_Ascent.Traj*". In this example we are going to focus on the ground take-off trajectory. The flight lasts for 89 seconds. The initial weight at ground take-off is 30,000 (lb) and its final weight at 76,000 (ft) altitude is 14,000 (lb). The final Mach number at engine cut-off is 4.5. The angle of attack in the beginning of flight is positive and it becomes negative at around Mach 1.

The basic aero-dynamic coefficients during ascent is in file "*Hyp_Ascent.Aero*". The data is a 3-dimentional array consisting of 12 Mach numbers ranging from Mach: 0.3 to Mach: 10, three angles of sideslip (-5°, 0°, +5°), and 16 angles of attack ranging between -6° and +30°. The aero-surface coefficients are in file "*Hyp-Ascent.Delt*". They represent increments contributed by the four aero-surface deflections in addition to the basic moments and forces. The aero-surfaces file consists of four sets of data for all four aero-surfaces. The aero-surface coefficients are 4-dimentional arrays. The first 3 array elements correspond to the same Mach, alpha and beta as in the basic aero coefficients array. The fourth element in the array corresponds to surface deflections which range between -20° to +20°. The surface increment angles are not the same in the aero-surfaces. The file "*Hyper.HMco*" contains the hinge moment coefficients data for all four aero-surfaces. It consists of a 4-dimentional array of coefficients (as a function of Mach, alpha, beta, and delta) for each surface and it looks very similar to the aero-surface coefficients file. It is used for sizing the control surface actuator torques. The file "*Hyp_Ascent.Unce*" contains the aerodynamic uncertainties data.

The mass properties are in file "*Hyper.Mass*". The first column contains the vehicle mass in (slugs). The remaining columns contain the corresponding moments and products of inertia, the CG location, and the vehicle length which is constant in this case. The main engine data is in file "*Hyper1.Engn*". It contains the nominal engine thrust, its location and orientation relative to vehicle coordinates, its maximum pitch and yaw deflections which are zero in this case because the engine is not gimbaling, and its maximum throttling capability which in this case is ±40% relative to its nominal thrust of 62,000 (lb). The file contains additional engine parameters (such as the engine mass, its moment of inertia about the gimbal, and its CG moment arm from the pivot point). These parameters are not used by "Trim" but they are transferred to the Flight Vehicle Modeling Program when the "*State-Space Modeling*" option is selected, and they are used for modeling the dynamic coupling between the engine nozzle and the vehicle (tail-wags-dog and actuator load-torque). This, obviously, does not apply in this case because the engine does not pivot. We have included, however, an alternative engine data file "*Hyper2.Engn*" which contains some engine data modifications as an optional design. When using this file, in addition to its

From the following menu you may select the type of aero data that you would like to plot. In the following case we select the basic pitch aero coefficients, and click "OK". In the plots below we see the pitch moment Cm, the normal force $C_Z$, and the aft force $C_A$ aero coefficients, as a function of (Mach and $\alpha$). Five separate Mach curves are shown for comparison.
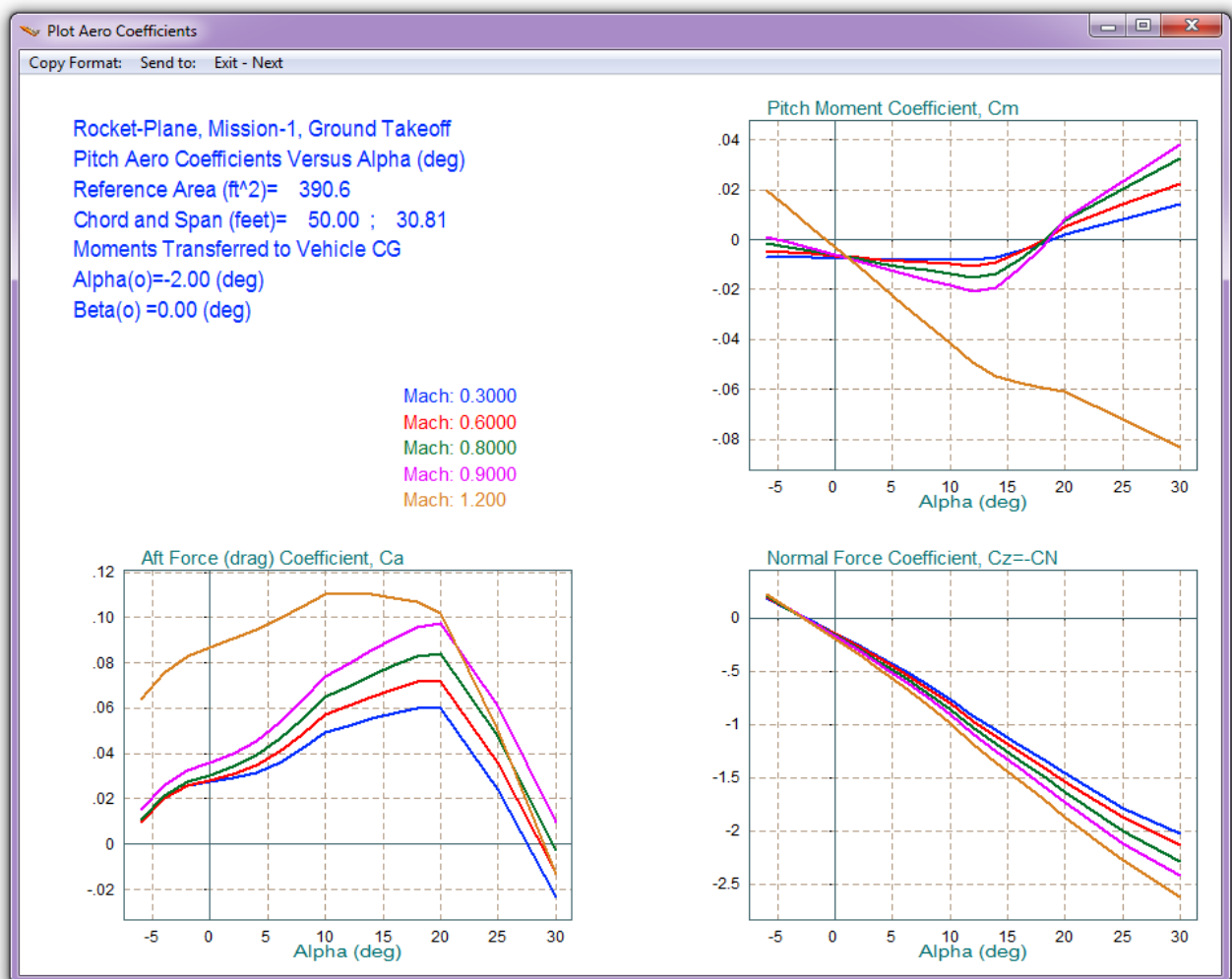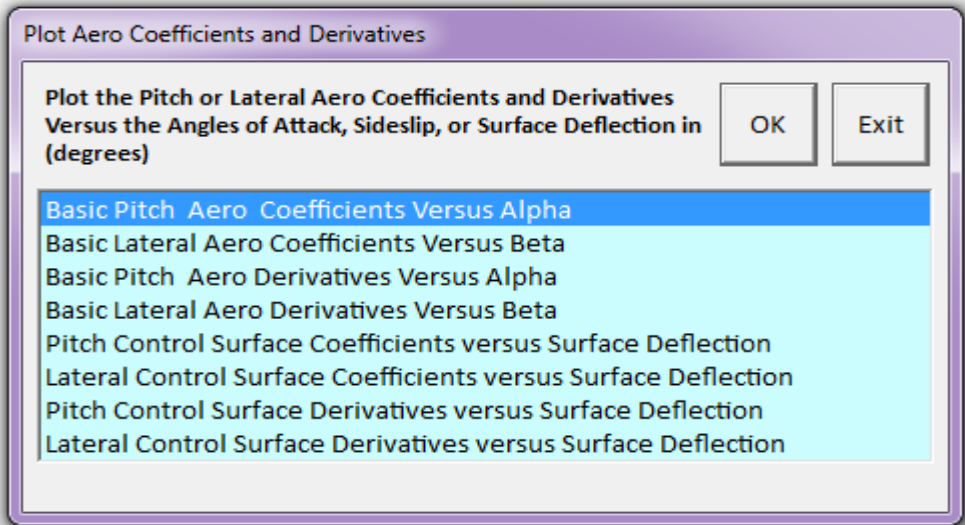
Next, select the pitch aero derivatives, and click "OK". The plot below shows the Cmα, CAα, and CZα derivatives as a function of (Mach and α). From the top menu bar (above the plot) click on "Exit/Next" to return to the aero coefficients plotting menu. From there you may either plot another set of aero data or you may return to the Main Trim Options menu. You may also try other options, such as, plotting the longitudinal and lateral aero-surface derivatives.
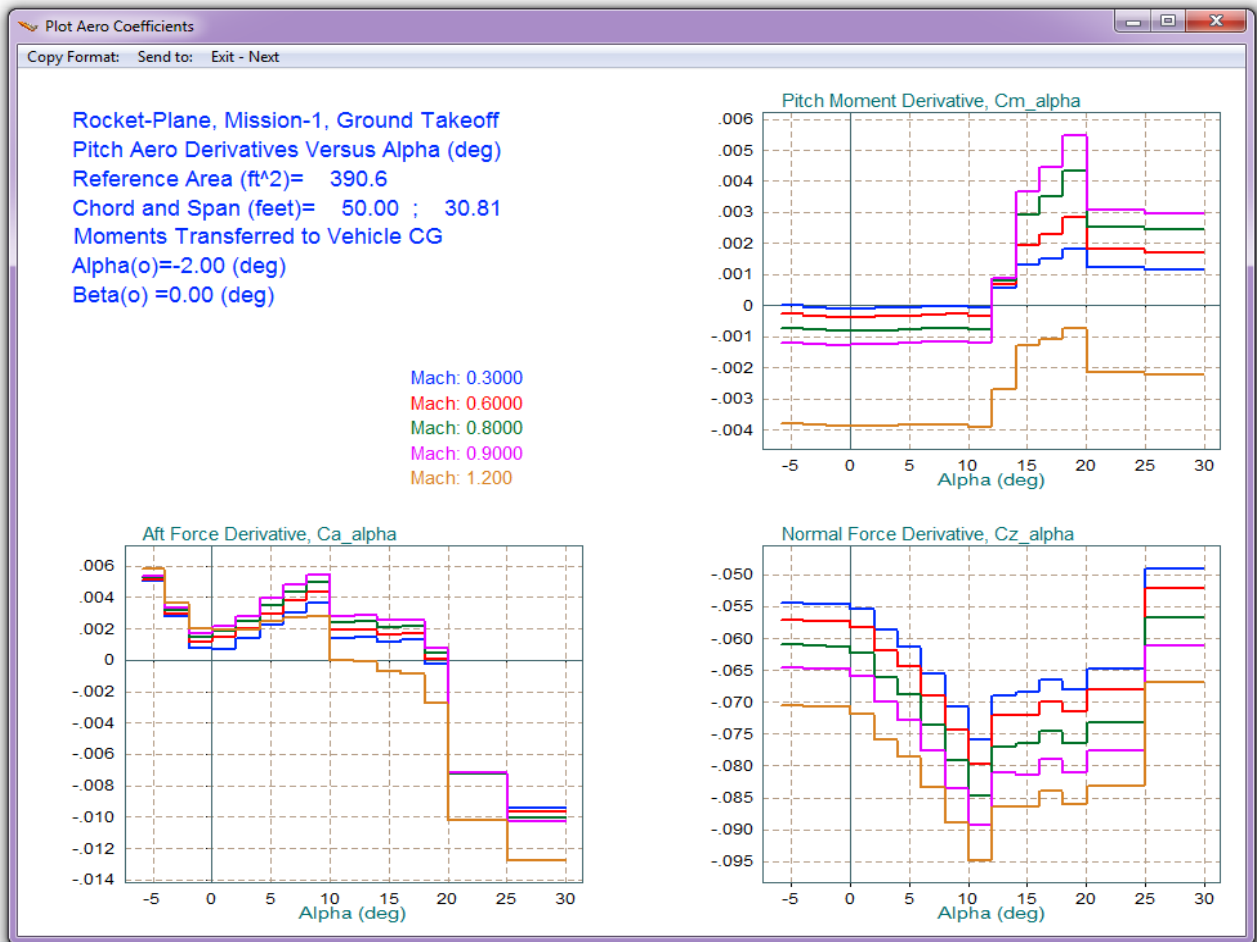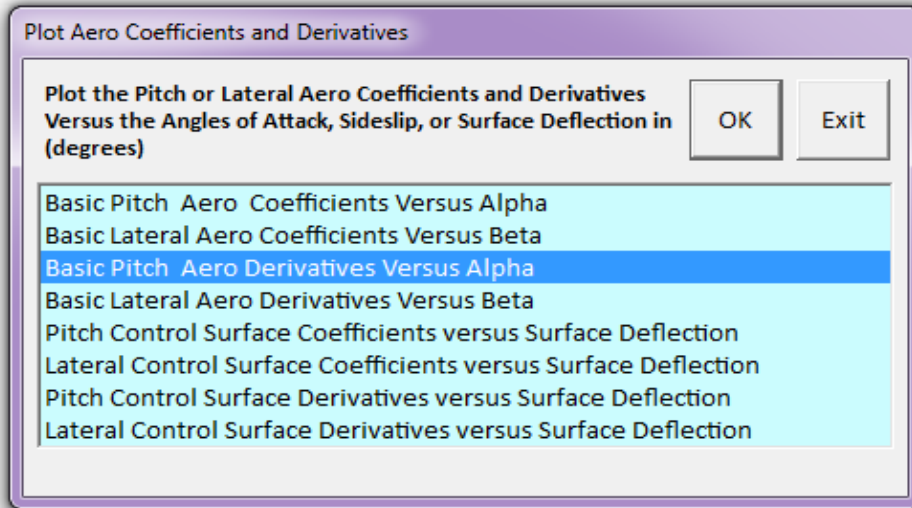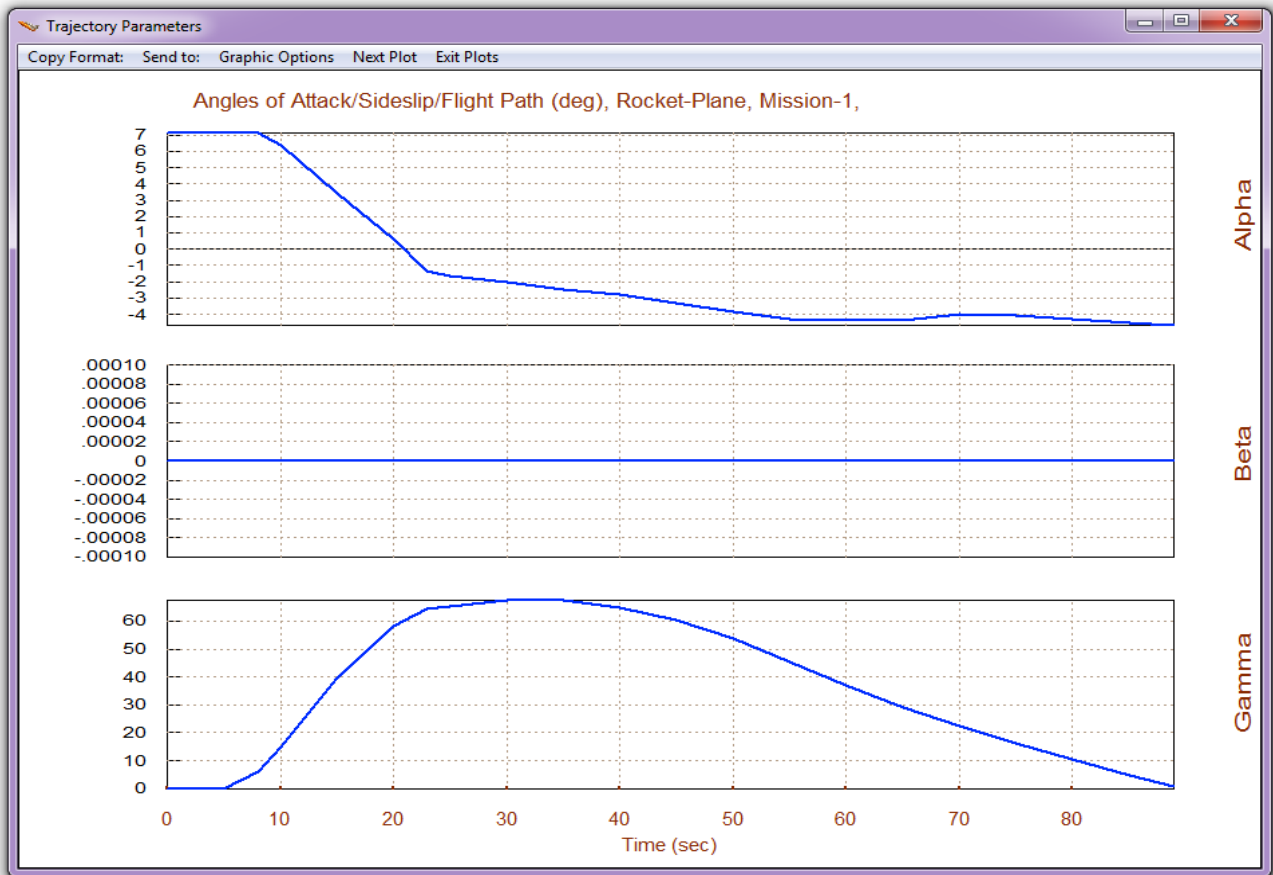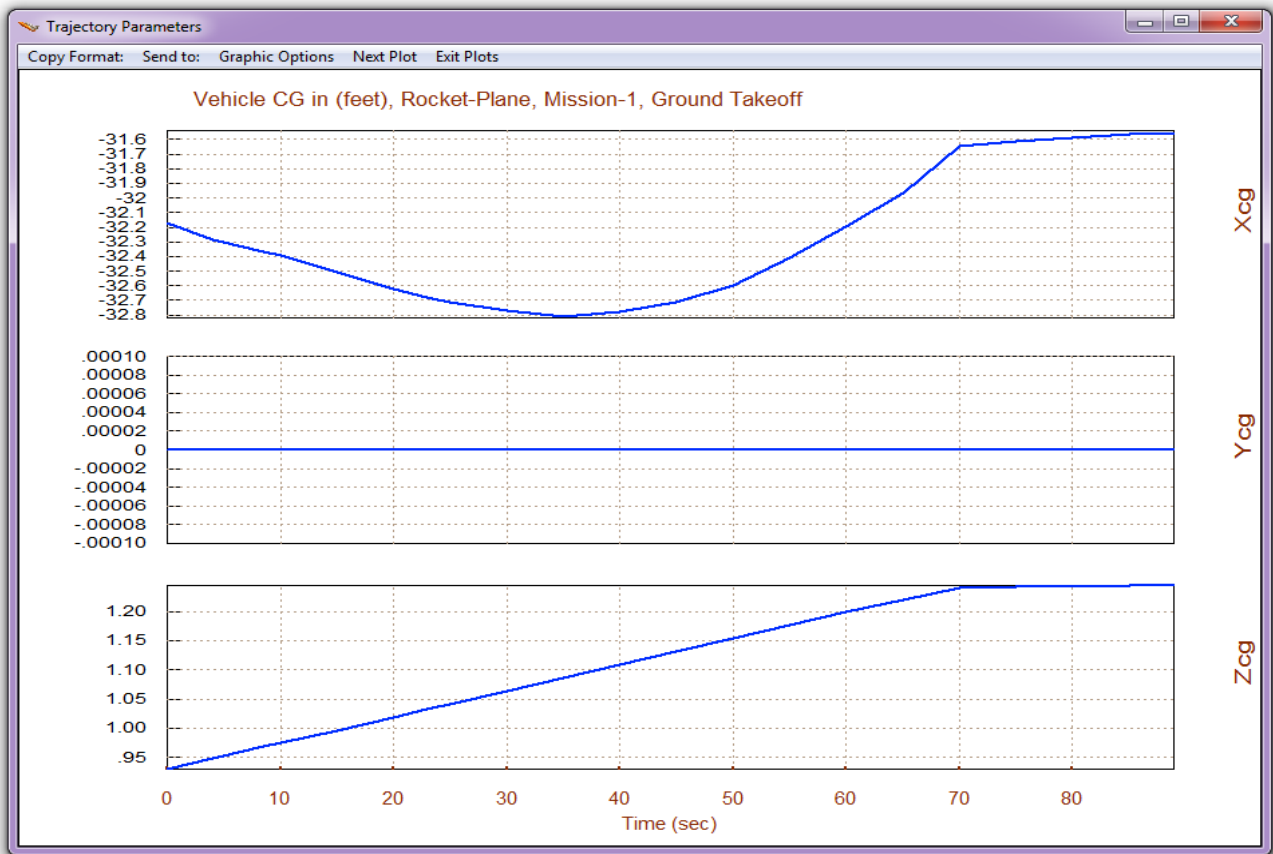
## 1.2 Checking the Trajectory Data

The next important thing to check before beginning the analysis is the trajectory data. Return to the Trim main menu, select the second option which is "*Plot the Trajectory Parameters Versus Time*" and click "OK".
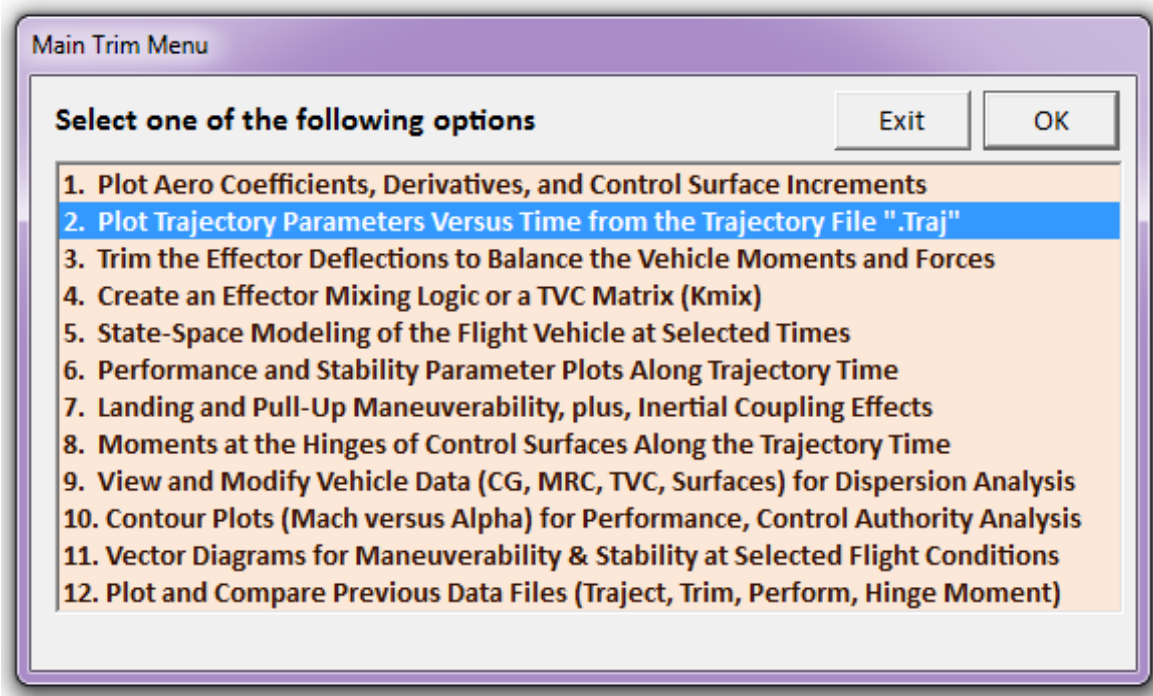


The following plots show the trajectory parameters, which are read by the program from file "*Hyp_Ascent.Traj*", as a function of time. The first plot shows the CG travel during the flight. The CG location is calculated from the vehicle mass versus time and also by the mass properties file "*Hyper.Mass*" that contains the CG coordinates as a function of mass. The next plot shows $\alpha$ which is initially positive and it changes sign after Mach 1. The sideslip $\beta$ is zero throughout the flight because there is lateral symmetry. The flight path angle $\gamma$ is positive during the climb and it decays to zero towards the end of the ascent phase. The dynamic pressure reaches a Max-Q of 970 (psf) at 26 seconds, just before Mach 1. There is an 8° bank angle between 10 and 22 sec for turning its heading direction. The engine thrust varies between 44,000 and 64,000 (lb). The normal and axial accelerations are also shown in (ft/sec$^2$). The axial x-acceleration reaches almost 3 g's. The normal z-acceleration towards the end of the flight is positive (down) as the vehicle is flying with a negative ($\alpha$).

On the top of each plotting window there is a horizontal menu bar that includes several options. If you click on "*Next Plot*" it will move to the next plot containing trajectory data. If you click on "*Exit Plots*" it will go back to the Trim main menu shown above.

## Vehicle CG in (feet), Rocket-Plane, Mission-1, Ground Takeoff

## Angles of Attack/Sideslip/Flight Path (deg), Rocket-Plane, Mission-1,

**Modify Trajectory Data**

Trajectory modifications are used for evaluating the vehicle performance under dispersed conditions. Some variables can be modified graphically using the mouse. This does not destroy the original trajectory which can be restored later.
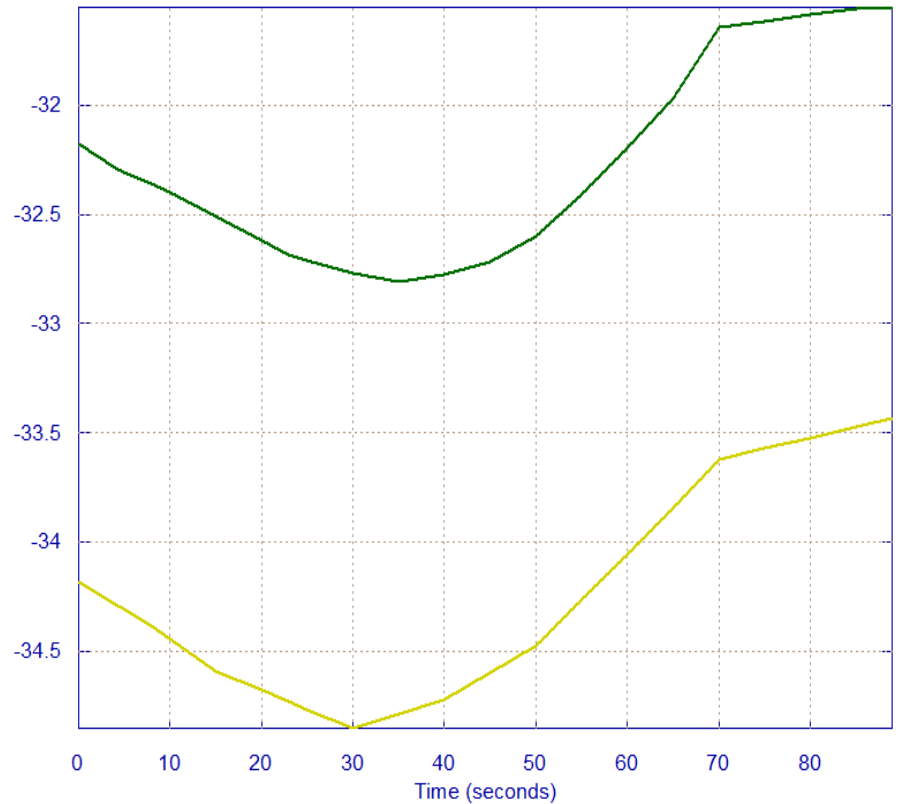
Angle of Attack, alpha, (deg)
Angle of Sideslip, beta, (deg)
Dynamic Pressure, Q_bar, (psf)
Mach Number, (Mach),  (---)
Roll Rate, (P), (deg/sec)
Pitch Rate, (Q),  (deg/sec)
Yaw Rate, (R),  (deg/sec)
Accelerat. Along X, (Ax)  (ft/sec^2)
Accelerat. Along Y, (Ay) (ft/sec^2)
Accelerat. Along Z, (Az)  (ft/sec^2)
CG Location Along-X (feet)
CG Location Along-Y (feet)
CG Location Along-Z (feet)
Total Engine Thrust, (Te), (lb)
Disturbance Force along X, Fd-x (lb)
Disturbance Force along Y, Fd-y (lb)
Disturbance Force along Z, Fd-z (lb)
Disturbance Moment about X, Ld-x (ft-lb)
Disturbance Moment about Y, Md-y (ft-lb)
Disturbance Moment about Z, Nd-z (ft-lb)

Select a Variable to Modify

Exit Menu

**Modify the Trajectory Variables**

## CG Location Along X, (Xcg),  (ft)

This curve shows the time history of the selected trajectory variable. This profile can be modified graphically by repeated adjustments using the mouse.

Specify a range between two points on the curve to be reshaped. Place the cursor at a point on the curve and click the mouse to select point (A), and then select point (B) shown by red dots. The mid-point (purple dot) is found and highlighted.

The mid-point may be shifted vertically to a new location. Click again to define a new shape between A and B. You may repeat several times as needed. When you have finished reshaping the curve you may "Continue With Another Variable to Modify".

Save the Modified Trajectory

Continue with Another Variable to Modify

**Main Trim Menu**

**Select one of the following options**          Exit     OK

1. Plot Aero Coefficients, Derivatives, and Control Surface Increments
2. Plot Trajectory Parameters Versus Time from the Trajectory File ".Traj"
3. Trim the Effector Deflections to Balance the Vehicle Moments and Forces
4. Create an Effector Mixing Logic or a TVC Matrix (Kmix)
5. State-Space Modeling of the Flight Vehicle at Selected Times
6. Performance and Stability Parameter Plots Along Trajectory Time
7. Landing and Pull-Up Maneuverability, plus, Inertial Coupling Effects
8. Moments at the Hinges of Control Surfaces Along the Trajectory Time
9. View and Modify Vehicle Data (CG, MRC, TVC, Surfaces) for Dispersion Analysis
10. Contour Plots (Mach versus Alpha) for Performance, Control Authority Analysis
11. Vector Diagrams for Maneuverability & Stability at Selected Flight Conditions
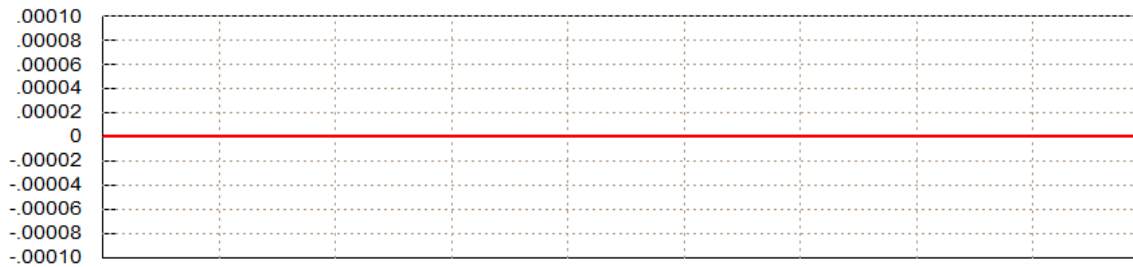12. Plot and Compare Previous Data Files (Traject, Trim, Perform, Hinge Moment)

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**Trajectory Parameters**

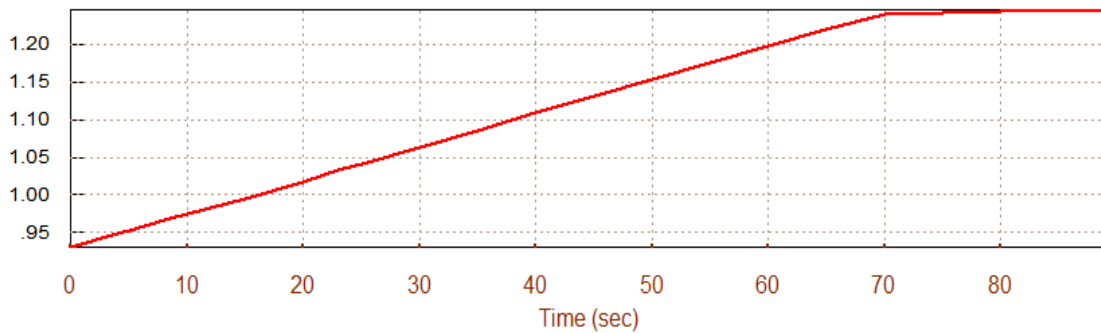Copy Format:   Send to:   Graphic Options   Next Plot   Exit Plots



Vehicle CG in (feet), User Modified Trajectory ...

Time (sec)
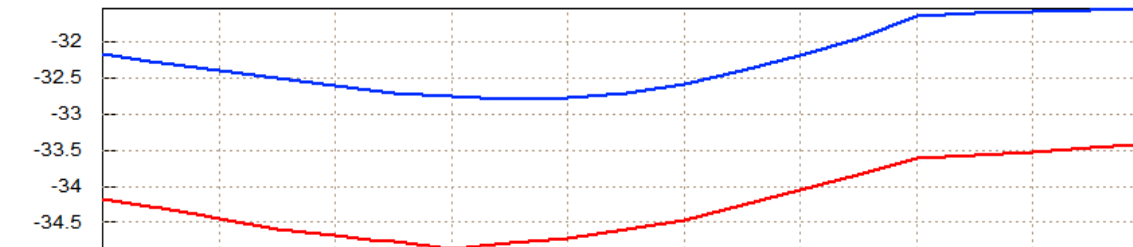
## 1.4 Trimming the Effectors and Comparing the Trim Results

Let us now trim the vehicle effectors along the trajectories. This time we do not constrain the Trim algorithm but allow the program to take into consideration the effectors maximum control capability without any feedback from the user. Further trimming adjustments are possible when the vehicle has multiple effectors in the same directions and in order to trade-off the activity of some effectors against others. In this example we have 5 effectors: a throttling main engine which is defined in file "*Hyper1.Engn*", and the four aero-surfaces which are defined in file "*Hyp-Ascent.Delt*". Later on we will try to introduce more controls by using a gimbaling engine and thruster jets and examine how this may improve the overall performance.

In section 1.3 we recently created a modified trajectory with an aft shifted $X_{CG}$. Let us continue using this modified trajectory to trim, and then we will restore the original CG trajectory, re-trim the effectors and compare trim results between the two cases: nominal and aft $X_{CG}$ trajectories. Return to the Trim menu and without restoring the original trajectory select option (3) to "*Trim the Effector Deflections*" and click "OK".
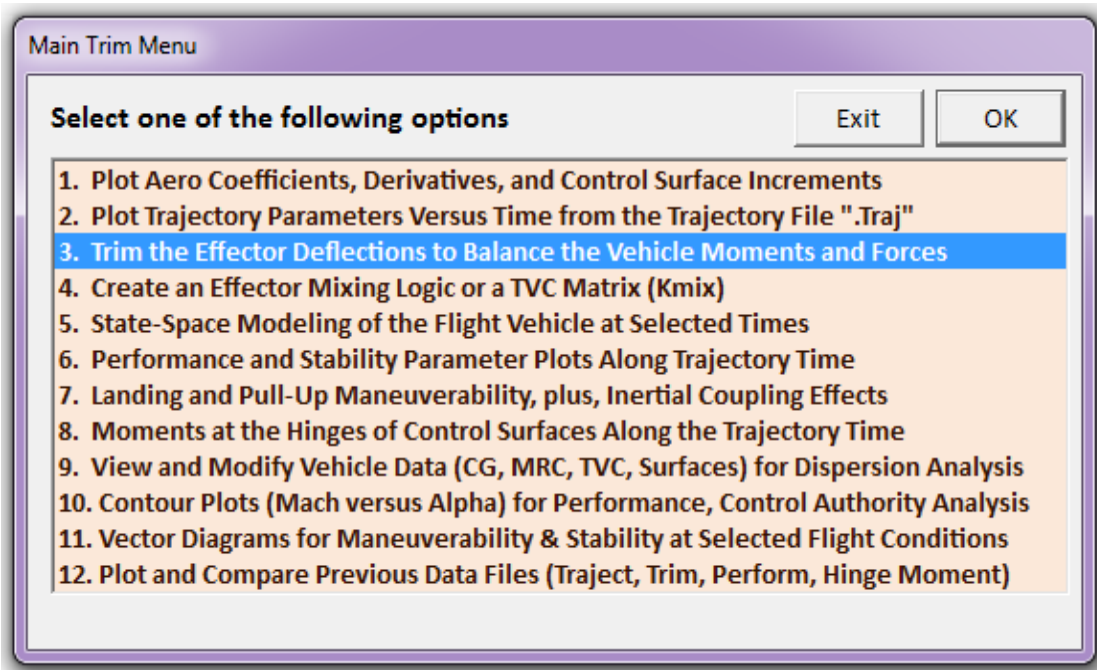


In the next menu the program wants to know how to initialize the trimming process. You may select a previous trim file to initialize the algorithm by selecting the file and clicking on "*Select File*" or you may start from zero deflection angles and throttle control by clicking on "*Do Not Select*" a file. Since we have not yet created a trim file for this configuration we must not select an initialization file. From the next menu the user may chose the directions along which to trim the vehicle. Since we have a throttlable engine, in addition to the 3 rotations, we would also like to trim in the x-direction. With this setup the flight control system will eventually be able to control 3 rotations plus velocity by varying the engine thrust (although this does not impose a velocity control requirement).

## 1.5 Effects of β disturbance and $Y_{CG}$ Variation on the Aileron and Rudder

This time we will analyze dispersion effects in the lateral axes. We will introduce a beta disturbance and a $Y_{CG}$ offset, re-trim, and evaluate their effects on the aileron and the rudder which control and trim the lateral directions. The disturbance effects will occur at d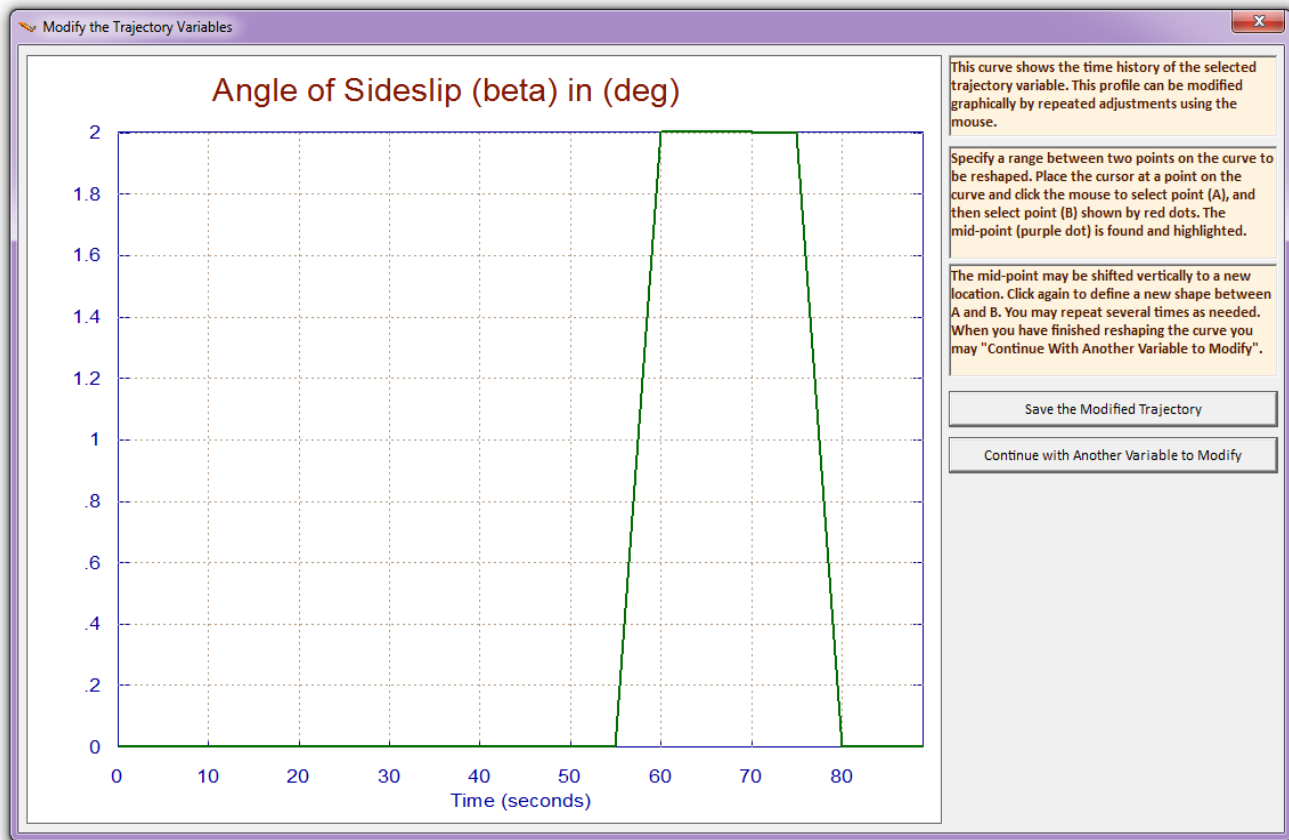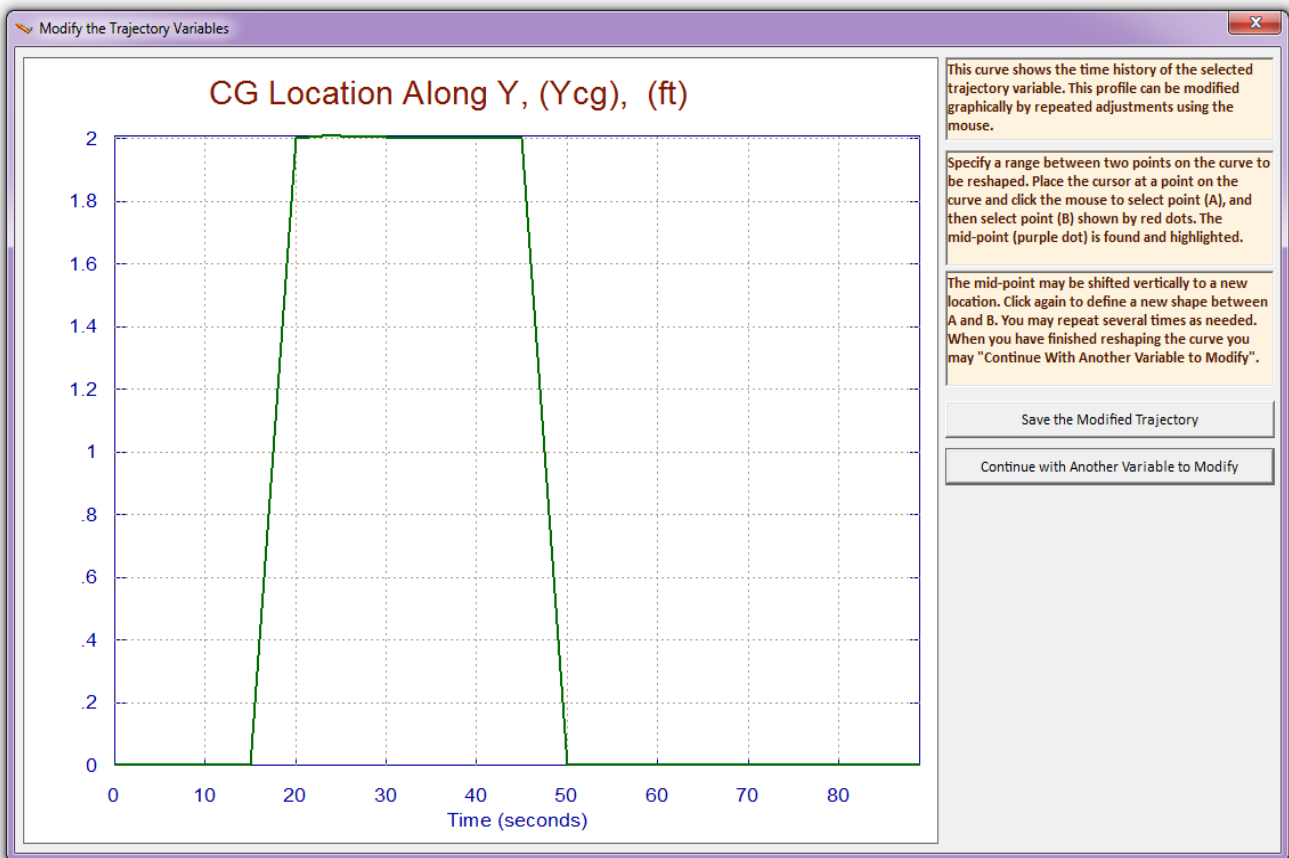ifferent flight times. We will first shift the $Y_{CG}$ location, which is originally at the center-line, from zero to 2 (ft) towards the right wing, for a period of 30 seconds, and then bring it back to zero. Then at a later time period we will apply a steady 2° β-disturbance for a period of 20 sec. Let us begin again by plotting the nominal trajectory, as in Section 1.2. From one of the trajectory plots, go to the horizontal top menu bar and click on "*Graphic Options*". Then from the vertical pop-up menu select the option "*Modify a Trajectory Plot Using the Mouse*".
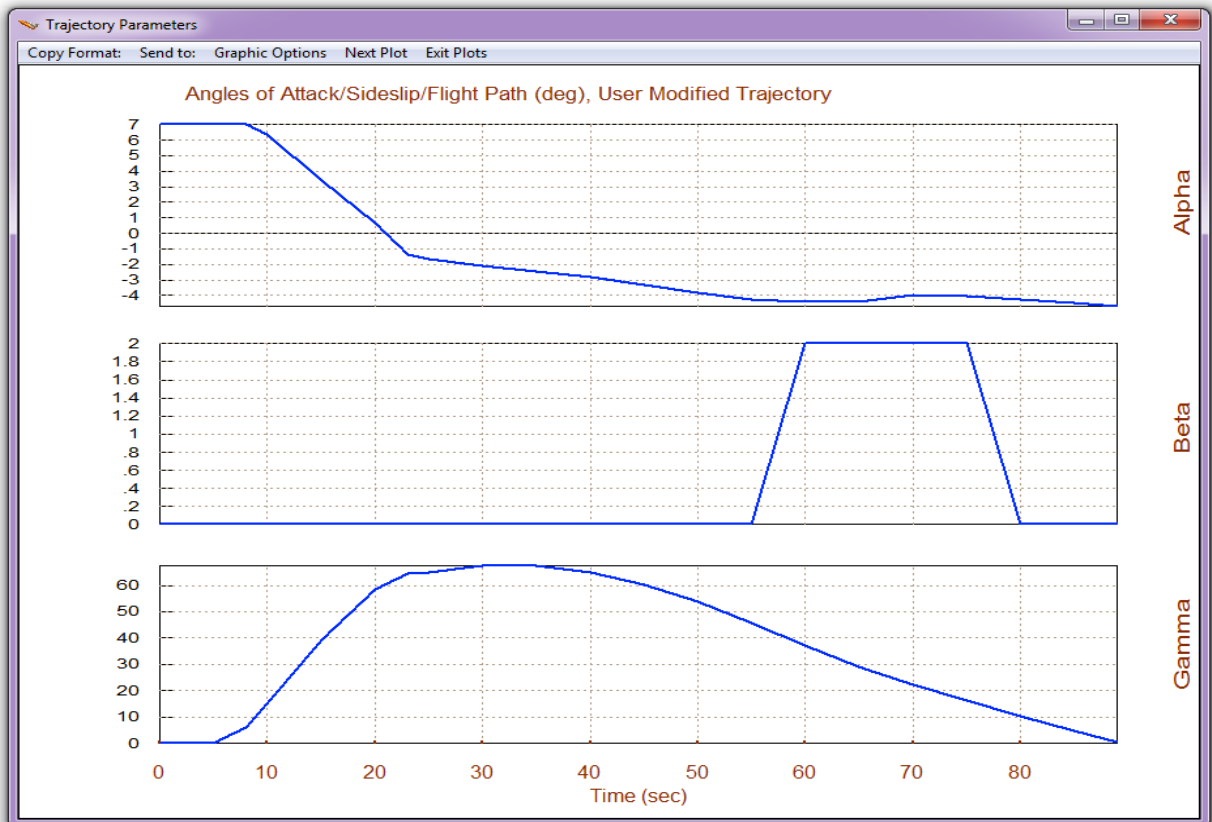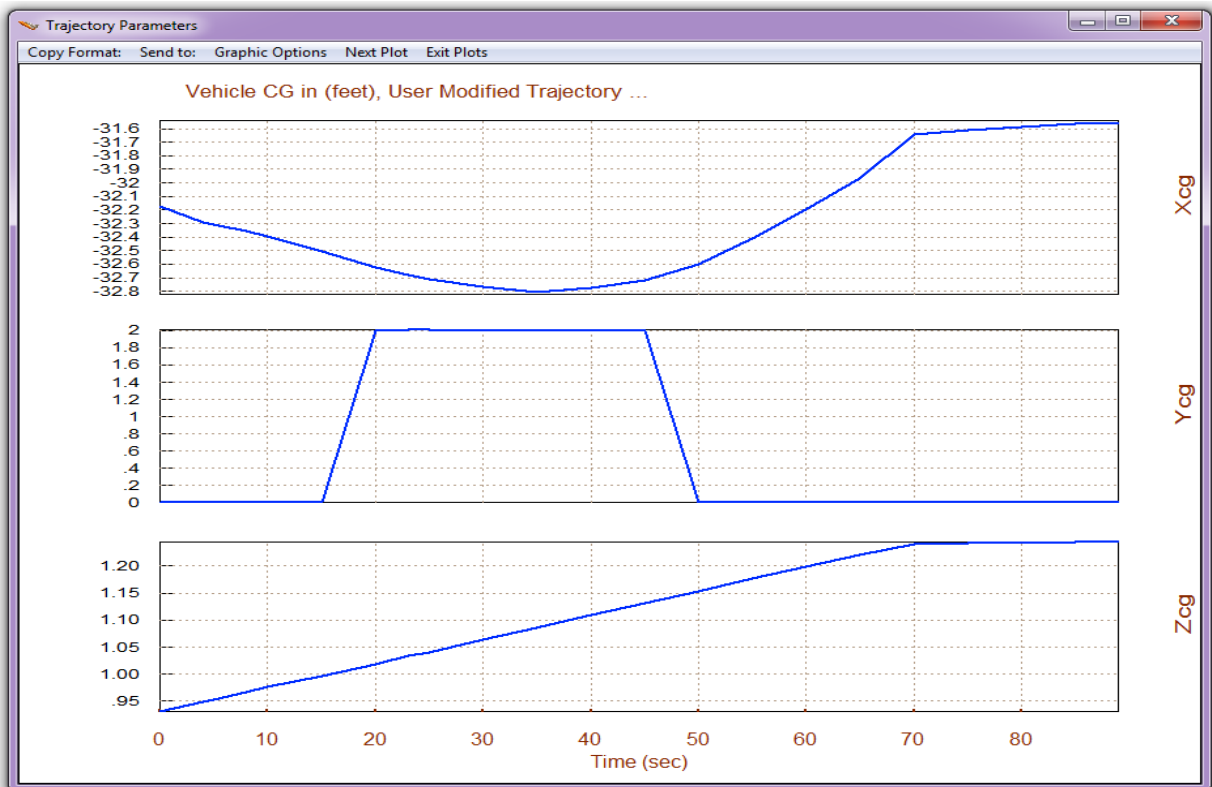
The following menu comes up and it is used for selecting trajectory variables to be modified by the analyst. First choose the CG location along the y-axis and click on "*Select Variable to Modify*" button. In the window-dialog that follows the $Y_{CG}$ can be modified interactively using the mouse, one small section at a time. It is finally shifted from zero to +2 (ft) for the time period between 20 and 50 (sec), as shown. When the modification is complete, press on "*Continue with Another Variable to Modify*" to return to the same menu.

Now select the angle of sideslip (β) to modify which originally is also zero. With a few mouse clicks this variable can be modified to the rectangular shape, as shown in the following dialog/ plot, with a 2° peak in the time period between 60 and 80 sec. Finally click on "*Save the Modified Trajectory*" to save it and it will remain active in memory until you remove it or restart the program. This user modified trajectory is also saved in file "*Hyp_Ascent1.Traj*" for further analysis. When you click on "*Exit Menu*" the program returns to the trajectory plots.

Modify Trajectory Data

Trajectory modifications are used for evaluating the vehicle performance under dispersed conditions. Some variables can be modified graphically using the mouse. This does not destroy the original trajectory which can be restored later.

Angle of Attack, alpha, (deg)
Angle of Sideslip, beta, (deg)
Dynamic Pressure, Q_bar, (psf)
Mach Number, (Mach), (---)
Roll Rate, (P), (deg/sec)
Pitch Rate, (Q), (deg/sec)
Yaw Rate, (R), (deg/sec)
Accelerat. Along X, (Ax) (ft/sec^2)
Accelerat. Along Y, (Ay) (ft/sec^2)
Accelerat. Along Z, (Az) (ft/sec^2)
CG Location Along-X (feet)
CG Location Along-Y (feet)
CG Location Along-Z (feet)
Total Engine Thrust, (Te), (lb)
Disturbance Force along X, Fd-x (lb)
Disturbance Force along Y, Fd-y (lb)
Disturbance Force along Z, Fd-z (lb)
Disturbance Moment about X, Ld-x (ft-lb)
Disturbance Moment about Y, Md-y (ft-lb)
Disturbance Moment about Z, Nd-z (ft-lb)

Exit Menu          Select a Variable to Modify

## Modify the Trajectory Variables

### CG Location Along Y, (Ycg), (ft)

Time (seconds)

This curve shows the time history of the selected trajectory variable. This profile can be modified graphically by repeated adjustments using the mouse.

Specify a range between two points on the curve to be reshaped. Place the cursor at a point on the curve and click the mouse to select point (A), and then select point (B) shown by red dots. The mid-point (purple dot) is found and highlighted.

The mid-point may be shifted vertically to a new location. Click again to define a new shape between A and B. You may repeat several times as needed. When you have finished reshaping the curve you may "Continue With Another Variable to Modify".

**Save the Modified Trajectory**

**Continue with Another Variable to Modify**

## Modify the Trajectory Variables

### Angle of Sideslip (beta) in (deg)

Time (seconds)

This curve shows the time history of the selected trajectory variable. This profile can be modified graphically by repeated adjustments using the mouse.

Specify a range between two points on the curve to be reshaped. Place the cursor at a point on the curve and click the mouse to select point (A), and then select point (B) shown by red dots. The mid-point (purple dot) is found and highlighted.

The mid-point may be shifted vertically to a new location. Click again to define a new shape between A and B. You may repeat several times as needed. When you have finished reshaping the curve you may "Continue With Another Variable to Modify".

**Save the Modified Trajectory**

**Continue with Another Variable to Modify**

The following plots show that the trajectory modifications were saved as a user modified trajectory which is now active in Trim. It will remain active until it is replaced with the original.



Vehicle CG in (feet), User Modified Trajectory ...



Angles of Attack/Sideslip/Flight Path (deg), User Modified Trajectory

The next step is to return to the Trim menu and repeat the trimming process using the latest modified trajectory. Select again option (3) for "*Trimming the Effector Deflections*". Select to trim 3 rotations plus x-acceleration, as before. The first plot shows the residual moments in roll, pitch, and yaw, which ideally they should be zero when trimming is performed perfectly. They are pretty close to zero except for the first 3 seconds in the pitch axis where the elevon hits its -20 (deg) limit at take-off. It requires a lot of pitching moment at take-off as it is maintaining a high ($\alpha$). The plot below shows the induced sideslip beta, initially due to the +YCG offset, and later due to the beta disturbance.
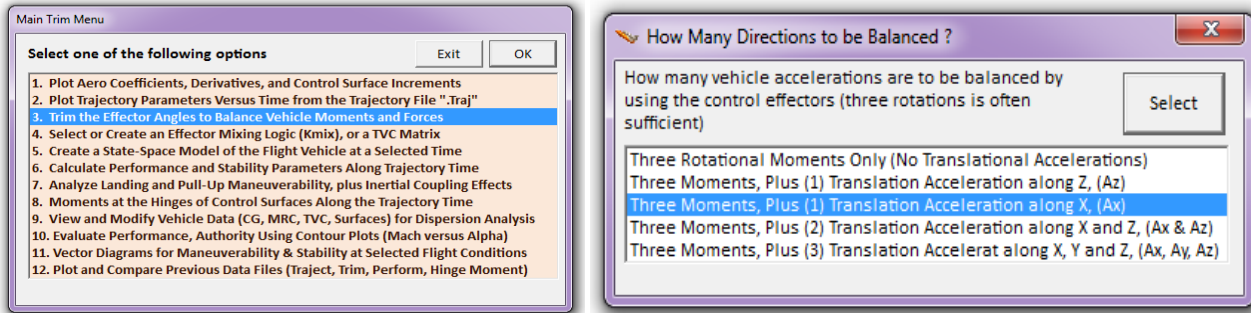


Alpha, Beta, and Bank Angle (phi), Assuming Effectors are Trimmed (deg)

Surface & Engine Deflections/ Thrusts, User Modified Trajectory ...
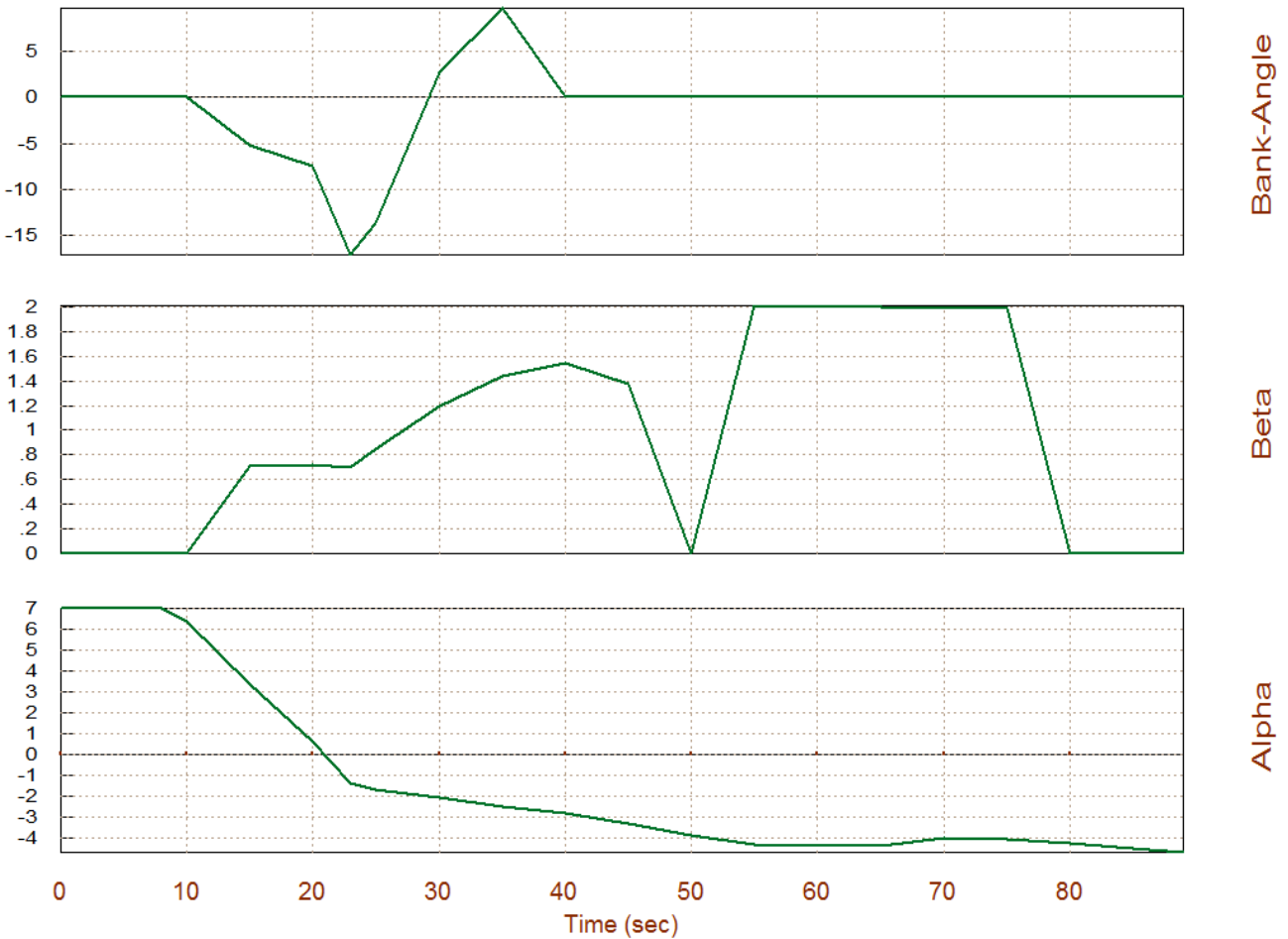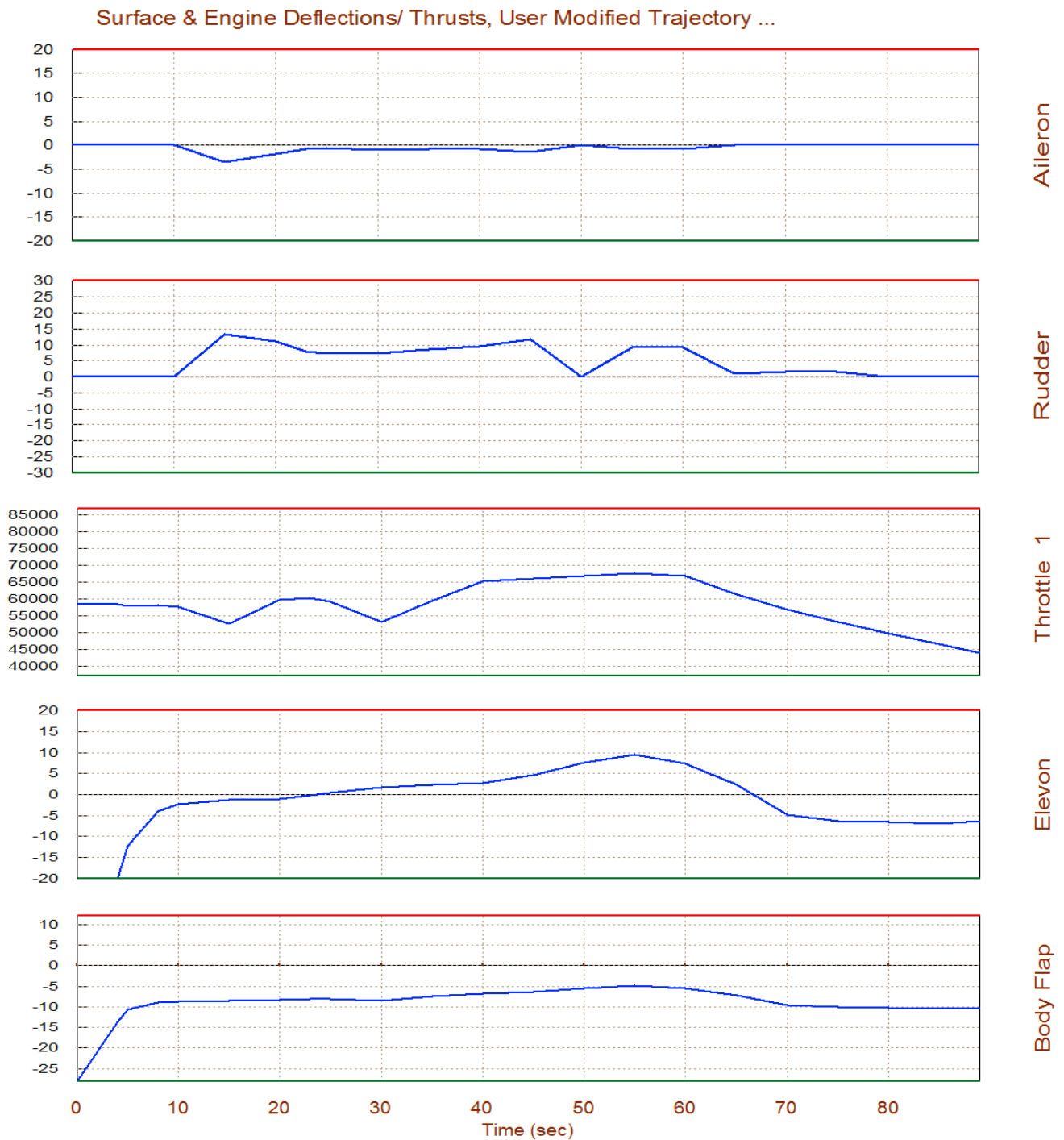
The plot above shows the aileron and rudder deflections required to trim the vehicle with the lateral trajectory dispersions described. The +YCG offset (in the time period between 10 and 50 sec) couples with the main engine thrust and induces a positive yawing moment on the vehicle. The rudder deflection is positive because it is generating a negative yaw torque trying to balance the positive yaw due to thrusting. The rudder also creates a positive roll torque and so the aileron deflection is negative trying to take it out. In the time period between 60 and 80 (sec) where the 2° of beta excitation is applied (the $Y_{CG}$ is back in the middle now), this excitation causes a positive yawing moment (because the vehicle is stable in yaw). The rudder deflection is positive again

creating a negative moment against it. It also creates a positive roll and the aileron deflection is negative trying to take it out. The Elevon and Body-Flap trim deflections are the same as in the nominal trajectory. We may now click on "Exit Plots" to return to the main Trim menu.

## 1.6 Analyzing Performance Parameters Along the Trajectory

We will now use option (6) to plot the performance and stability parameters along the trajectory time. These parameters were discussed in Section 3. However, in order to calculate the vehicle static performance the program needs to know how the effectors combine together to control the four directions specified. The mixing logic matrix defines the effector control allocation along the four trimming directions. Roll, pitch, yaw, and axial directions will independently be controlled. Although at this point we are not designing a control system an effector combination matrix is required to define the four control loops and to decouple the accelerations between the four axes. This matrix is also needed for the calculation of control authority. It was not required for trimming because in trimming we are only balancing moments and are not distributing control. The analyst may use a constant pre-calculated mixing matrix and the program will select it from a systems file (.Qdr) and use it. In this case we do not have a mixing matrix picked and will let the program calculate it along the trajectory. The matrix must have 4 inputs for the 4 control directions (3 rotations plus x-acceleration), and 5 outputs (4 control surfaces plus the engine throttle control).
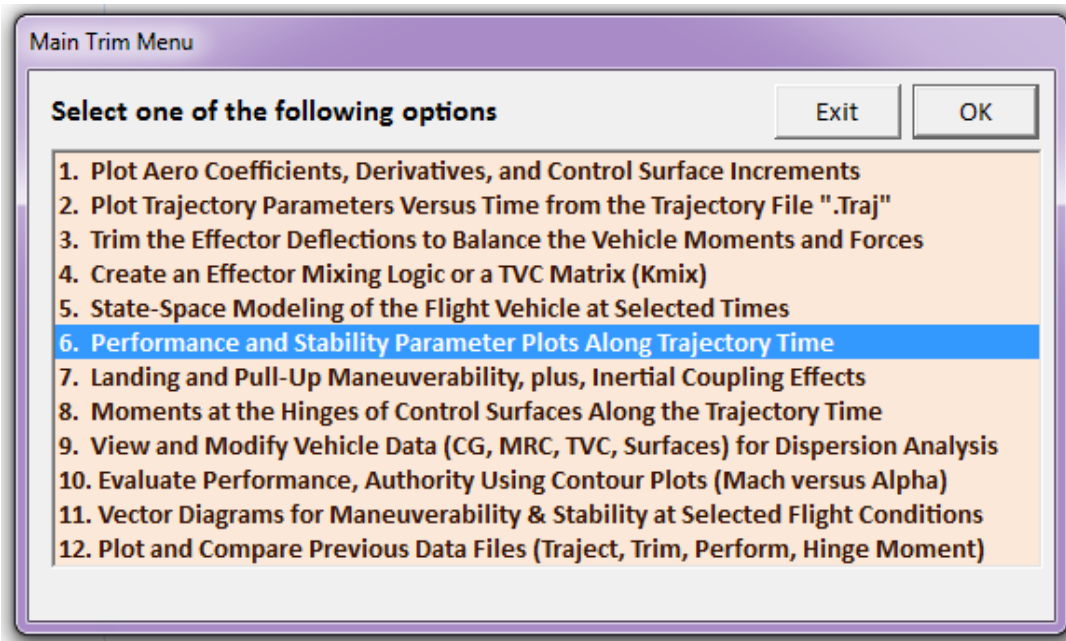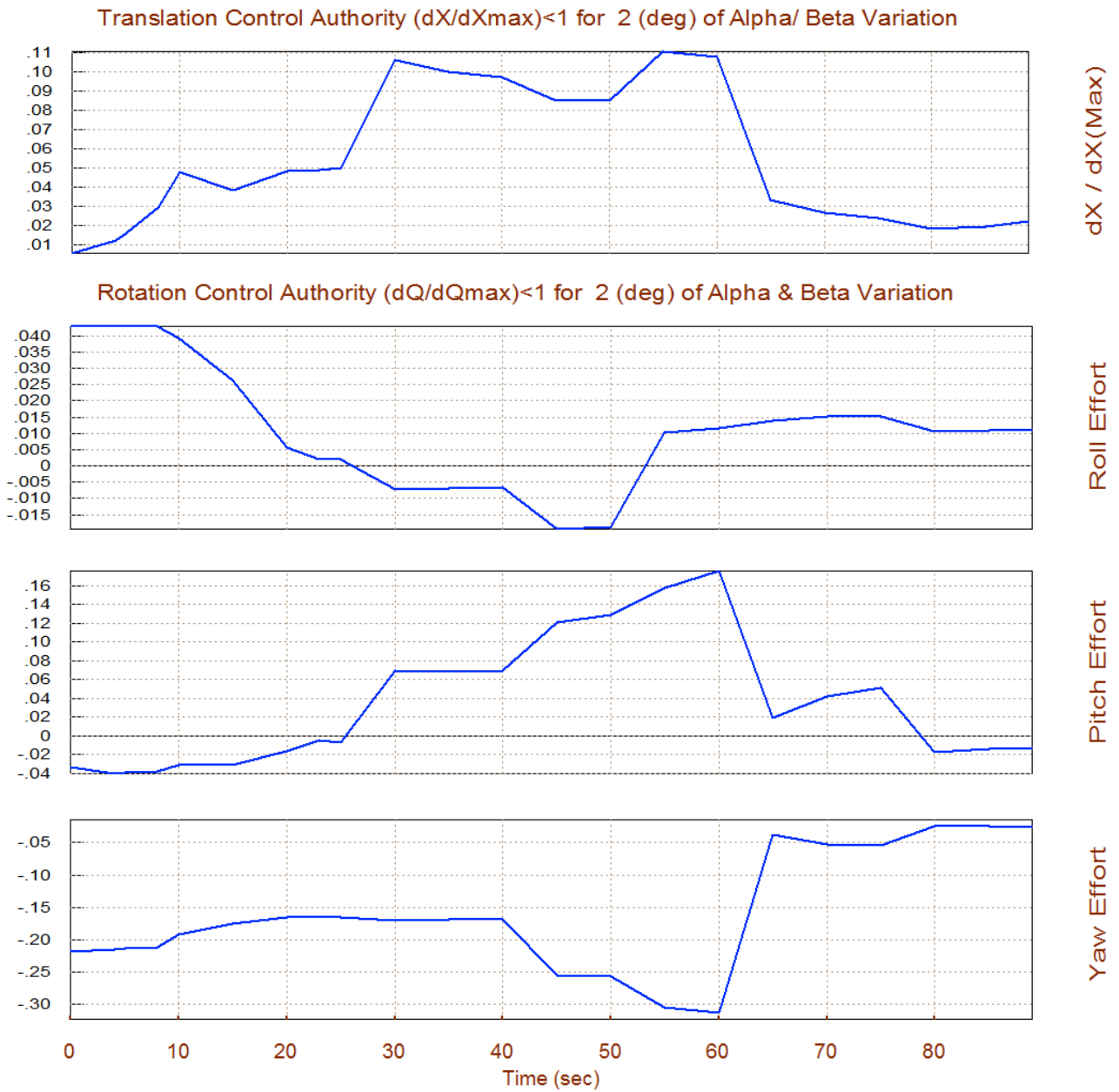


From the effector combination menu below, select the second option to let the program create a mixing matrix using 100% effector participation. Also, as we described in Section (3) we must also specify a worst case wind-shear environment which at steady-state is defined by the maximum dispersion angles ($\alpha_{max}$ and $\beta_{max}$) from trim. Enter 2° for both angles and the program will plot the performance parameters, as shown below.

## Short-Period (w)/ Time-to-Double-Ampl-Inverse (/sec), Q_alpha_beta (deg-lb/ft^2)
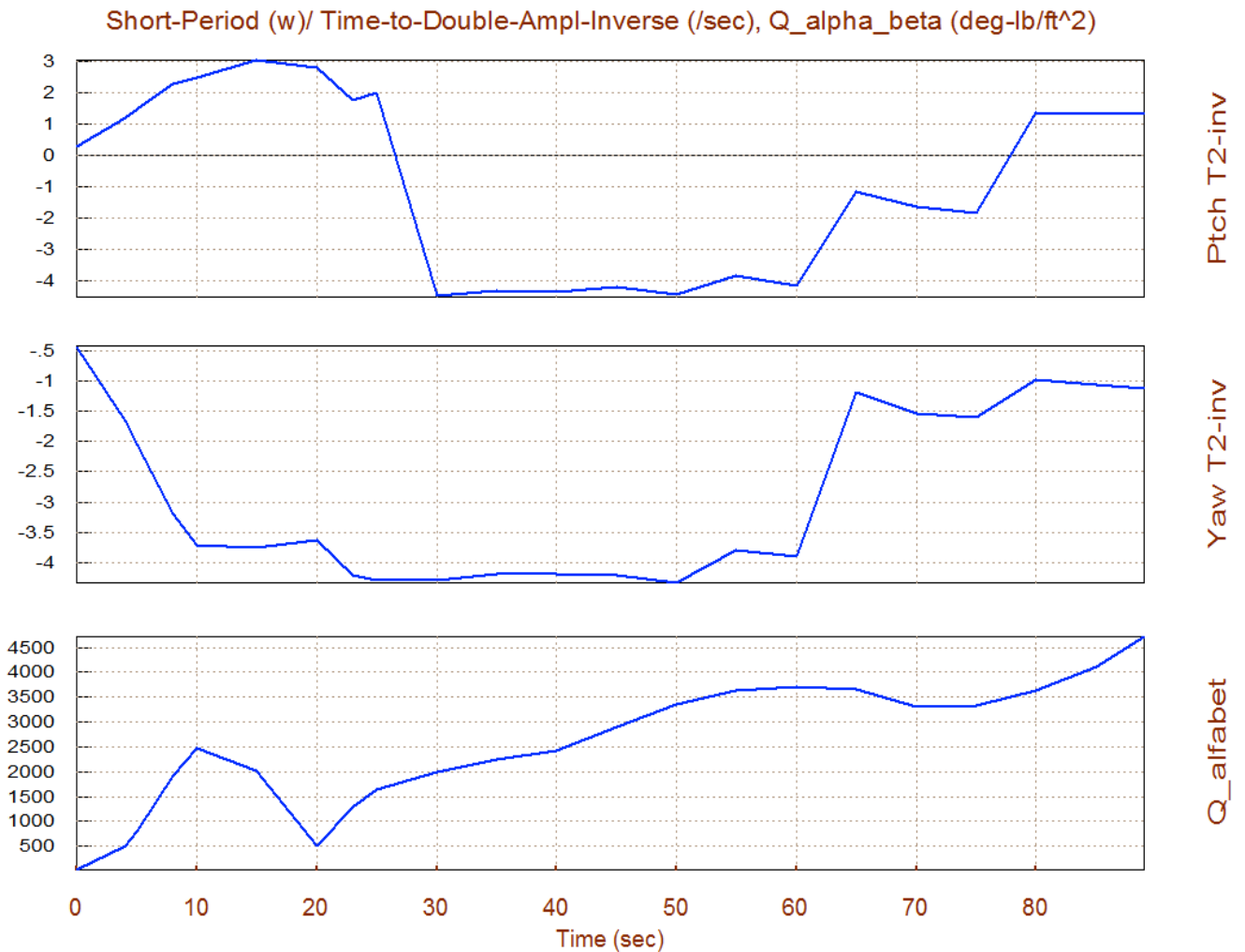


The pitch instability is also confirmed by the T2-inverse parameter which shows that at 15 seconds the time-to-double amplitude is short, T2=0.33 sec. It implies that the vehicle actuators must be fast enough in order to be able to catch-up with this fast rate of divergence. During the stable region the short period resonance reaches $\omega_p$=4.3 (rad/sec) at T=30 sec. Towards the end of boost the pitch axis becomes unstable again with a T2=0.59 sec. In the lateral direction the vehicle is stable during the entire boost phase with the "Dutch-Roll" resonance reaching to $\omega_d$=4.5 (rad/sec) at 24 sec, and remaining at about the same level for a period of 40 seconds.

The combined (Q$\alpha$ & Q$\beta$) normal and lateral load indicator parameter is shown in the bottom plot. It is indicative of normal and lateral loading when the vehicle is flying at a constant $\alpha_{max}$ =2° and $\beta_{max}$ =2° dispersions. Its value is pretty high 4500 (psf-deg) towards the end of boost where the dynamic pressure is very high. This of course is conservative because the vehicle is not intended to fly at such a high angle of attack and sideslip during high Qbar.

Translation Control Authority (dX/dXmax)<1 for 2 (deg) of Alpha/ Beta Variation

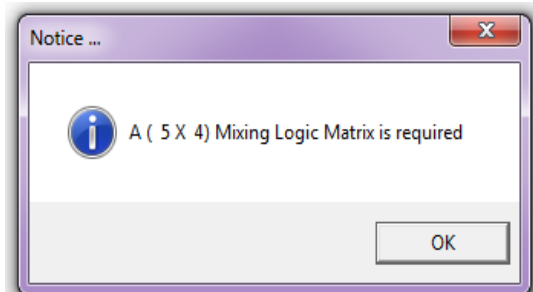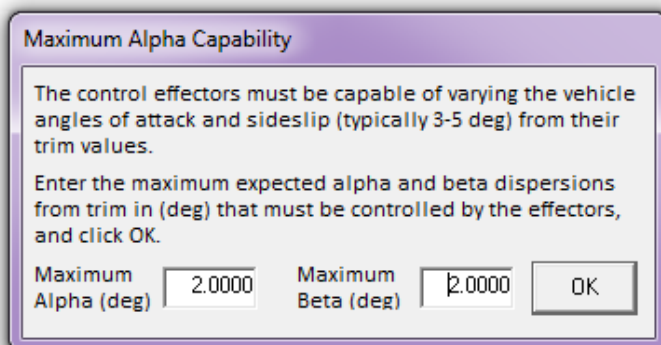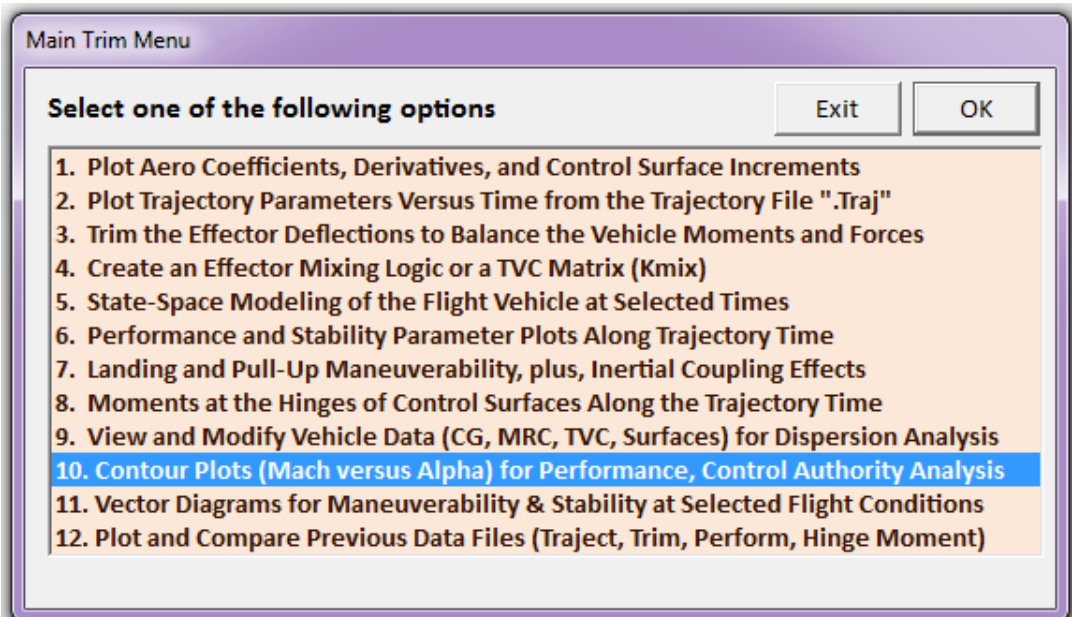Rotation Control Authority (dQ/dQmax)<1 for 2 (deg) of Alpha & Beta Variation

The above plot shows the flight control effort in the four controlled directions, as it was defined in equations (3.24) through (3.26). The control effort is well below the 0.5 limit in all 3 rotational directions and also in the x translation. This means that the control authority of the FCS is more than sufficient for resisting against the moments and forces originating from the 2° of $\alpha_{max}$ and $\beta_{max}$ dispersions, plus it has additional control authority left for other functions (gusts, commands).
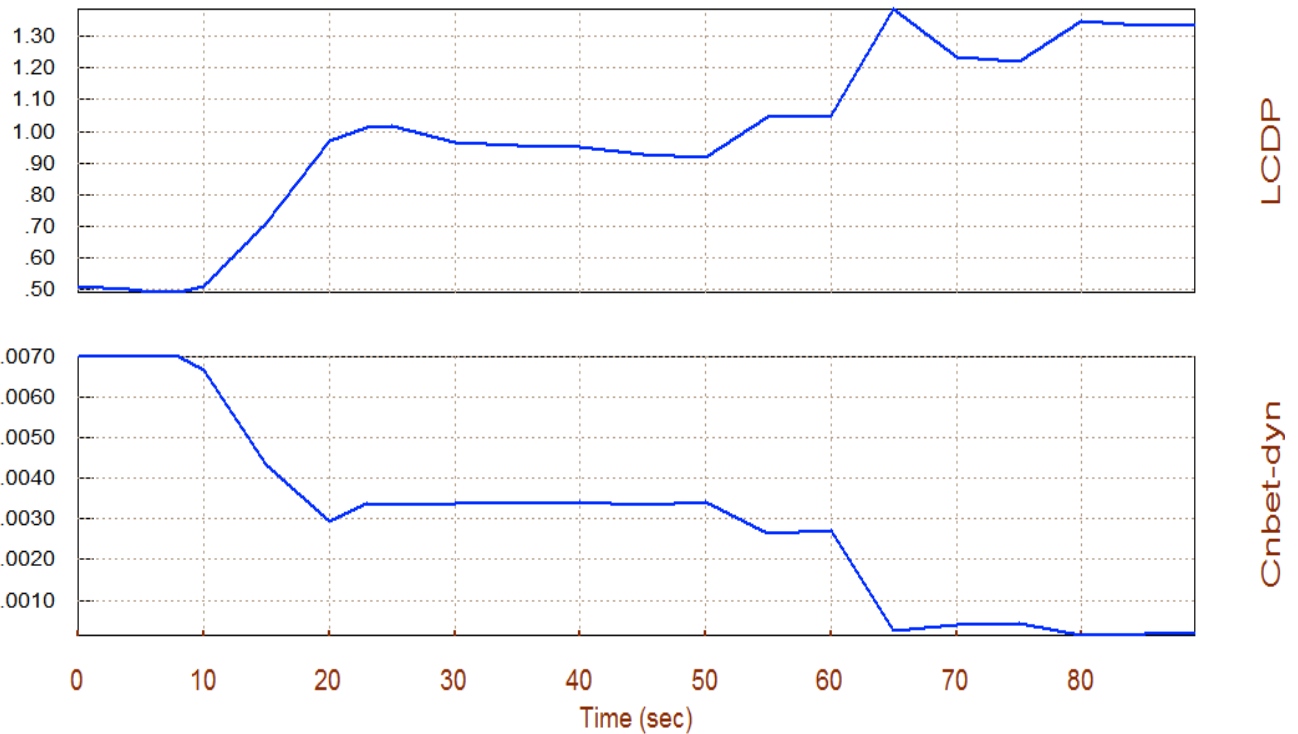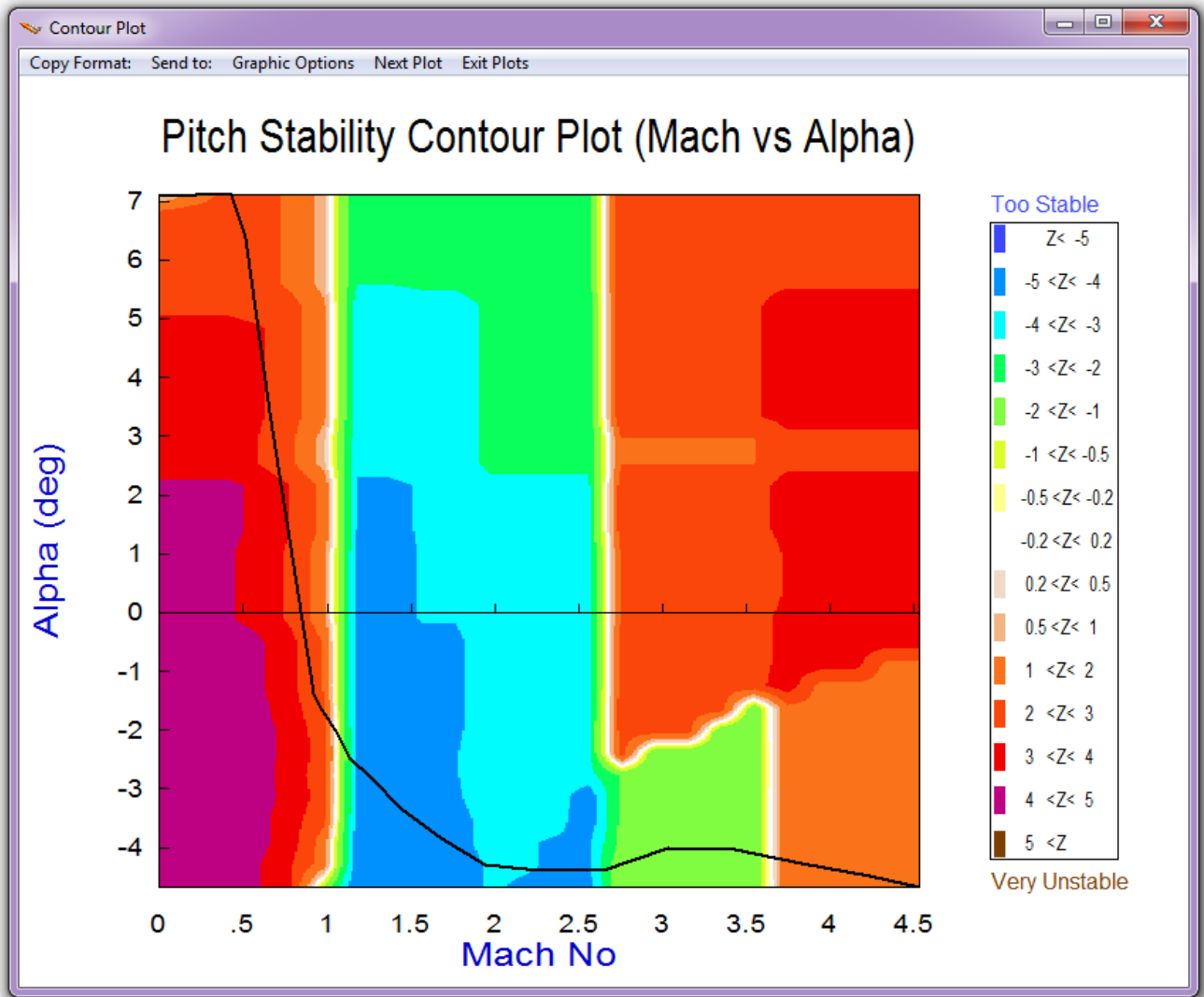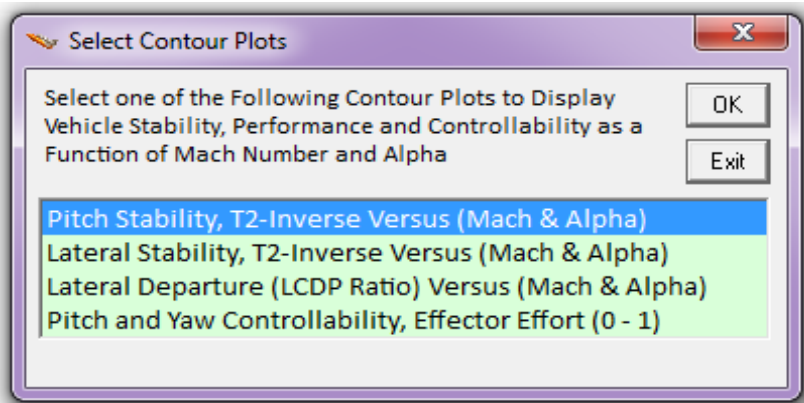
The bank angle with cross-wind is defined in equation (3.24) and it has no significant meaning in this case, it is only useful near landing. The LCDP ratio is described in equation (3.28) and in Figure (3.3). It is an indicator of roll axis dynamic controllability and in this case it looks pretty good because (a) it is positive and its magnitude is reasonably close to one, which is an ideal value, and (b) it does not change sign which causes roll reversals and unreliable situations. In fact it proves that the vehicle has an excellent turn coordination. The Cnβ-dynamic parameter, defined in equation (3.16), is positive throughout the boost phase which also shows that the vehicle is directionally stable. It drops pretty low in magnitude towards the end of the boost phase near neutral stability. This causes reduction in the yaw control effort.

## 1.7 Analyzing Stability & Performance During Ascent Using Contour Plots

Contour plots are 3-dimensional plots that allow us to visualize vehicle performance over the entire Mach versus Alpha range rather than in the vicinity of the trajectory. Occasionally the vehicle may deviate from its expected alpha versus Mach trajectory and we would like to make sure that the planned trajectory is not close to regions of unacceptable performance. Contour plots are also useful by giving us insight on how to reshape trajectories and improving performance. From the main menu select option (10) to create contour plots of some important parameters against Mach and $\alpha$. Similar to our previous analysis in section (1.6) we must specify the aero disturbance environment which at steady-state is defined by the maximum dispersion angles ($\alpha_{max}$ and $\beta_{max}$) from trim. Enter 2° for both $\alpha_{max}$ and $\beta_{max}$, as shown below. Also, the program requires an effector combination matrix, and since we do not have one selected we will allow the program to calculate it along the trajectory by selecting the second option in the Mixing Logic selection dialog. The matrix has 4 inputs corresponding to the control directions and 5 outputs corresponding to the effector commands. From the contour plot selection menu we must select one of the performance parameters to be presented in color-coded form in a 2-dimensional array versus Mach and Alpha.

---

**Main Trim Menu**

**Select one of the following options**       Exit       OK

1. Plot Aero Coefficients, Derivatives, and Control Surface Increments
2. Plot Trajectory Parameters Versus Time from the Trajectory File ".Traj"
3. Trim the Effector Deflections to Balance the Vehicle Moments and Forces
4. Create an Effector Mixing Logic or a TVC Matrix (Kmix)
5. State-Space Modeling of the Flight Vehicle at Selected Times
6. Performance and Stability Parameter Plots Along Trajectory Time
7. Landing and Pull-Up Maneuverability, plus, Inertial Coupling Effects
8. Moments at the Hinges of Control Surfaces Along the Trajectory Time
9. View and Modify Vehicle Data (CG, MRC, TVC, Surfaces) for Dispersion Analysis
10. Contour Plots (Mach versus Alpha) for Performance, Control Authority Analysis
11. Vector Diagrams for Maneuverability & Stability at Selected Flight Conditions
12. Plot and Compare Previous Data Files (Traject, Trim, Perform, Hinge Moment)

---

**Maximum Alpha Capability**

The control effectors must be capable of varying the vehicle angles of attack and sideslip (typically 3-5 deg) from their trim values.

Enter the maximum expected alpha and beta dispersions from trim in (deg) that must be controlled by the effectors, and click OK.

Maximum Alpha (deg)   2.0000     Maximum Beta (deg)   2.0000     OK

---

**Notice …**

(i) A ( 5 X 4) Mixing Logic Matrix is required

OK

The first parameter is the pitch stability parameter which was described in equations (3.14 & 3.15). The trajectory is represented by a black line across the Mach versus Alpha array. It starts at T=0 (sec) in the upper left hand corner where (α=7°, Mach=0) and it ends at T=89 (sec) in the lower right corner where (α=-4.5°, Mach=4.5). The color represents the value of the performance parameter, which in this case it is T2-inverse for pitch stability. The color variations across the field present us a good indication on what pitch stability is like over the entire of Mach and alpha range.

We also observe how our trajectory (black line) travels across the (Mach vs Alpha) field and compare our trajectory's stability relative to the overall stability of the array and decide if there is a need to change course in order to improve it. In our example, we see that initially the vehicle is statically unstable as it crosses through the red orange region, with a time to double, T2=1/Z, reaching 0.33 (sec). Then the trajectory crosses through the neutrally stable (white) region and passes in the stable region (blue, cyan, green) where the short period resonance reaches $\omega_p$= 4.3 (rad/sec). Then it crosses through the neutral white region again and it ends up in the unstable (orange) region towards the end of the boost phase. In the entire course, however, the stability parameter remains within acceptable range avoiding the "Too Stable" or "Too Unstable" regions, and it does not pass near any threatening regions that should be avoided.

In lateral direction the vehicle stability parameter is in the stable region through the entire trajectory with the Dutch-Roll resonance reaching $\omega_d$= 4.5 (rad/sec), see equations (3.17 & 3.18). There are a couple of mildly unstable island regions at ($\alpha$=-2°, Mach=3, and $\alpha$=-1°, Mach=4) which are not crossed or positioned near the trajectory.
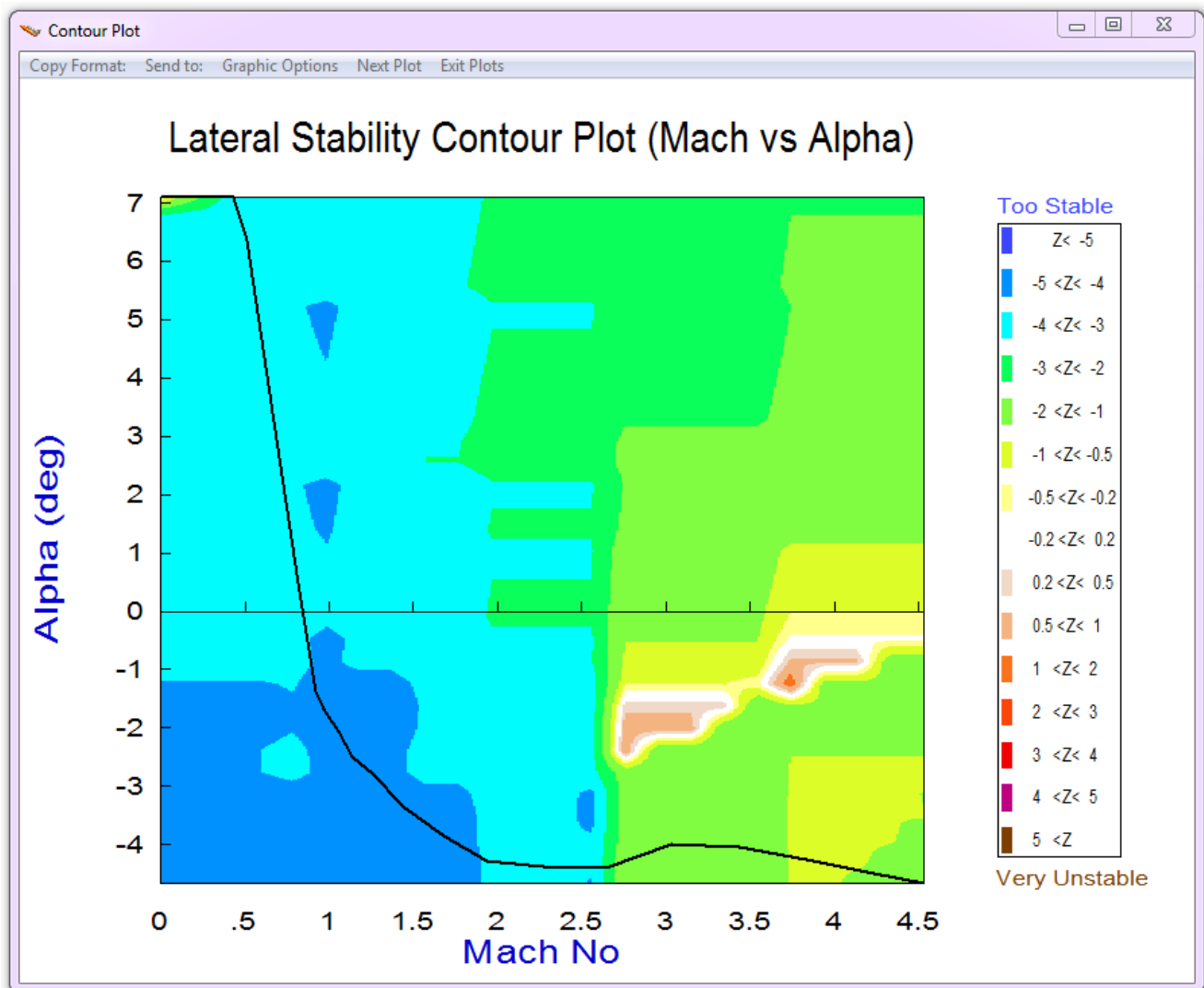
The roll departure LCDP-ratio looks very good also, see Figures (3.3 & 4.2). It is entirely in the positive region (bright colors) and mostly in the white region indicating that it has an excellent turn coordination. There are a couple of bad regions to be avoided. In one of them ($\alpha$=-2°, Mach=3) the LCDP-ratio changes sign (dark colors surrounded by a red band). If the trajectory would cross this region the roll control gain should be reversed twice making it vulnerable to aero uncertainties. There is also a region around ($\alpha$=-0.6°, Mach=4) where the LCDP ratio exceeds 10 meaning that roll maneuvers in that region would induce too much beta transints. Fortunately, these regions are not crossed by the trajectory line and, in fact, our trajectory is safely distant from those regions.



Roll Departure (LCDP) Contour Plot (Mach vs Alpha)
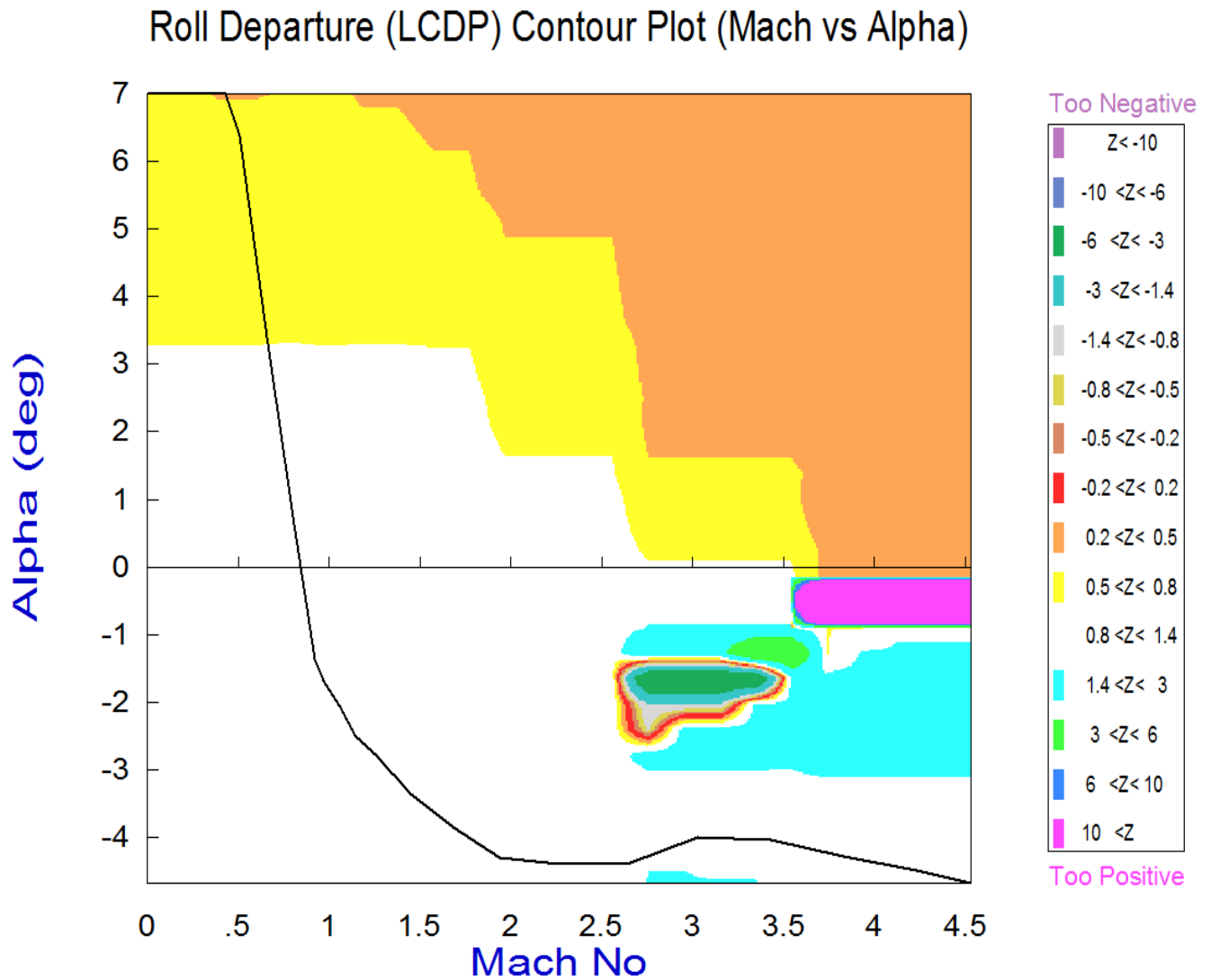
The next 3 plots show the control authority of the effectors system against wind disturbances defined as ($\alpha_{max}$= 2° and $\beta_{max}$= 2°). As it was already discussed in Section 3, the control effectors will prevail over the disturbances when the control effort parameter is less than one, or even better, less than 0.5. In this case, the effector system satisfies the control effort requirement in all 3 axes. The worst control authority happens to be in yaw, where the control effort reaches 0.3. In pitch, the control direction changes sign a couple of times because the stability parameter also changes sign and the control direction reverses.
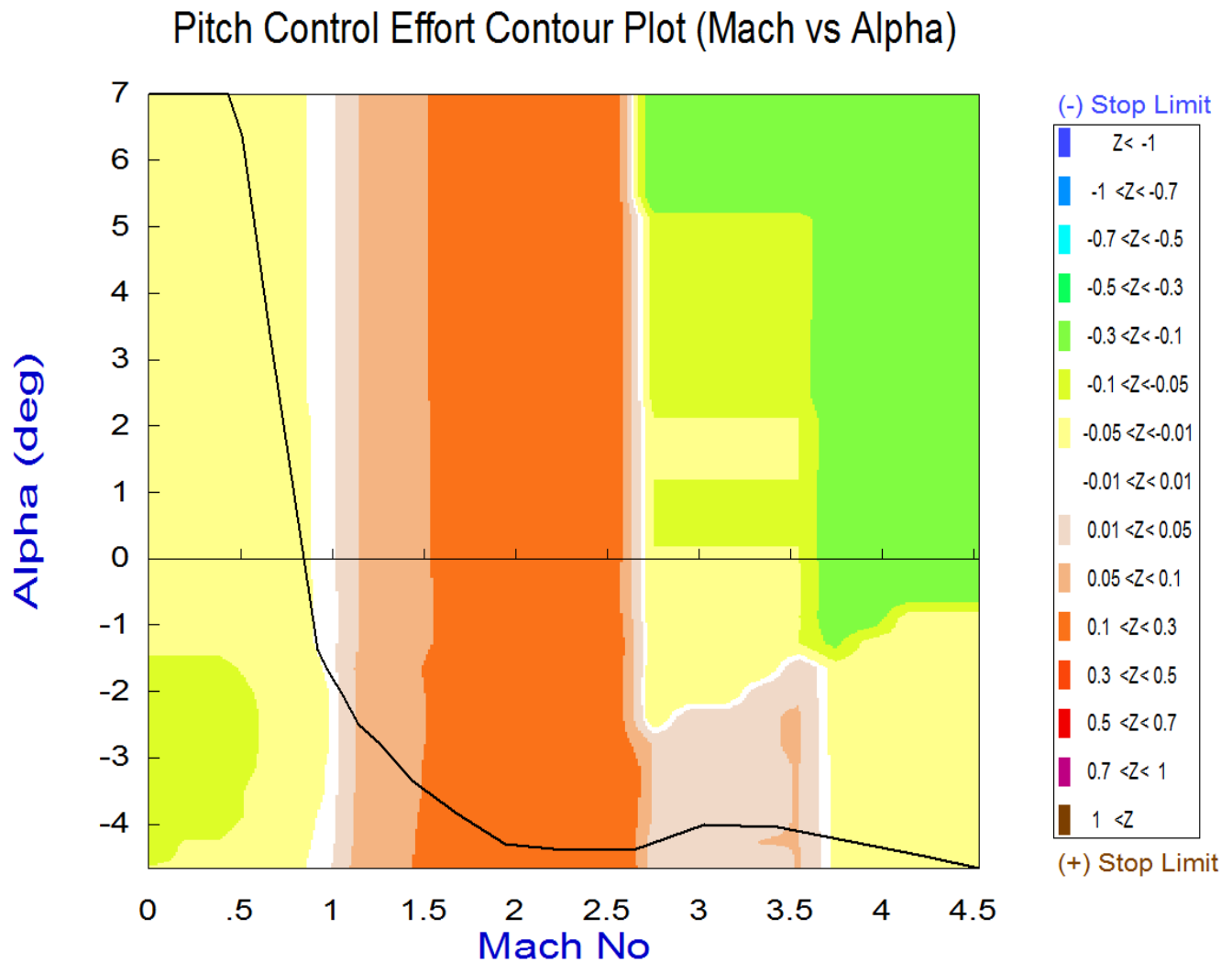
## Pitch Control Effort Contour Plot (Mach vs Alpha)

Yaw Control Effort Contour Plot (Mach vs Alpha)
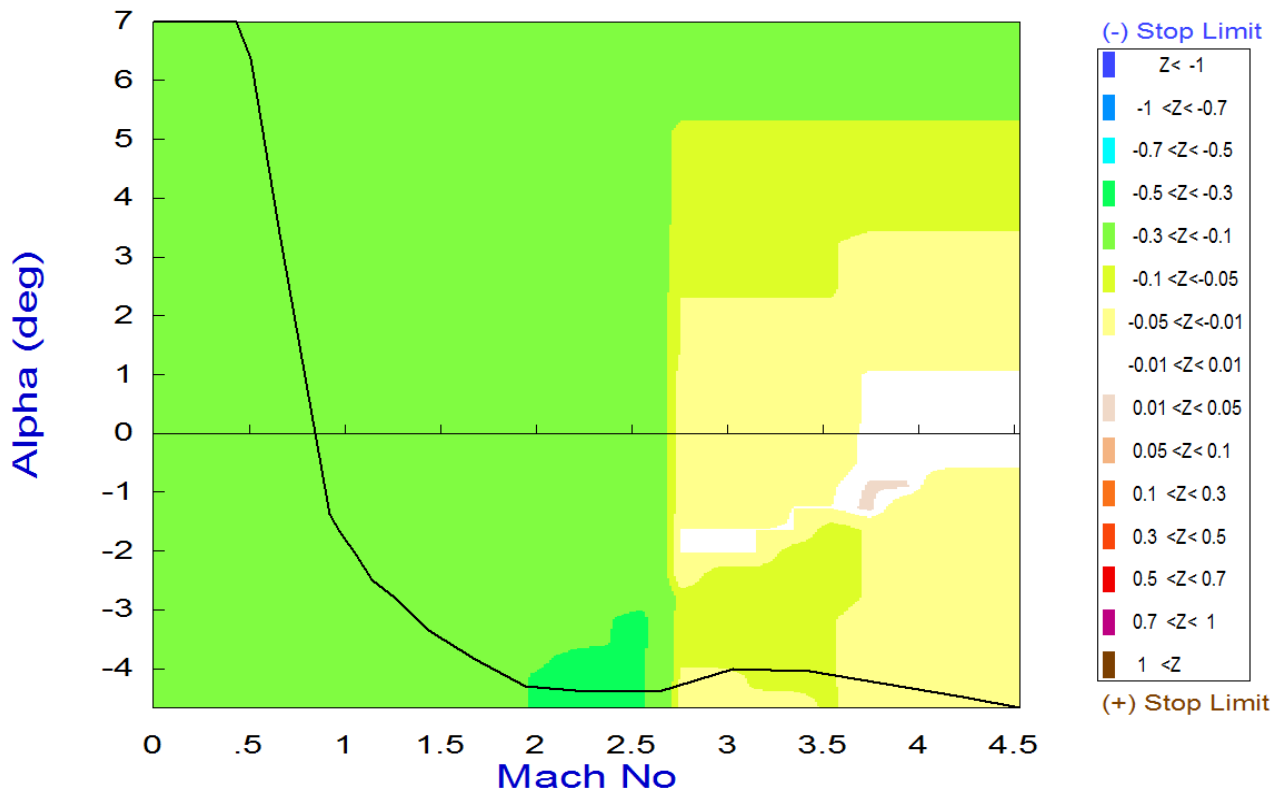
Roll Control Effort Contour Plot (Mach vs Alpha)
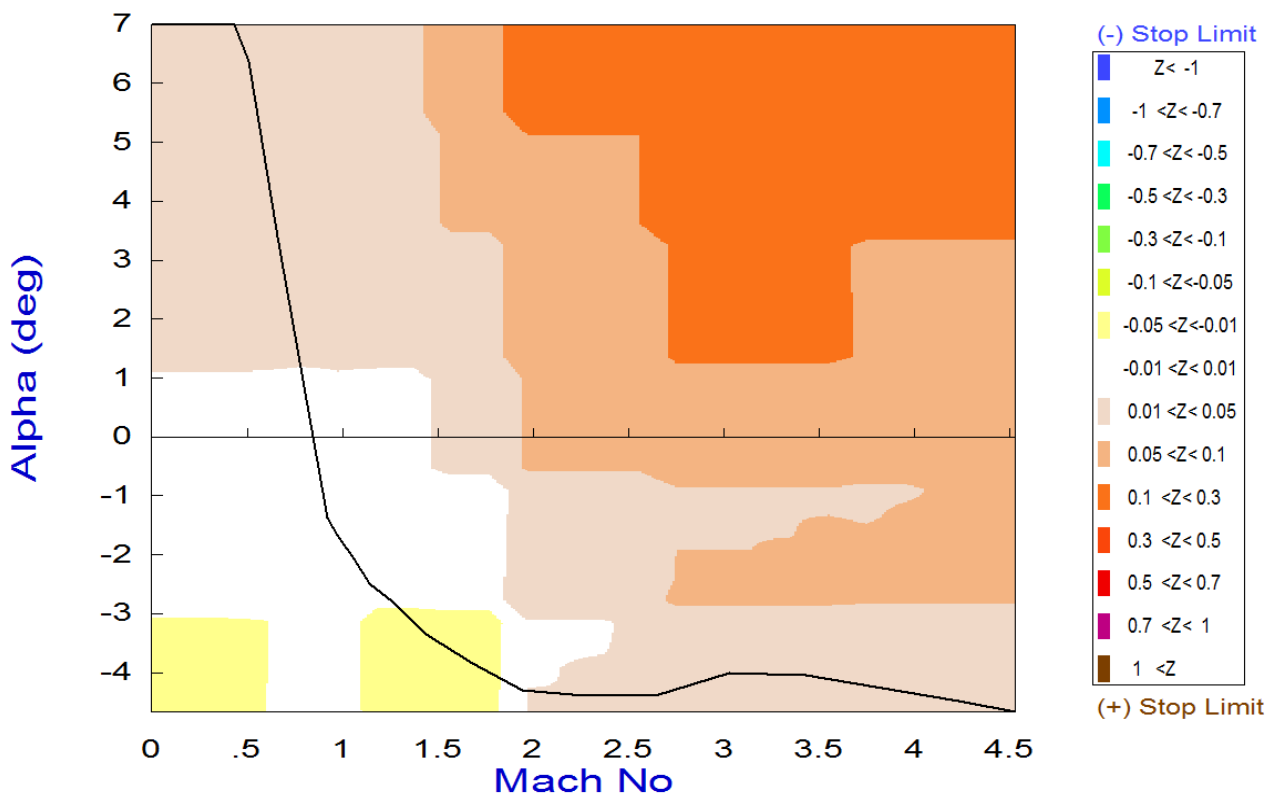
## 1.8 Analyzing Maneuverability Using Vector Diagrams

Vector diagrams are 2-dimensional diagrams used for analyzing vehicle controllability at a specified flight condition. We compare the control capability of the effector system in two directions against the effects on the vehicle of a wind-shear disturbance which is expressed by the dispersions it causes in the angles of attack and sideslip, and we determine if the vehicle provides sufficient control authority to counteract against the disturbance moments and forces. This is not just a magnitude comparison but it also allows us to examine the directions of the controls against the disturbance. It helps us evaluate the orthogonality of the control system, compare the acceleration magnitudes due to the controls and wind disturbance, and determine if the controls are powerful enough and their directions are capable of counteracting the disturbance moments along the roll, pitch, yaw, and axial directions in this case. Assuming that the initialization files and trim conditions are the same as before, from the Trim menu select option (11) for plotting vector diagrams, and then an arbitrary flight condition at t=80 sec. The following dialog consists of menus used for selecting the vehicle mass, Mach number, alpha, and beta. The default values correspond to the selected flight time. You may keep those parameters or change them to something different. In this case we select the default values and click "*Select*". The disturbances are caused by wind-shear defined by the maximum alpha and beta produced. In the following dialog enter the maximum disturbance angles ($\alpha_{max}$ and $\beta_{max}$)=2°.
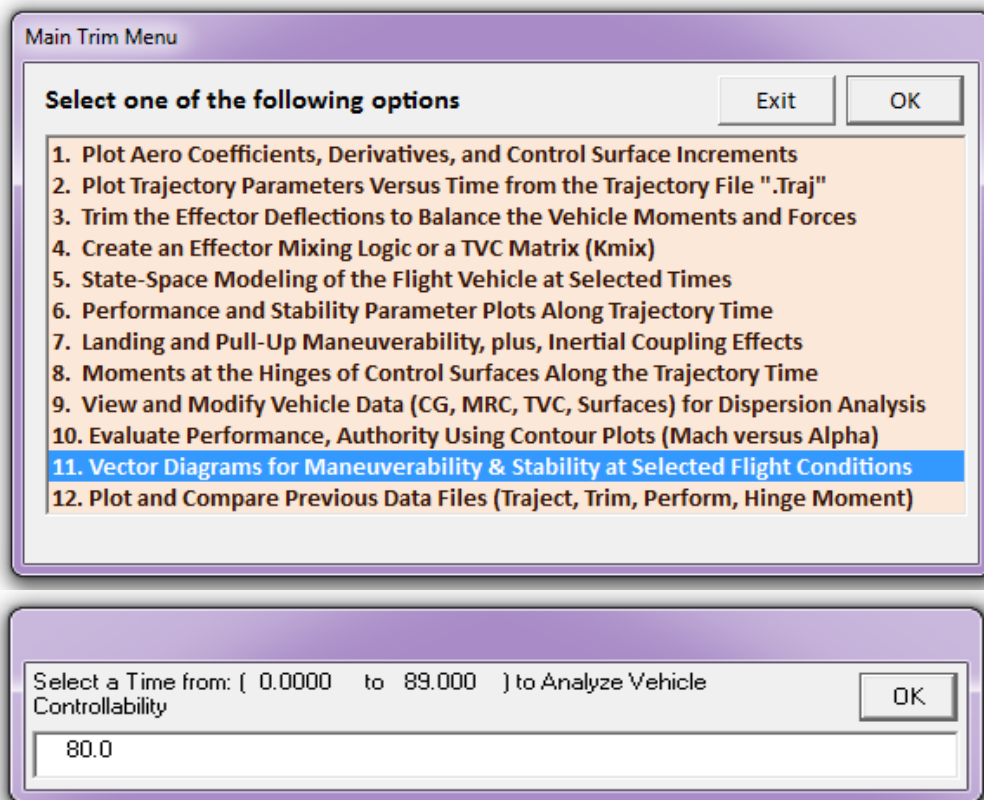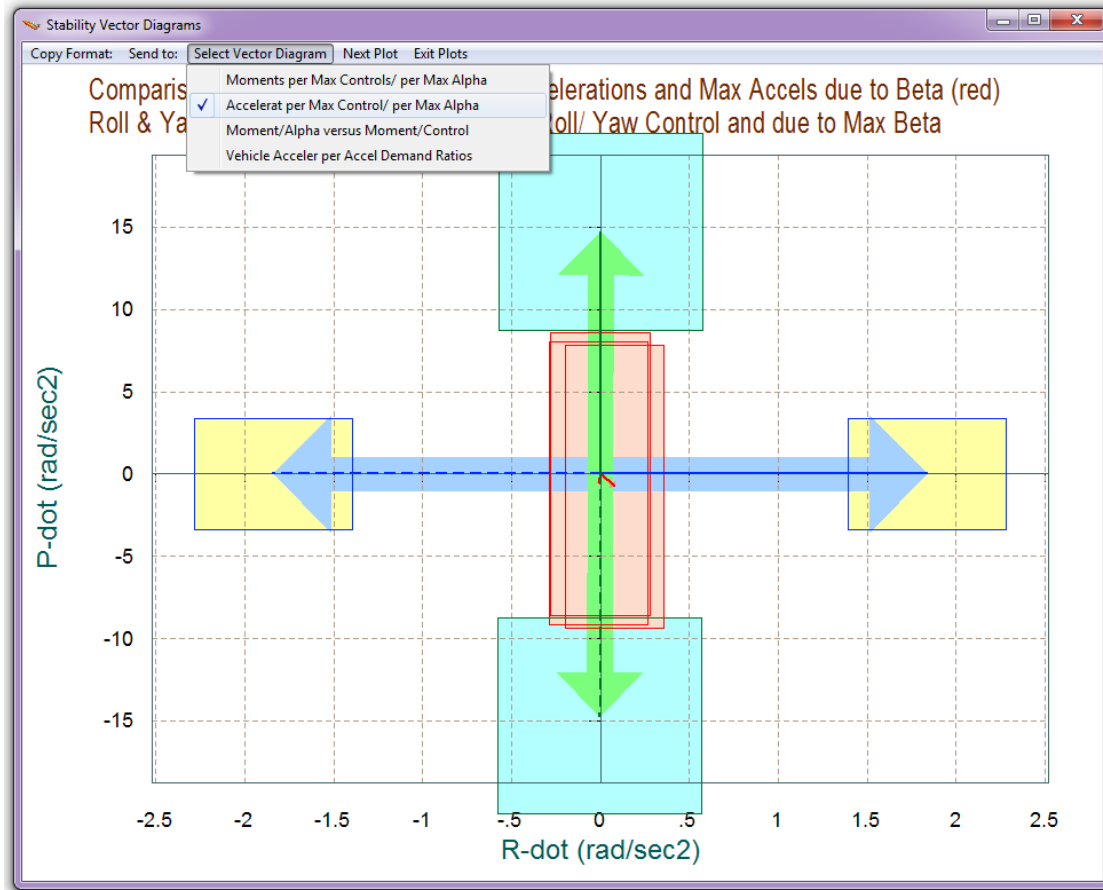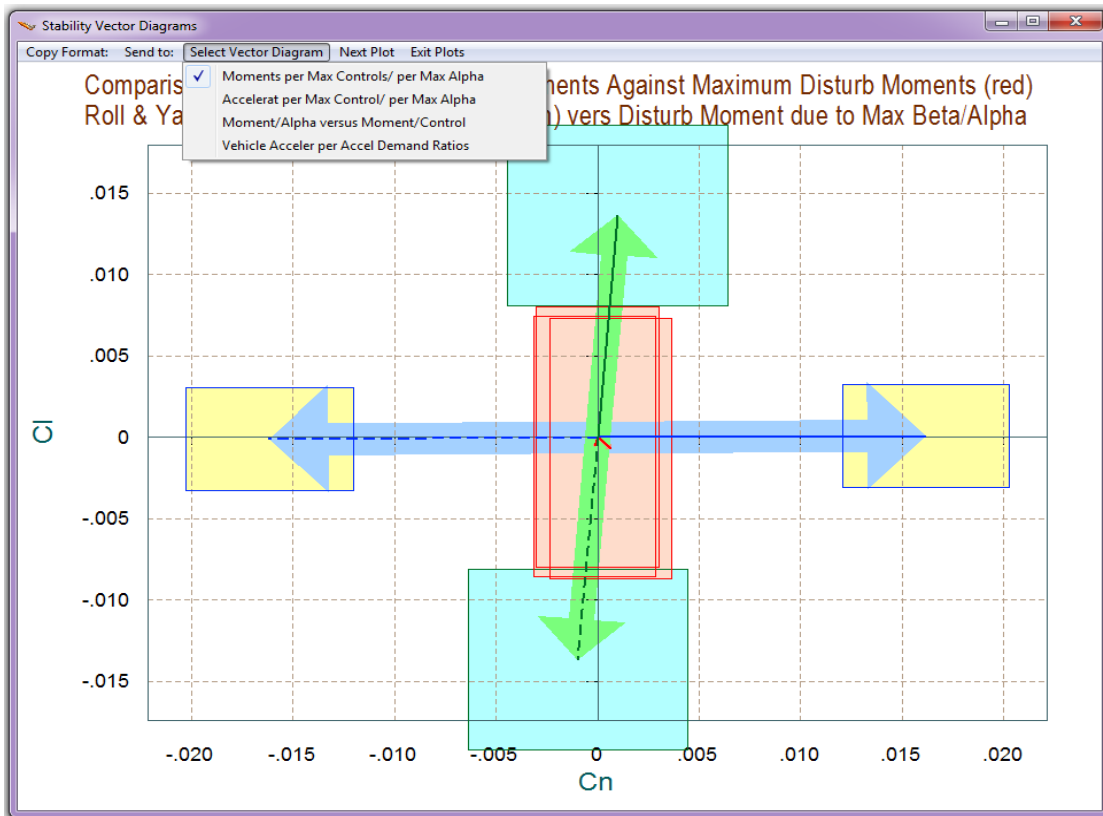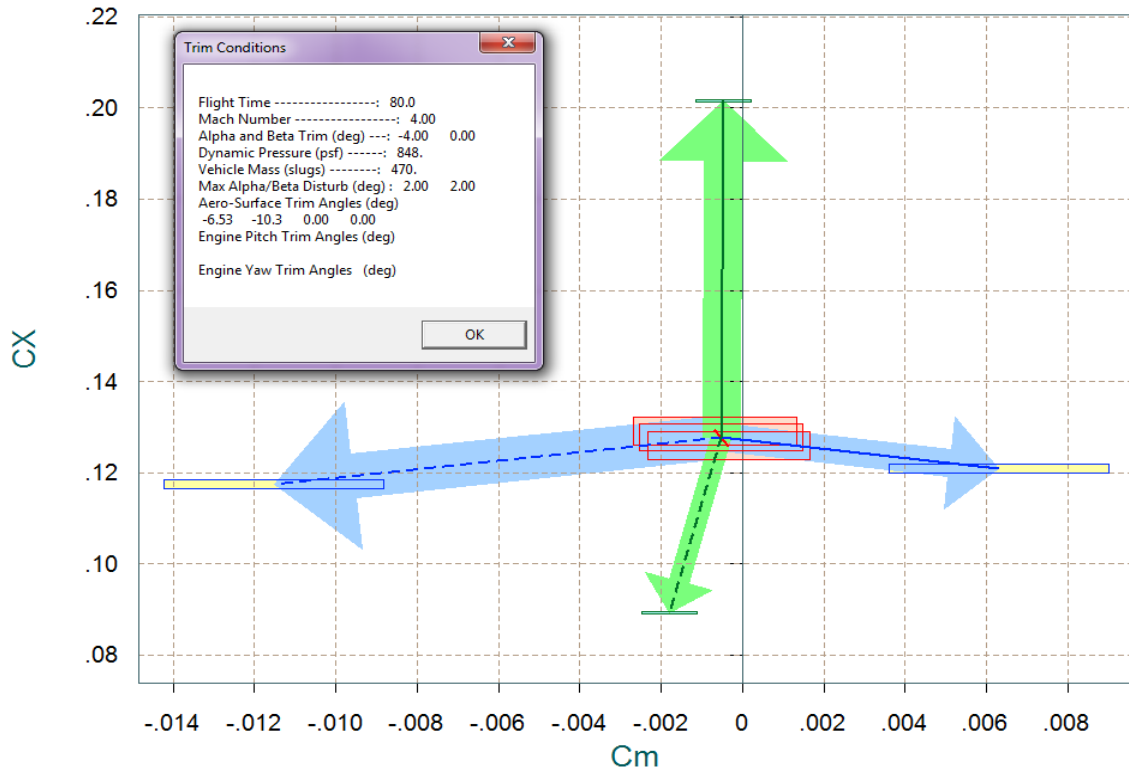
**Figure 1.8.1 Maximum Roll & Yaw Moments and Accelerations Vector Diagrams Including Uncertainties**

## Comparison Between Maximum Control Moment/Forces Against Maximum Disturbance (red Pitch Control Moment and Axial X-Force (non-dimens) versus Disturb due to Max Alpha



**Trim Conditions**

```
Flight Time ------------------:  80.0
Mach Number ------------------:  4.00
Alpha and Beta Trim (deg) ---:  -4.00    0.00
Dynamic Pressure (psf) ------:  848.
Vehicle Mass (slugs) --------:  470.
Max Alpha/Beta Disturb (deg) :  2.00    2.00
Aero-Surface Trim Angles (deg)
  -6.53    -10.3    0.00    0.00
Engine Pitch Trim Angles (deg)

Engine Yaw Trim Angles   (deg)
```

OK

## Comparison Between Maximum Control Accelerations and Max Accels due to Alpha (red) Pitch Accelerat and Axial X-Accelerations due to Max Control and due to Max Alpha
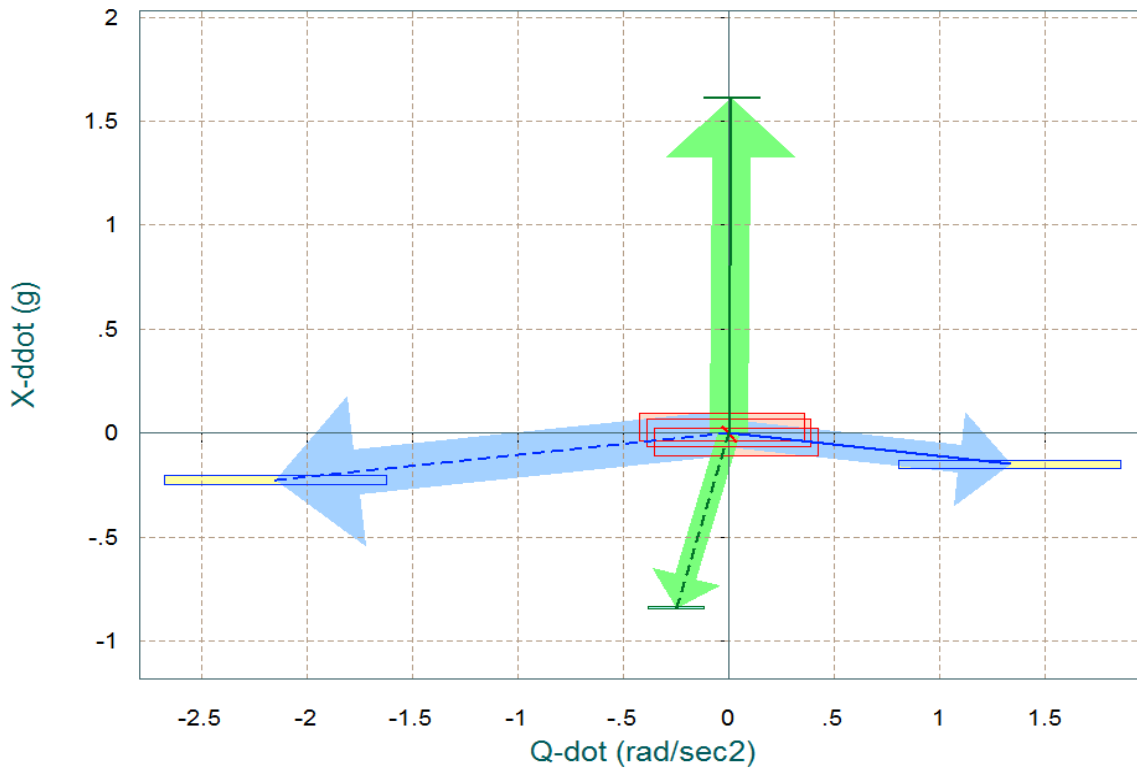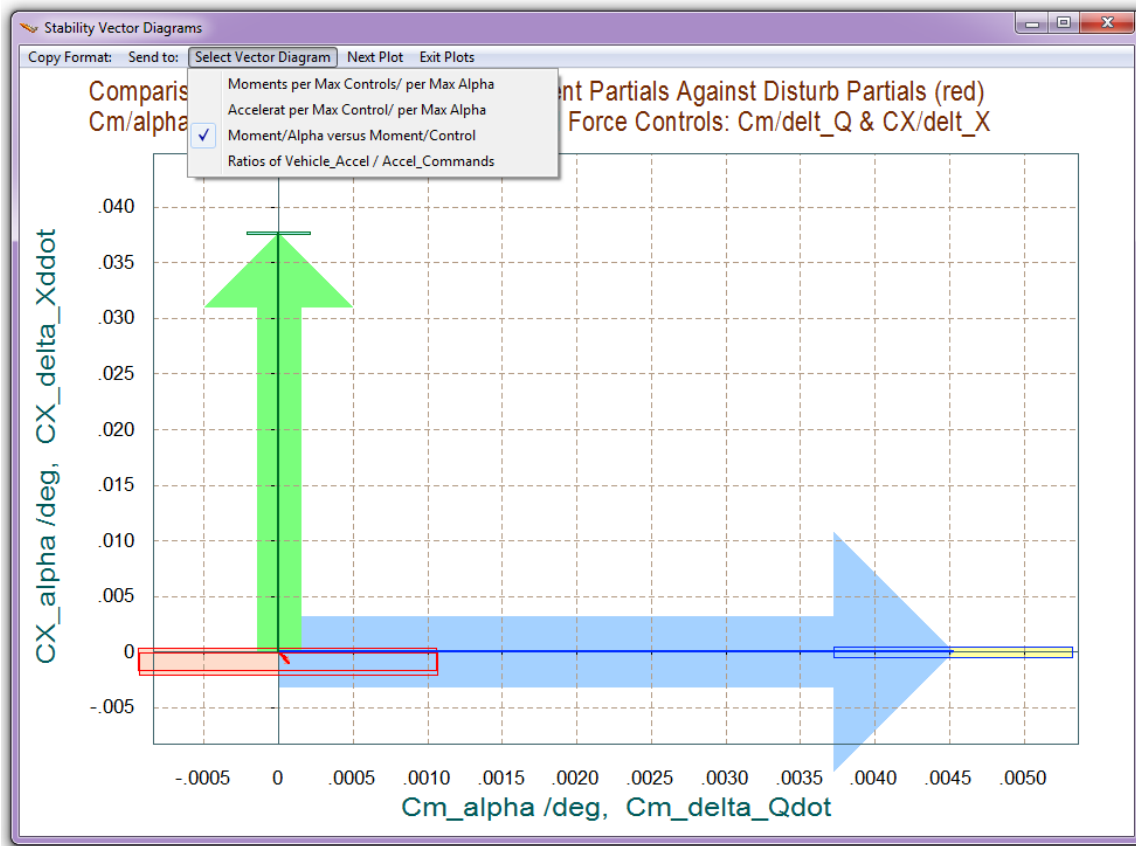


**Figure 1.8.2 Maximum Pitch Moment, Axial Force, and Accelerations Vector Diagram Including Uncertainties**

**Stability Vector Diagrams**

Copy Format:   Send to:   Select Vector Diagram   Next Plot   Exit Plots

Moments per Max Controls/ per Max Alpha
Accelerat per Max Control/ per Max Alpha
✓ Moment/Alpha versus Moment/Control
Ratios of Vehicle_Accel / Accel_Commands

Comparison Between Control Force & Moment Partials Against Disturb Partials (red)
Cm/alpha & CX/alpha versus Pitch and Force Controls: Cm/delt_Q & CX/delt_X

Comparison Between Control Force & Moment Partials Against Disturb Partials (red)
Cm/alpha & CZ/alpha versus Pitch and Norml Z-Force Controls: Cm/delt_Q & CZ/delt_Z

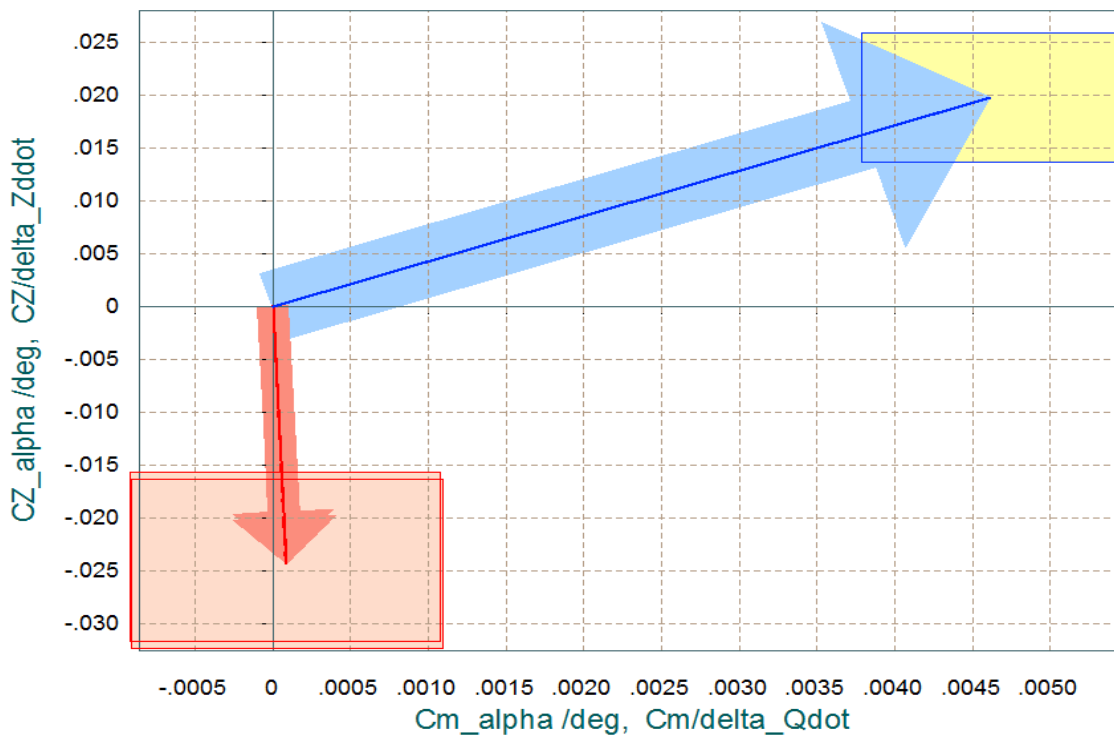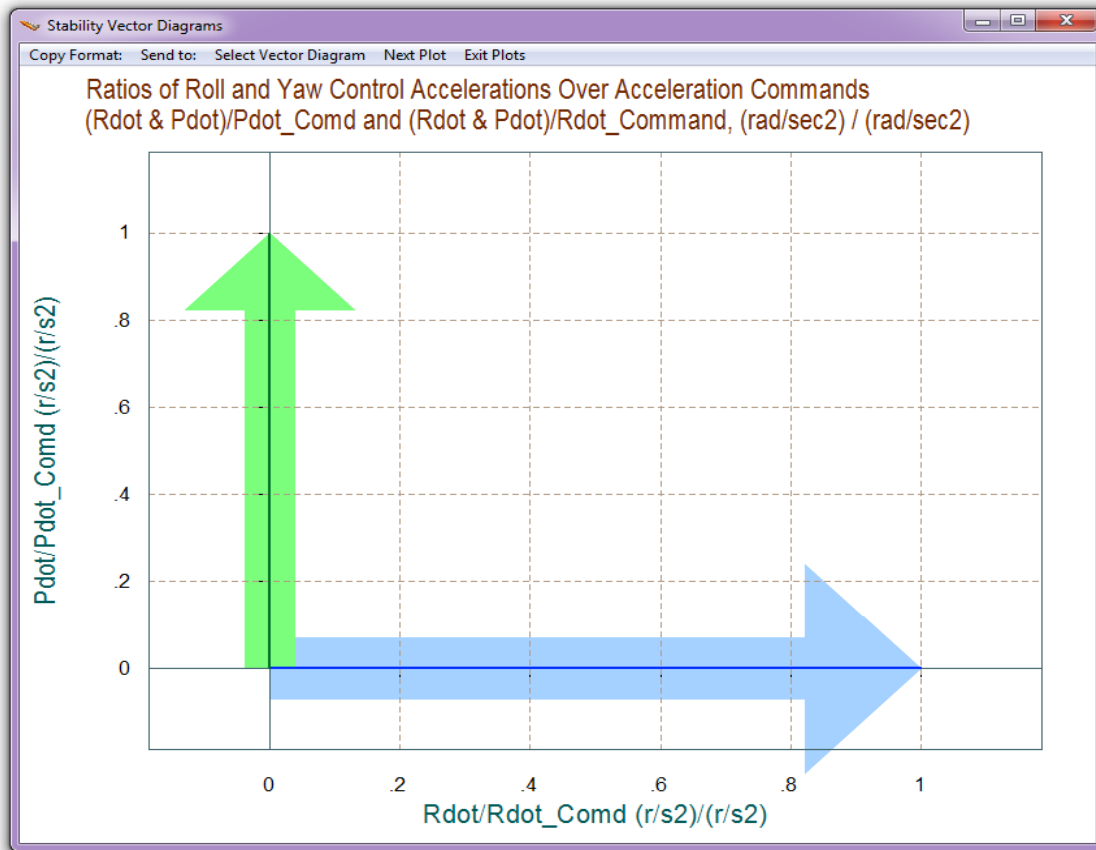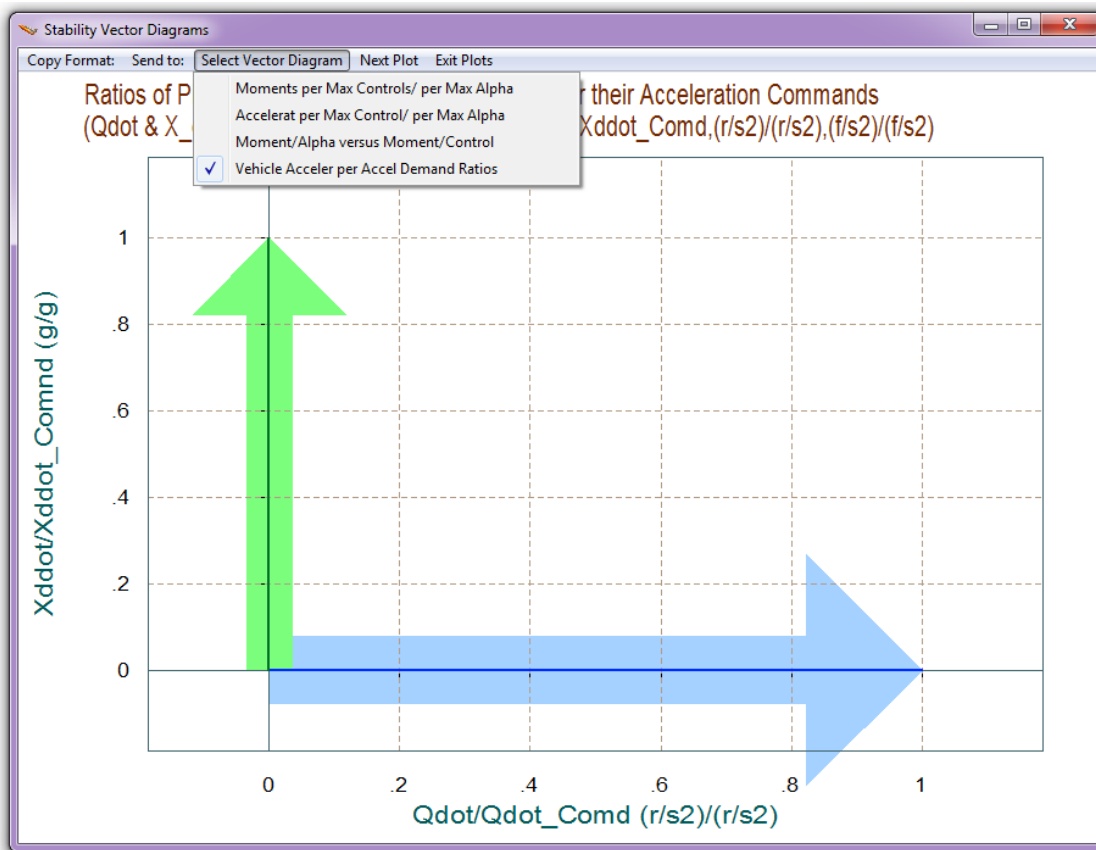**Figure 1.8.3 Moment and Force Partials in the Longitudinal Directions**

**Figure 1.8.5 Acceleration Partials with respect to Acceleration Demands**

## 1.9 Manually Adjusting the Trimming Conditions

The Trim program calculates the effector trim positions by taking into consideration the control capability of each effector in the trim directions, at each point along the trajectory. When multiple effectors are present this makes it more likely that multiple directions will be affected and trimming will be easier. The pseudo-inverse algorithm in the Trim program favors the effectors with greater control authority in a certain direction to contribute more in that direction than those who have less authority. There are situations, however, where the analyst may wish to bypass this automatic allocation of control authority and to trade the activity of one effector against another. In the longitudinal direction, for example, a vehicle may have an elevon, a body-flap, a speed-brake, and thrusters. It may be possible to eliminate, or to reduce activity of some of the controls, like for example the body-flap, by keeping it at a fixed position and trimming with the other effectors. In the example that follows we will show how to modify the trimming conditions in the program. This of course is only possible when the vehicle is equipped with other effectors that can provide sufficient controllability in the trim directions. If the effectors system is insufficient or barely sufficient to span all directions the program will not allow the user to make any adjustments in the effector trim positions or permit very small adjustments. So in order demonstrating this trimming adjustment feature we must first include some more effectors in our vehicle so that we may be able to trade controls.

The additional effectors are specified in a new propulsion file "*Hyper2.Engn*". We will replace the previous file "*Hyper1.Engn*" that used only one fixed but throttling main engine with this new file that includes the same main engine but it is also allowed to gimbal it nozzle ±5° in the pitch and yaw directions relative to its mounting position, which is along the x-axis. This is in addition to the ±40% throttling capability that we already demonstrated. This provides us some additional pitch and yaw control capability. Furthermore, in addition to the main engine gimbaling we have also included two reaction control thrusters located near the front end of the vehicle, firing in the ±Z and in the ±Y directions and are capable of producing up to 4,000 (lb) of thrust. These thrusters will give us some acceleration control in the normal and lateral directions and since we already have x-acceleration control provided by the main engine, with the additional effectors we should be able to trim in all 6 directions. This means that we should be able to define our trimming conditions not only for balancing 3 rotational moments but also matching all 3 translational accelerations defined in the trajectory, assuming of course that the trajectory is reasonable.
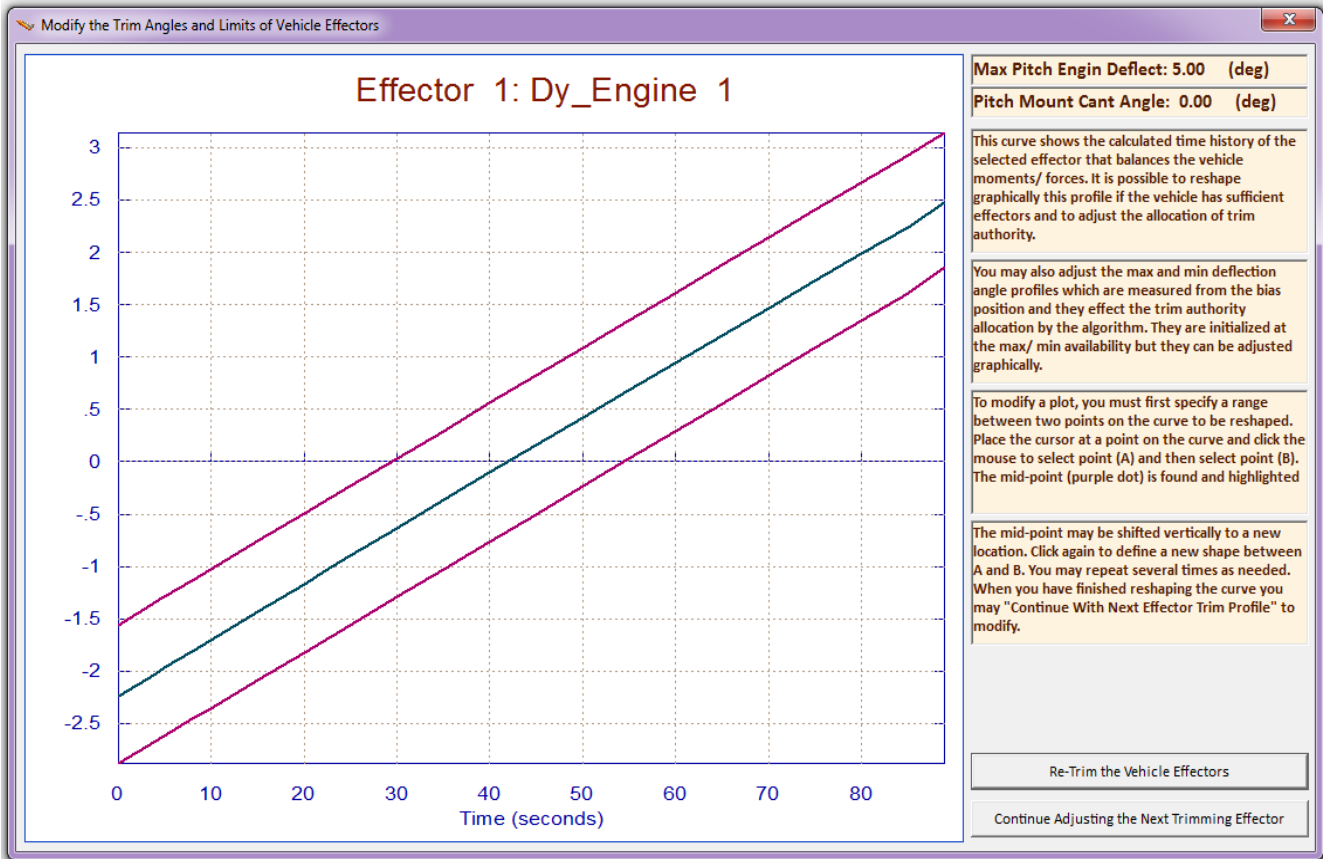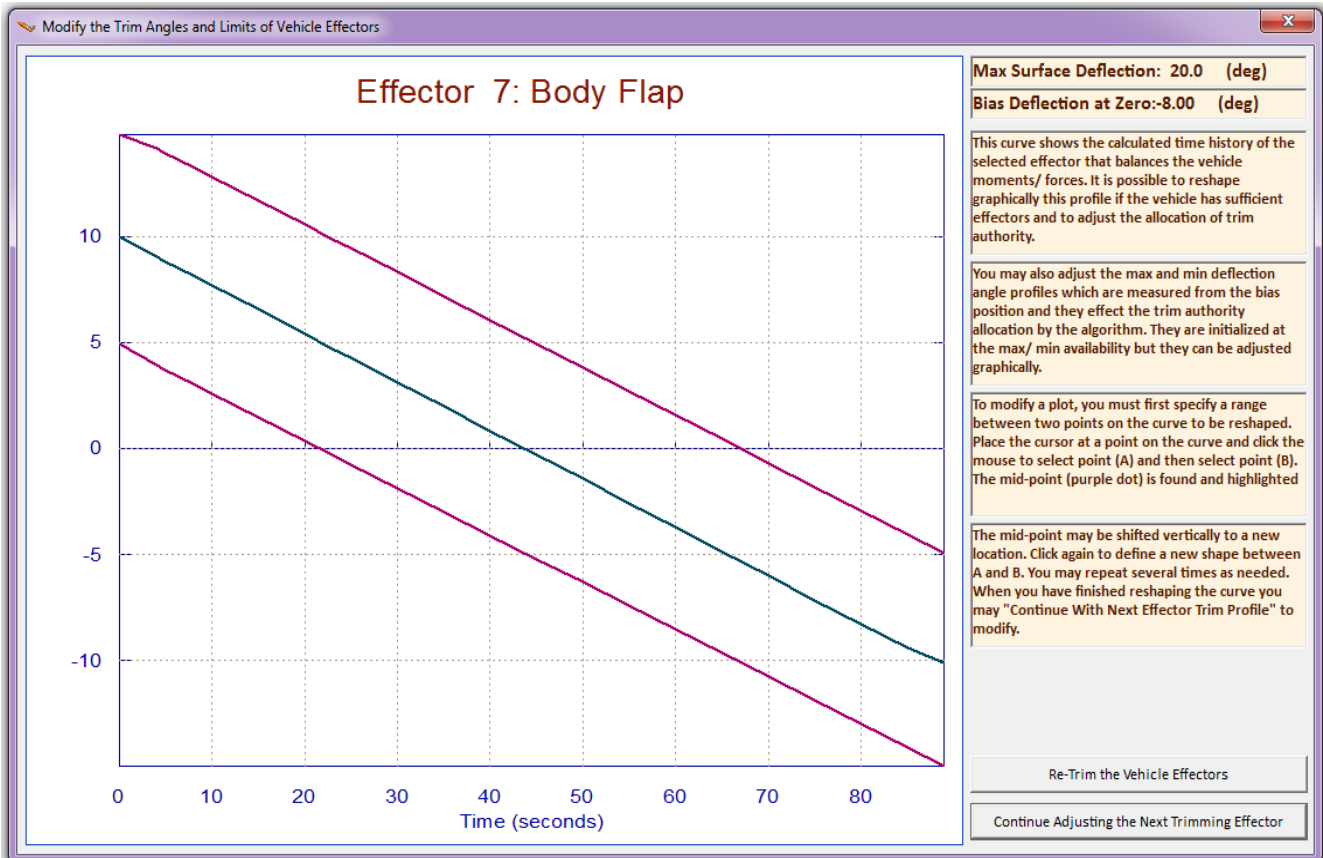
Notice that there is a difference in the throttle control definition between the main engine and the lateral jets and how it is interpreted by the program. In file "Hyper2.Engn" the max throttle control for the main engine is 0.4 (which is less than 1). It means that the nominal engine thrust is 62,000 (lb) and it varies from 37,000 (lb) to 89,000 (lb) depending on the throttle control variable that is calculated when trimming along the trajectory, and the engine thrust is always positive. In the case of the ±Z and ±Y RCS jets the max throttle parameter in the engines file is defined to be exactly 1. This means that the nominal jet thrusts are zero and as the throttle control varies between zero
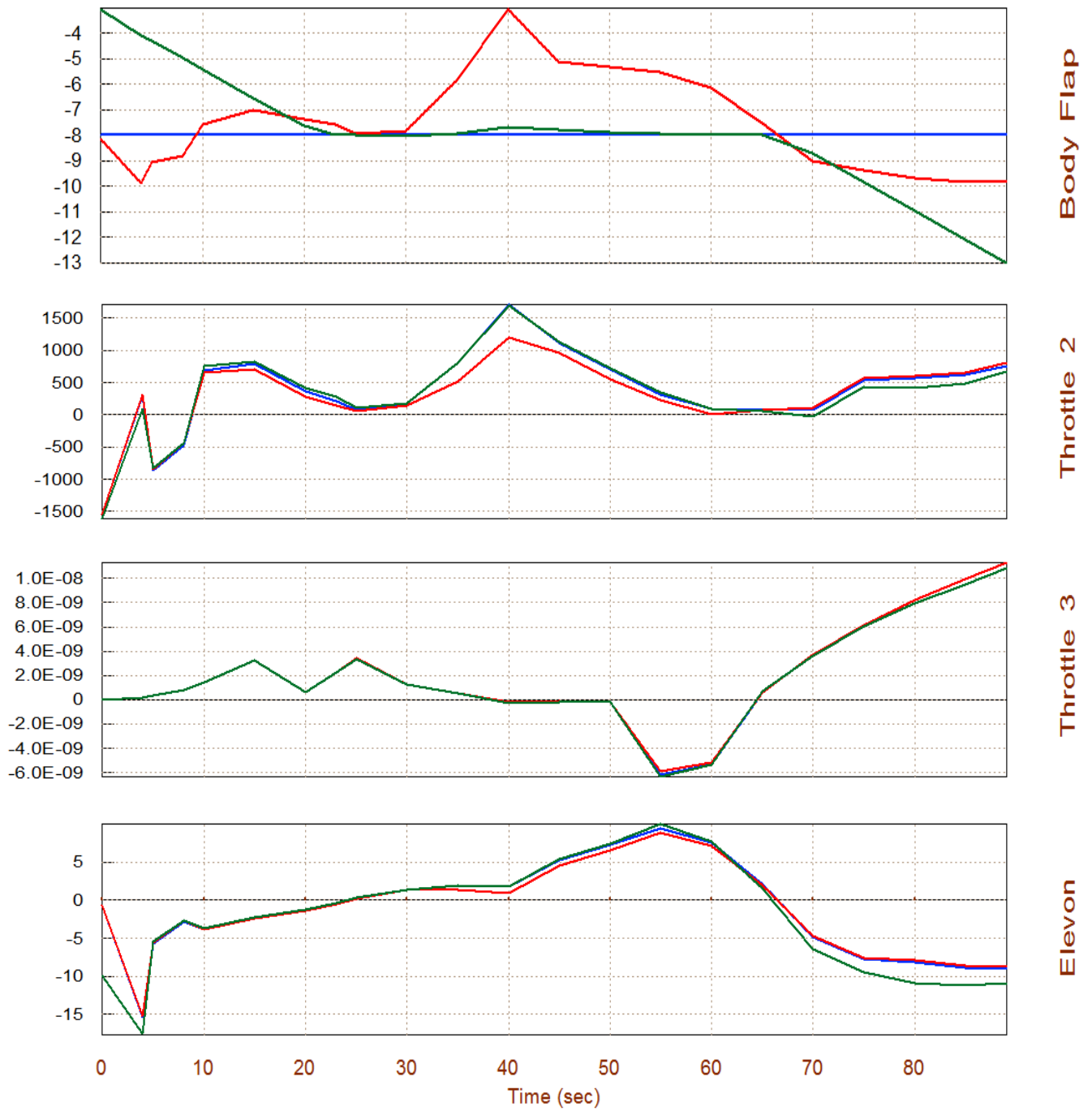
**Scheduling the Effectors**

It is also possible to use the same process in order to schedule (vary as a function of time) the positions of more than one effectors simultaneously by constraining their deflections graphically, as shown below, assuming of course that there are other effectors that can be manipulated by the Trim algorithm in order to balance the moments and forces. In this example we are still using the same effectors and after the first trim we go to the top menu, select "*Modify a Trajectory Plot*", but instead of fixing the position of the Body-Flap we are now scheduling it as shown in the next figure (green line). We also modify the upper and lower limits (magenta lines) parallel to the schedule line allowing ±5° space for adjustment. When the Body-Flap modification is complete we do not re-trim yet but click on "*Continue Adjusting the Next Effector*". From the effectors menu this time choose the pitch TVC deflection of the main engine "*Dy_Engine 1*". The dialog-plot shows the pitch engine deflection from the previous (unconstrained) trim. Using the mouse and the dialog graphic capabilities this plot can be modified and scheduled as shown in the next figure. The upper and lower limits are also set parallel to the scheduled pitch engine deflection line with a small space in between. When the adjustments are complete, click on "*Re-Trim*".
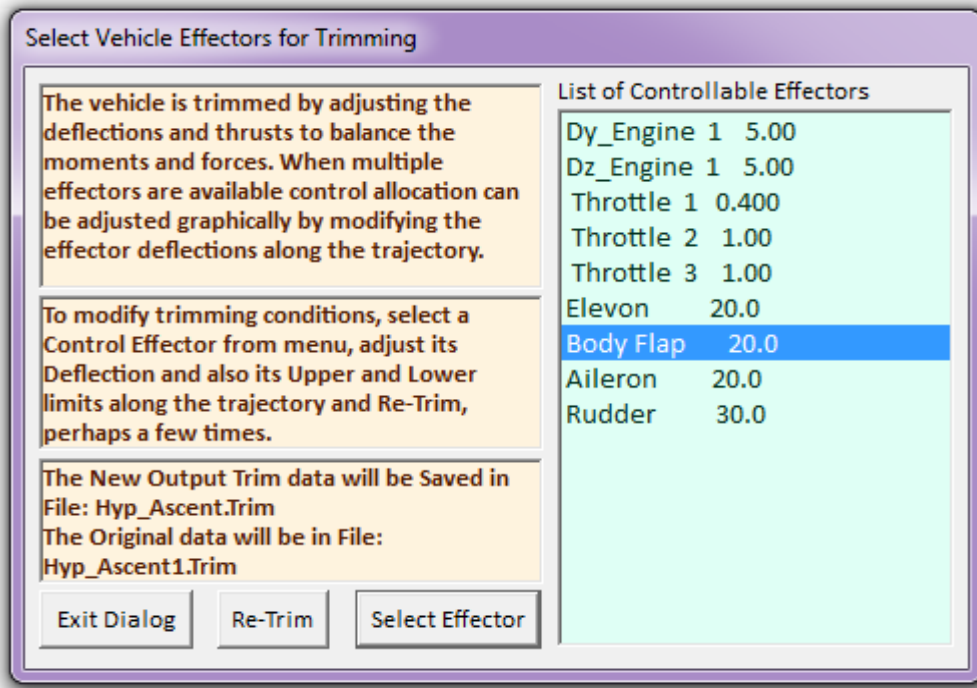


After re-trimming the following plots show the new trim results with the bounded Body-Flap and pitch engine deflections. The new trim positions (blue lines) are now constrained to lie in between the upper and lower limits (red and green lines). To summarize, let us plot all 3 cases together and compare them using option (12), as shown in the following plots. The red curves are from the original unconstrained file "Hyp_Ascent.Trim". The blue curves correspond to the previous case where we fixed the Body-Flap at its bias and saved it in file "*Fixed_BF.Trim*". The green curves are from our latest trim where we scheduled the pitch TVC and the Body-Flap and saved it in file "*Scheduled.Trim*".

## Modify the Trim Angles and Limits of Vehicle Effectors

### Effector 7: Body Flap

**Max Surface Deflection: 20.0   (deg)**

**Bias Deflection at Zero:-8.00   (deg)**

This curve shows the calculated time history of the selected effector that balances the vehicle moments/ forces. It is possible to reshape graphically this profile if the vehicle has sufficient effectors and to adjust the allocation of trim authority.

You may also adjust the max and min deflection angle profiles which are measured from the bias position and they effect the trim authority allocation by the algorithm. They are initialized at the max/ min availability but they can be adjusted graphically.

To modify a plot, you must first specify a range between two points on the curve to be reshaped. Place the cursor at a point on the curve and click the mouse to select point (A) and then select point (B). The mid-point (purple dot) is found and highlighted

The mid-point may be shifted vertically to a new location. Click again to define a new shape between A and B. You may repeat several times as needed. When you have finished reshaping the curve you may "Continue With Next Effector Trim Profile" to modify.

Re-Trim the Vehicle Effectors

Continue Adjusting the Next Trimming Effector

(x-axis: Time (seconds), y-axis values: 10, 5, 0, -5, -10)

---

## Modify the Trim Angles and Limits of Vehicle Effectors

### Effector 1: Dy_Engine 1

**Max Pitch Engin Deflect: 5.00   (deg)**

**Pitch Mount Cant Angle: 0.00   (deg)**

This curve shows the calculated time history of the selected effector that balances the vehicle moments/ forces. It is possible to reshape graphically this profile if the vehicle has sufficient effectors and to adjust the allocation of trim authority.

You may also adjust the max and min deflection angle profiles which are measured from the bias position and they effect the trim authority allocation by the algorithm. They are initialized at the max/ min availability but they can be adjusted graphically.

To modify a plot, you must first specify a range between two points on the curve to be reshaped. Place the cursor at a point on the curve and click the mouse to select point (A) and then select point (B). The mid-point (purple dot) is found and highlighted

The mid-point may be shifted vertically to a new location. Click again to define a new shape between A and B. You may repeat several times as needed. When you have finished reshaping the curve you may "Continue With Next Effector Trim Profile" to modify.

Re-Trim the Vehicle Effectors

Continue Adjusting the Next Trimming Effector

(x-axis: Time (seconds), y-axis values: 3, 2.5, 2, 1.5, 1, .5, 0, -.5, -1, -1.5, -2, -2.5)

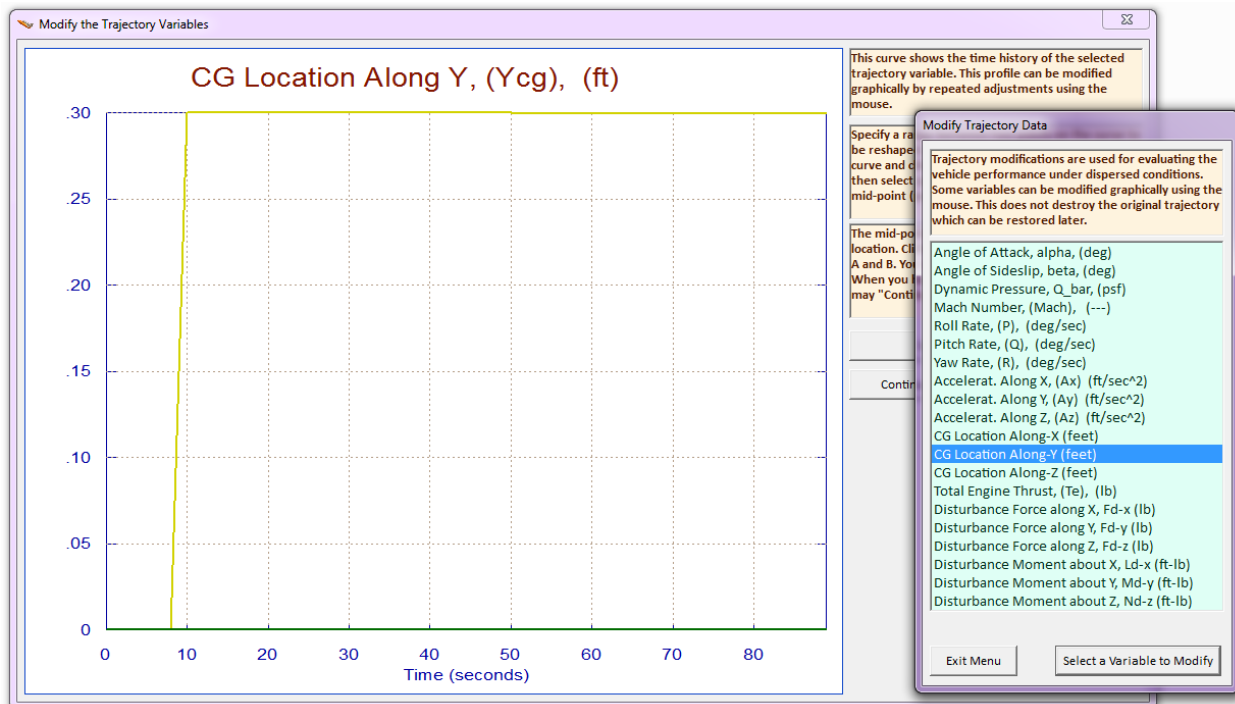Surface & Engine Deflections/ Thrusts, Rocket-Plane, Mission-1, Ground Takeoff

The Body-Flap and the pitch engine deflection (green lines) are now within the schedule limits. It appears that the Elevon and the ±Z RCS jet "Throttle 2" are now compensating in pitch for the scheduling of the two effectors.

## 1.10 Lateral Trimming with a YCG Offset

In our previous example, using file "Hyper2.Engn", the vehicle was trimmed in all 6 directions, but we did not really examine the effects of an excitation in the lateral directions. Being able to trim in all 6 directions means that the vehicle has the control authority to counteract not only disturbance moments but also forces along the x, z and y directions. It means that it can maintain the accelerations required by the trajectory even in the event of a disturbance or a CG shift because it has the effectors authority to compensate against the disturbances directly without having to modify its trajectory and its $\alpha$ and $\beta$ angles. One way to upset the balance of moments and forces in the lateral axes is to shift the CG in the y direction towards the right wing. The main engine thrust through the vehicle center axis will create a positive yawing moment and the other effectors must react against it.

The first thing to do is modify the YCG. Continuing from section 1.9 and after returning to the Trim main menu, go to option (2) which plots the trajectory data and then modify the CG as it was described in Section (1.3), using the "Graphic Options", and then from the vertical pop-up menu select "Modify a Trajectory Plot Using the Mouse". From the menu select "CG Location Along Y". The YCG initially is at zero (green line at the bottom) but its time history can be modified using the mouse, by shifting it 0.3 (ft) to the right, starting at 10 sec. Then click on "Save the Modified Trajectory" and the modification will remain in memory for the trim analysis. Click on "Exit Menu" and return to the trajectory plots to check that the YCG has been changed and also the trajectory title is now "User Modified Trajectory". The original trajectory may later be restored by going back to the "Graphic Options" menu and clicking on "Restore the Original Trajectory".

Surface & Engine Deflections/ Thrusts,  User Modified Trajectory ...
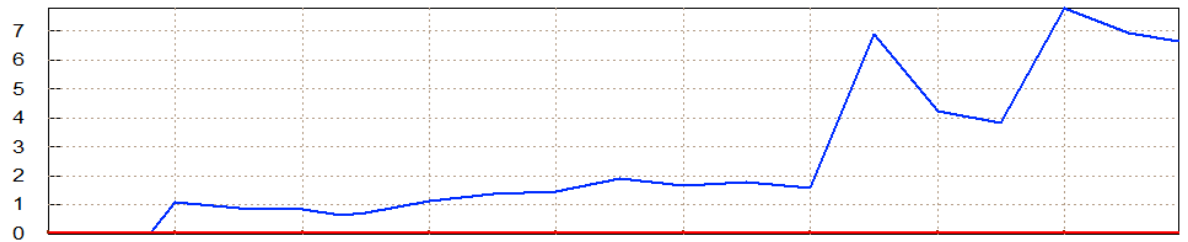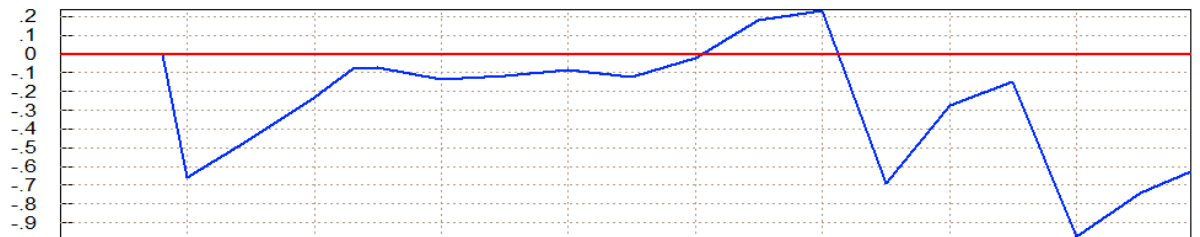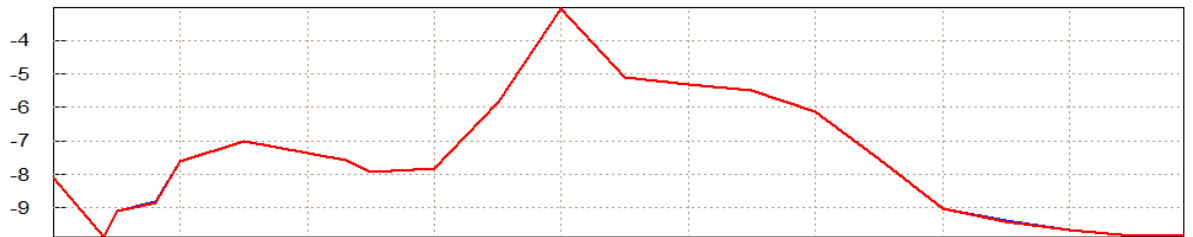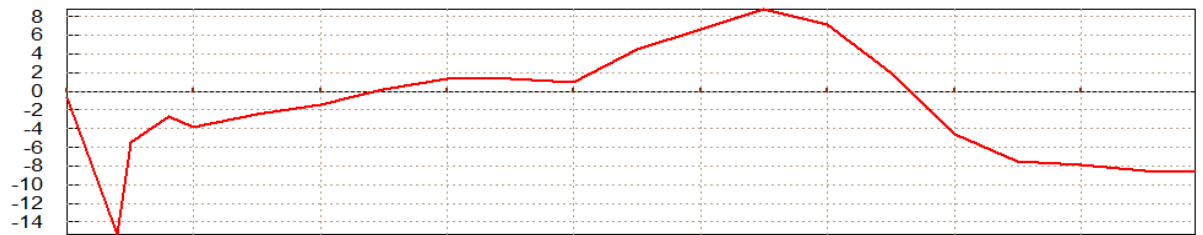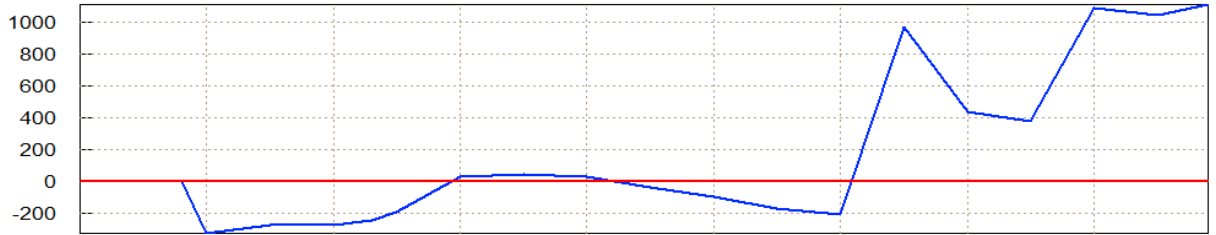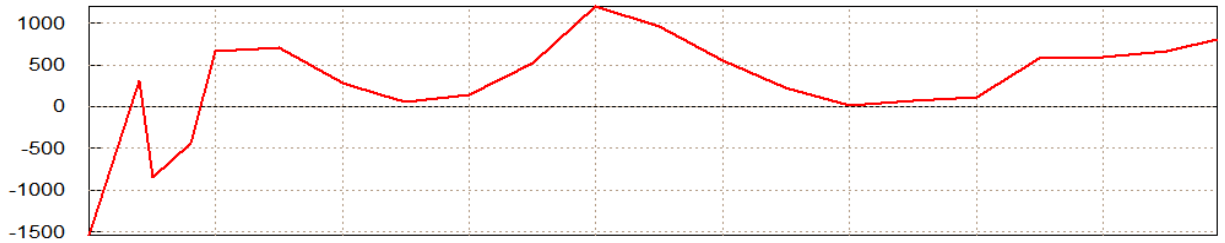
## 2.0 Descent Phase



When the vehicle reaches an altitude of 76,000 (ft) the descent phase begins. During descent the engine and RCS jets are turned off, the vehicle banks to the left and it glides and lands on a runway like a space shuttle controlled by the four aero-surfaces. We are going to use the "Trim" program to analyze the descent part of the trajectory, calculate the trim angles, and evaluate the vehicle performance along the trajectory by using also contour plots and vector diagrams. We will also show how to use Trim to schedule the position of the Body-Flap as a function of time. Dynamic modeling and control design will also be performed at a selected flight condition. The control gains will be used in a 6-dof non-linear simulation model.

### 2.1 Descent Data Files

The data for this hypersonic vehicle example during descent is in folder "*C:\Flixan\Trim\ Examples\ Hypersonic Vehicle \Descent*". This folder includes data files which are inputs to Trim, and also the files which are generated by Trim. Let us first describe the input data files.

- The descent trajectory file is "*Hyp_Desc.Traj*". It starts from an altitude of 76,000 (ft), at Mach: 4.5, and with a negative alpha= -1.7°. Then it banks to the left to change direction and align itself with the landing site. It maintains a positive $\alpha$= 5° during most of the descent flight. The max dynamic pressure is 1000 (psf) and the flight duration is approximately 1100 sec. There is also a modified version of this trajectory that has the $Y_{CG}$ off-centered towards the right "*Hyp+Ycg.Traj*".
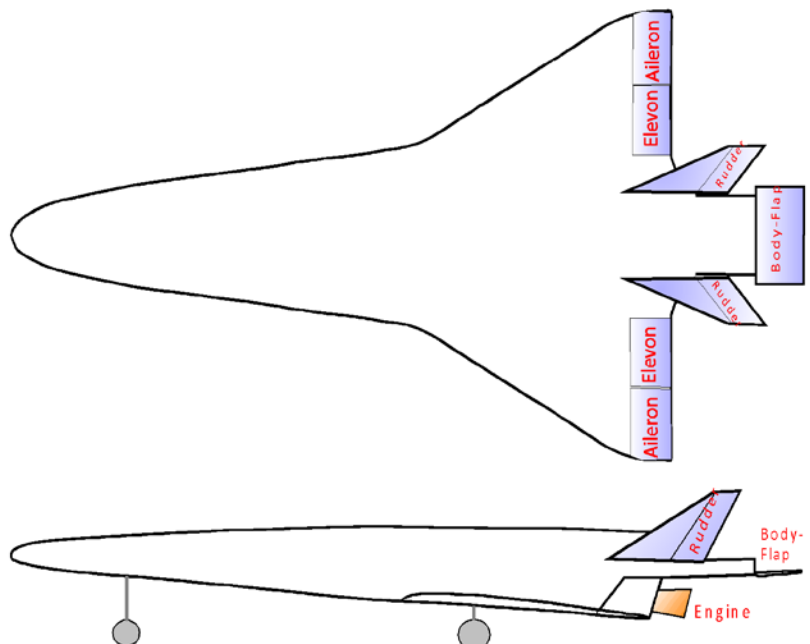- The basic aero data is in file "*Hyp_Desc.Aero*".

- The four aero-surface coefficient increments are in file "*Hyp_Desc.Delt*".
- The file "*Hyper.HMco*" contains the hinge moment coefficients data for the four aero-surfaces. It contains a 4-dimentional array of coefficients for each surface (as a function of Mach, alpha, beta, and delta) and it is very similar to the aero-surface coefficients file.
- The file "*Hyp_Desc.Unce*" contains the aerodynamic uncertainties data.
- The mass properties are in file *"Hyper.Mass"*. The first column contains the vehicle mass in (slugs), and the remaining columns contain the corresponding inertias and CG location. Only the last mass point is used from this file (450 slugs) because the weight is not changing during descent.

There is no engine and jet data file in this example because the vehicle does not use any gimbaling engines or jets during descent. The files generated by "Trim" are:

- "*Hyp_Desc.Trim*" that contains the control surface trim positions along the trajectory. When creating multiple trims from the same trajectory file, the two previous trim trajectories are saved in files "*Hyp_Desc1.Trim*" and "*Hyp_Desc2.Trim*".
- "*Hyp_Desc.Perf*" contains the performance parameters data along the trajectory. The two previous performance parameter data are saved in files: "*Hyp_Desc1.Perf*" and "*Hyp_Desc2.Perf*".
- "*Hyp_Desc.HiMo*" that contains the moments at the control surface hinges in (ft-lb)

## 2.1 Checking the Trajectory Data

Let us begin the analysis by checking out the trajectory first. Start the Flixan program and select the descent project folder "*C:\Flixan\Trim\Examples\Hypersonic Vehicle\Descent*", that contains the analysis files. Then, from the Flixan main menu select "*Analysis Tools*", from the drop-down menu select "*Flight Vehicle/ Spacecraft Modeling Tools*", and then "*Trim/ Static Performance Analysis*". From the following dialog select the input files that will be used by the Trim program, as shown below. From the input/ output filenames menu select the default "*NewFile*" names because we are not using any specific files for the time being.

Angles of Attack/Sideslip/Flight Path (deg), Rocket-Plane Mission-1, D



Velocity, Dynamic Pressure, Rocket-Plane Mission-1, Descent Phase
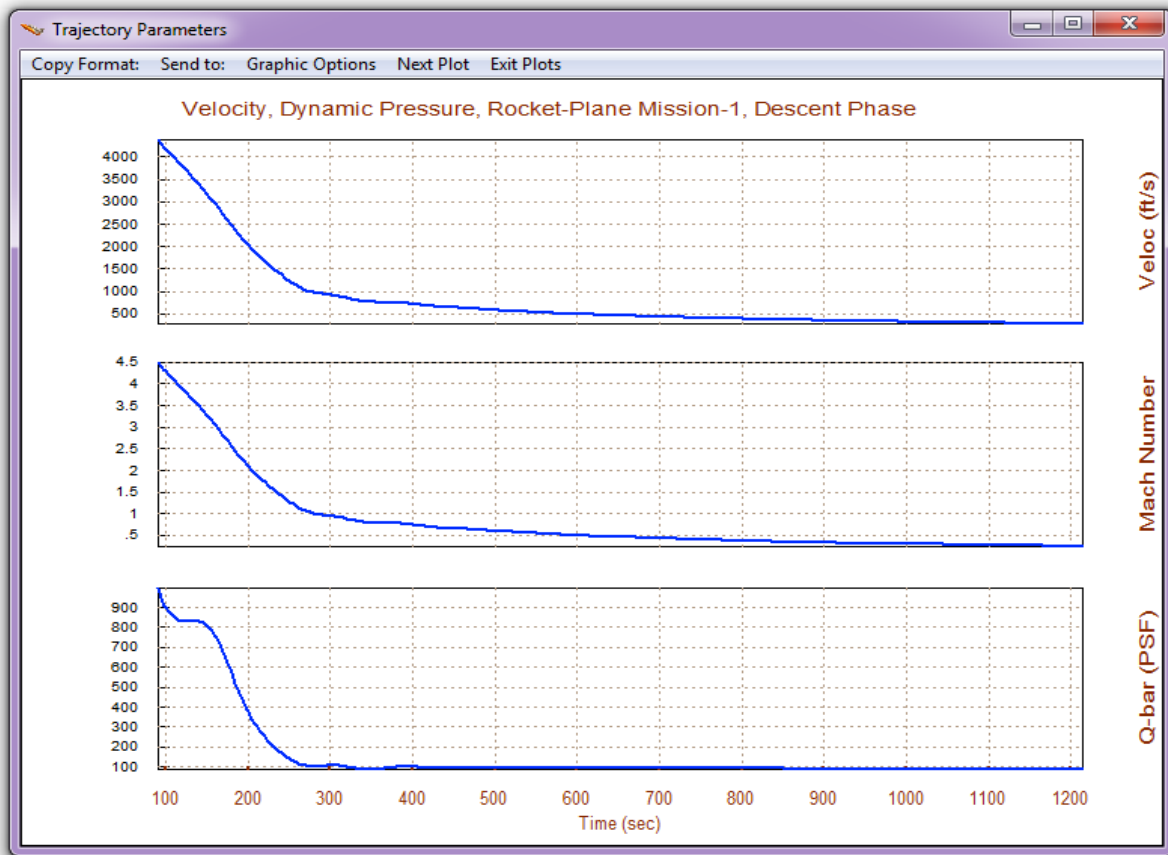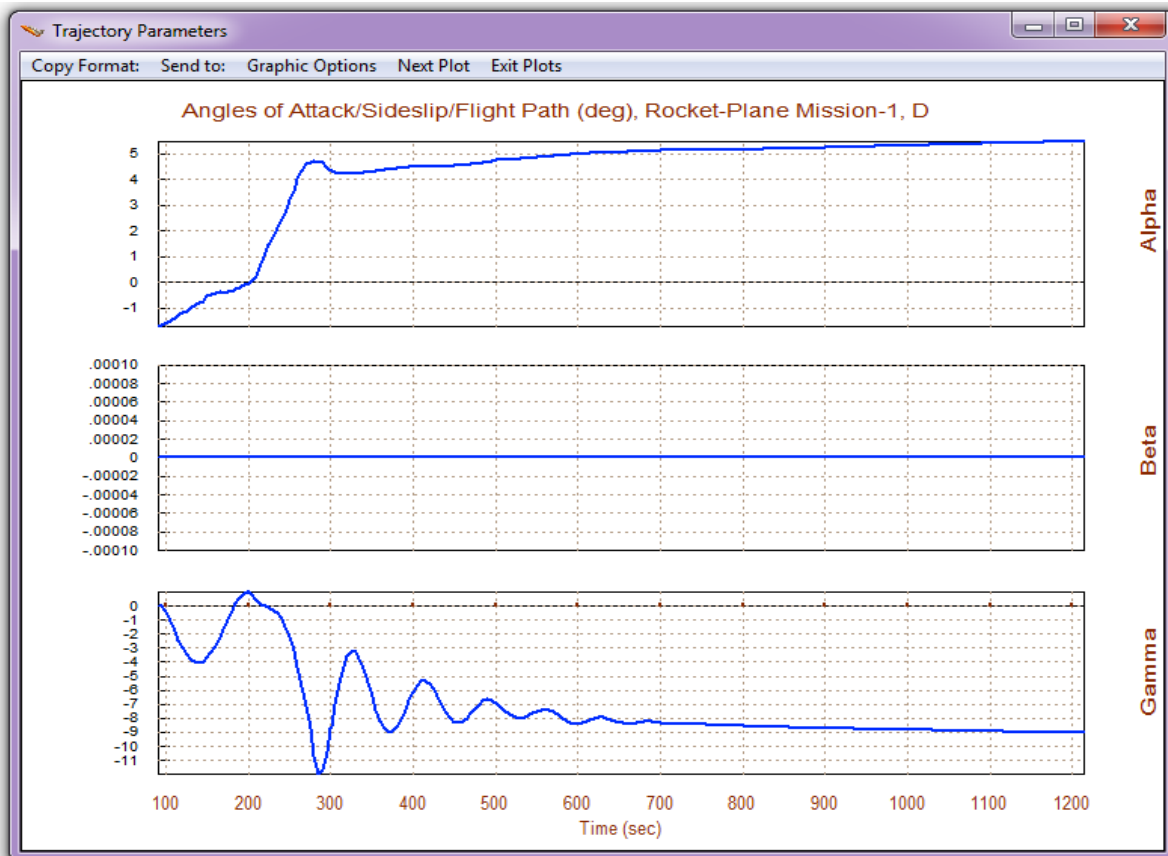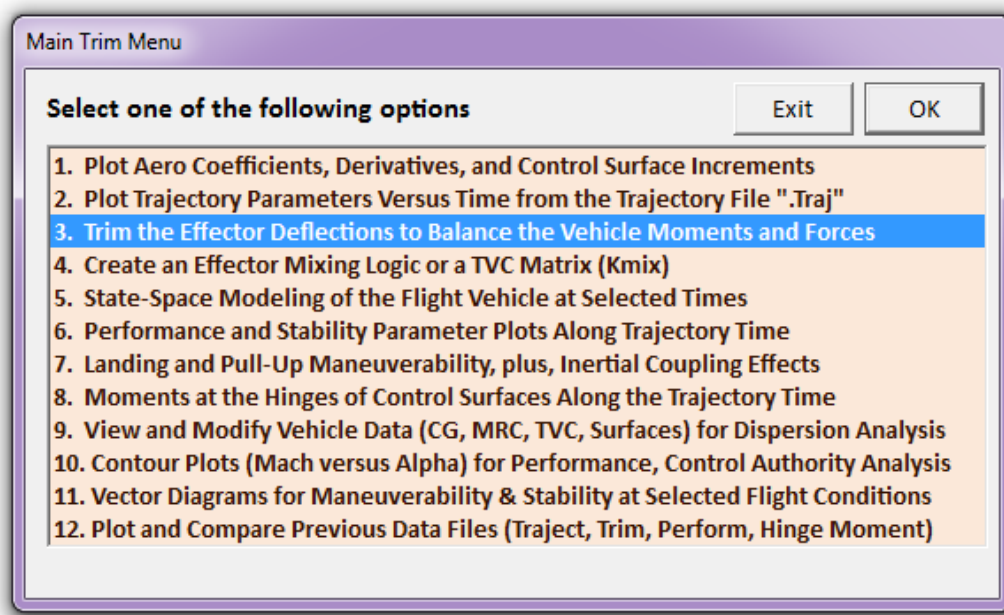
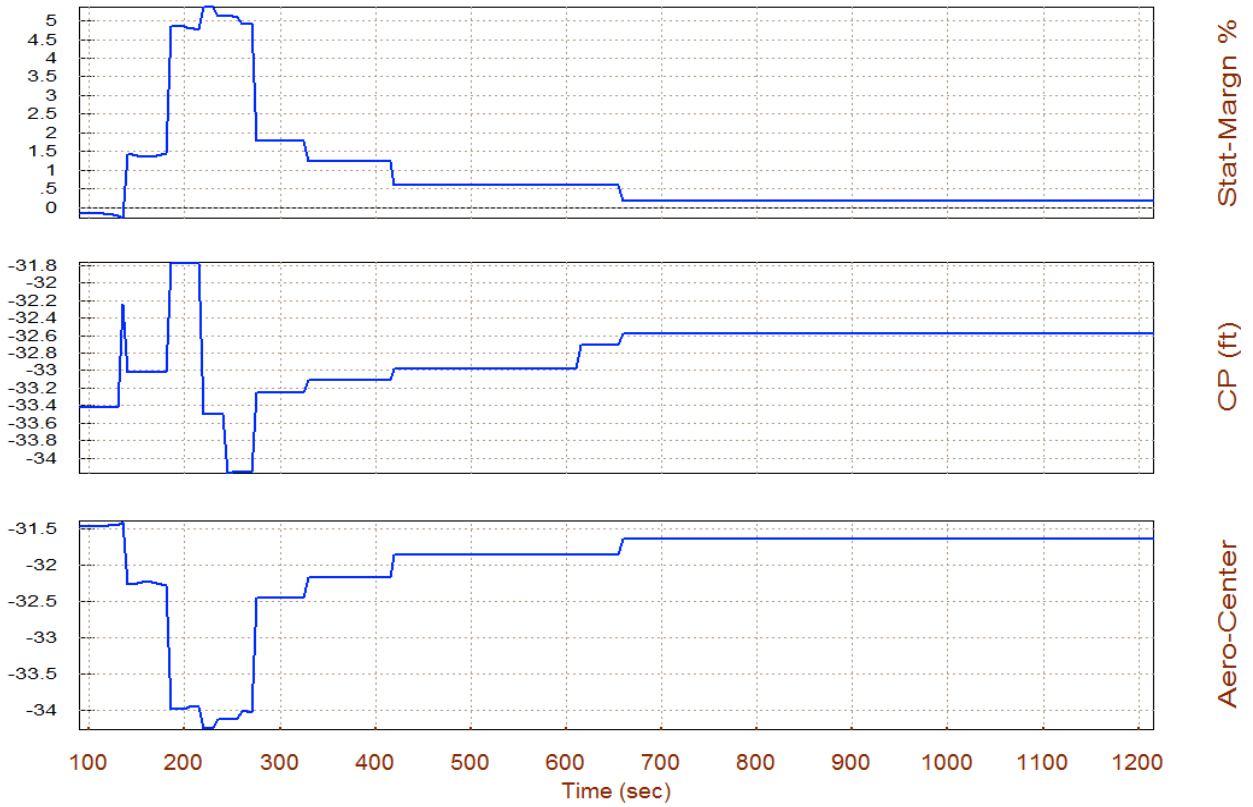## 2.2 Trimming and Performance Analysis During Descent

The vehicle during descent uses the Elevon and the Body-Flap for trimming and control in the longitudinal directions. With two aero-surfaces it may be possible to independently control and to trim not only in pitch but also in z-acceleration. However, before we decide on how to allocate longitudinal controllability we must first evaluate if the Body-Flap (BF) can provide sufficient controllability to trim the vehicle in the z-acceleration direction, independently of pitch which is trimmed mainly by the Elevon. If the BF is sufficiently effective in the z-direction we may decouple the pitch and z-acceleration directions and to control them both independently. Otherwise, the only use for the BF would be to assist the Elevon in pitch trimming and possibly also for pitch control. It may be used, for example, to bias the trim position of the Elevon so that it is more centered and improve its effectiveness. Plus controlling and trimming the normal acceleration independently of pitch may not be a very valuable feature because normal acceleration can also be achieved by means of alpha pitching. In such a case the BF can be used along side with the Elevon by scheduling it to increase the Elevon effectiveness and also assisting parallel with the Elevon in pitch control by deflecting it relative to its scheduled position. We must, therefore, trim and analyze the vehicle controllability against aero-disturbances in both cases: (a) when the BF is operating in parallel with the Elevon to trim and control in pitch, and (b) when the BF is used independently of the Elevon to trim and control normal acceleration. We must examine both cases to determine which approach is better fitting.

**Trimming and Controlling Only 3 Moments**

Let us first trim along the 3 rotations. During descent the bias in the Body-Flap aero-surface data file "*Hyp_Desc.Delt*", is set to zero. Return to the Trim main menu and select the third option for trimming the control surfaces along the selected trajectory.



Main Trim Menu

Select one of the following options     Exit     OK

1. Plot Aero Coefficients, Derivatives, and Control Surface Increments
2. Plot Trajectory Parameters Versus Time from the Trajectory File ".Traj"
3. Trim the Effector Deflections to Balance the Vehicle Moments and Forces
4. Create an Effector Mixing Logic or a TVC Matrix (Kmix)
5. State-Space Modeling of the Flight Vehicle at Selected Times
6. Performance and Stability Parameter Plots Along Trajectory Time
7. Landing and Pull-Up Maneuverability, plus, Inertial Coupling Effects
8. Moments at the Hinges of Control Surfaces Along the Trajectory Time
9. View and Modify Vehicle Data (CG, MRC, TVC, Surfaces) for Dispersion Analysis
10. Contour Plots (Mach versus Alpha) for Performance, Control Authority Analysis
11. Vector Diagrams for Maneuverability & Stability at Selected Flight Conditions
12. Plot and Compare Previous Data Files (Traject, Trim, Perform, Hinge Moment)

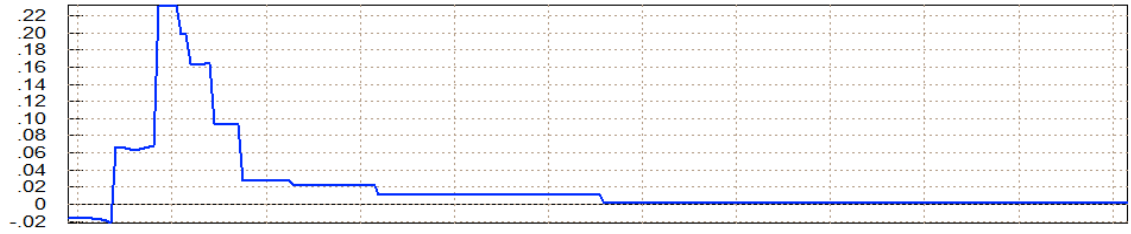Static Margin, Center of Pressure, Aero-Center (ft), Rocket-Plane Mission-1, Des

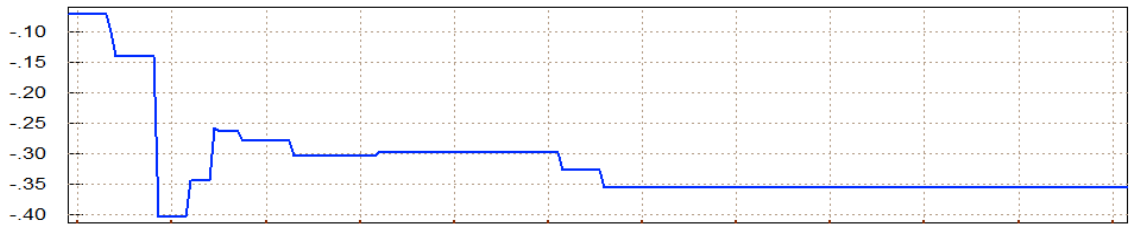Short-Period (w)/ Time-to-Double-Ampl-Inverse (/sec), Q_alpha_beta (deg-lb/ft^2)
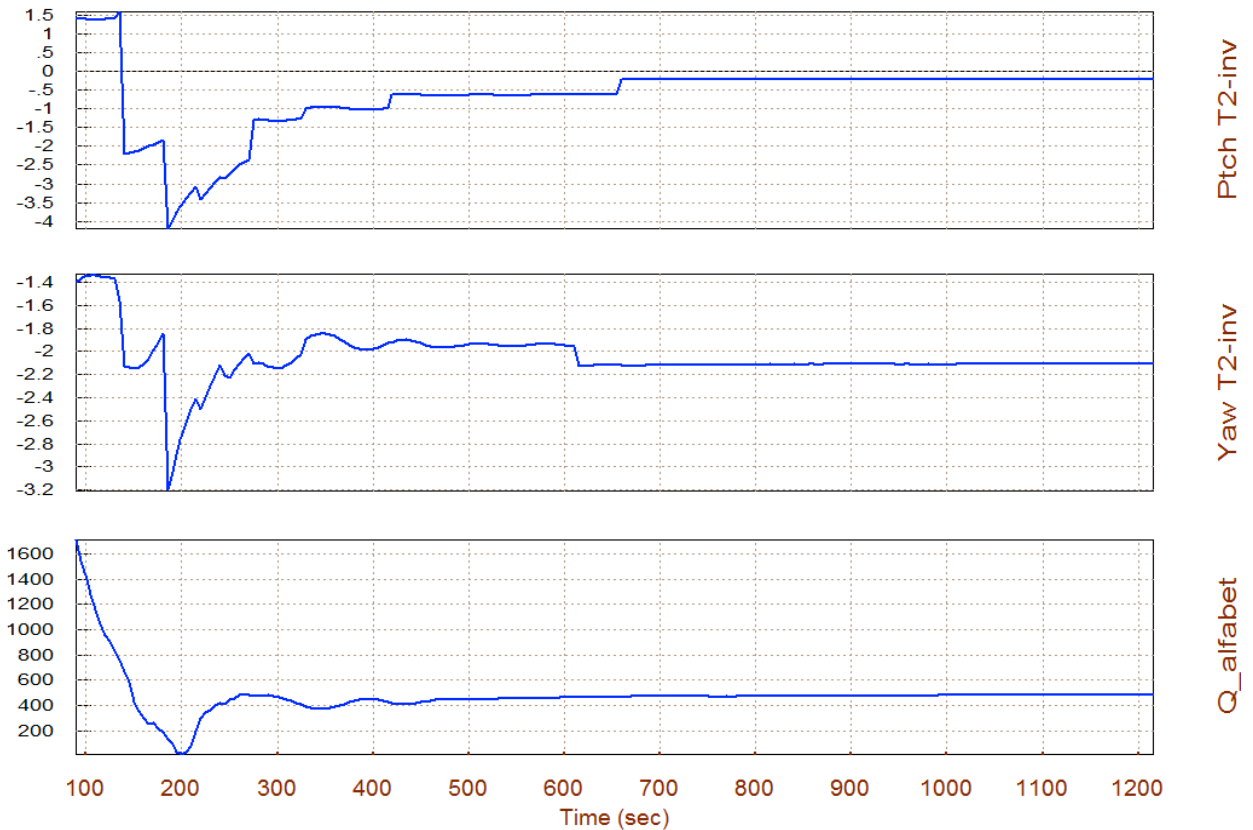
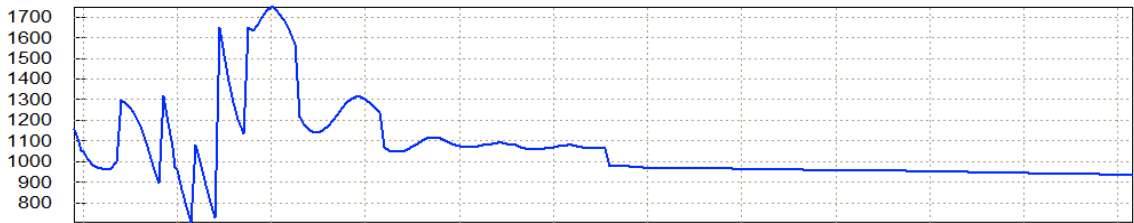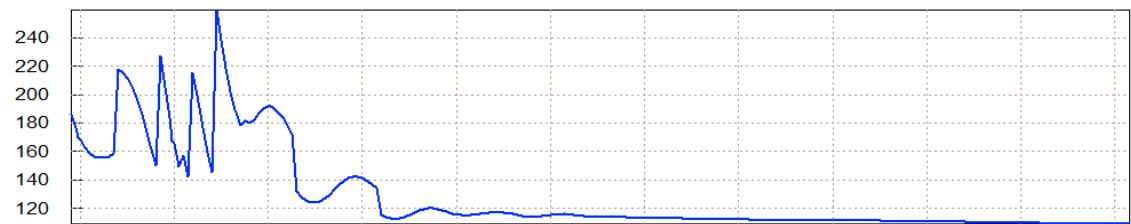# Rotation Control Authority (dQ/dQmax)<1 for  3 (deg) of Alpha & Beta Variation
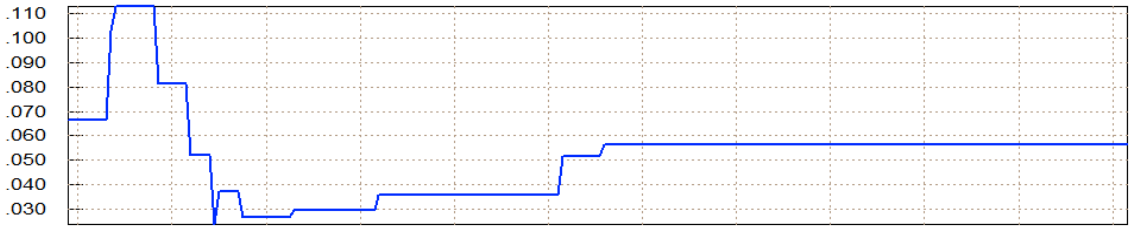


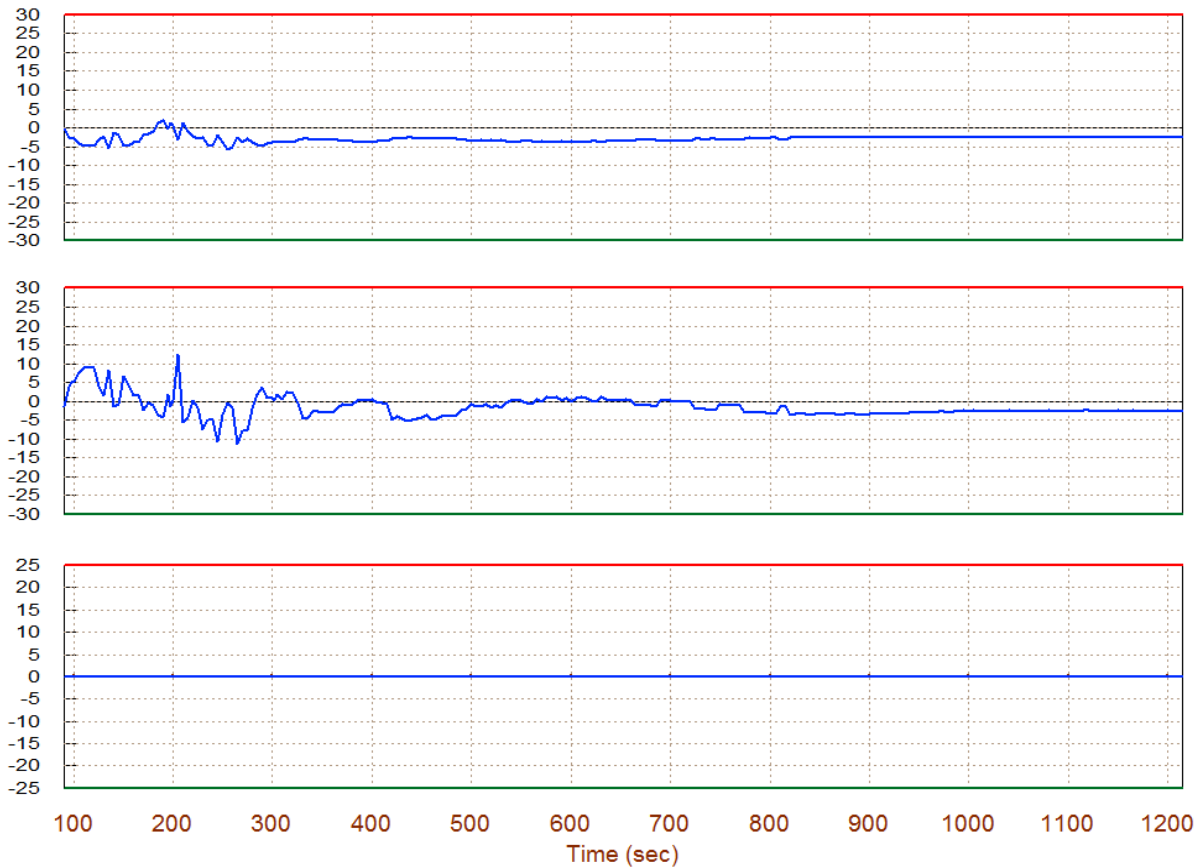# Max Angular Accelerat (deg/sec^2), at Max Control Demand Rocket-Plane Mission-1,
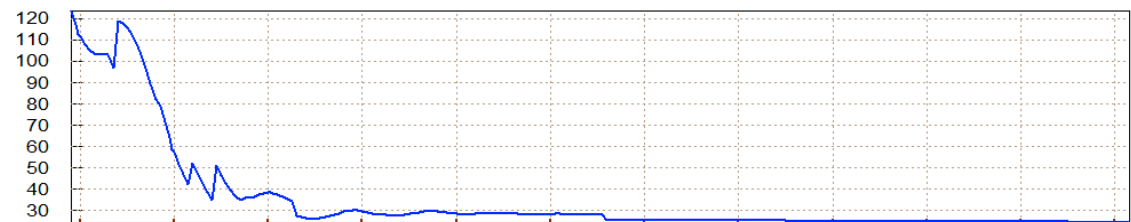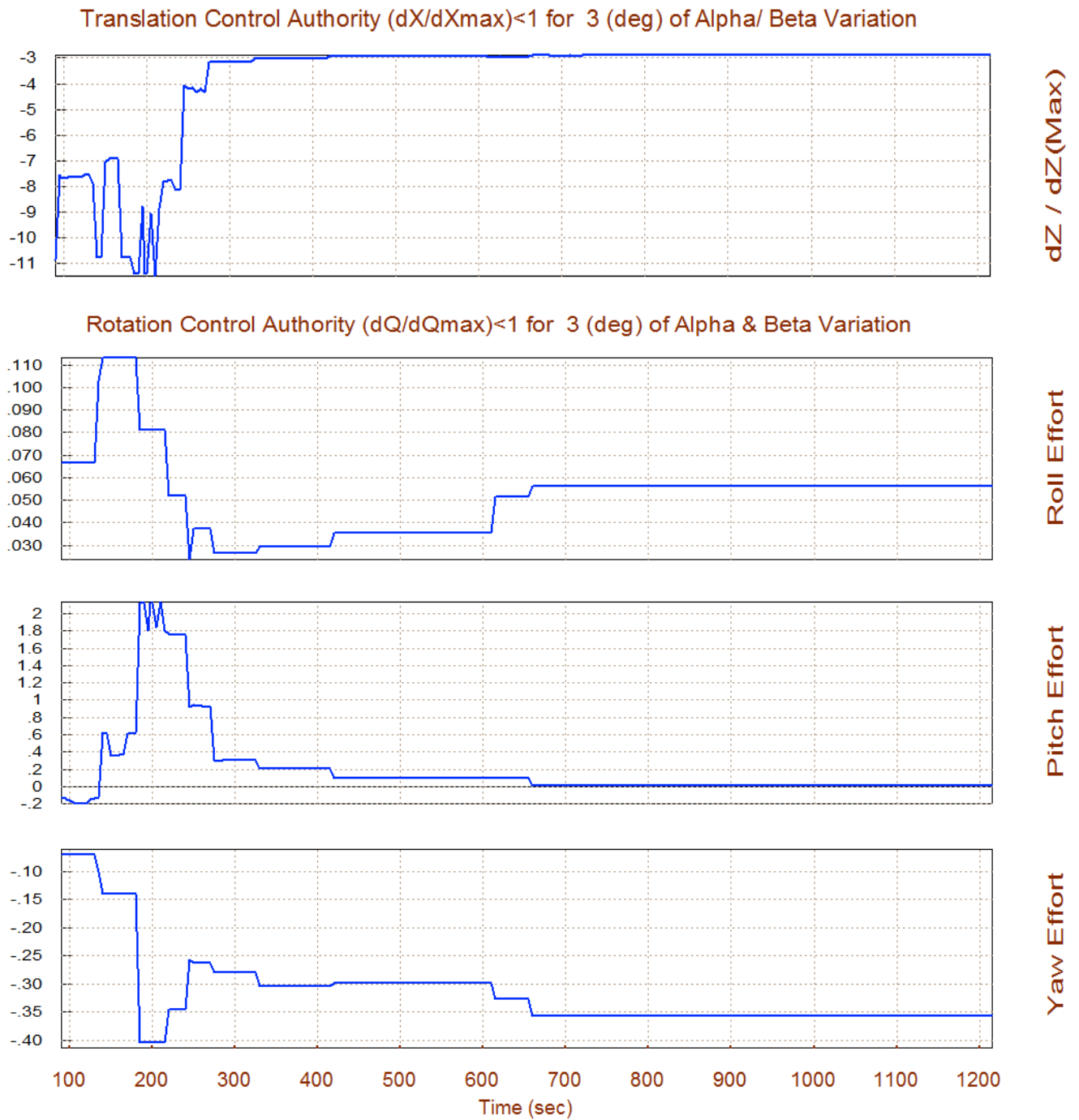
**Trimming and Controlling 3 Moments plus the Z-Acceleration**

Now let us return to the Trim main menu and select option (3) again to re-trim. This time we will include the Z-acceleration, in addition to the three rotations, to see if it is possible to trim along the 4 directions. It means that the vehicle will use the Elevon and mainly the BF to also trim along the normal acceleration which is defined in the trajectory.
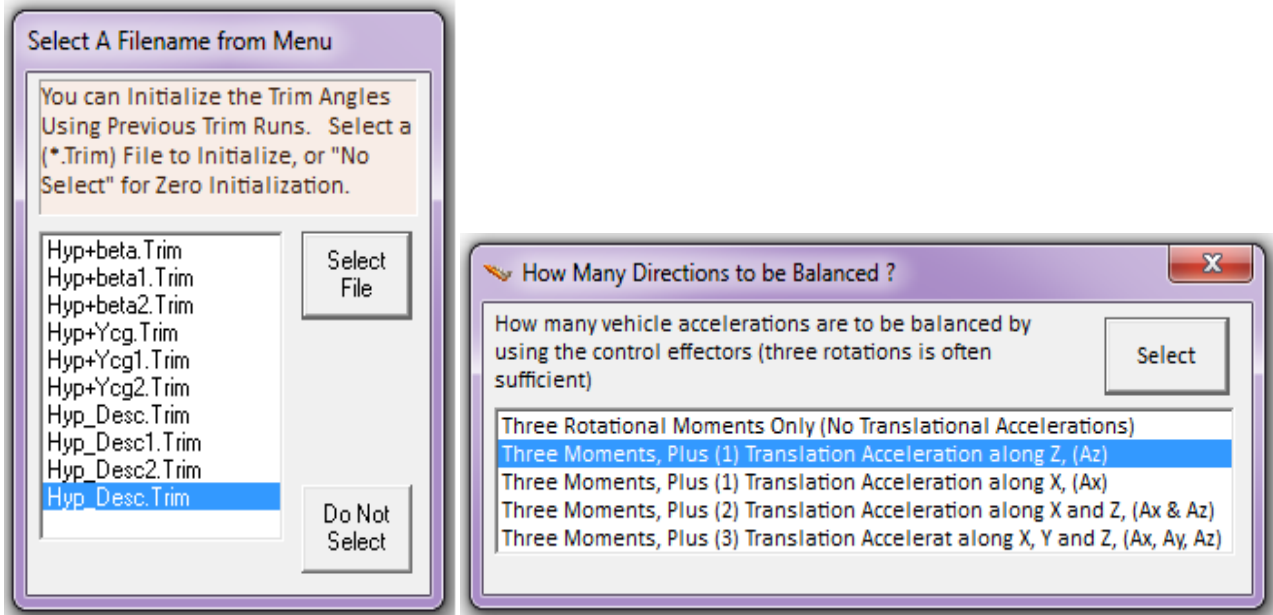




Surface & Engine Deflections/ Thrusts, Rocket-Plane Mission-1, Descent Phase

Translation Control Authority (dX/dXmax)<1 for 3 (deg) of Alpha/ Beta Variation

Rotation Control Authority (dQ/dQmax)<1 for 3 (deg) of Alpha & Beta Variation

The longitudinal controllability is greatly reduced when attempting to independently control Nz and pitch directions. The control effort in the pitch axis is violated at times between 180 to 250 sec. It means that 3° of alpha disturbance is too much for pitch control system to handle. The authority in the Nz control loop, shown in the top plot is even worst. The magnitude of the z-axis control effort ($\delta\ddot{z}/\delta\ddot{z}_{max}$) greatly exceeds the max allowable effort that should be less than 1. When operating independently, the two controllers are incapable of producing the control authority required to overcome wind disturbances in neither direction. They can, however, join forces together to control the pitch direction, which also controls Nz indirectly via $\alpha$.

## 2.4 Additional Performance Parameters

We now return to the Trim main menu and select option (7) where we will analyze some conditions which are mostly applicable to a maneuvering aircraft. This analysis is performed at specific flight conditions and not along the trajectory.
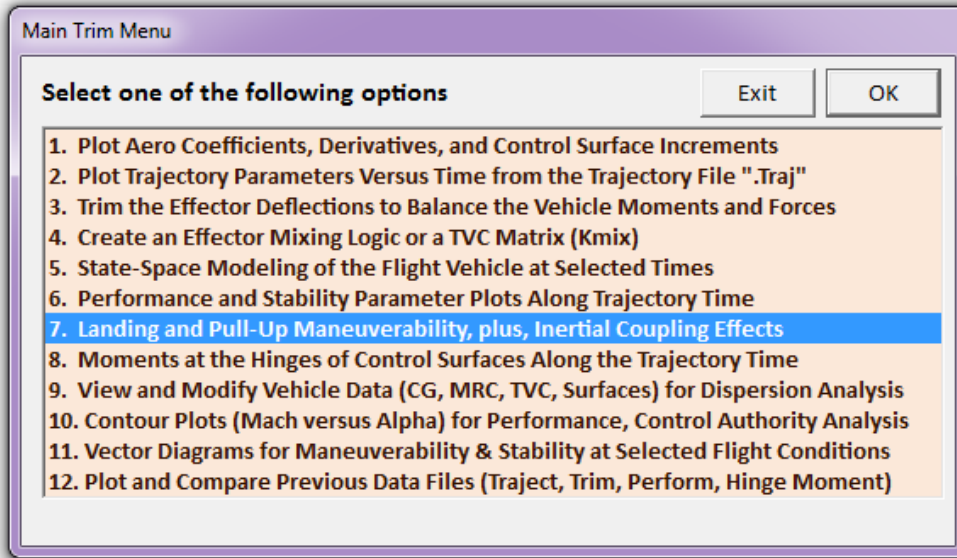


The following menu consists of 4 options that allows the user to select one of the maneuverability analysis conditions, as described in Section 3. You must enter a flight time in the data input field within the trajectory time range, and click on "Select Time". You may then select the first option in the menu which analyzes the control authority of the vehicle's pitch effectors to perform a pull-up maneuver from level flight, as described in equations (3.41 and 3.42), and then click on "Select Option".
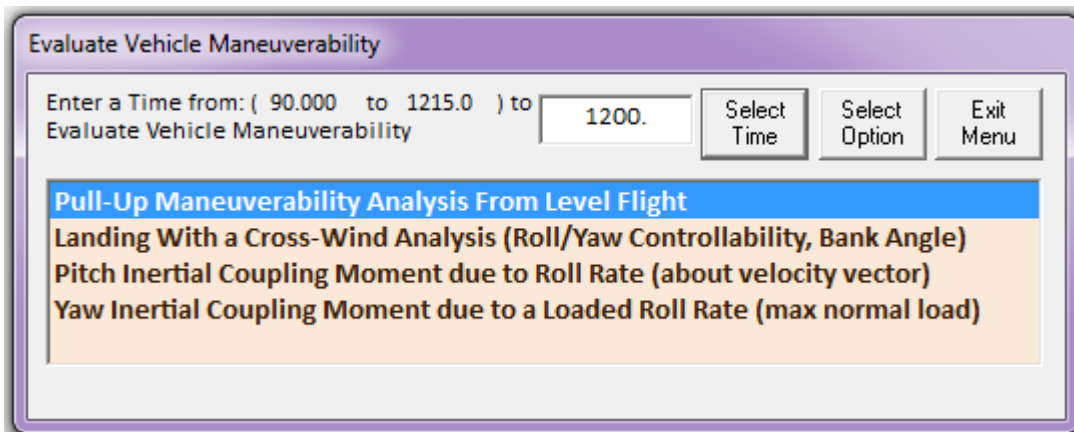


The program requires a (4x3) effector combination matrix, like before, so we let the program calculate a matrix by clicking on the second option in the next dialog "*Create a Mixing Matrix Using All Effectors at 100% Participation*". Another dialog comes up that is specific to the pull-up maneuverability option and where the user enters and receives data. In the light cyan fields you

In the fourth test we assume that the vehicle is capable of maneuvering like a fighter aircraft and it is rolling under max normal load. During such a loaded pullout maneuver there is a significant amount of yawing moment generated due to the inertial pitch/roll coupling that has to be taken out by the rudder (or yaw control). We are interested to calculate what is the max roll rate under peak normal load that the vehicle is able to perform without saturating the rudder. Select the fourth option from the menu and the corresponding dialog comes up on the right. In the cyan fields we must enter the angle of attack (5°), the yaw controllability margin factor (0.7), and the max normal load factor (4-g's). Click on the "*Update Inputs*" button and the yellow field at the bottom is showing us that a maximum roll rate of 36 (deg/sec), is a safe pullout roll rate under max loading that will keep the rudder within range. This completes the additional performance tests.
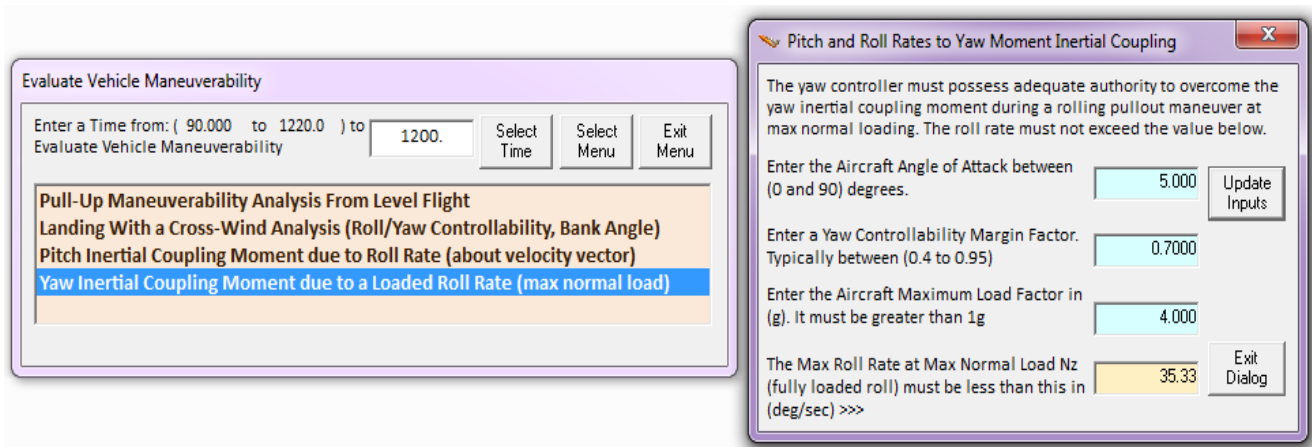


## 2.5 Scheduling the Body-Flap

In section (2.2) we attempted to use the Body-Flap as a means to control and trim the vertical acceleration independently of alpha. We concluded that although this is theoretically possible, it is not feasible, however, because it does not produce the controllability required to counteract the normal forces generated when the vehicle deviates a couple of degrees from nominal $\alpha_0$. The pitch axis controllability was also degraded because the effectors control authority was divided in two directions. We also concluded that normal acceleration can be controlled indirectly by controlling $\alpha$ via pitch control and, therefore, we compromised by trimming 3-rotational dof and a flight control system using 3 control loops. So is there any use for the Body-Flap? Yes, it can be used for pitch control in combination with the Elevon. We can design a mixing logic matrix that combines them in parallel for pitch control. But in addition and most importantly it can be biased or scheduled to offer better trimming conditions
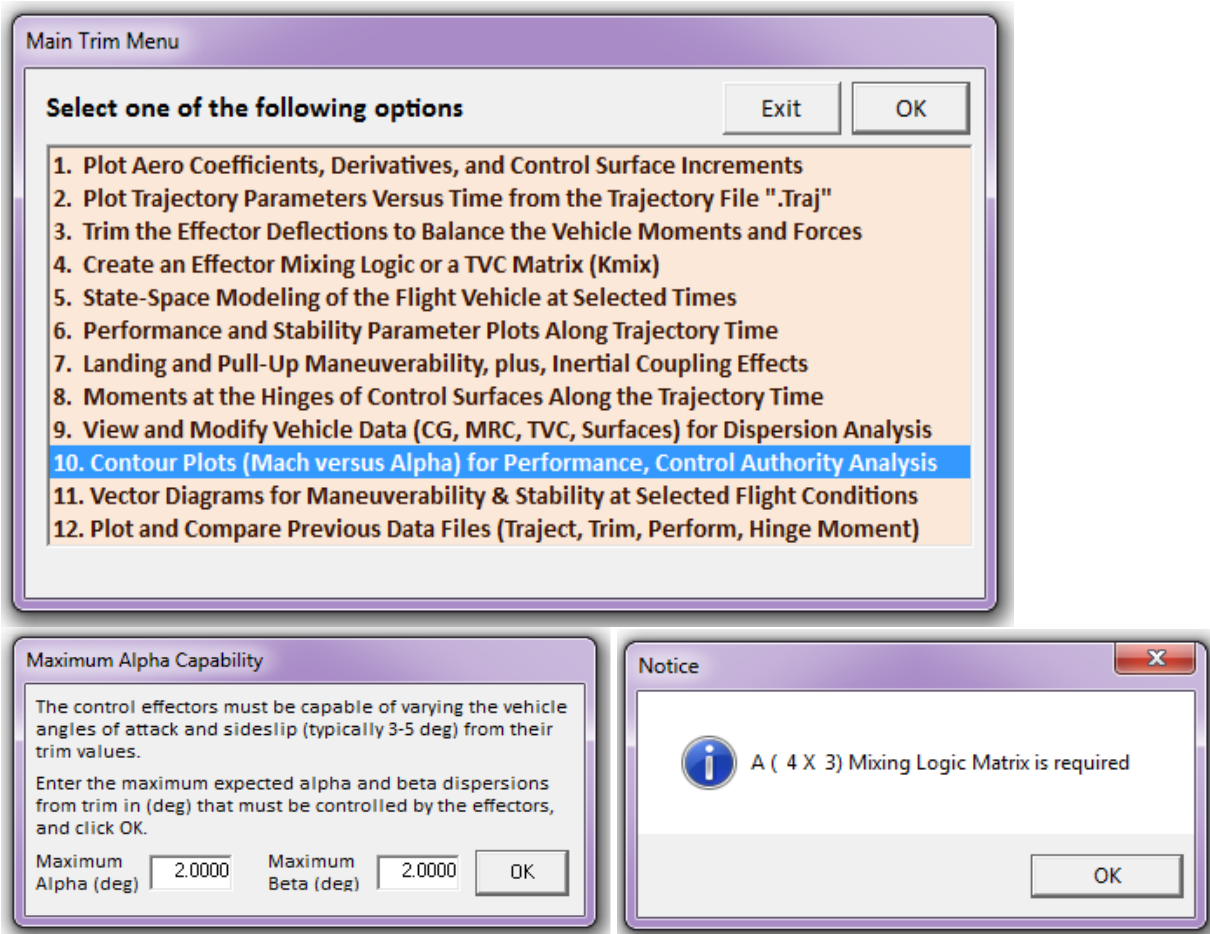
**Effector 2: Body Flap**



**Effector 2: Body Flap**
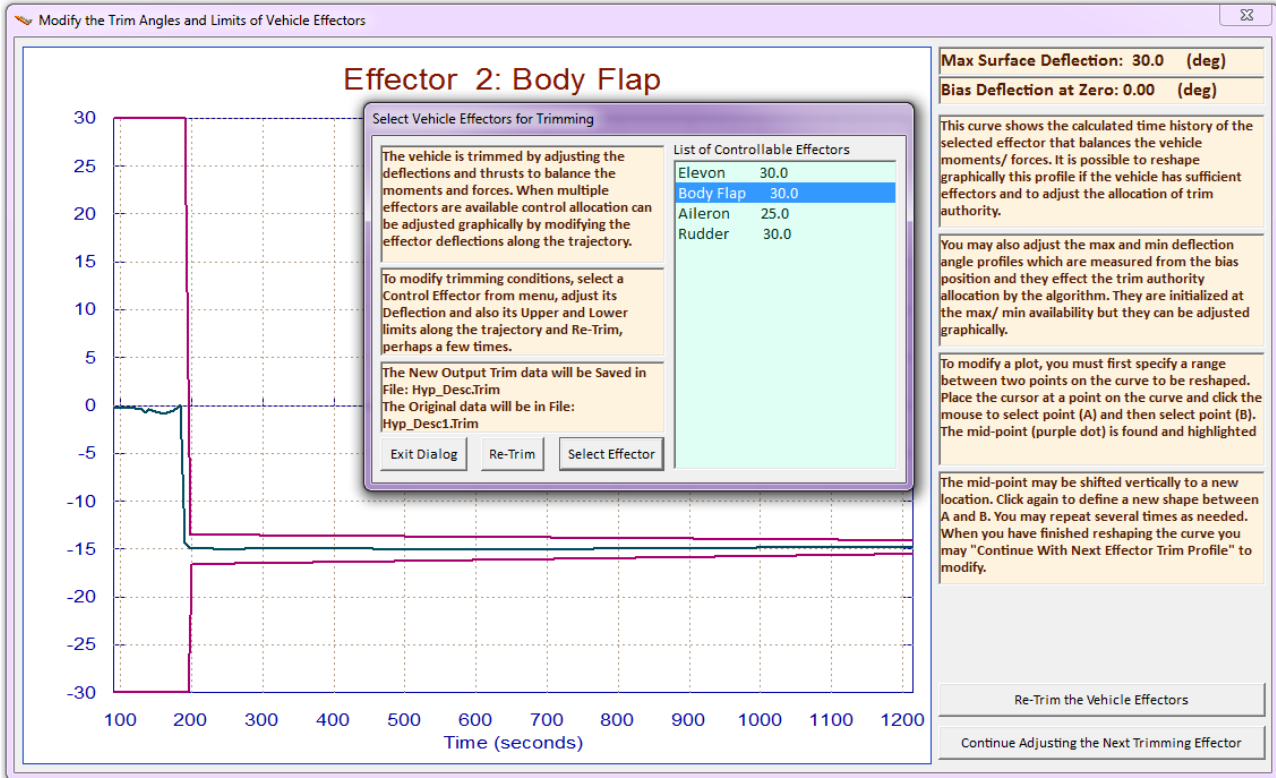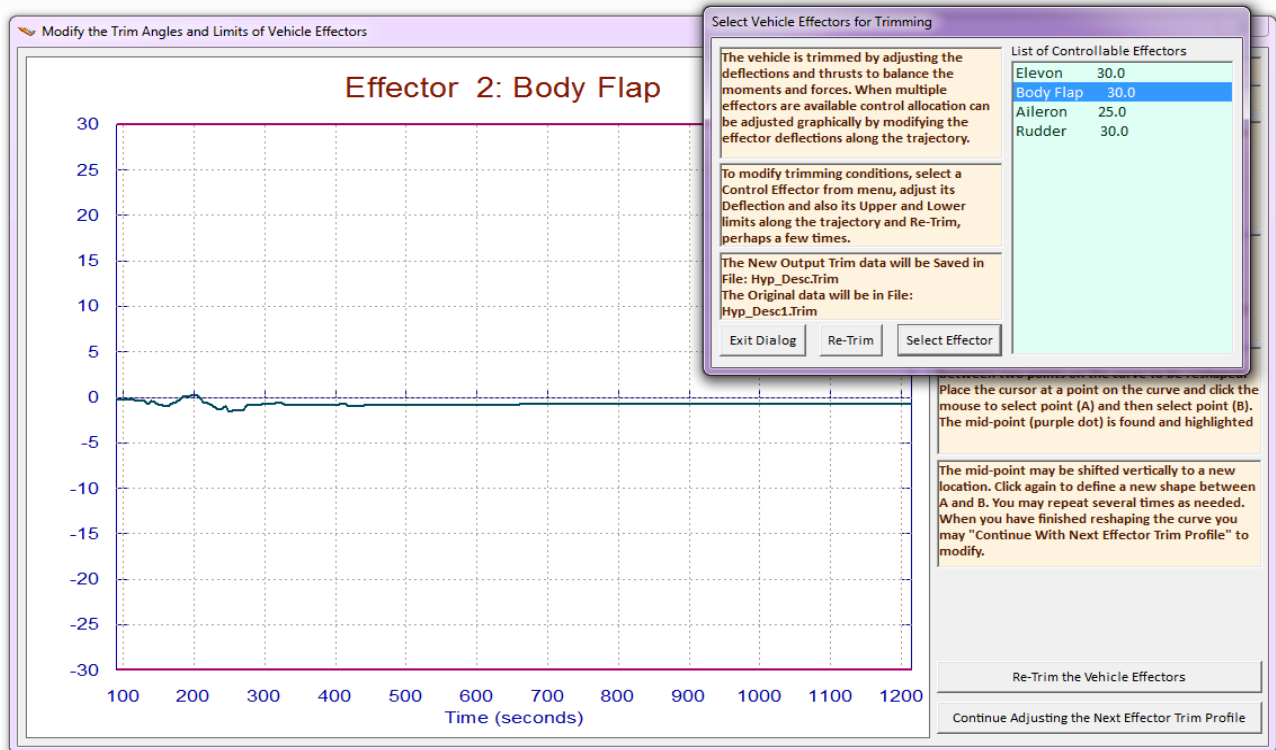
A menu comes up showing a list of the four aero-surfaces. We select the Body-Flap to modify and click on the "*Select Effector*" button. The dialog/plot on the top shows the original trim history of the Body-Flap (green) as it was calculated by the Trim program the first time. It also shows its upper and lower limits (magenta lines) which initially they are ±30° according to the aero-surface data file. We would like to re-schedule this trim trajectory and make the Body-Flap more negative which, hopefully, will force the Elevon

to trim closer to zero. The second plot shows the modified Body-Flap trajectory after it was reshaped by the analyst using the mouse. The modified BF deflection was set at a constant value -14°, after t=200 seconds, as an attempt to reduce the Elevon deflection in that range. The upper and lower BF limits are also reduced from ±30° to smaller values, because reducing the limits it bounds and de-emphasizes the effector priority during trim and the deflection will not change as much from its set value after re-trimming. When the user modifications are complete, click on "*Re-Trim*" on either dialog, and the program will generate a new trim history, as shown below. As you can see, the user modified Body-Flap trim history was not altered by re-trimming, but the Elevon deflection was reduced to zero, as planned. The next plot is for evaluating the trim success, showing that a perfect moments balance was achieved without any residuals. We should now compare the latest trim results against the original trim, so we should save the latest trim file before it gets over-written by another re-trim and we rename it from "*Hyp_Desc.Trim*" to "*Sched_BF.Trim*".



Surface & Engine Deflections/ Thrusts, Rocket-Plane Mission-1, Descent Phase
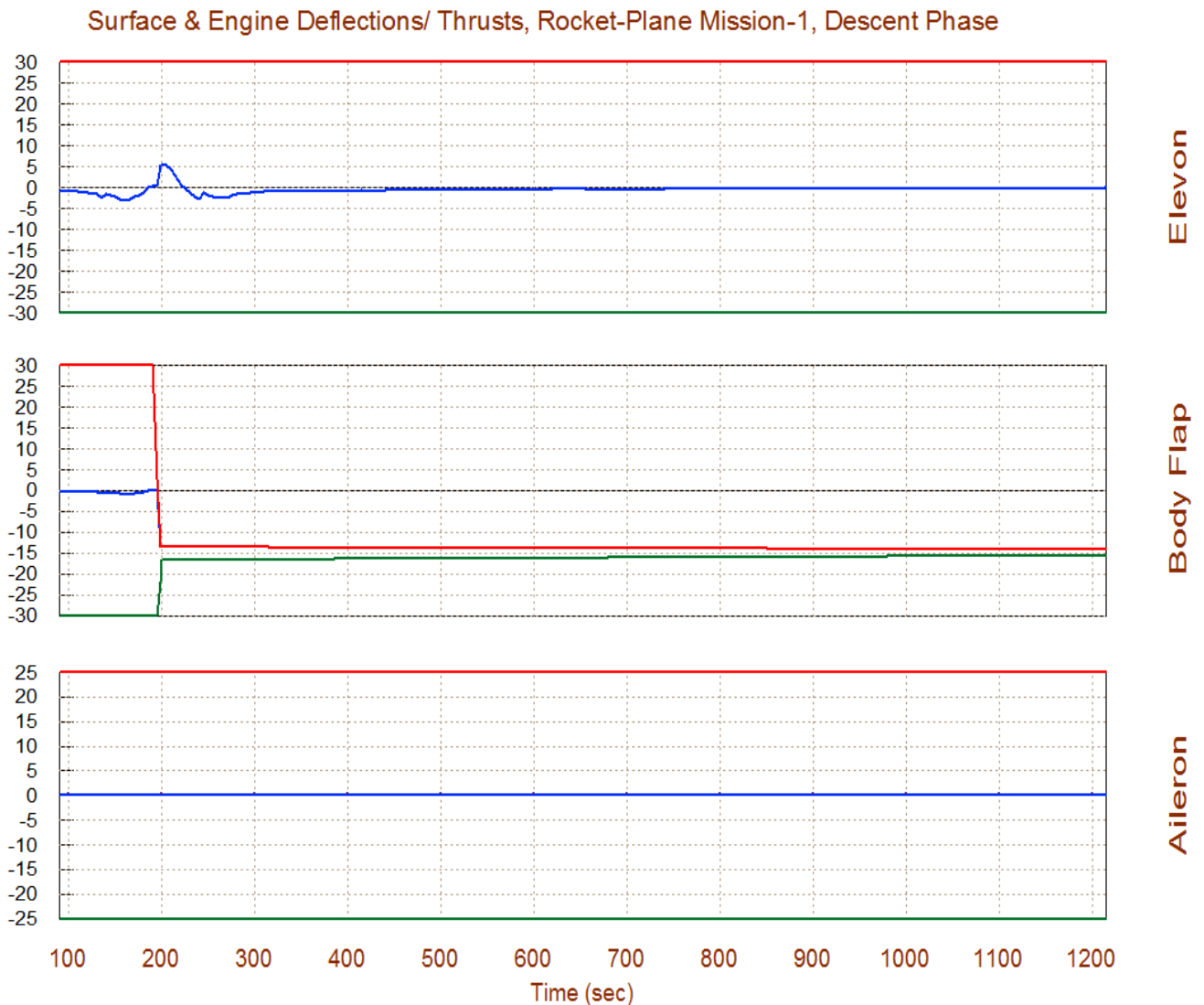
## 2.6 Stability & Performance Analysis Using Contour Plots

As we discussed earlier, contour plots are 3-dimensional plots that provide a broader perspective of vehicle performance versus Mach and alpha rather than in the vicinity of the trajectory. From the main menu select option (10) to create contour plots of some important parameters against Mach and Alpha. In the next dialog we must specify the aero disturbance environment which in steady-state is defined by the maximum angles ($\alpha_{max}$ and $\beta_{max}$) relative to the wind. Enter 2° for both $\alpha_{max}$ and $\beta_{max}$, as shown below. The next step is to create a control surface combination matrix. We will again allow the program to calculate a mixing matrix along the trajectory by selecting the second option in the Mixing Logic selection dialog (all effectors at 100% participation). The (4x3) effector combination matrix converts the 3 flight control (roll, pitch, yaw) demands to 4 aero-surface commands.

**Main Trim Menu**

Select one of the following options          Exit      OK

1. Plot Aero Coefficients, Derivatives, and Control Surface Increments
2. Plot Trajectory Parameters Versus Time from the Trajectory File ".Traj"
3. Trim the Effector Deflections to Balance the Vehicle Moments and Forces
4. Create an Effector Mixing Logic or a TVC Matrix (Kmix)
5. State-Space Modeling of the Flight Vehicle at Selected Times
6. Performance and Stability Parameter Plots Along Trajectory Time
7. Landing and Pull-Up Maneuverability, plus, Inertial Coupling Effects
8. Moments at the Hinges of Control Surfaces Along the Trajectory Time
9. View and Modify Vehicle Data (CG, MRC, TVC, Surfaces) for Dispersion Analysis
10. Contour Plots (Mach versus Alpha) for Performance, Control Authority Analysis
11. Vector Diagrams for Maneuverability & Stability at Selected Flight Conditions
12. Plot and Compare Previous Data Files (Traject, Trim, Perform, Hinge Moment)

**Maximum Alpha Capability**

The control effectors must be capable of varying the vehicle angles of attack and sideslip (typically 3-5 deg) from their trim values.

Enter the maximum expected alpha and beta dispersions from trim in (deg) that must be controlled by the effectors, and click OK.

Maximum Alpha (deg)  `2.0000`     Maximum Beta (deg)  `2.0000`     OK

**Notice**

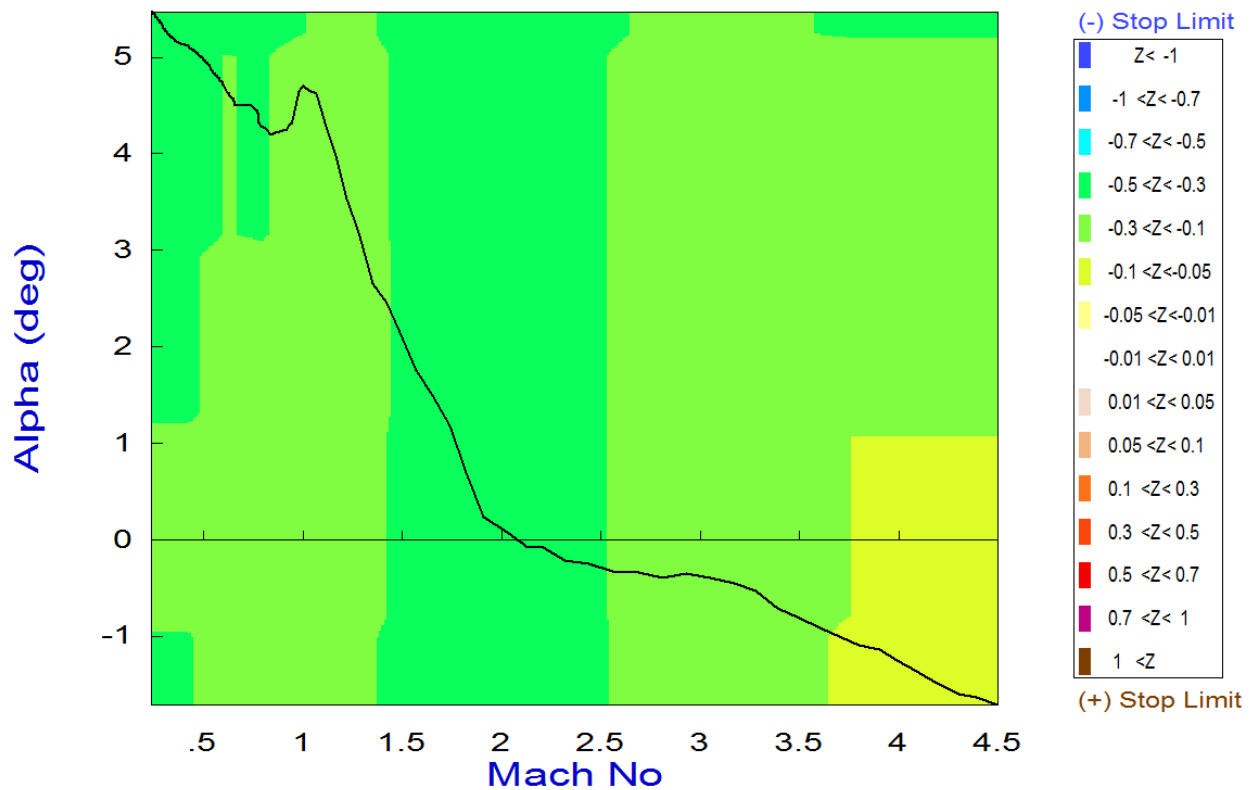ⓘ  A ( 4 X 3) Mixing Logic Matrix is required

OK

The following menu selects the type of contour plot and the performance parameter to be plotted versus Mach and Alpha. The first parameter in the menu is the pitch stability parameter that was described in equations (3.14 & 3.15).
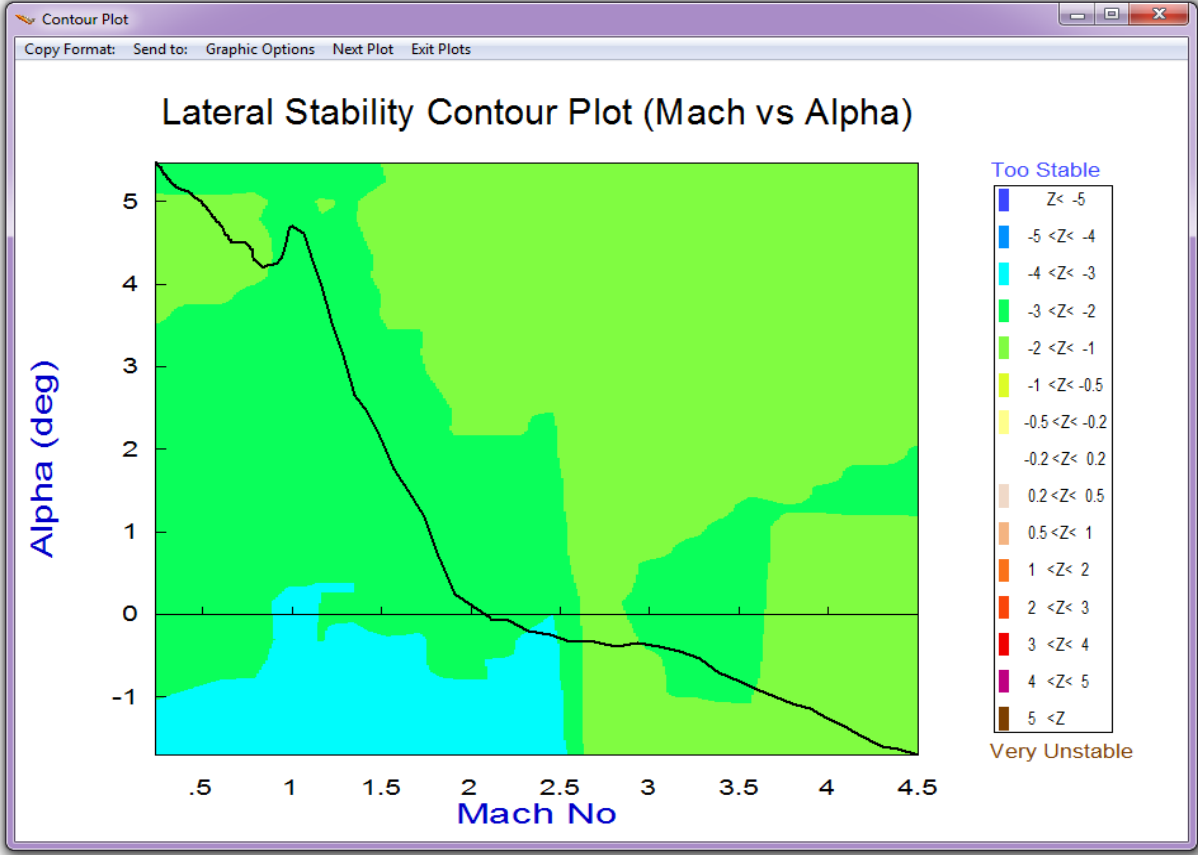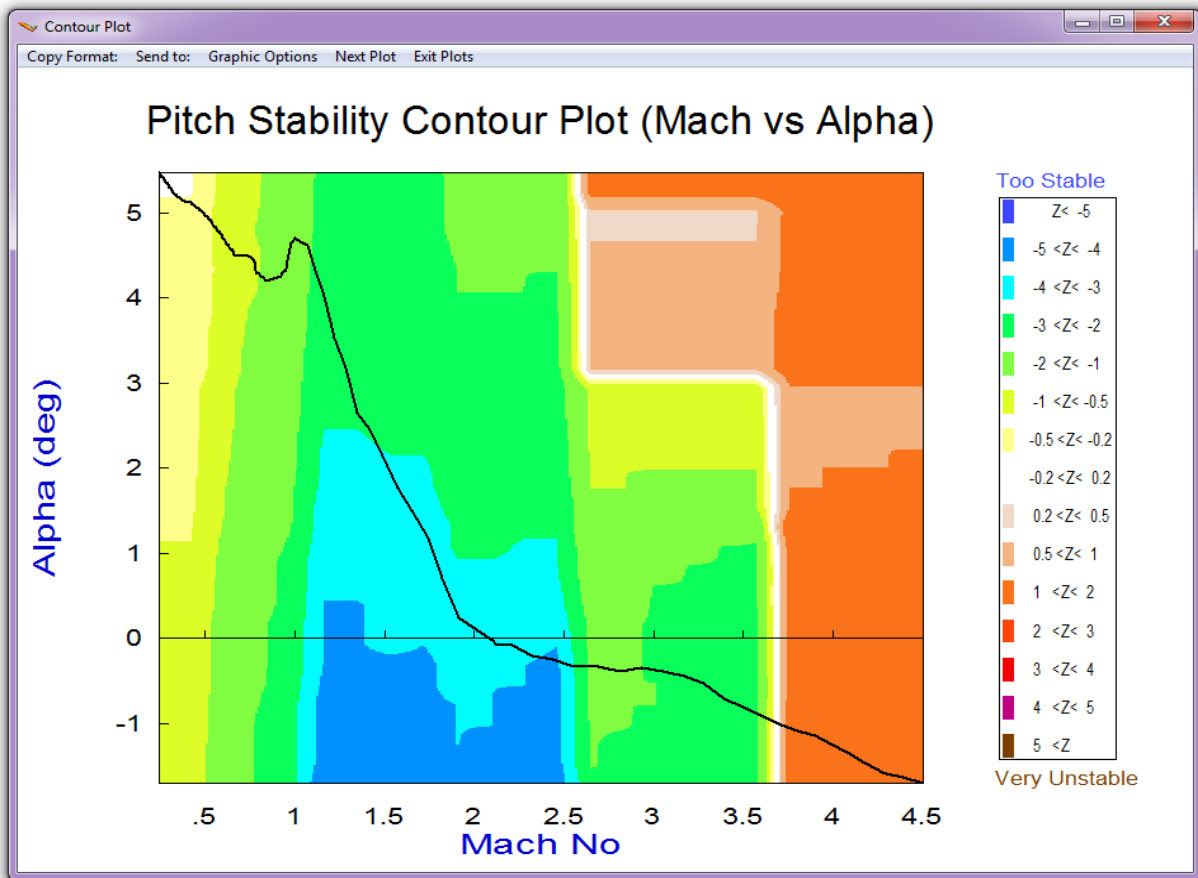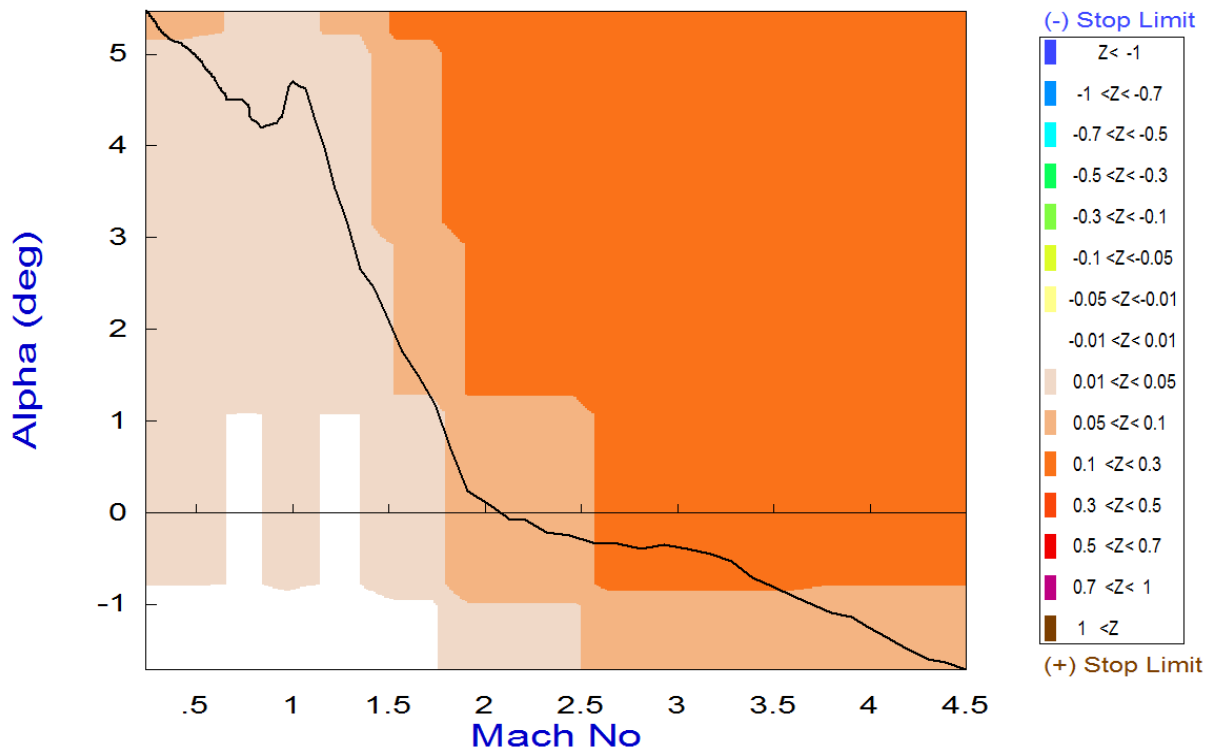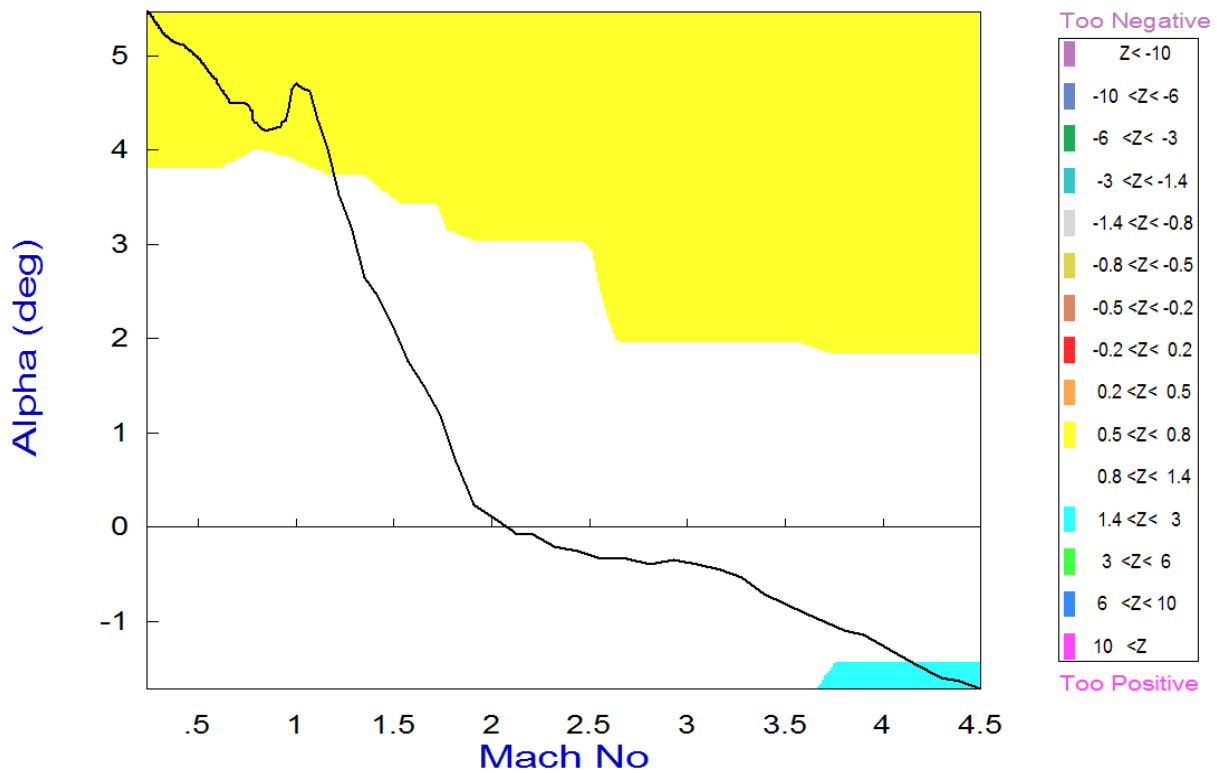
Pitch Stability Contour Plot (Mach vs Alpha)
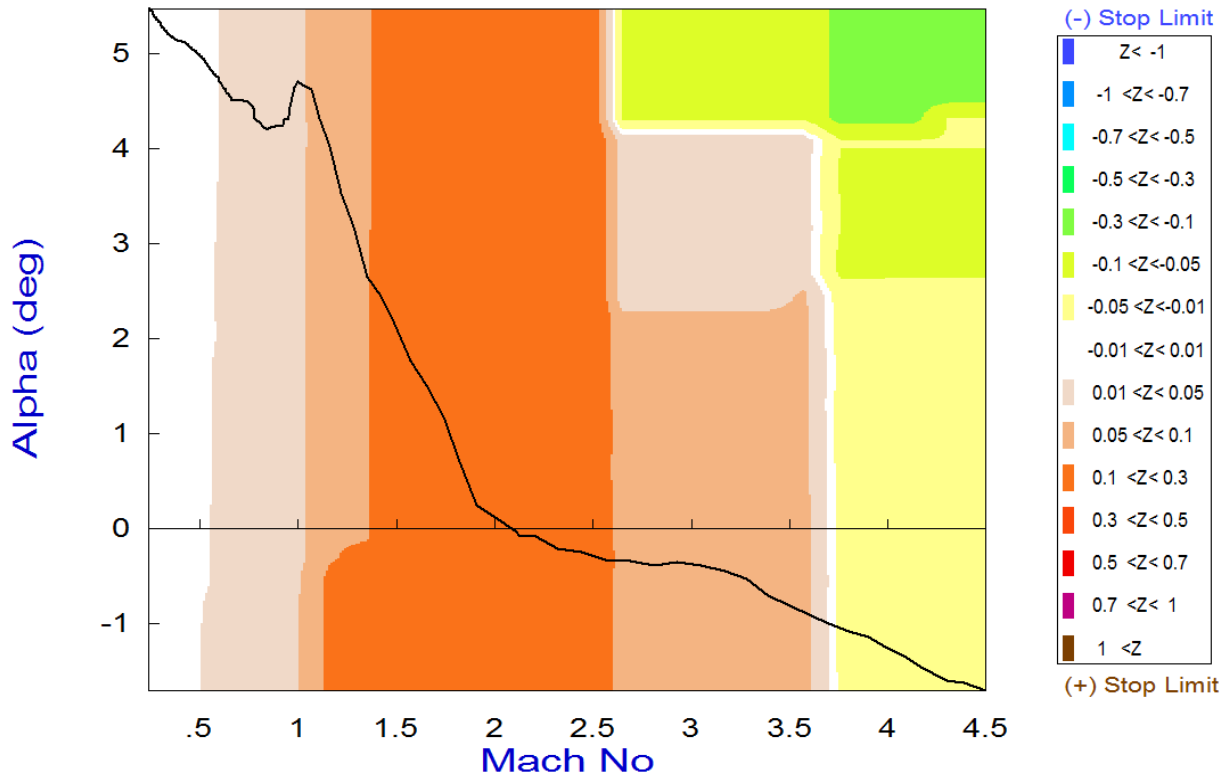


Lateral Stability Contour Plot (Mach vs Alpha)

# Pitch Control Effort Contour Plot (Mach vs Alpha)



# Yaw Control Effort Contour Plot (Mach vs Alpha)

# Roll Control Effort Contour Plot (Mach vs Alpha)



**(-) Stop Limit**

| | |
|---|---|
| | Z< -1 |
| | -1 <Z< -0.7 |
| | -0.7 <Z< -0.5 |
| | -0.5 <Z< -0.3 |
| | -0.3 <Z< -0.1 |
| | -0.1 <Z< -0.05 |
| | -0.05 <Z< -0.01 |
| | -0.01 <Z< 0.01 |
| | 0.01 <Z< 0.05 |
| | 0.05 <Z< 0.1 |
| | 0.1 <Z< 0.3 |
| | 0.3 <Z< 0.5 |
| | 0.5 <Z< 0.7 |
| | 0.7 <Z< 1 |
| | 1 <Z |

**(+) Stop Limit**

# Roll Departure (LCDP) Contour Plot (Mach vs Alpha)



**Too Negative**

| | |
|---|---|
| | Z< -10 |
| | -10 <Z< -6 |
| | -6 <Z< -3 |
| | -3 <Z< -1.4 |
| | -1.4 <Z< -0.8 |
| | -0.8 <Z< -0.5 |
| | -0.5 <Z< -0.2 |
| | -0.2 <Z< 0.2 |
| | 0.2 <Z< 0.5 |
| | 0.5 <Z< 0.8 |
| | 0.8 <Z< 1.4 |
| | 1.4 <Z< 3 |
| | 3 <Z< 6 |
| | 6 <Z< 10 |
| | 10 <Z |

**Too Positive**

## 2.7 Vector Diagrams Analysis

Vector diagrams are 2-dimensional diagrams used for analyzing vehicle controllability at a specified flight condition. We compare the control capability of the aero-surfaces in two directions against the effects on the vehicle of a wind-shear disturbance which is expressed by the dispersions it causes in the angles of attack and sideslip and we determine if the vehicle provides sufficient control authority to counteract against the disturbance moments and forces. This is not just a magnitude comparison but it also allows us to examine the directions of the controls against the disturbance. It helps us evaluate the orthogonality of the control system, compare the acceleration magnitudes due to the controls and wind disturbance and to determine if the controls are powerful enough and their directions are capable of counteracting the disturbance moments along the roll, pitch, and yaw, in this case.
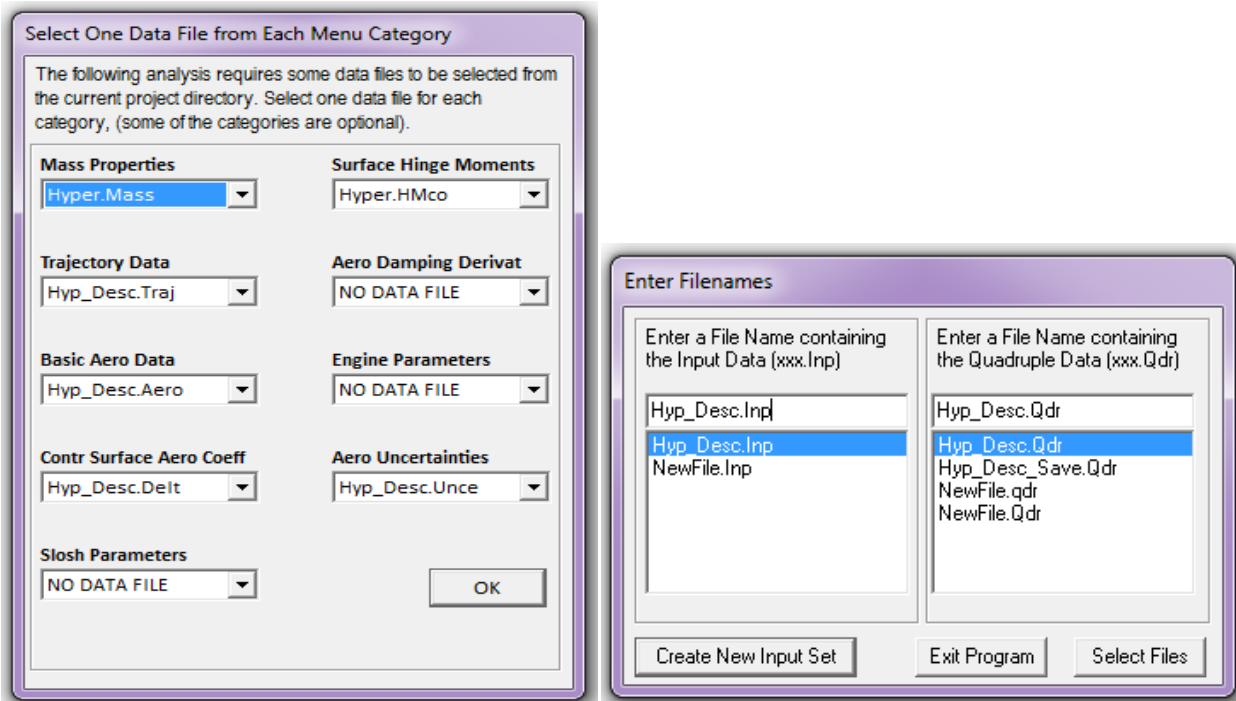


Assuming that the initialization files and trim conditions are the same as before, from the Trim menu select option (11) for plotting vector diagrams, and then an arbitrary flight condition at t=1000 sec. The following dialog consists of four menus for selecting the vehicle mass, Mach number, alpha, and beta. The default values correspond to the selected flight time, but they may be changed if the user desires to analyze a different flight condition, such as, a combination of parameters that does not correspond to a trajectory time. You may keep those parameters or change them to something different. In this case we select the default values and click "*Select*". The wind disturbances are defined by the maximum alpha and beta excursions from trim. In the next dialog enter the maximum disturbance angles ($\alpha_{max}$ and $\beta_{max}$)=2°. The program also requires an aero-surface combination matrix. This time we will select a pre-calculated mixing logic matrix. We will use the matrix "Kmix_1000" from file "*Hyp_Desc.Qdr*" which was created in Section (2.8).

The matrix selection menu below shows the only gain matrix that is saved in systems file "*Hyp_Desc.Qdr*". Select it and click on "*View Matrix*" to take a look at the matrix in color coded form. Its inputs are: roll, pitch and yaw FCS demands, and its outputs are: Elevon, Body-Flap, Aileron, and Rudder deflections. Click on "Exit" to return to the menu and then click on "*Select Matrix*" to continue. The program uses the mixing matrix plus other inputs to calculate the vector diagrams, some of which are shown in the next few pages.

## Comparison Between Maximum Control Moments Against Maximum Disturb Moments (red)
## Roll & Yaw Control Moments (non-dimension) vers Disturb Moment due to Max Beta/Alpha



Trim Conditions

Flight Time ------------------; 0.100E+04
Mach Number ------------------; 0.300
Alpha and Beta Trim (deg) ---; 6.00    0.00
Dynamic Pressure (psf) ------; 90.2
Vehicle Mass (slugs) --------; 433.
Max Alpha/Beta Disturb (deg) : 2.00    2.00
Aero-Surface Trim Angles (deg)
 -3.06   -0.659   0.00   0.00
Engine Pitch Trim Angles (deg)

Engine Yaw Trim Angles  (deg)

OK
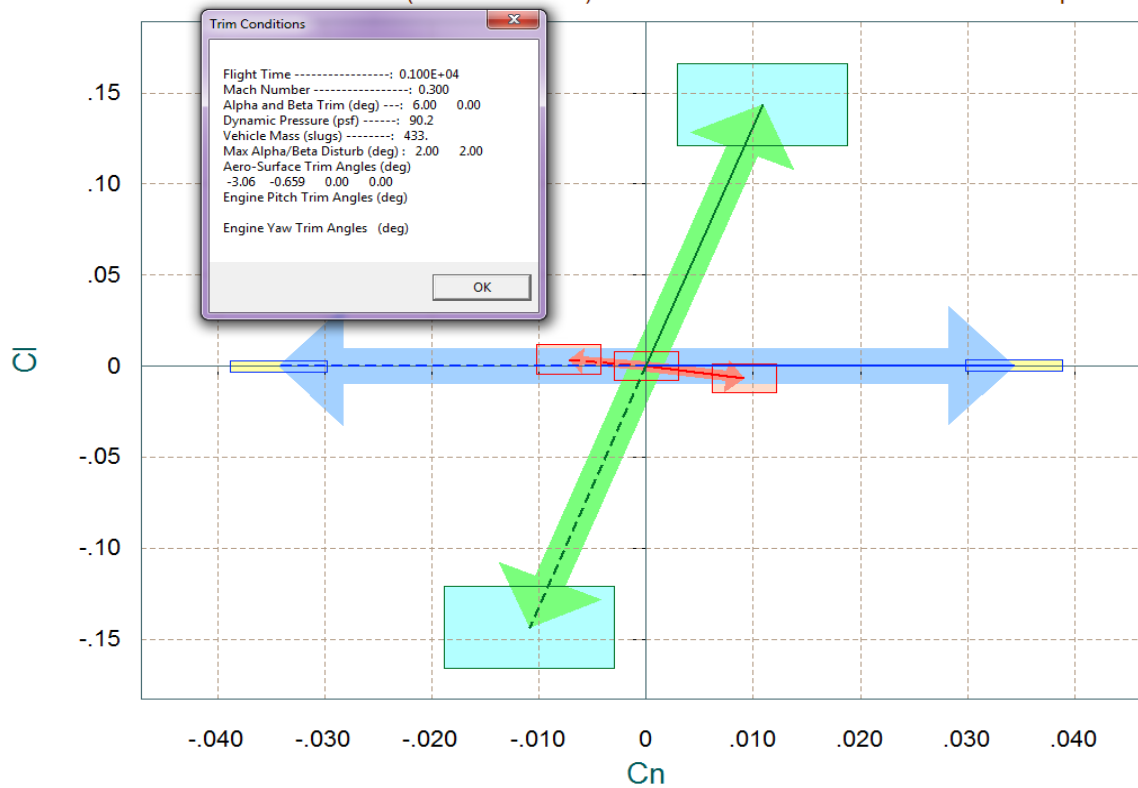
## Comparison Between Lateral Maxim Moment & Forces, Control Versus Beta Forces (red)
## Control Yaw Moment & Side-Force against Moment and Force due to Max Beta Variation



Trim Conditions

Flight Time ------------------; 0.100E+04
Mach Number ------------------; 0.300
Alpha and Beta Trim (deg) ---; 6.00    0.00
Dynamic Pressure (psf) ------; 90.2
Vehicle Mass (slugs) --------; 433.
Max Alpha/Beta Disturb (deg) : 2.00    2.00
Aero-Surface Trim Angles (deg)
 -3.06   -0.659   0.00   0.00
Engine Pitch Trim Angles (deg)

Engine Yaw Trim Angles  (deg)

OK

**Figure 2.7.1 Maximum Roll and Yaw Moments and Side-Force produced due to ±$\beta_{max}$ and Controls**

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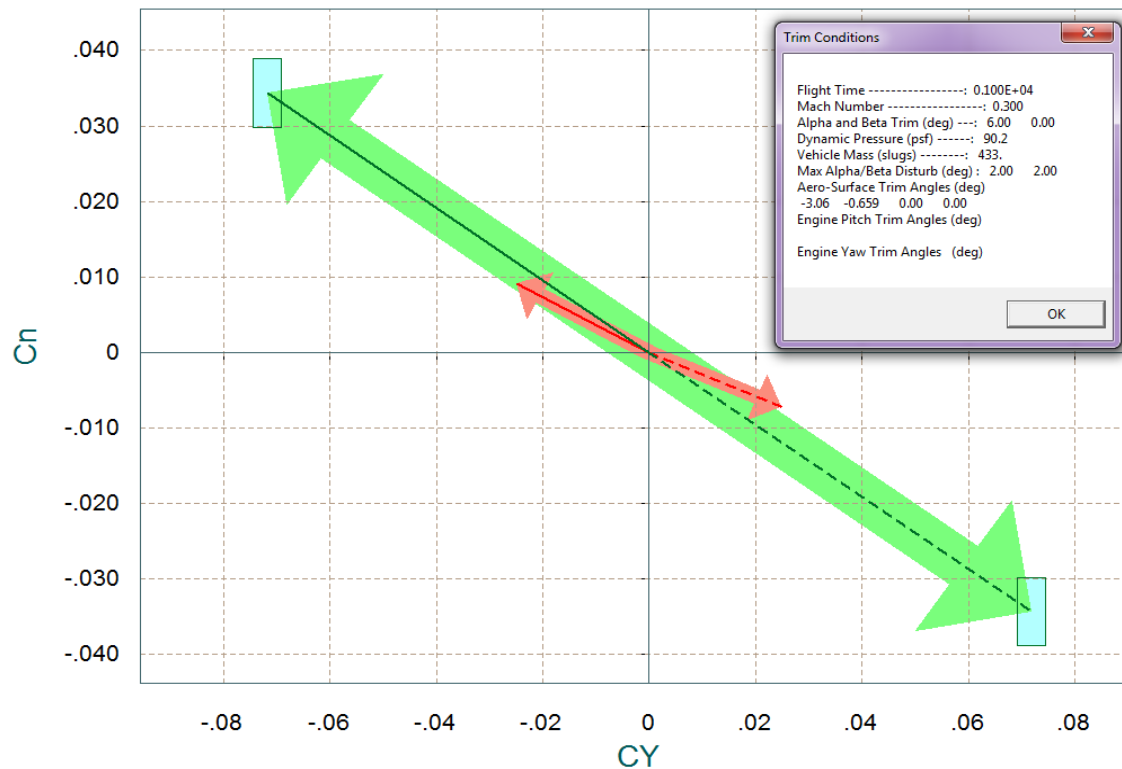Comparison Between Maximum Control Moment/Forces Against Maximum Disturbance (red)
Pitch and Normal-Z Control Moments and Forces versus Disturb due to Alpha Variation

Trim Conditions

Flight Time ------------------: 0.100E+04
Mach Number ------------------: 0.300
Alpha and Beta Trim (deg) ---: 6.00    0.00
Dynamic Pressure (psf) ------: 90.2
Vehicle Mass (slugs) --------: 433.
Max Alpha/Beta Disturb (deg) : 2.00    2.00
Aero-Surface Trim Angles (deg)
  -3.06  -0.659   0.00    0.00
Engine Pitch Trim Angles (deg)

Engine Yaw Trim Angles  (deg)

OK

Copy Format:   Send to:   Select Vector Diagram   Next Plot   Exit Plots

✓  Moments per Max Controls/ per Max Alpha
   Accelerat per Max Control/ per Max Alpha
   Moment/Alpha versus Moment/Control
   Vehicle Acceler per Accel Demand Ratios

Comparison Between Maximum Control Moment/Forces Against Maximum Disturbance (red)
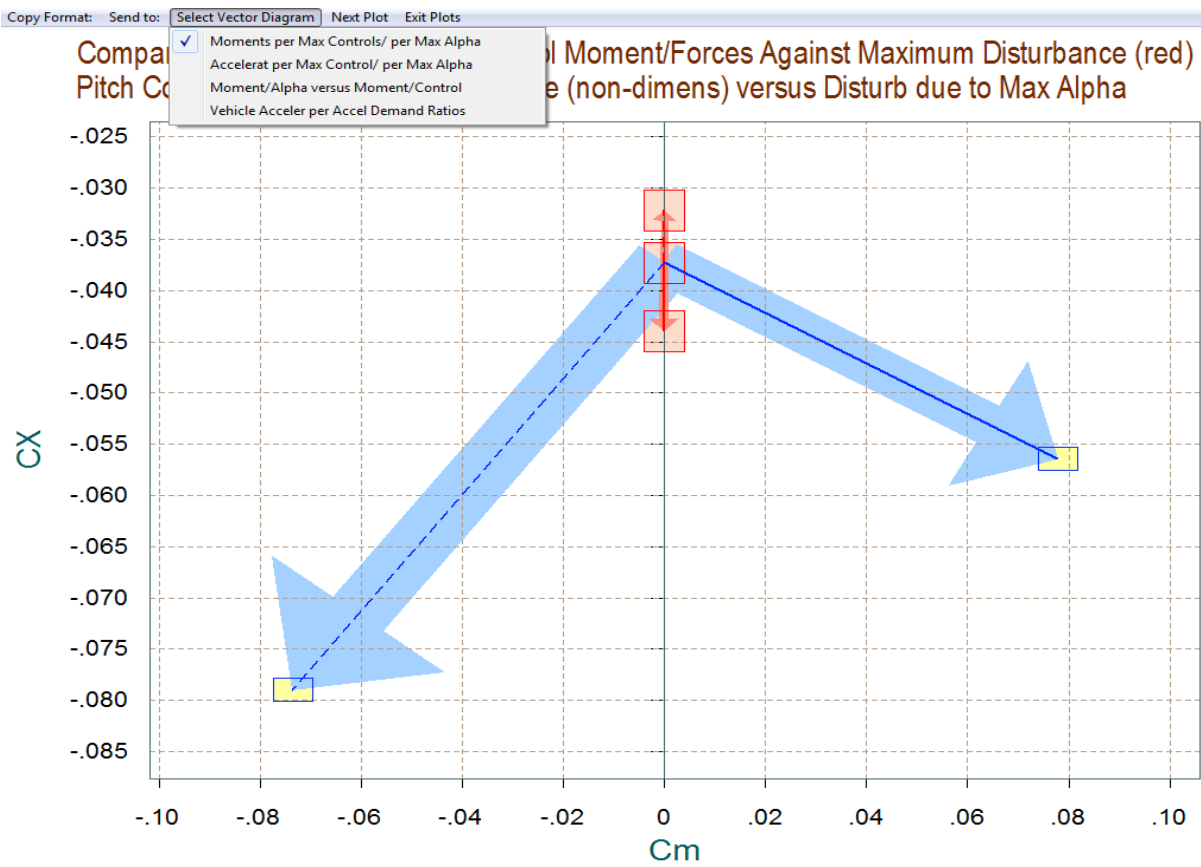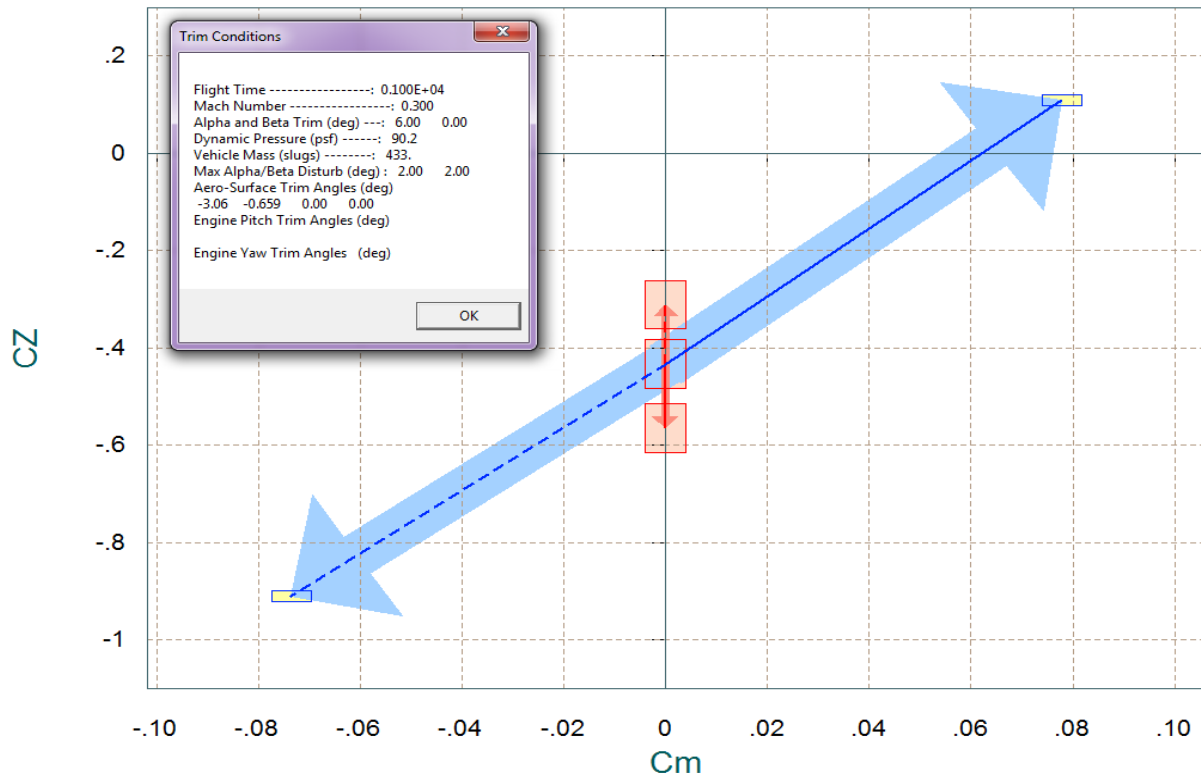Pitch Control Moment/Forces (non-dimens) versus Disturb due to Max Alpha

Figure 2.7.2 **Maximum Pitch Moment, Normal and Axial Forces produced due to ±$\alpha_{max}$ and Controls**

1-104

Comparison Between Control Moment Partials Versus Aero-Disturb Moment Partials (red)
Yaw & Roll Control Partials Cn/delta_R and Cl/delta_P versus Cn/beta & Cl/beta

Copy Format:  Send to:  Select Vector Diagram  Next Plot  Exit Plots

Moments per Max Controls/ per Max Alpha
Accelerat per Max Control/ per Max Alpha
Moment/Alpha versus Moment/Control
✓ Vehicle Acceler per Accel Demand Ratios

Ratios of ... over Corresp Control Acceleration Demands
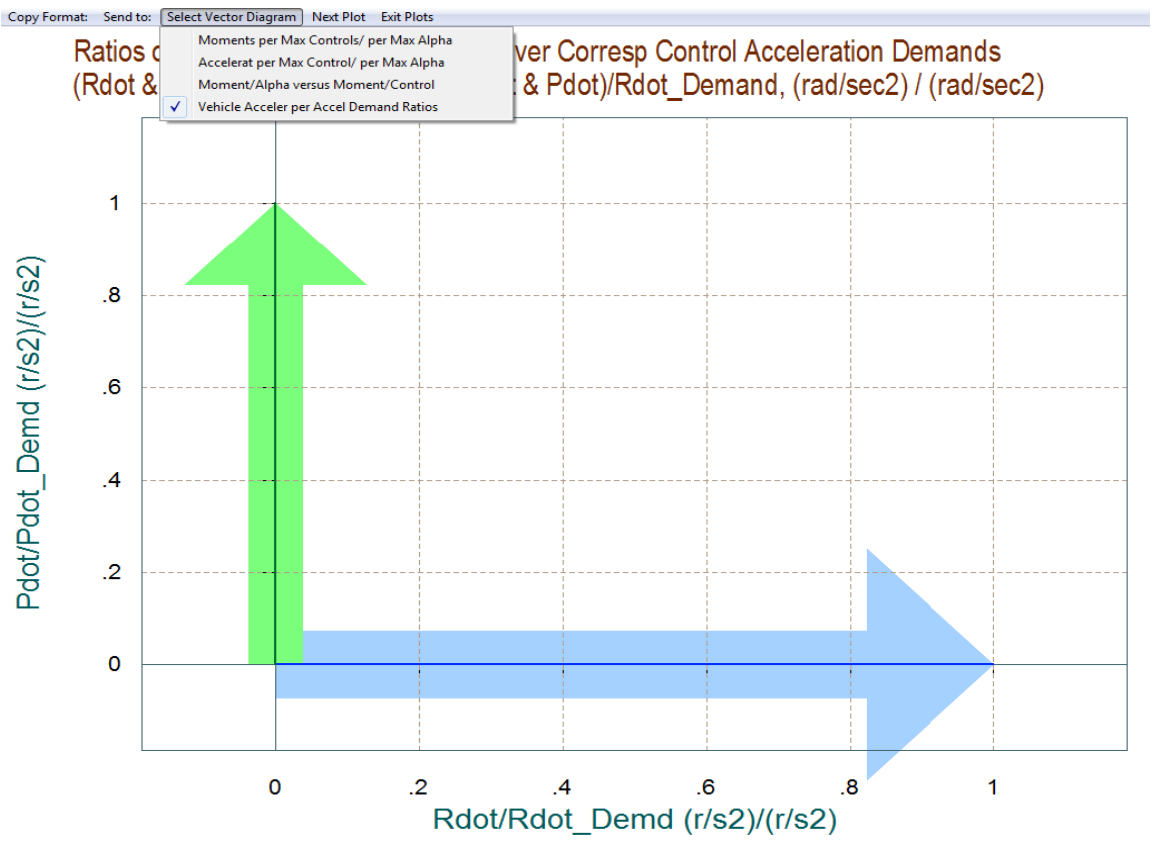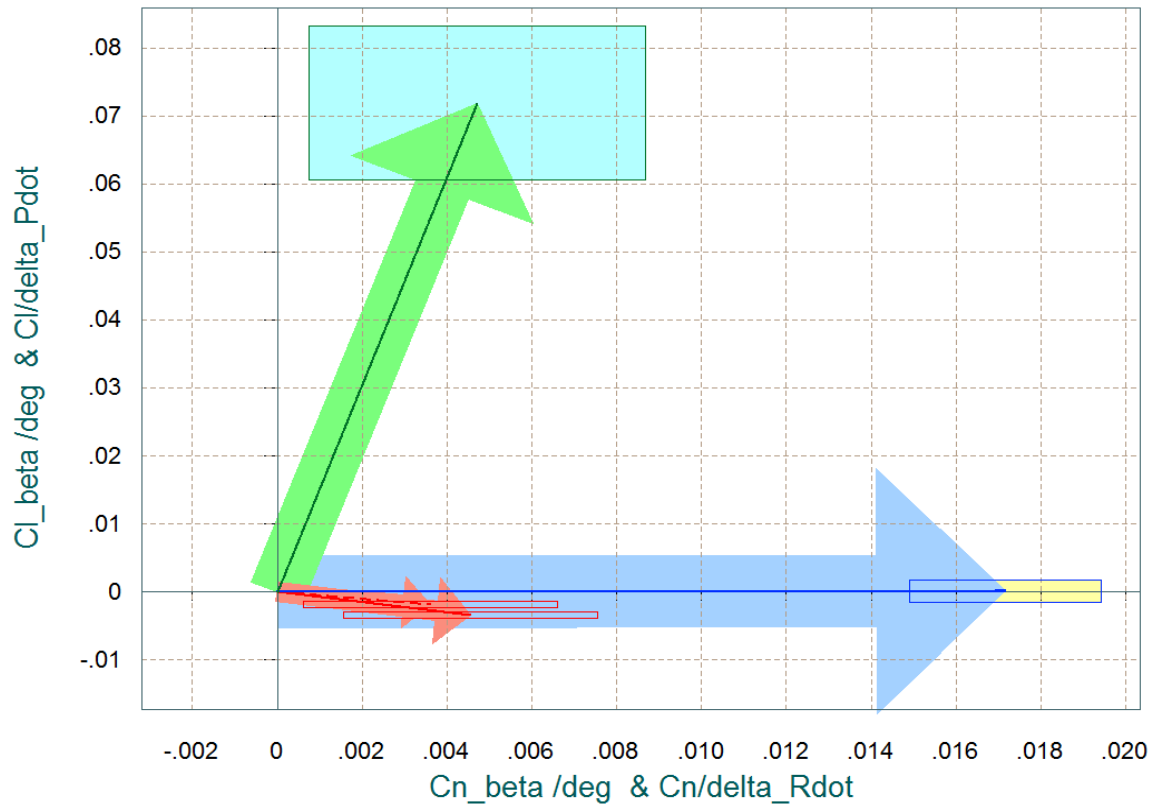(Rdot & ... & Pdot)/Rdot_Demand, (rad/sec2) / (rad/sec2)

**Figure 2.7.4 Roll and Yaw Moment Partials and Acceleration Partials**

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## 2.8 Preliminary Dynamic Analysis

We will now demonstrate how to use the Trim program to create linear models for control design and analysis. We will select an arbitrary flight time 1000 (sec) from take-off and generate a linear state-space system using the Flixan flight vehicle modeling program (FVMP). We will also generate a control surface mixing matrix that corresponds to the same flight condition. We will separate the pitch and lateral subsystems, check their open-loop eigenvalues, and see how they compare with our static analysis. So let us begin the Trim program, as before, select the Trim data files, select also the Flixan input data file "*Hyp_Desc.Inp*" and the systems file "*Hyp_Desc.Qdr*".
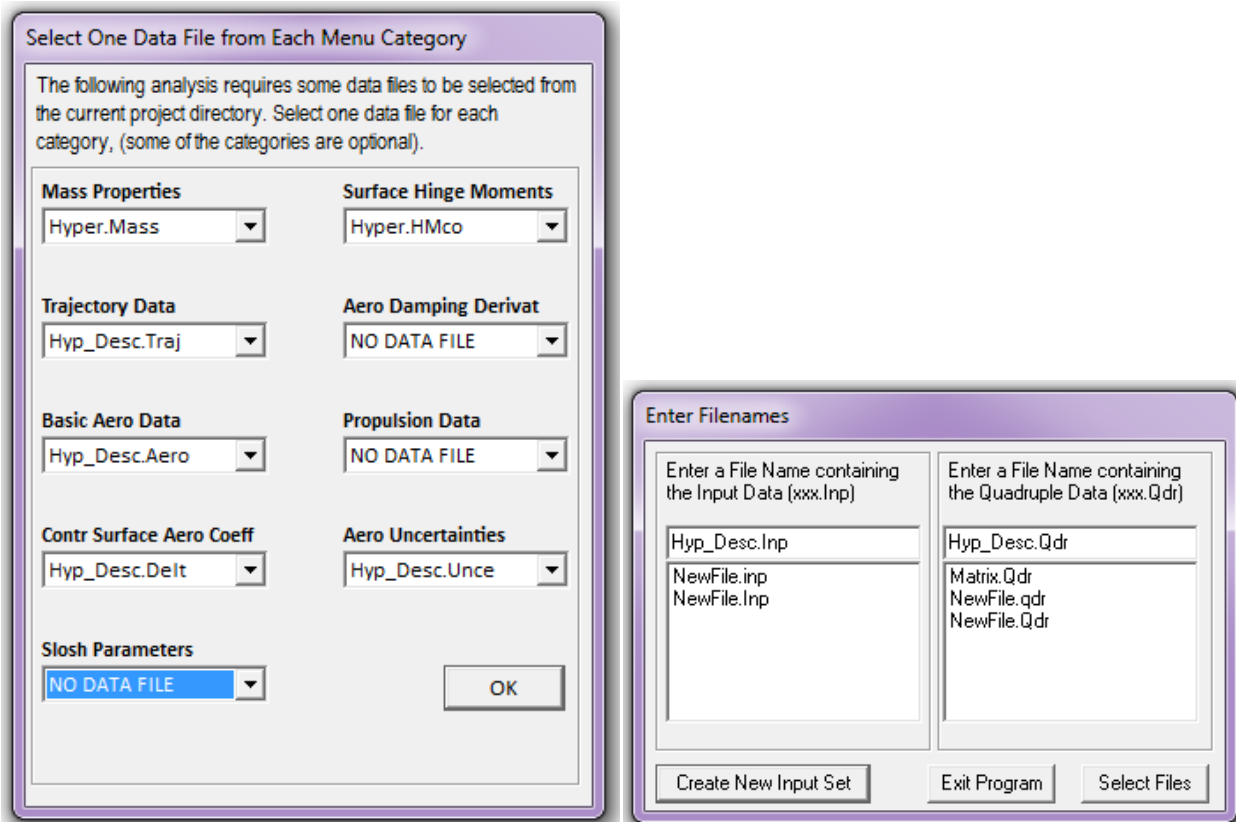


The Trim program will generate the flight vehicle input data in file "*Hyp_Desc.Inp*", and the vehicle dynamic model will be created by the FVMP and it will save it in "*Hyp_Desc.Qdr*". This input data file has already been prepared, and in addition to the vehicle data it contains also data sets that will be used by Flixan to further refine the vehicle modeling and analysis. The Trim program first generates the vehicle input data in file "*Hyp-Desc.Inp*". The additional data-sets in that file are introduced by the analyst and processed by other Flixan utilities, for the purpose of separating the pitch and lateral systems, generating a surface combination matrix (Kmix), and re-formatting the systems for Matlab.

From the Trim main menu we select Option (5) to generate the vehicle data for the dynamic model. The program activates the trajectory plotting option from where the user is prompted to select a flight time across the plot using the mouse. From the top menu bar, click on "*Graphic*
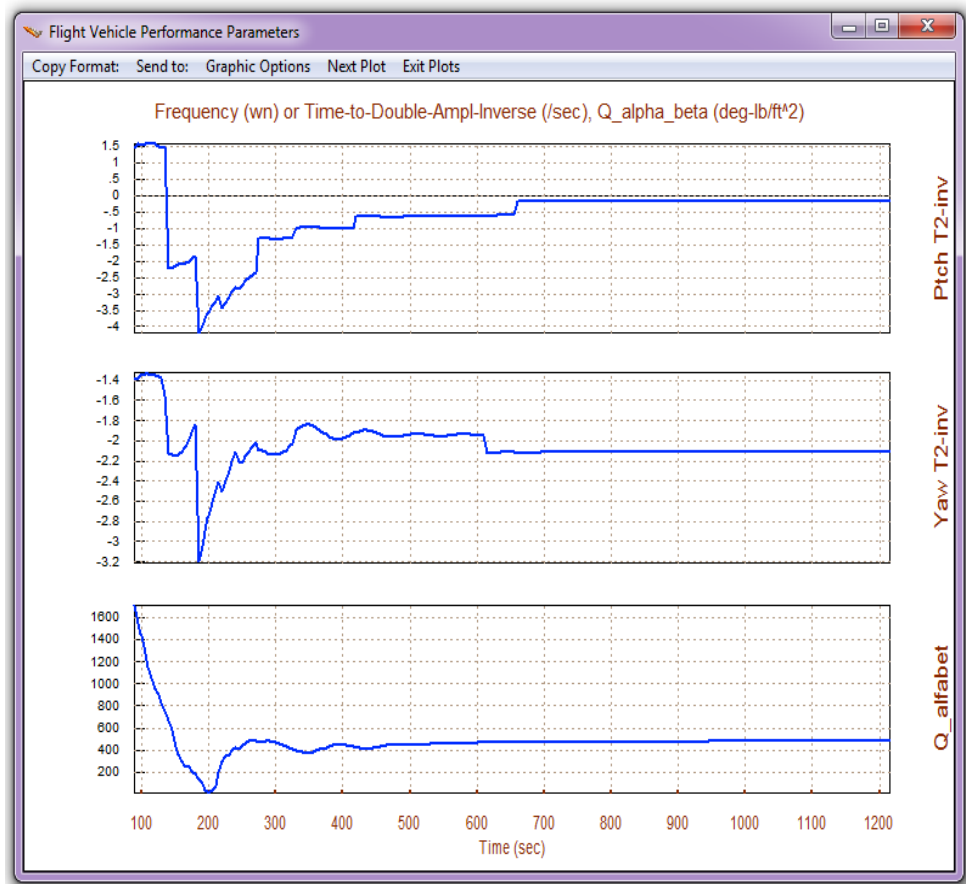
Flight Vehicle Performance Parameters

Copy Format:   Send to:   Graphic Options   Next Plot   Exit Plots

Frequency (wn) or Time-to-Double-Ampl-Inverse (/sec), Q_alpha_beta (deg-lb/ft^2)

Pitch_Eigen =

  -0.8358
   0.3610
  -0.2375 + 0.2476i
  -0.2375 - 0.2476i
   0.0000

Lateral_Eigen =

     0
   0.1109 + 2.0965i
   0.1109 - 2.0965i
  -0.3985
  -0.0000

## 2.9 Flight Control Design

The directory: "*C:\Flixan\Trim\Examples\Hypersonic Vehicle\Descent\Mat_Design*" contains the data used for a preliminary flight control design using vehicle models generated at some critical flight conditions along the trajectory. The folder contains input data files used by Flixan at 7 flight conditions and the corresponding system files. The LQR method was used for the creation of state-feedback gains. Each input data file, for example "*T95_Design.Inp*", contains the following:

- A batch data-set which calls the various Flixan utilities and creates the various matrices and systems.
- A set of flight vehicle data, title: "*Rocket-Plane Mission-1, Descent Phase/ T= 95.0 sec*", which is processed by the flight vehicle modeling program.
- A mixing logic set, title: "*Mixing Logic Matrix for the Rocket-Plane during Descent*" which generates an effector combination matrix "Kmix_95", corresponding to the same flight condition.
- A systems interconnection set "*Rocket-Plane Mission-1, Descent Phase (Including Mixing Matrix)*" which combines the vehicle model "*Rocket-Plane Mission-1, Descent Phase/ T= 95.0 sec*" with the mixing matrix "*Kmix_95*", and creates a new system.
- A data set "*Rocket-Plane Mission-1, Descent Phase (Pitch Model)*" which extracts only the pitch dynamics from the combined system.

- A data set "*Rocket-Plane Mission-1, Descent Phase (Lateral Model)*" which extracts only the roll/yaw dynamics from the combined system.
- Two data sets which convert the pitch and lateral subsystems to Matlab m-function format, "*pitch_95.m*" and "*later_95.m*".

**Pitch Design**

The pitch design is performed in sub-directory "*Mat_Design\Pitch LQR*" by running the file "*pitch_design.m*". The Matlab script loads the pitch state-state system "*Pitch_95.m*" and augments the state-vector by introducing one additional state, alpha-integral. The state-vector consists of: theta ($\theta$), pitch rate (q), $\alpha$, change in altitude (h), change in velocity (v), and $\alpha$-integral. The design script uses LQR to calculate the state-feedback gain Kq_95.mat. Then it performs a frequency response analysis to show stability margins.

```
% Pitch LQR Design for Rocket-Plane
[Ap,Bp,Cp,Dp]= pitch_95;                          % Load the Pitch Design Model
[Api,Bpi,Cpi,Dpi]= linmod('Pitch_Design');        % Augment Pitch Simulink model
sys=SS(Api,Bpi,Cpi,Dpi);
% Weights[thet,   q,  alpha,  h,   V,   alf_int]
Q= diag( [0.0001, 0.2, 0.001, 0.03,0.01,0.0001]); % Weights(thet,q,alpha,h,v,alf-int)
R=12;                                             % Control Weights R=2
[Kq,S,E] = LQR(sys,Q,R)
save Kq_95.mat Kq -ascii                          % Save the LQR gains in Kpqr.mat

w=logspace(-3, 3, 4000);                          % Define Frequ Range
[A1,B1,C1,D1]= linmod('Freq_Anal');               % Linearize Open-Loop model
sys= SS(A1,B1,C1,D1);                             % Create SS System
%figure(1); Nyquist(sys,w)                         % Plot Nichol's Chart
figure(2); Nichols(sys,w)                         % Plot Nichol's Chart
figure(3); Bode(sys,w); grid on                   % Plot Bode Plot
```

**Lateral Design**

Similarly, the lateral design is performed in sub-directory "Mat_Design\Lateral LQR" by executing file "Later_design.m". The Matlab script loads the lateral state-state system "*Later_95_lqr.m*" and augments the state-vector by introducing one additional state, phi-integral. The lateral state-vector consists of: roll attitude ($\phi$), roll rate (p), yaw rate (r), sideslip ($\beta$), and $\phi$-integral. The design script uses LQR to calculate the lateral state-feedback gain Kpr_95.mat. Then it performs a frequency response analysis to show stability margins.

```
% LQR Lateral Design for Hypersonic Rocket-Plane
[Al,Bl,Cl,Dl]= later_95_lqr;                      % Load the Lateral Design Model
[Ali,Bli,Cli,Dli]= linmod('Lat_des');             % Linearize Lateral Simulink model
sys=SS(Ali,Bli,Cli,Dli);
% states: phi, p,   r,    beta,  phi_int
Q=diag([ 8, 2, 2,  0.01,  0.05]);
R=diag([1  1]*3);                                 % Control Weights
[Kpr,S,E] = LQR(sys,Q,R)
save Kpr_95.mat Kpr -ascii                         % Save the LQR gains in Kpqr.mat
```

```matlab
% Frequency Domain Analysis for Hypersonic Vehicle
w=logspace(-3, 3, 4000);                % Define Frequ Range
[A1,B1,C1,D1]= linmod('Freq_Anal');     % Linearize Open-Loop model uses combined plant
sys= SS(A1,B1,C1,D1);                   % Create SS System
% %figure(1); Nyquist(sys,w)            % Plot Nichol's Chart
figure(2); Nichols(sys,w)               % Plot Nichol's Chart
figure(3); Bode(sys,w); grid on         % Plot Bode Plot
```

## 2.10 Non-Linear 6-dof Descent Simulation

The LQR gains and the aero-surface mixing matrices from section 2.9 are used in a 6-dof Matlab simulation. The simulation files are in subdirectory "C:\Flixan\Trim\Examples\Hypersonic Vehicle\Descent\Descent 6-dof Matlab Simulation", and the Simulink model is "Descent_Sim.mdl", shown below.
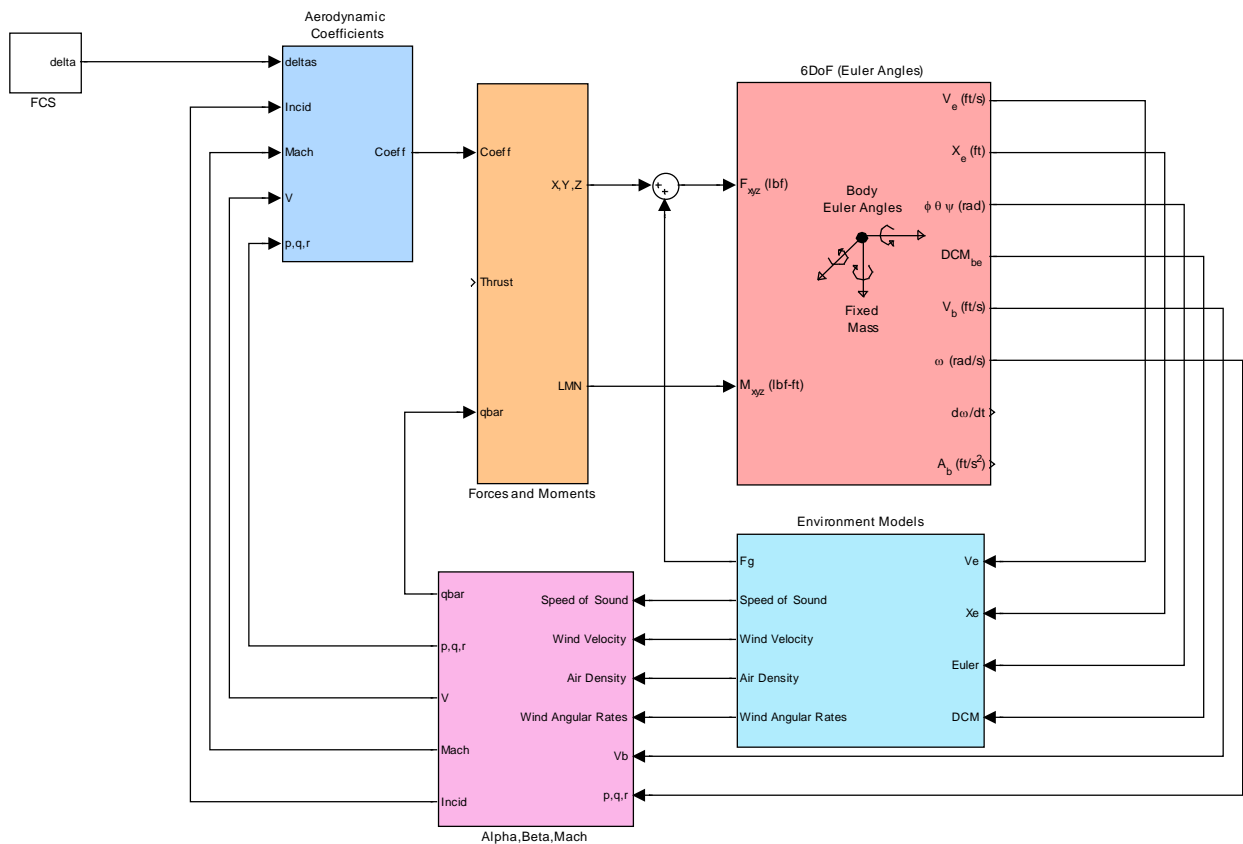


The simulation is initialized from file "init.m", which loads the aero data from file "Base_Aero_Coeff.m" and the aero-surface aero coefficients from files "Elevon.m", "BodyFlap.m", "Aileron.m", and "Rudder.m". It also loads the control gains and mixing matrices that were computed in section 2.9, from file FCS.mat. The simulation tracks a pre-calculated altitude versus time trajectory which is slightly different from the point-mass trajectory.