Rocket Plane

In this example we will analyze a rocket vehicle during level flight. It is controlled by five aero-surfaces and a fixed engine, similar to the vehicle shown in Figure (1). The control surfaces are: two flaps which are almost horizontal used mostly for roll control, two rudders (forming a 45°V-tail) used for pitch and yaw control, and a body-flap in the back used mainly for trimming, and also for pitch control at high angles of attack when the rudders are not very effective. The engine nominal thrust is 2,000 (lb), it does not gimbal and used mainly for controlling speed, but its thrust can vary from zero to 4,000 (lb) as needed for acceleration and speed control. In this analysis we will create dynamic models for this vehicle at a single flight condition, at around mach 0.85, altitude 45,000 (ft), and dynamic pressure of 150 (psf). We will begin with the rigid-body modeling, pitch and lateral control systems design and analysis, introduce structural flexibility, and then evaluated the overall vehicle system stability and performance with respect to commands and gusts.

The example files are located in directory *"C:\Flixan\Examples\Rocket Plane"*. The analysis is separated in three parts. In Section 1 we create rigid-body models and perform control system design, stability analysis and simulations. In Section 2 we include flexibility in the model using modal data without GAFD. We select the dominant modes, combine them with the vehicle dynamics, analyze flex stability, and perform simulations. In Section 3 we go one step further in the flexibility analysis and include also aero-elastic GAFD data and h-parameters. We finally repeat the flex analysis and simulations using the more refined aero-elastic model.

1.0 Rigid-Body Analysis and Control System Description

The rigid-body modeling and control analysis files are in directory "*C:\Flixan\ Examples\ Rocket Plane\Rigid Body Design*". It has three subdirectories containing Matlab files for separate analysis: pitch axis analysis, lateral axes analysis, and coupled pitch and lateral axes analysis. The vehicle data for this flight condition are in file "*RocketPlane_RB.Inp*". This input file creates several state-space systems and gain matrices which are saved in the systems file "*RocketPlane_RB.Qdr*".

1.1 Creating the Rigid-Body Vehicle Model

The title of the vehicle model in the input data file is "*Rocket Plane at Mach=0.85, Q=150, Rigid Body*". The top part of the vehicle data contains the mass properties, trajectory, and basic aero data coefficients. The direction of the wind-gust disturbance vector is defined relative to the vehicle at an elevation angle of 90°, that is, perpendicular to the body x-axis. The azimuth angle is at 45°, that is, between the +z and the +y axes, causing an increase in both the angles of attack (α) and sideslip (β). The vehicle has five control surfaces and in this initial design phase the tail-wags-dog/ hinge moments option is turned off (NO TWD). The tail-wags-dog dynamics will be included later when we will analyze the structural flexibility. The input file also contains data for the five control surfaces which are: the left and right flaps, the left and right rudders, and the body-flap. The surface data include: trim angles, hinge vector orientation angles, surface mass, inertia about the hinge, distance between surface cg to hinge line, control surface area, hinge moment coefficients, locations of the hinge lines in vehicle coordinates, and aerodynamic force and moment increment coefficients for each aero-surface.

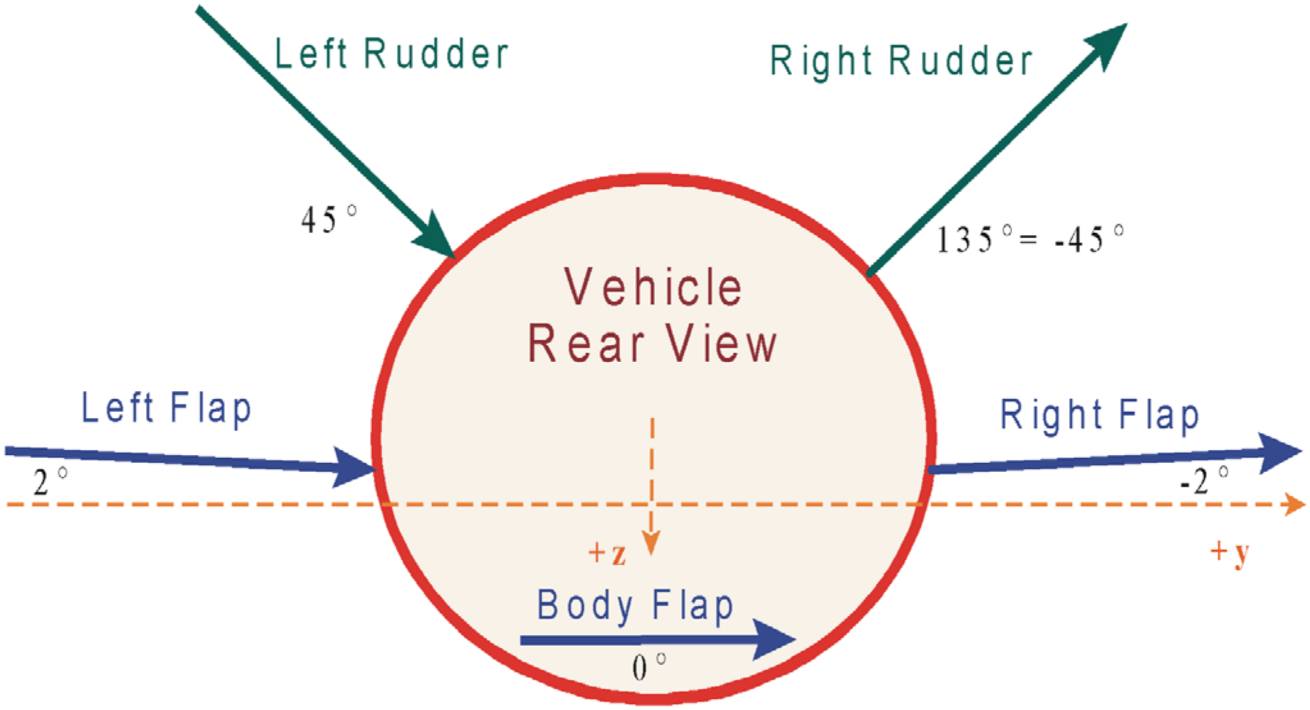
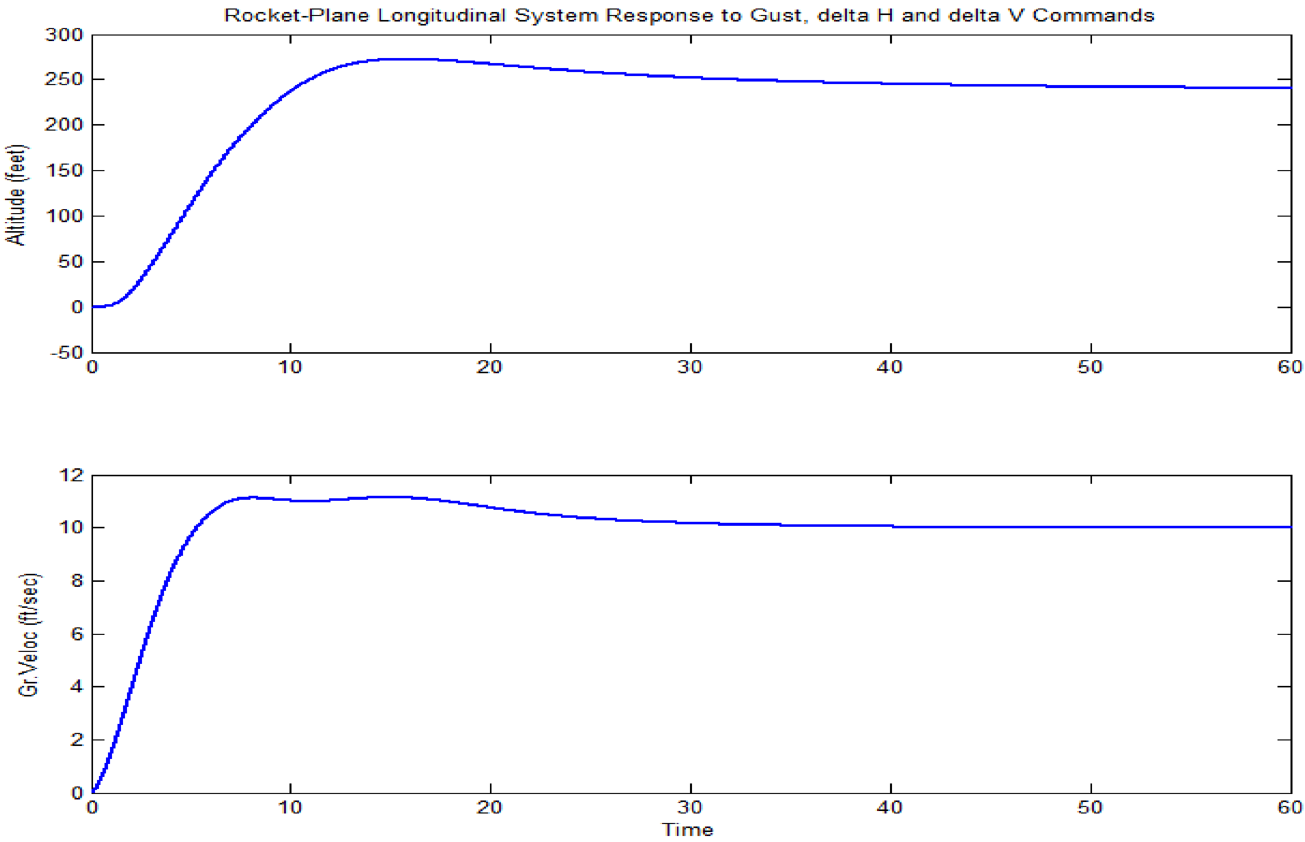
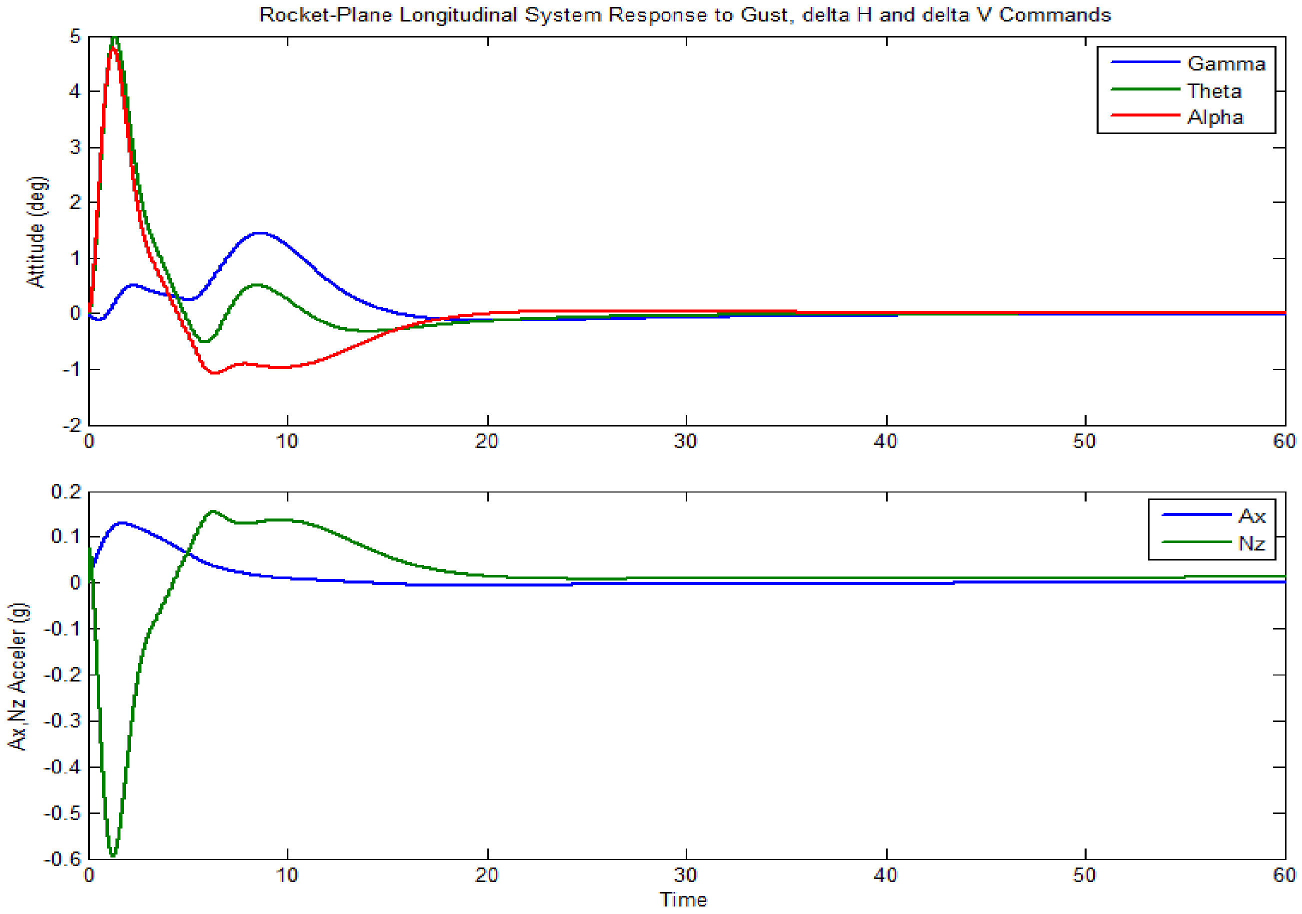
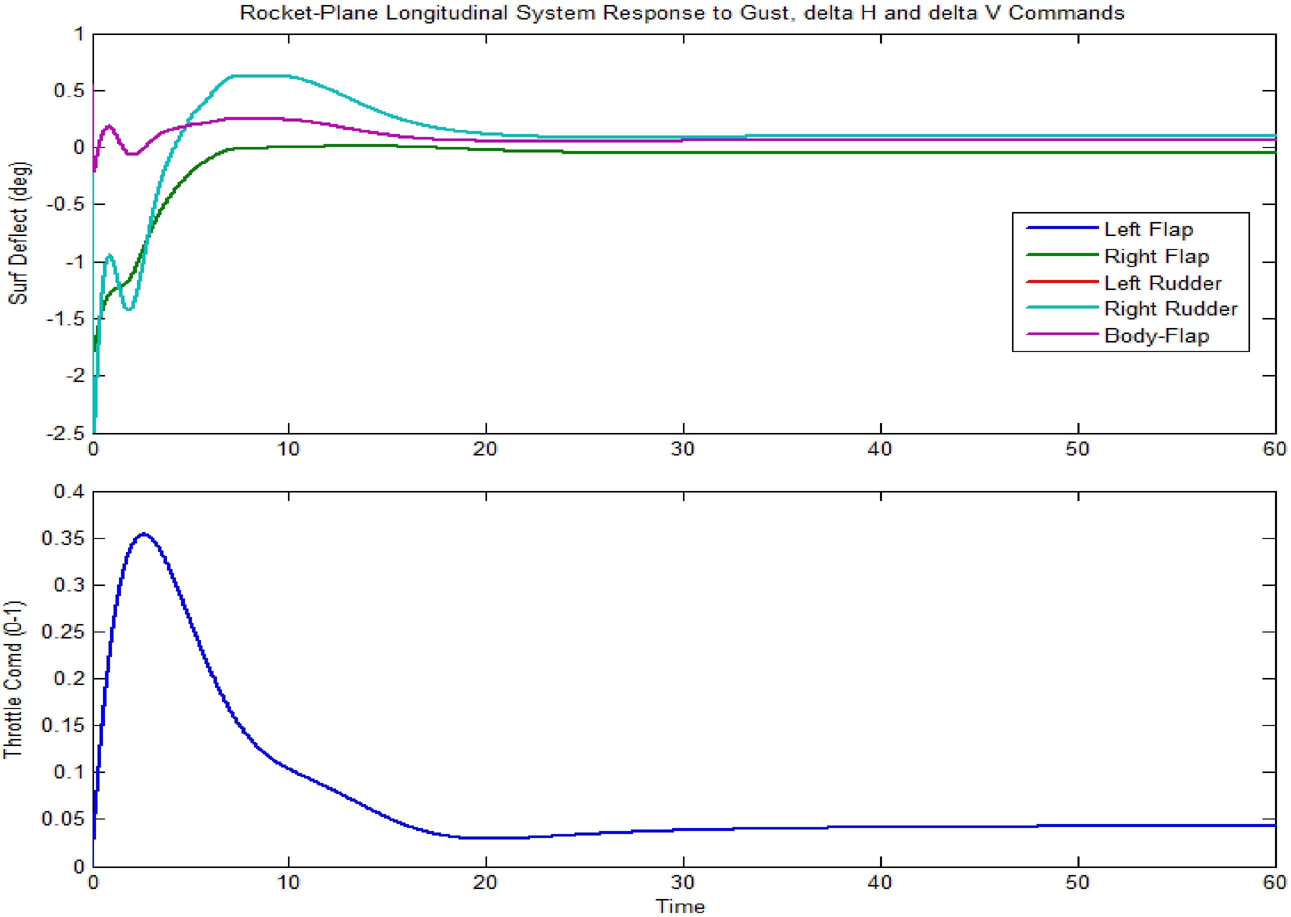


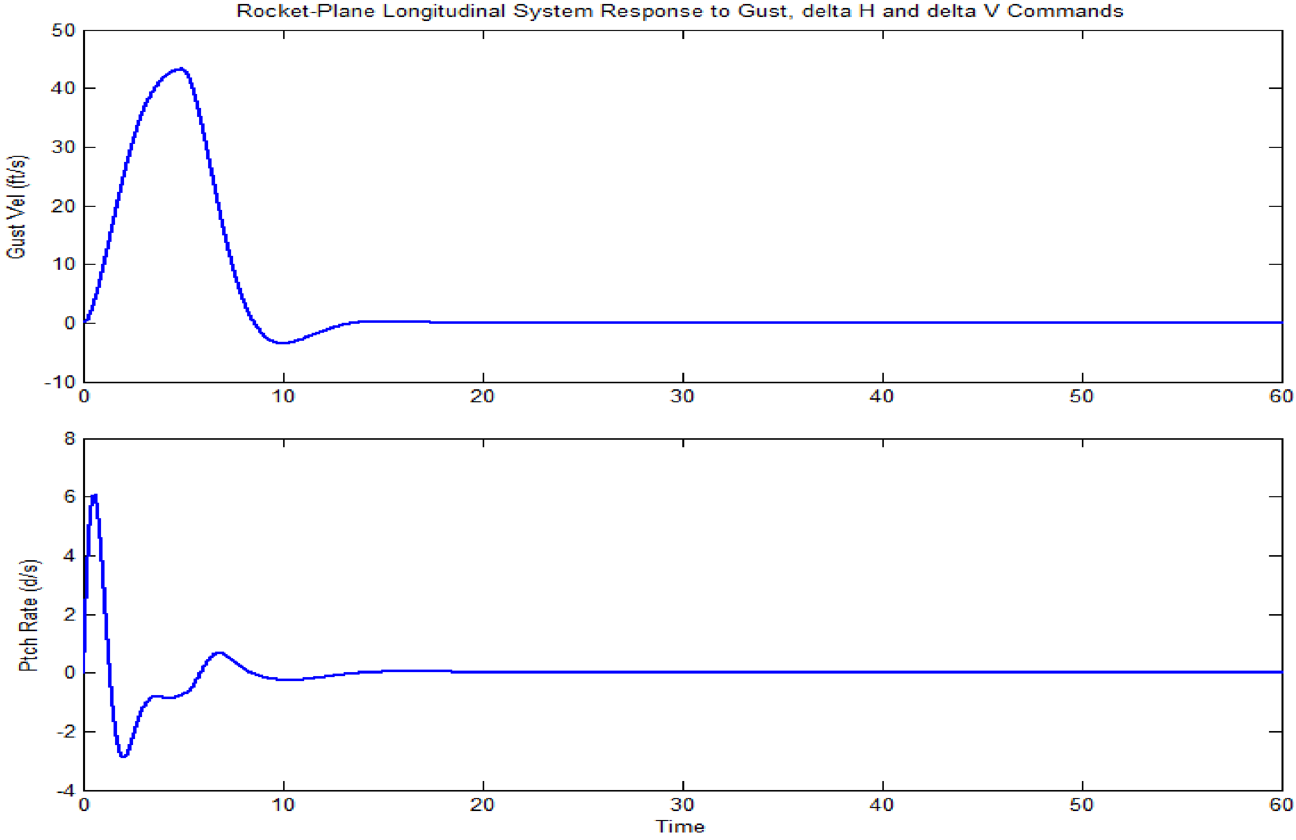
Figure (2) The arrows show the directions of the hinge vectors and the hinge orientation angles (ϕ_h)

Figure (2) shows the roll orientation angles of the hinge vectors (ϕ_h) which are defined positive clockwise about the vehicle x axis. All hinge lines are perpendicular to the vehicle x-axis, i.e. the (λ_h) angles are zero. See Figure (2.5.3) in vehicle equations. The two flaps are located near the trailing edge


Rocket-Plane Longitudinal System Response to Gust, delta H and delta V Commands
Altitude (feet)
300
250
200
150
100
50
0
-50
0
10
20
30
40
50
60
Gr.Veloc (ft/sec)
12
10
8
6
4
2
0
0
10
20
30
40
50
60
Time


Rocket-Plane Longitudinal System Response to Gust, delta H and delta V Commands
Gamma
Theta
Alpha
Attitude (deg)
5
4
3
2
1
0
-1
-2
0
10
20
30
40
50
60
Ax
Nz
Ax,Nz Acceler (g)
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
0
10
20
30
40
50
60
Time


Rocket-Plane Longitudinal System Response to Gust, delta H and delta V Commands
Surf Deflect (deg)
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
10
20
30
40
50
60
Left Flap
Right Flap
Left Rudder
Right Rudder
Body-Flap
Throttle Comd (0-1)
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Time


Rocket-Plane Longitudinal System Response to Gust, delta H and delta V Commands
Gust Vel (ft/s)
50
40
30
20
10
0
-10
0
10
20
30
40
50
60
Ptch Rate (d/s)
8
6
4
2
0
-2
-4
Time

Longitudinal Control System Stability Analysis

The Simulink model "Open_Pitch.mdl", in Figure (1.7) is used for stability analysis. It uses the same subsystems as the simulation model. The system has two control loops: the elevon loop and throttle control loop. When checking the elevon stability we open the elevon loop and close the throttle loop, and vise-versa.

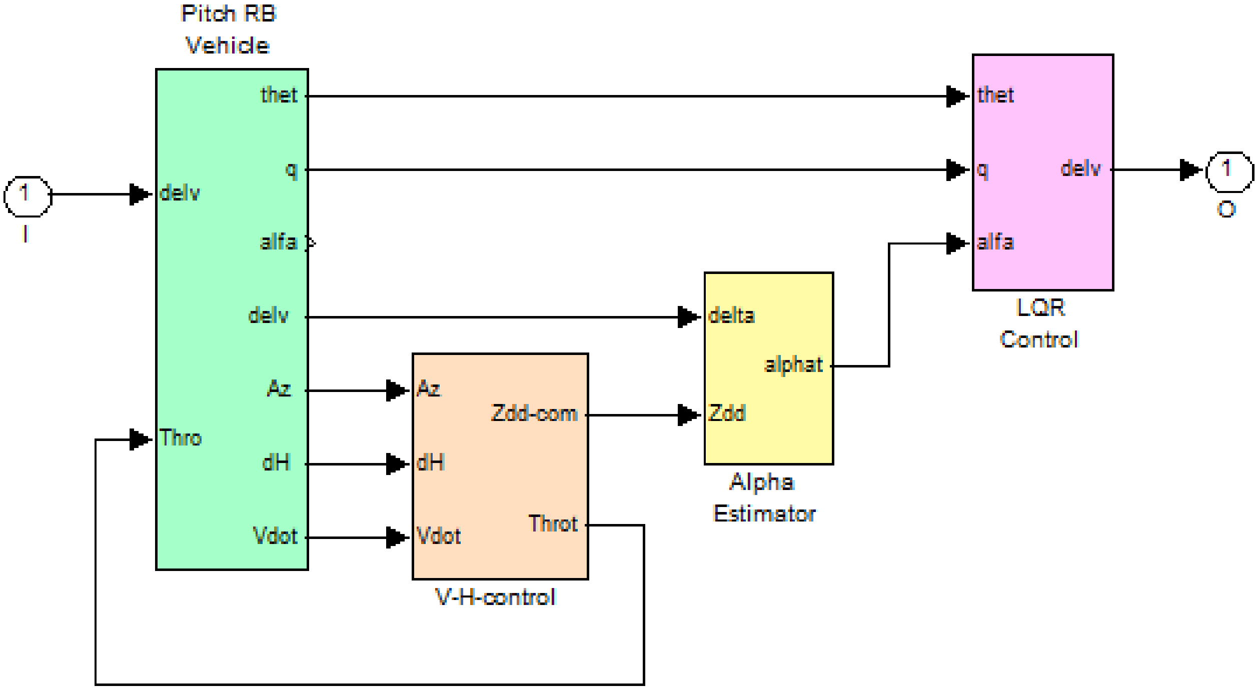
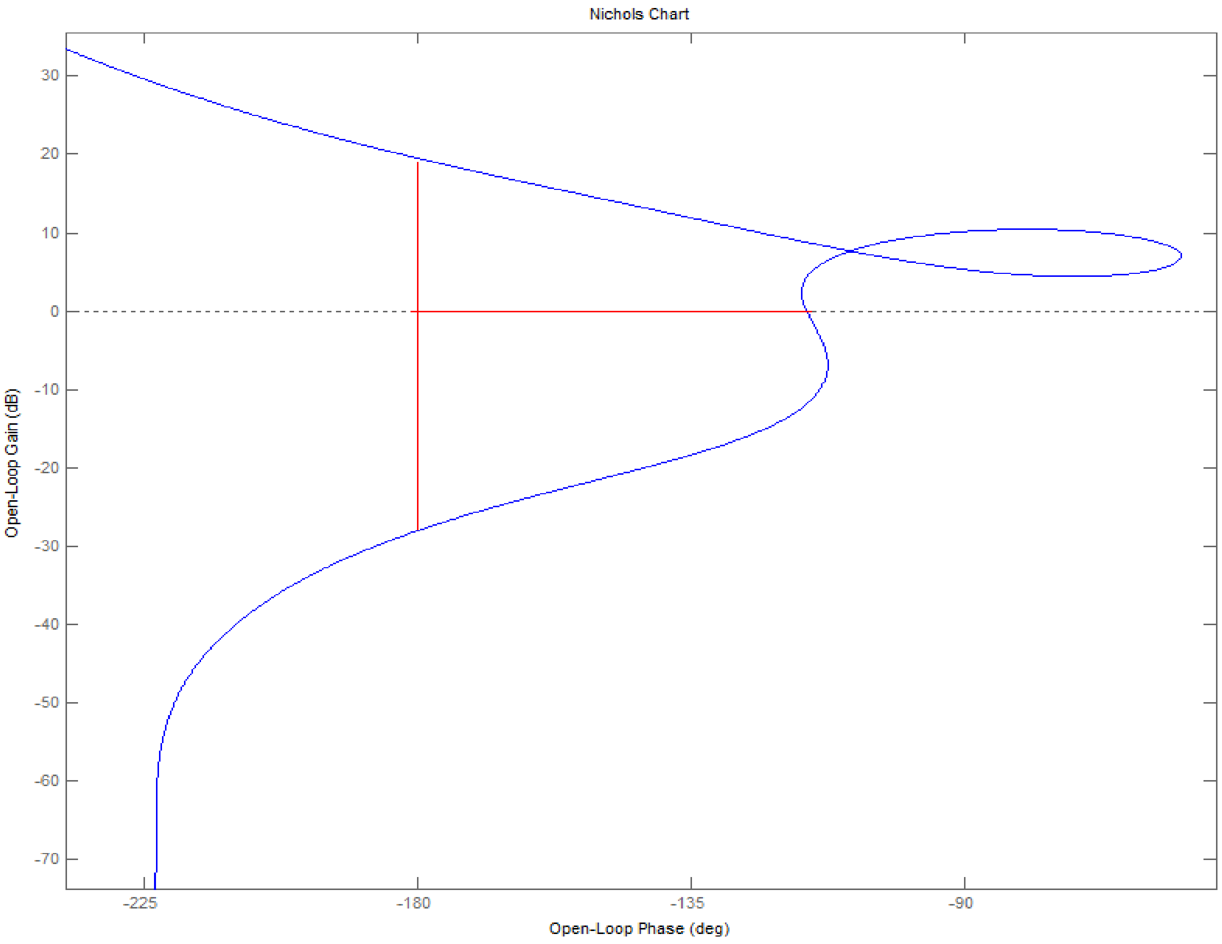


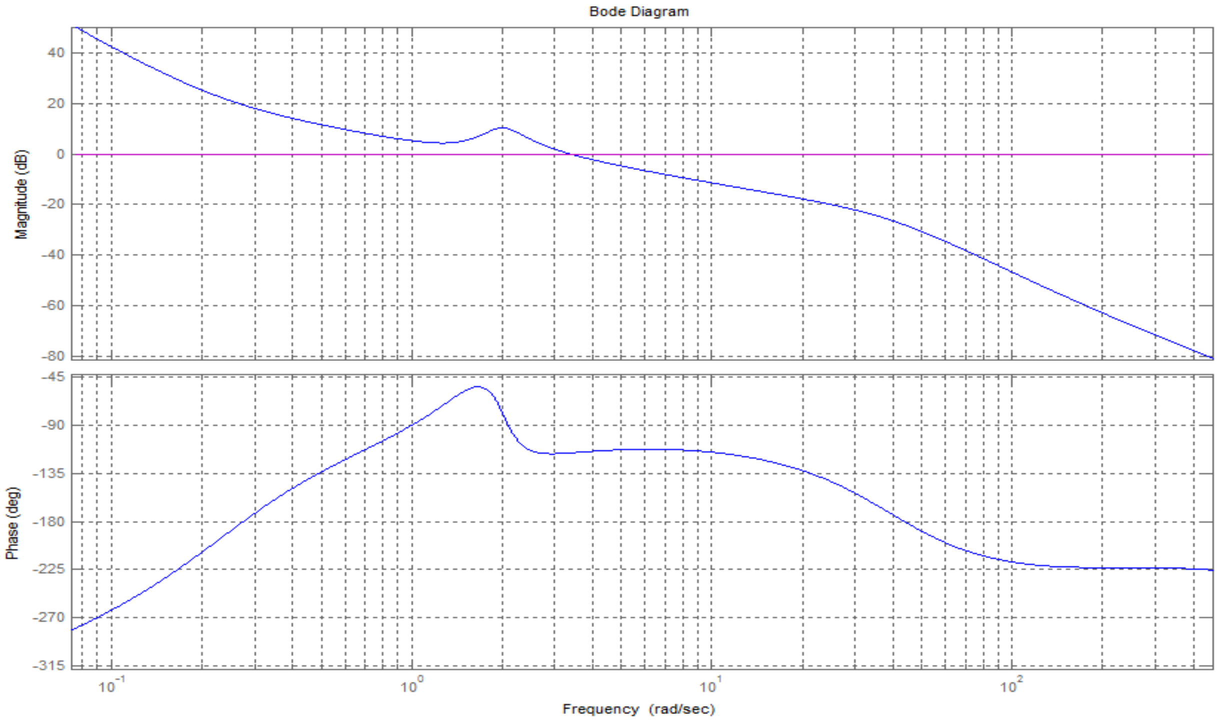
Figure (1.7) Simulink model "Open_Pitch.Mdl" used for Stability Analysis

The Matlab file "frequ.m" calculates the frequency response of this open-loop system and plots the Bode and Nichols plots for analyzing the system's stability. The following plots in Figure (1.8) show that the system has good phase and gain margins and a cross-over frequency of 3.5 (rad/sec). The alpha integrator provides high gain at low frequencies needed for command tracking.

Pitch Axis Stability Margins



Pitch Axis Bode Plot



1.2 Lateral System Analysis

From the coupled axes rigid-body vehicle model will now extract two lateral axes systems: an analysis system and a smaller system for lateral LQR control design. We will calculate a lateral mixing logic matrix that combines the control surface deflections to decouple the control in roll and yaw. We will also create a model for LQR control design, design the lateral guidance system, an angle of sideslip estimator, perform stability analysis in the frequency domain, and use a simulation to evaluate the system performance to guidance commands and its response to wind-gusts.

Processing the Input File in Batch Mode

We will now extract the lateral state-space systems by processing the input data file: "*RocketPlane-RB.Inp*" in batch mode. On the top of this file there is a batch data-set "*Batch Set for Creating Rigid-Body Models for the Rocket-Plane*" that can be processed to create the lateral axes models and also the pitch models that were created in Section (1.1). Batch processing is a much faster way to create the dynamic models in comparison with the interactive processing presented in Section (1.1). The batch creates the rocket-plane rigid-body model "*Rocket Plane at Mach=0.85, Q=150, Rigid Body*". It performs some of the functions already described in the longitudinal axis analysis. Then it creates the lateral axes model "*Rocket Plane at Mach=0.85, Q=150, Rigid Body, Lateral Axes*", by extracting the lateral states and outputs using the systems modification program.

Lateral Mixing-Logic: The batch also creates three types of mixing-logic matrices: a pitch only matrix Kpitmix (already used in the previous section), a lateral only matrix Klat, and a combined pitch and lateral 4-axes matrix K4axmix. The lateral control system uses only the two flaps and the two rudders to control the vehicle yaw and roll attitude. It does not use the body-flap and the engine throttling. The lateral mixing logic matrix "Klat" translates the roll and yaw acceleration demands to two flap and two rudder deflections and minimizes the interaction between the two lateral loops. It pre-multiplies the lateral vehicle model attempting to decouple the roll and yaw axes, and to achieve the commanded accelerations in the open-loop sense, before feedback. The mixing logic program executes in batch to process the data-set title: "*Lateral axes Mixing Matrix for Rocket-Plane [Klat: PR-dot]*" and to derive the (4x2) lateral mixing-logic matrix "Klat". It uses mass properties from a vehicle model "*Rocket Plane at Mach=0.85, Q=150, Rigid Body (for computing the lateral mixing matrix Klat)*", that includes only the four aero-surfaces. Only two control directions are defined at the matrix input: the roll acceleration (p-dot), and the yaw acceleration (r-dot). These are the directions to be decoupled by the mixing logic matrix.

Lateral Design Model: A second lateral rigid-body model is also extracted from the coupled system, using the systems modification program, to be used for LQR control design. Its title is "*Lateral Design Model*". The lateral design model is further augmented using the systems interconnection program to include one additional state: the integral of roll rate. This is to improve the tracking of the bank angle command. This system has only two inputs: the roll and yaw control demands (δ_{ailer} and δ_{rudder}) from the FCS, because it includes the lateral mixing-logic matrix "Klat" that converts the demands to four deflections. This reduces the number of control inputs from four surfaces to two. The system outputs are equal to the state-vector. The title of the augmented design model is "*Lateral Design Model with p-Integral (4-states)"*, shown in Figure (1.9).

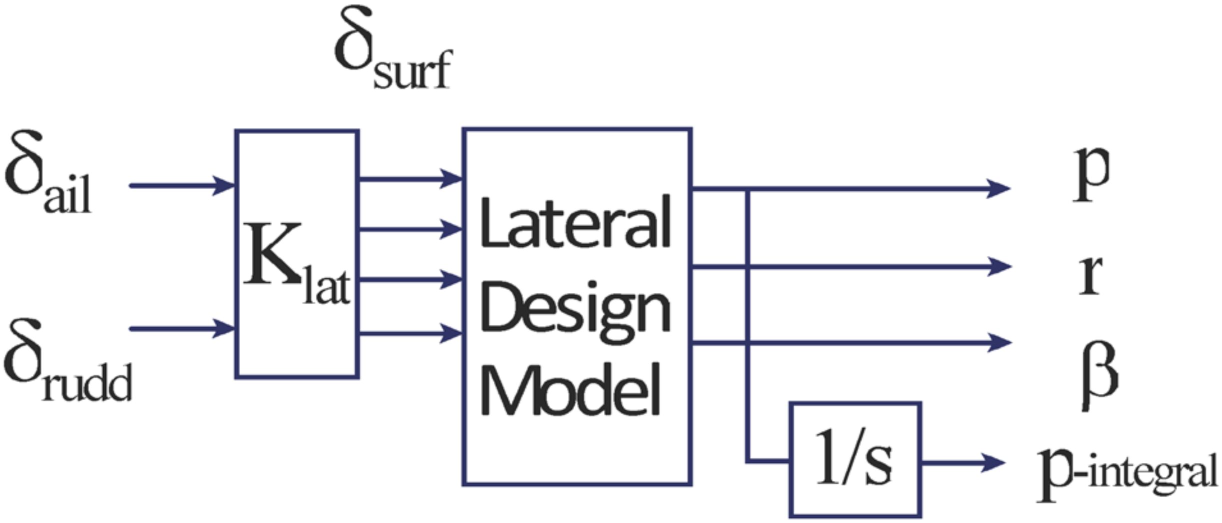


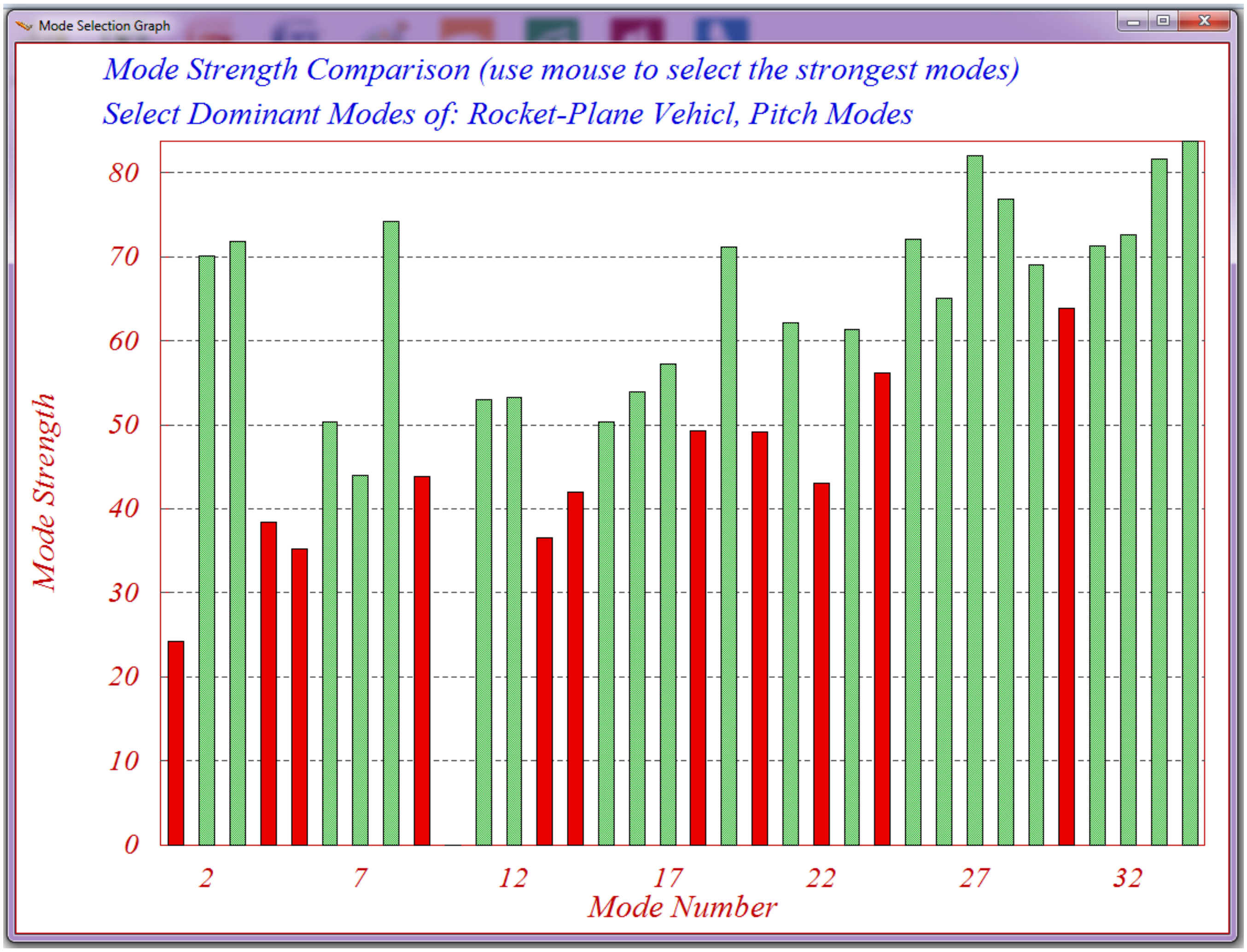
Figure (1.9) Lateral Design Model with p-Integral (4-states)

<u>Exporting Data to Matlab</u>: The batch finally saves the systems and mixing matrices to Matlab function format that can be loaded into Matlab for further analysis.

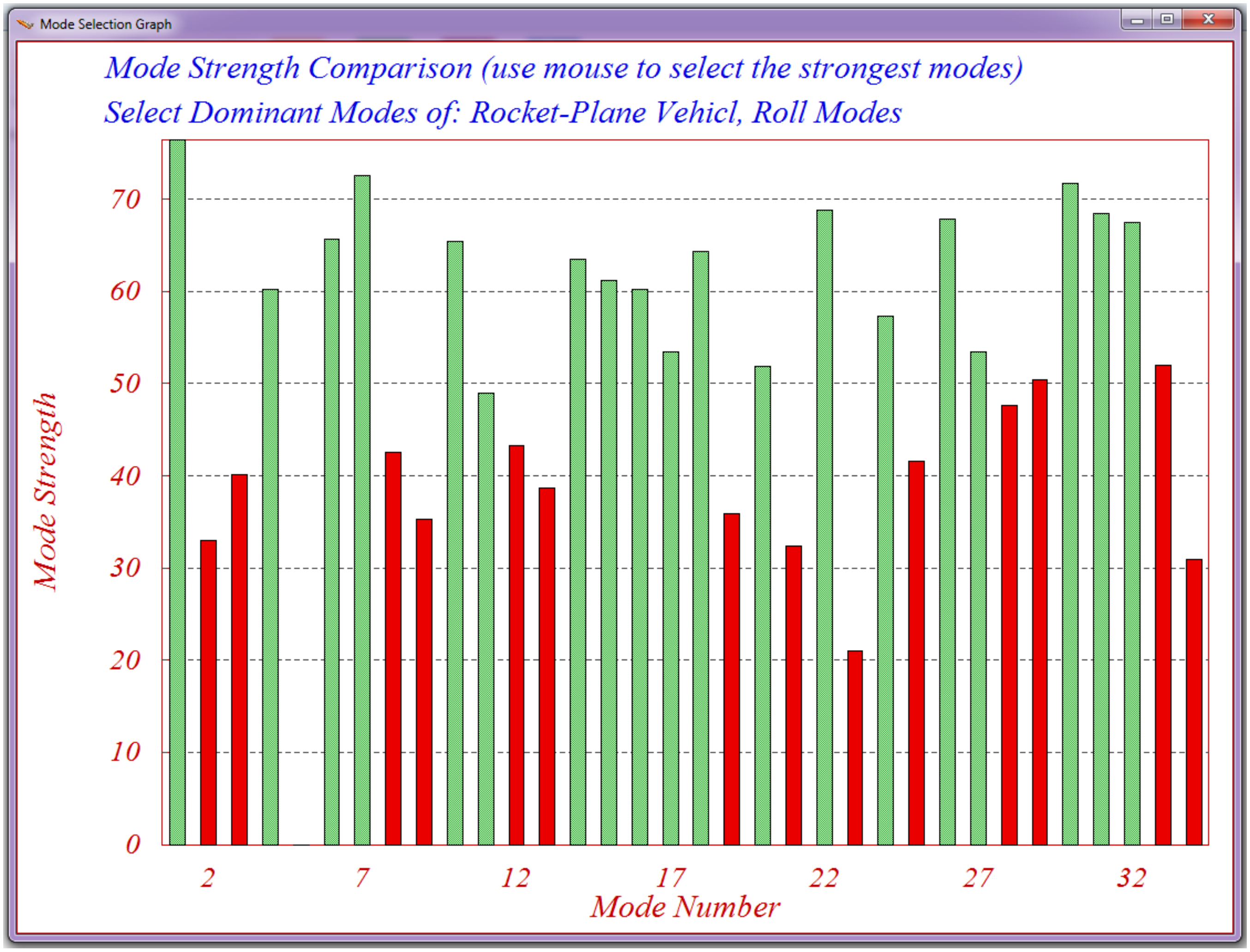
1. The original coupled vehicle model "*Rocket Plane at Mach=0.85, Q=150, Rigid Body*" is saved in function file "*Vehi_Coupled_rb.m*".
2. The pitch axis analysis model "*Rocket Plane at Mach=0.85, Q=150, Rigid Body, Pitch Axis*" is saved in file "*Vehi_Pitch_rb.m*".
3. The pitch design model "*Pitch Design Model with Alpha Integral (1-input, 4-states)*" is saved in file "*Vehi_pdes_4x.m*".
4. The lateral axes analysis model "*Rocket Plane at Mach=0.85, Q=150, Rigid Body, Lateral Axes*" is saved in file "*Vehi_RBlat.m*".
5. The lateral axes design model "*Lateral Design Model with p-Integral (4-states)*" is saved in file "*Vehi_Lat_4x.m*".
6. The pitch mixing-logic matrix "*Pitch axis Mixing Matrix for Rocket-Plane [Kpitmix: Q-dot, Ax]*" is saved in file "*Kpitmix.Mat*".
7. The (4x2) lateral mixing-logic matrix "*Lateral axes Mixing Matrix for Rocket-Plane [Klat: PR-dot]*" is saved in file "*Klat.Mat*".
8. The coupled axes mixing-logic matrix "*Coupled axes Mixing Matrix for Rocket-Plane [K4axmix: PQR-dot, Ax]*" is saved in file "*K4axmix.Mat*".

The lateral system files and matrices are transferred to directory "*C:\Flixan\Examples\Rocket Plane\Rigid Body Design\Mat Later*" for further analysis using Matlab.

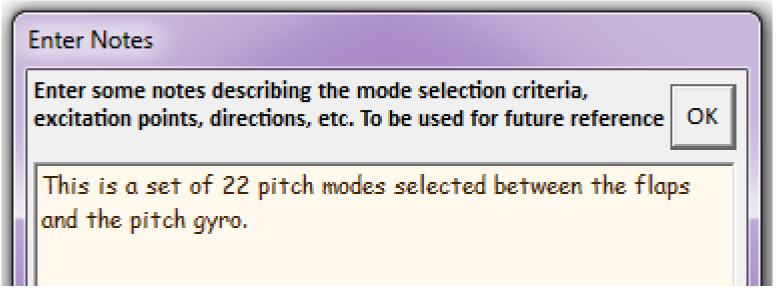
The program finally shows a bar-chart that compares the mode strength for each mode versus mode number in the selected pitch direction. The color of all bars is initially red before selection. The analyst manually selects each mode to be included in the set from the bar-chart by clicking with the mouse on the selected bar. When a mode is selected the color of the bar changes after selection from red to green. Press “Enter” when the selection is complete to save the modes set. The next two bar-charts show the relative mode strengths resulting from two types of mode selection cases: a pitch selection that was just described and a roll selection.



In the roll mode selection case, shown below, the two flaps were excited anti-symmetrically with opposite forces in the ±Z direction and also with anti-symmetric pitch torques at the hinges of the two flaps. A rotational sensor was included measuring roll. Notice that the modes that are strong in pitch are not strong in roll, and vise versa.



In the field below the user may finally write a short description that provides information about the mode selection process. Explain, for example, the rationale behind the choices made, such as, conditions, actuator and sensor locations, forcing directions, etc. The user notes will be included as comment lines below the title in the selected modes set. The selected set of mode frequencies and shapes is finally saved in file "*RocketPlane_Flx.Inp*" and ready to be processed by the vehicle modeling program, as already described in Section (2.2). The default title of the modal set was modified to: "*Rocket Plane at Mach=0.85, Q=150, Flex Model, (22 Pitch Modes)*".



Enter some notes describing the mode selection criteria, excitation points, directions, etc. To be used for future reference

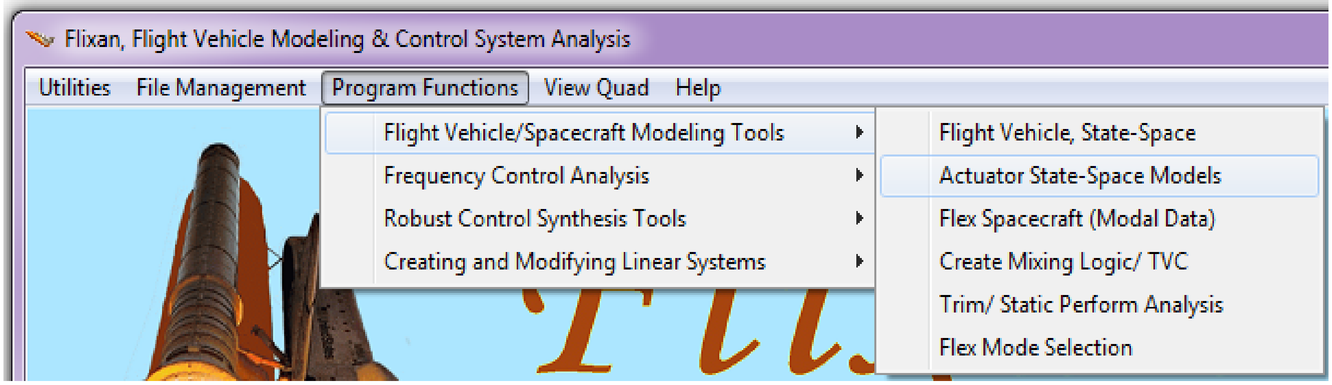
This is a set of 22 pitch modes selected between the flaps and the pitch gyro.

2.5 Actuator Model

In the rigid-body analysis and without TWD dynamics a simple 2nd order actuator model was sufficient at this level. With flexibility, however, and the TWD included in this model we will need a more refined actuator model that provides aero-surface rate and acceleration output in addition to the surface deflection in order to implement the TWD dynamics. The three actuator model outputs: control surface position, rate, and acceleration are used as inputs to drive the vehicle state-space model. The accelerations of the control surfaces about the hinge is what creates the TWD reaction forces and moments on the vehicle and also excites flexibility. The rotation of the control surface relative to its hinge is the result of two torques which are applied to the control surface: a torque provided by the actuator which in this case is an electro-mechanical actuator, and also an external load-torque caused by the vehicle motion, which is mainly due to rotational and translational accelerations at the hinge.

The actuator model in addition to the actuator dynamics and the actuator position control system it includes also the rotational dynamics of the aero-surface load and the associated stiffnesses. The load acceleration relative to the hinge is a function of the control torque and also the load-torque. The control torque is an internal variable within the actuator model, but the load-torque has to come as an external input to the actuator model. The load-torque is a mechanical feedback from the vehicle's hinge moment outputs to the actuator load-torque input. The actuator model in this example is an electro-mechanical type (b) actuator, which is fully documented in the actuator modeling program. The actuator parameters are already in file "*RocketPlane-Flx.Inp*" and ready to be processed by the actuator modeling program. Ideally, each surface should have a separate actuator with separate parameters, but in this case we have a single actuator model for all 5 surfaces and its title is "*Electro-Mechanical Actuator with an Extendable Push Rod*".

Start the Flixan program and select the "*\Rocket Plane\ Flex Model*" directory. From the main menu go to "*Flight Vehicle/ Spacecraft Modeling*", and then go to "*Actuator State-Space Modeling*" program. From the filenames selection menu select the input file "*RocketPlane_Flx.Inp*" and the systems file "*RocketPlane_Flx.Qdr*". From the actuator selection menu select the only one title and click "Run".



2.6 Closed-Loop Simulation Model with Flexibility

The Matlab analysis files for the flex vehicle are in subdirectory “Mat Anal”. The Simulink model with flexibility and TWD is in file "*Closed_Loop_Flx.Mdl*", shown in Figure (2.1), and it is similar to the rigid-body simulation model, but with some modifications. From the top level it looks almost identical to the rigid-body model described in Section 1, consisting of four control loops: roll, pitch, yaw and the axial acceleration loop that controls the vehicle speed. The vehicle (green block), however, includes the flex system with the 30 flex modes and includes TWD, that is loaded into Matlab from file “*vehi_30flx.m*”.

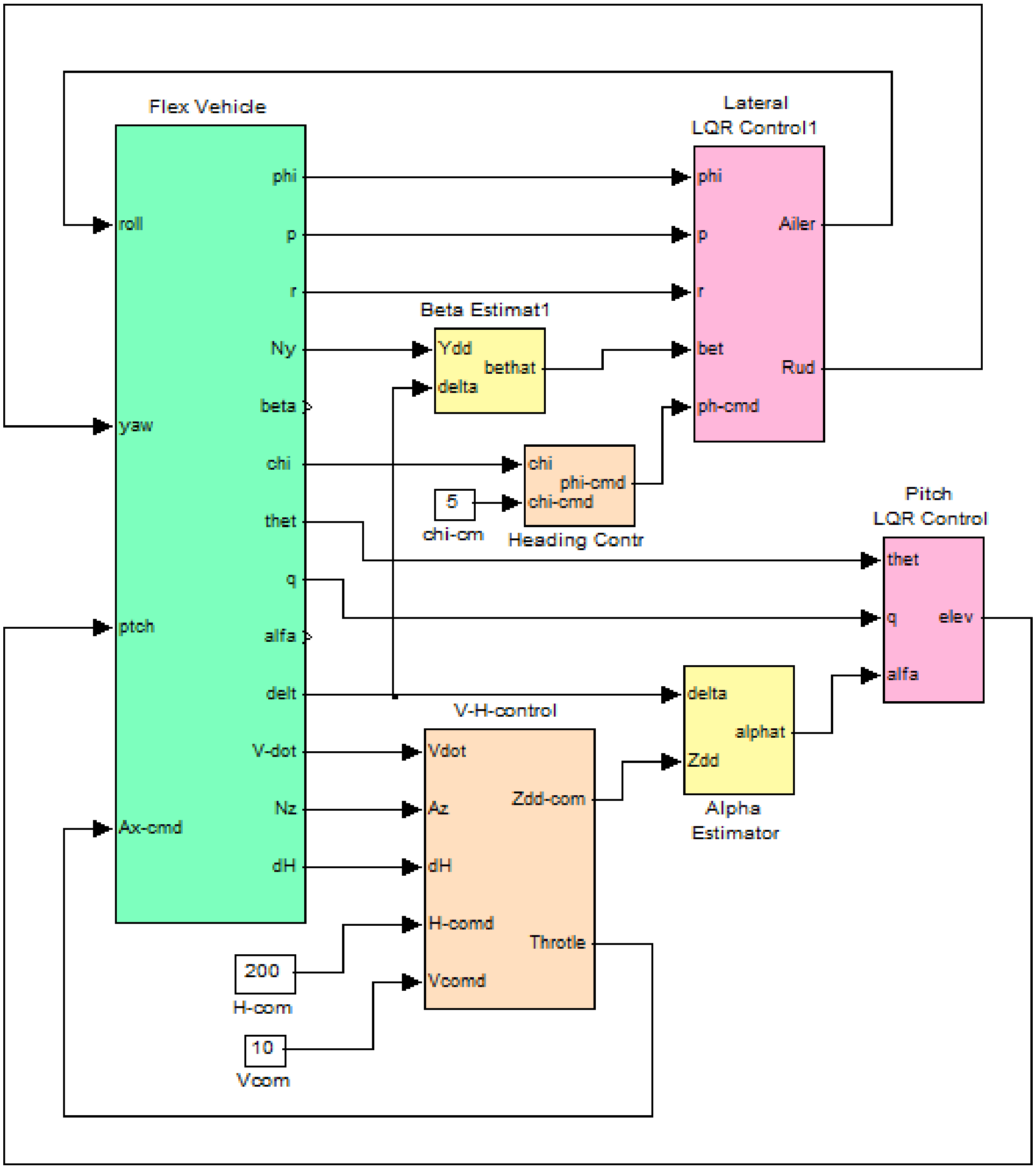
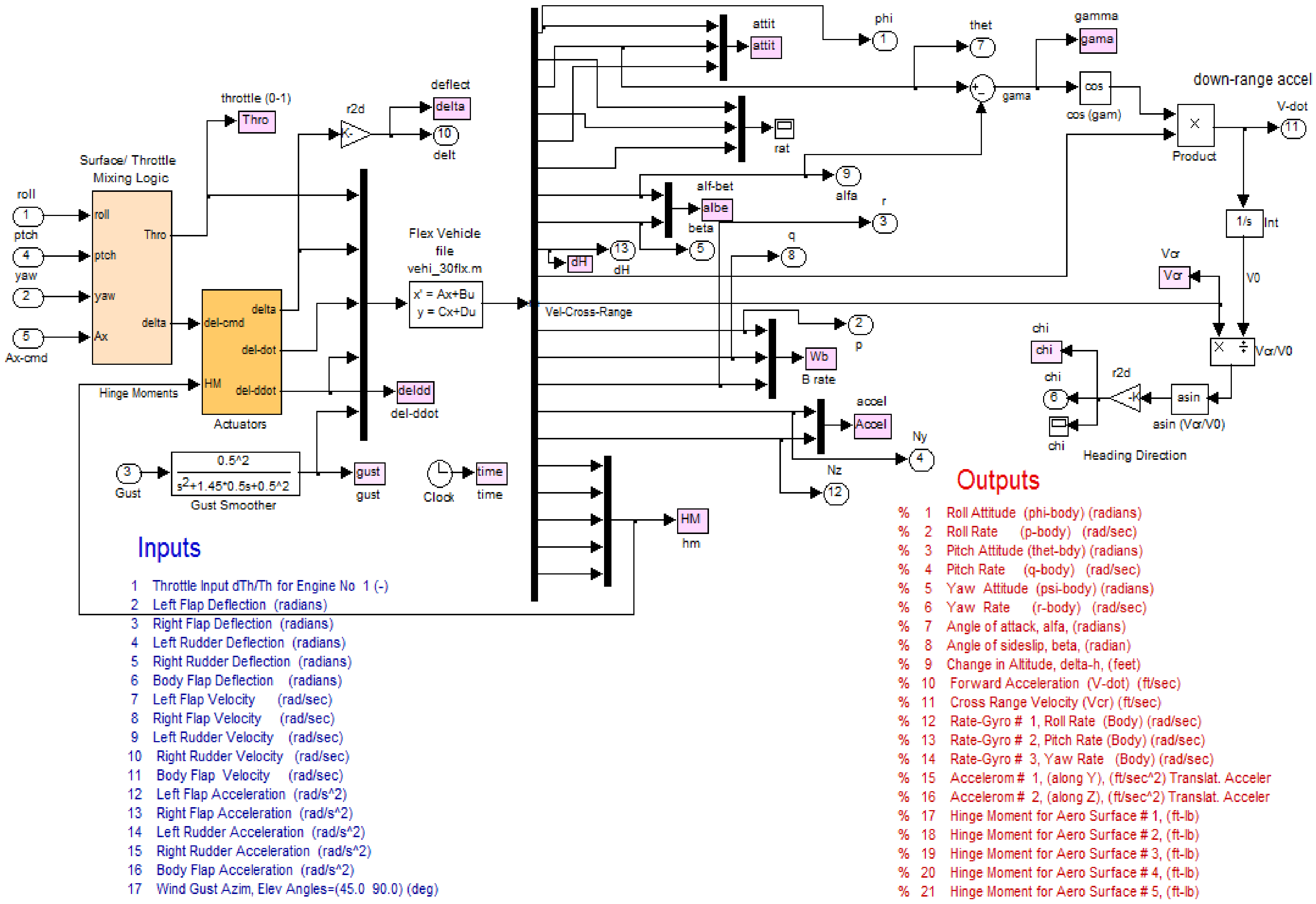


Figure (2.1) Closed-Loop Simulation Model with Flexibility "Closed_Loop_Flx.Mdl"



Inputs

1 Throttle Input dTh/Th for Engine No 1 (-)
2 Left Flap Deflection (radians)
3 Right Flap Deflection (radians)
4 Left Rudder Deflection (radians)
5 Right Rudder Deflection (radians)
6 Body Flap Deflection (radians)
7 Left Flap Velocity (rad/sec)
8 Right Flap Velocity (rad/sec)
9 Left Rudder Velocity (rad/sec)
10 Right Rudder Velocity (rad/sec)
11 Body Flap Velocity (rad/sec)
12 Left Flap Acceleration (rad/s^2)
13 Right Flap Acceleration (rad/s^2)
14 Left Rudder Acceleration (rad/s^2)
15 Right Rudder Acceleration (rad/s^2)
16 Body Flap Acceleration (rad/s^2)
17 Wind Gust Azim, Elev Angles=(45.0 90.0) (deg)

Outputs

% 1 Roll Attitude (phi-body) (radians)
% 2 Roll Rate (p-body) (rad/sec)
% 3 Pitch Attitude (thet-bdy) (radians)
% 4 Pitch Rate (q-body) (rad/sec)
% 5 Yaw Attitude (psi-body) (radians)
% 6 Yaw Rate (r-body) (rad/sec)
% 7 Angle of attack, alfa, (radians)
% 8 Angle of sideslip, beta, (radian)
% 9 Change in Altitude, delta-h, (feet)
% 10 Forward Acceleration (V-dot) (ft/sec)
% 11 Cross Range Velocity (Vcr) (ft/sec)
% 12 Rate-Gyro # 1, Roll Rate (Body) (rad/sec)
% 13 Rate-Gyro # 2, Pitch Rate (Body) (rad/sec)
% 14 Rate-Gyro # 3, Yaw Rate (Body) (rad/sec)
% 15 Accelerom # 1, (along Y), (ft/sec^2) Translat. Acceler
% 16 Accelerom # 2, (along Z), (ft/sec^2) Translat. Acceler
% 17 Hinge Moment for Aero Surface # 1, (ft-lb)
% 18 Hinge Moment for Aero Surface # 2, (ft-lb)
% 19 Hinge Moment for Aero Surface # 3, (ft-lb)
% 20 Hinge Moment for Aero Surface # 4, (ft-lb)
% 21 Hinge Moment for Aero Surface # 5, (ft-lb)

The green vehicle block is shown in detail in Figure (2.2). It contains the (6x4) mixing logic matrix "K4AXMIX.mat" that transforms the roll, pitch, yaw, and axial acceleration demands into control surface and throttle commands. The following initialization file "run.m" loads into Matlab the flex vehicle, the actuator, the surfaces mixing matrix, the LQR gain matrices Kgp and Kgl, the (α, β) estimator parameters, and the flexibility (notch) filter coefficients.

```
% Initialization File:  run.m
d2r=pi/180; r2d=180/pi;
[Avf,Bvf,Cvf,Dvf]=vehi_30flx;                          % Load Flex Vehicle Model
[Aema,Bema,Cema,Dema]=ema_actuator;                    % Load Electro-Mechan (B) Actuator
load K4AXMIX.mat K4AXMIX -ascii                        % Control Surface Mixing Logic

% Load LQR gains from Rigid-Body Design
Kgp=[0.0000    2.6440    4.0785    7.0711];            % Pitch Gains (thet,q,alf,alf-int)
Kgl=[2.8186    2.4399  -14.5092    6.1944; ...         % Later Gains (p,r,phi,beta)
     0.5102    3.5601   -7.5662    0.5709];

% Notches for Flex Modes
zn1=0.01;  wn1=79;  zd1=0.2; wd1=74;  Kf1=wd1^2/wn1^2; %  79 rad Notch
zn2=0.015; wn2=116; zd2=0.2; wd2=110; Kf2=wd2^2/wn2^2; % 116 rad Notch
zn3=0.015; wn3=132; zd3=0.2; wd3=125; Kf3=wd3^2/wn3^2; % 130 rad Notch (3)
zn4=0.015; wn4=112; zd4=0.2; wd4=109; Kf4=wd4^2/wn4^2; % 112 rad Notch (4)
zn5=0.015; wn5=140; zd5=0.2; wd5=135; Kf5=wd5^2/wn5^2; % 140 rad Notch (5)
zn6=0.015; wn6=211; zd6=0.2; wd6=205; Kf6=wd6^2/wn6^2; % 211 rad Notch (6)

% Alpha-Beta Estimator Parameters
Qbar=150; Mass=198.32; Sref=79.069; V0=822.8;
Cza=-0.073; Cyb=-0.0245;
Cydel= [-0.24668,0.24668, 0.47413,-0.47413,0]/100;     % Cy due to surf deflect
Czdel= [0.51664,0.51664,0.60478,0.60478,0.06]/100;     % Cz due to contrl surface
```

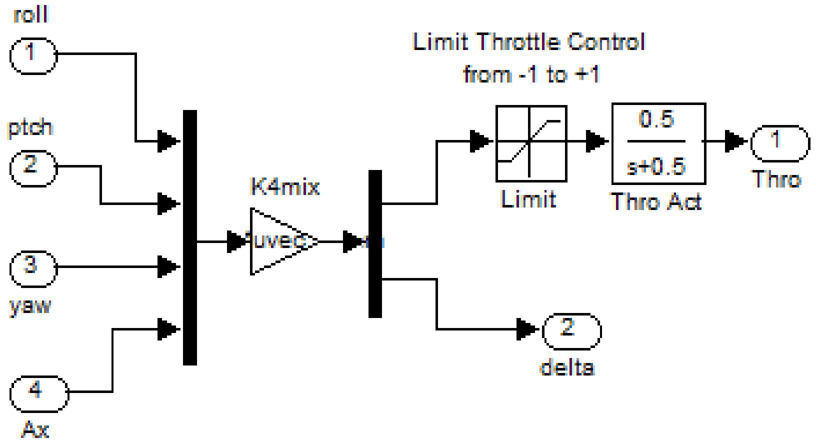


Figure (2.3) Throttle Control and Aero-Surface Mixing Logic

The vehicle and actuator are included as continuous state-space systems. The Mixing-Logic matrix converts the roll, pitch, yaw, and axial acceleration demands to thrust variations and surface deflections. The engine throttle control is the first vehicle input that varies between (-1) to (+1), with zero corresponding to the nominal 2000 (lb) thrust, (-1) corresponding zero thrust, and (+1) corresponding to the maximum 4000 (lb) thrust. Five separate actuators are used, one for each of the five control surfaces. The 5 position, 5 velocity, and 5 acceleration outputs from the actuators are driving the corresponding inputs in the flexible vehicle model.

The vehicle system has five additional outputs which are the hinge moment at the five control surfaces. These are the external load-torques that we talked about representing the mechanical external loading on the actuators caused by the vehicle acceleration. The direction of a hinge moment is defined to be positive when the torque is in the clockwise direction about the hinge vector. The hinge moments are fed-back to the actuator hinge moment inputs. The last input to the vehicle model is the gust disturbance. A gust filter is included in the vehicle gust input to smooth out the rectangular wind-gust impulse and make it more realistic.

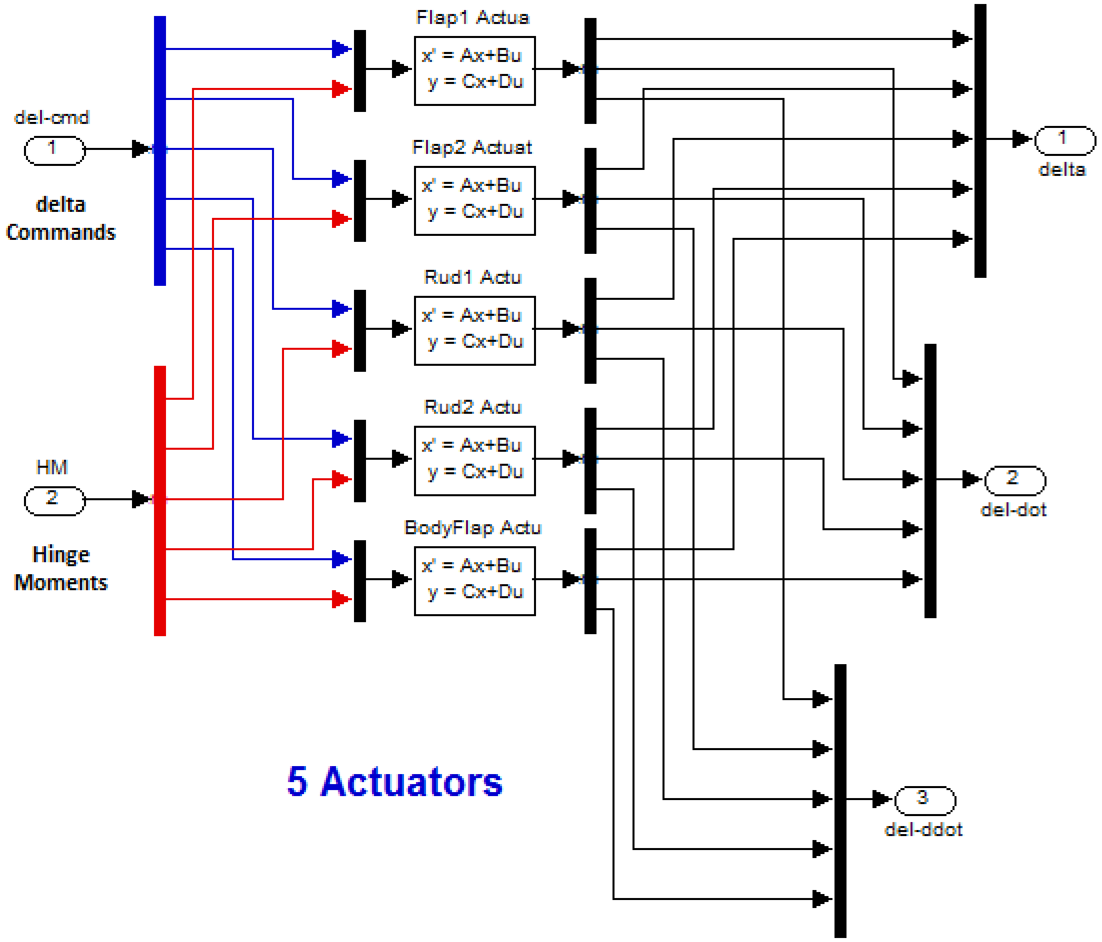


Figure (2.4) System of 5 Aero-Surface Actuators

The remaining of the flexible Simulink model is very similar to the rigid-body. The cross-range velocity (V_{cr}) is divided by the total ground speed to compute the heading direction angle (chi) which needs to be controlled by performing bank maneuvers in the lateral system. The V-dot integrator which computes the vehicle speed is initialized at the nominal vehicle speed which is 823 (ft/sec). The heading direction controller is part of the lateral control system. It receives a change in heading direction command (chi-cmd) and commands a momentary change in bank angle to achieve the required change in the heading direction. The bank angle returns to zero after the change in heading direction is achieved.

Several notch and low-pass filters have been included in the control blocks to prevent flex mode instabilities. The initialization file loads also the alpha & beta estimator parameters, because the estimated alpha and beta replace the real alpha and beta angles required by the LQR controller.

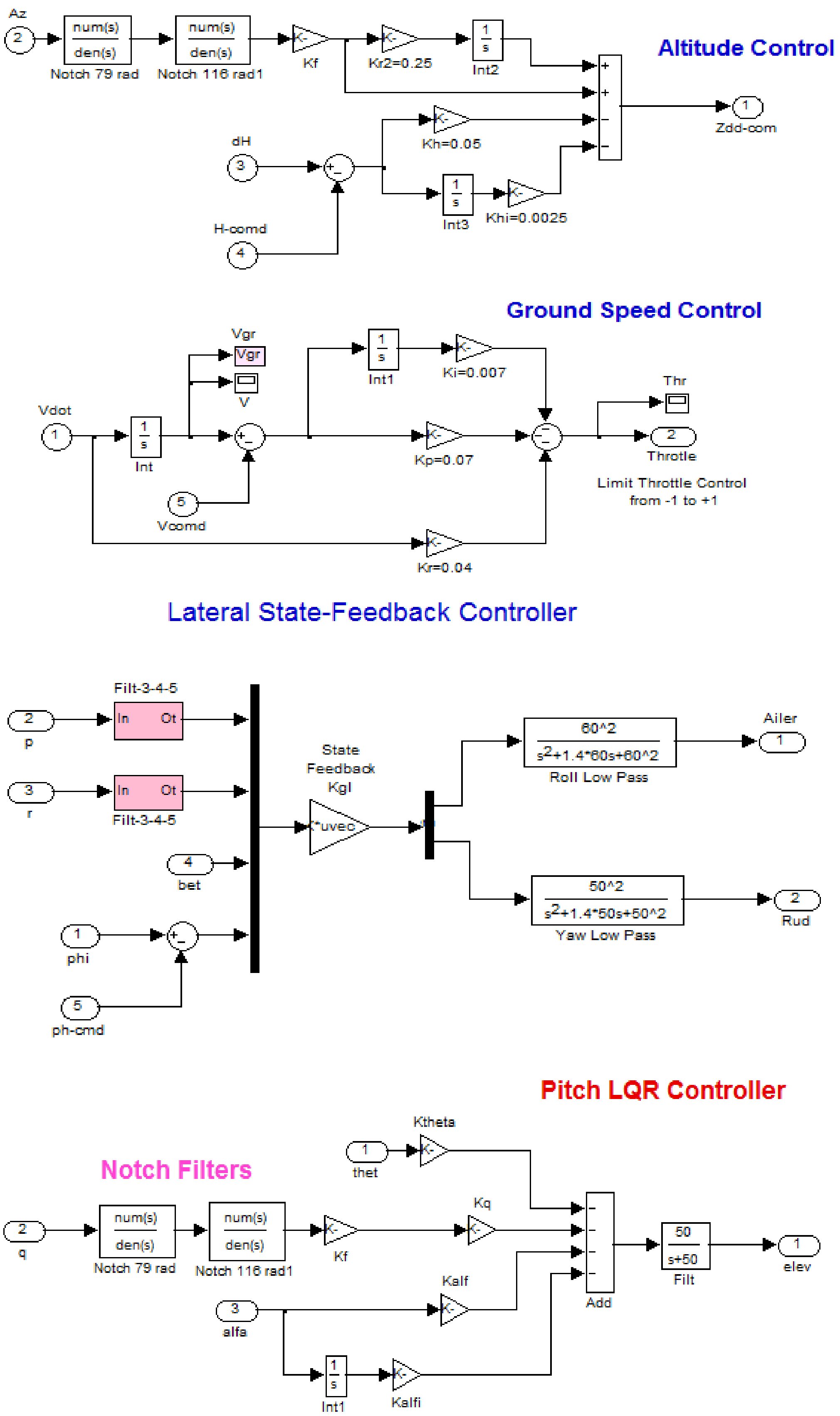
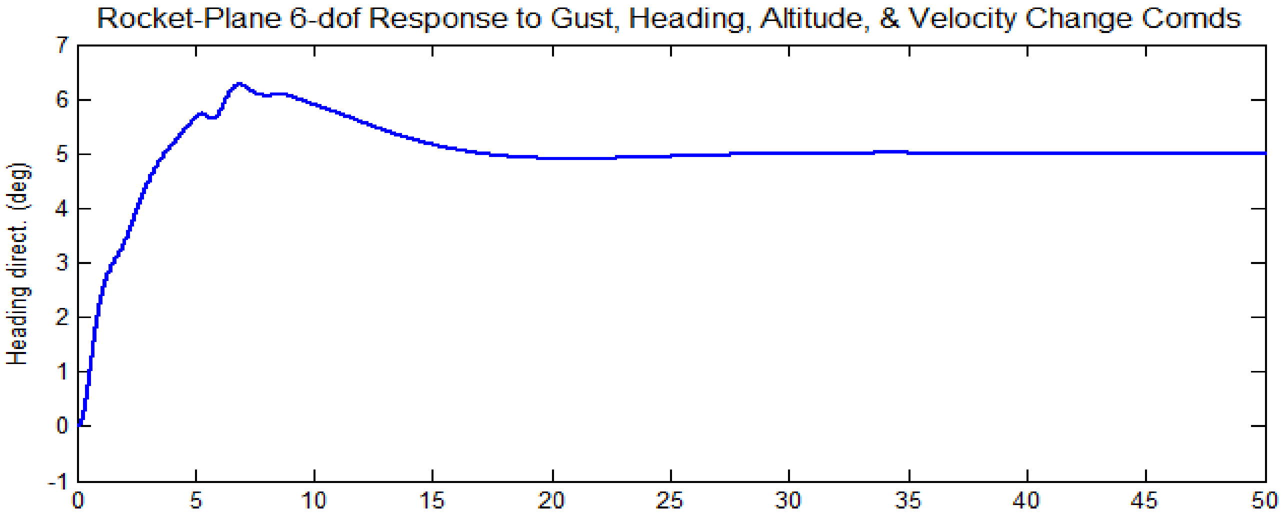
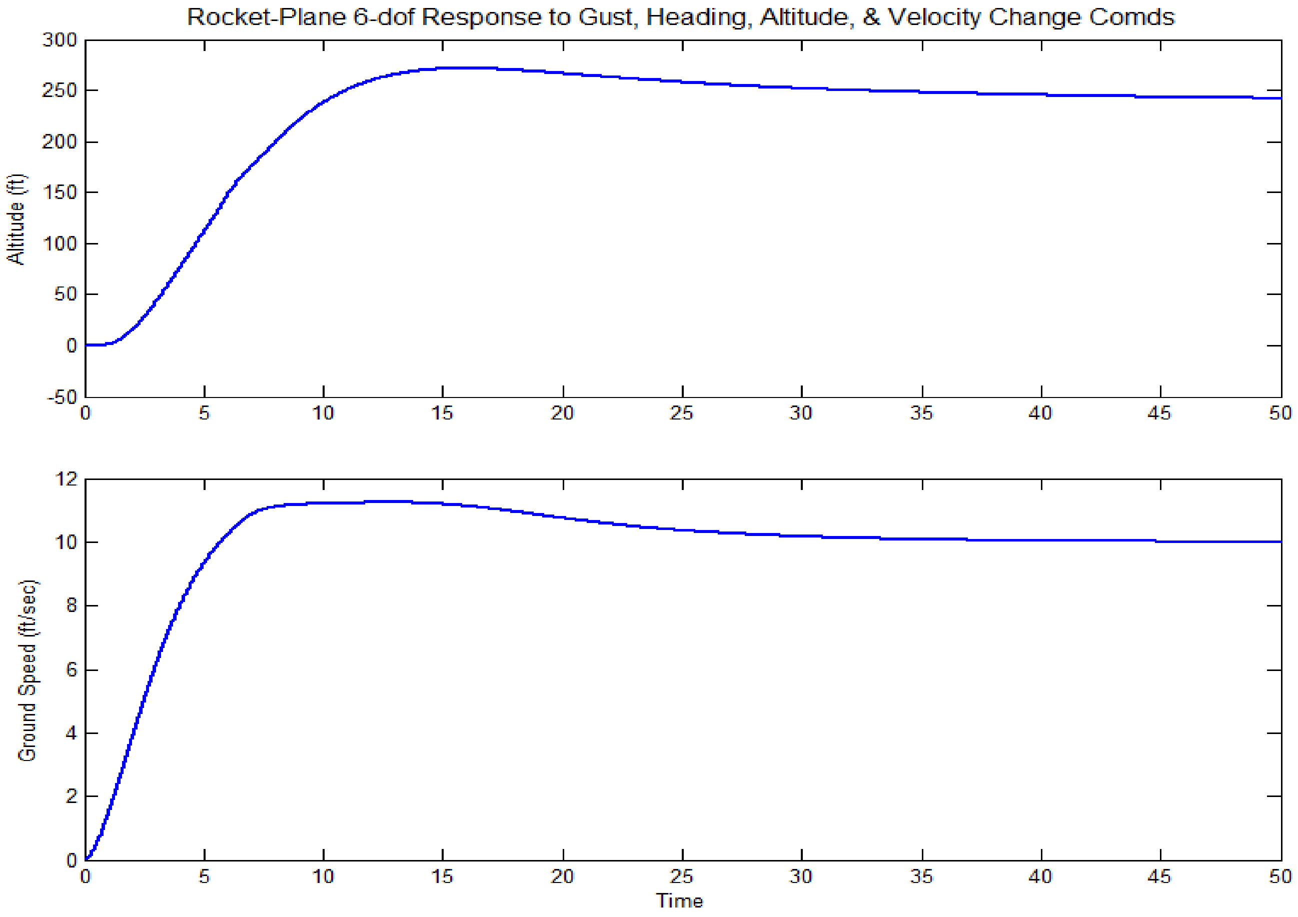


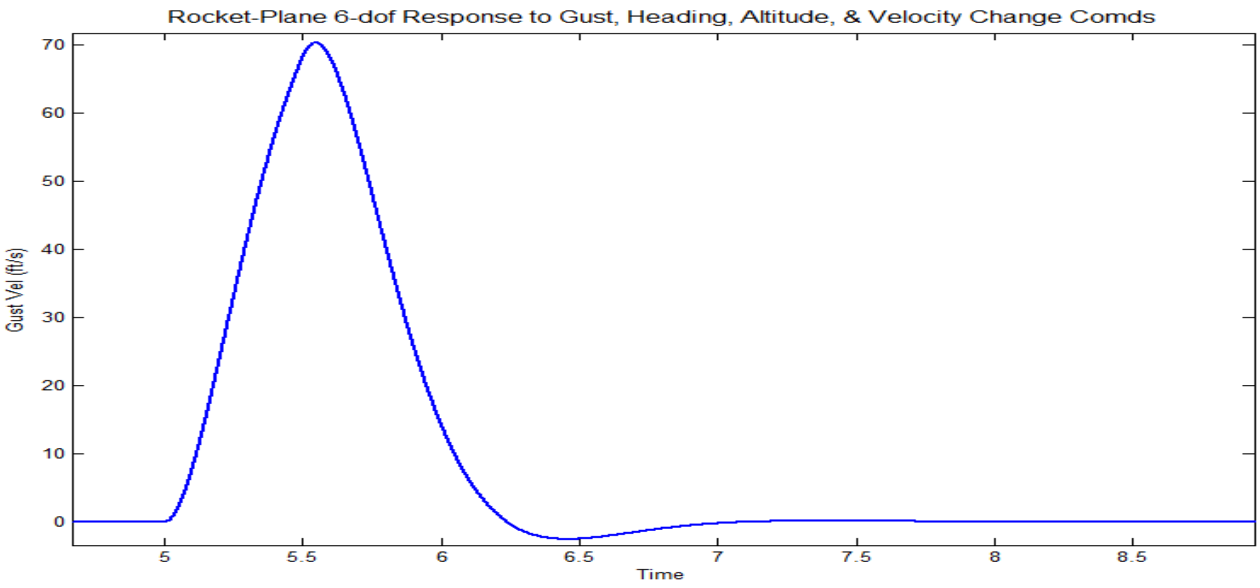
Figure (2.5) The Flight Control Systems are now modified to include Filters

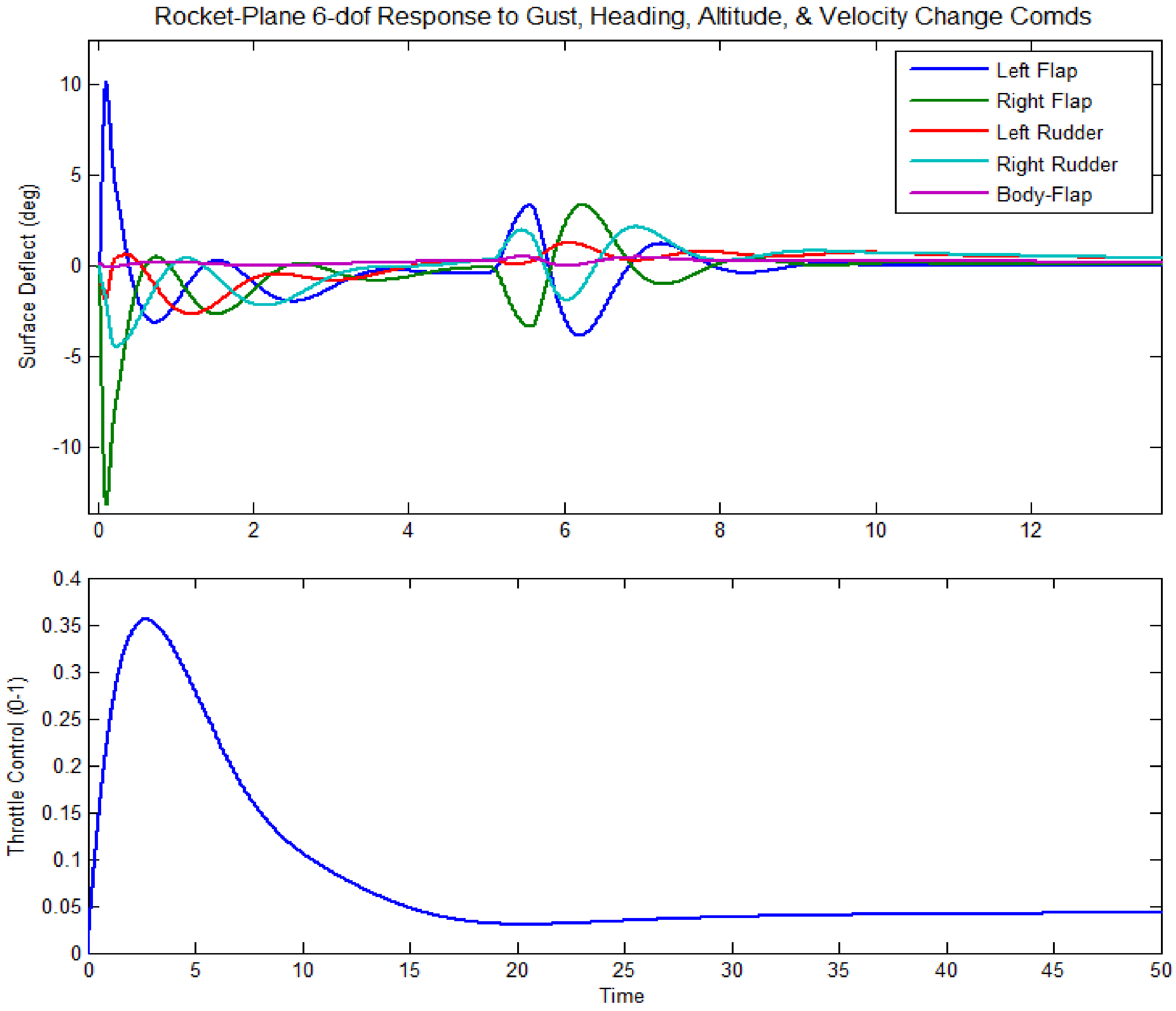
2.7 Simulation Results

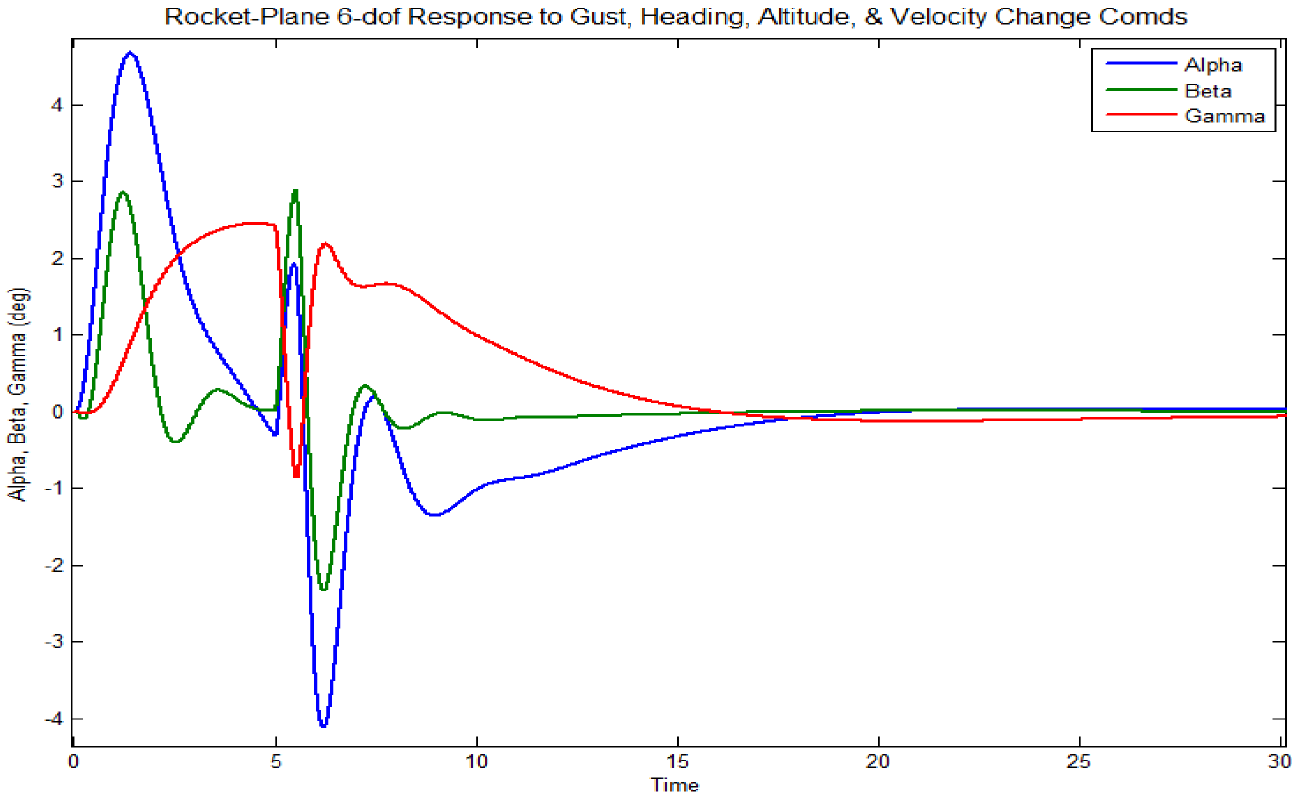
The Simulink model "*Closed_Loop_Flx.Mdl*" that was described in Section (2.6) will be used to perform a simulation similar to the rigid-body simulation of Section 1. This time the vehicle model includes flexibility and "tail-wag-dog" dynamics which is visible in the accelerations. The gust impulse is applied at 5 (sec) and its duration is shortened to 0.5 (sec) to excite flexibility. Two additional plots are included showing the control surface accelerations (from the actuator outputs) and the hinge moments due to vehicle motion. The Matlab file “Pl.m” is used to plot the simulation data.

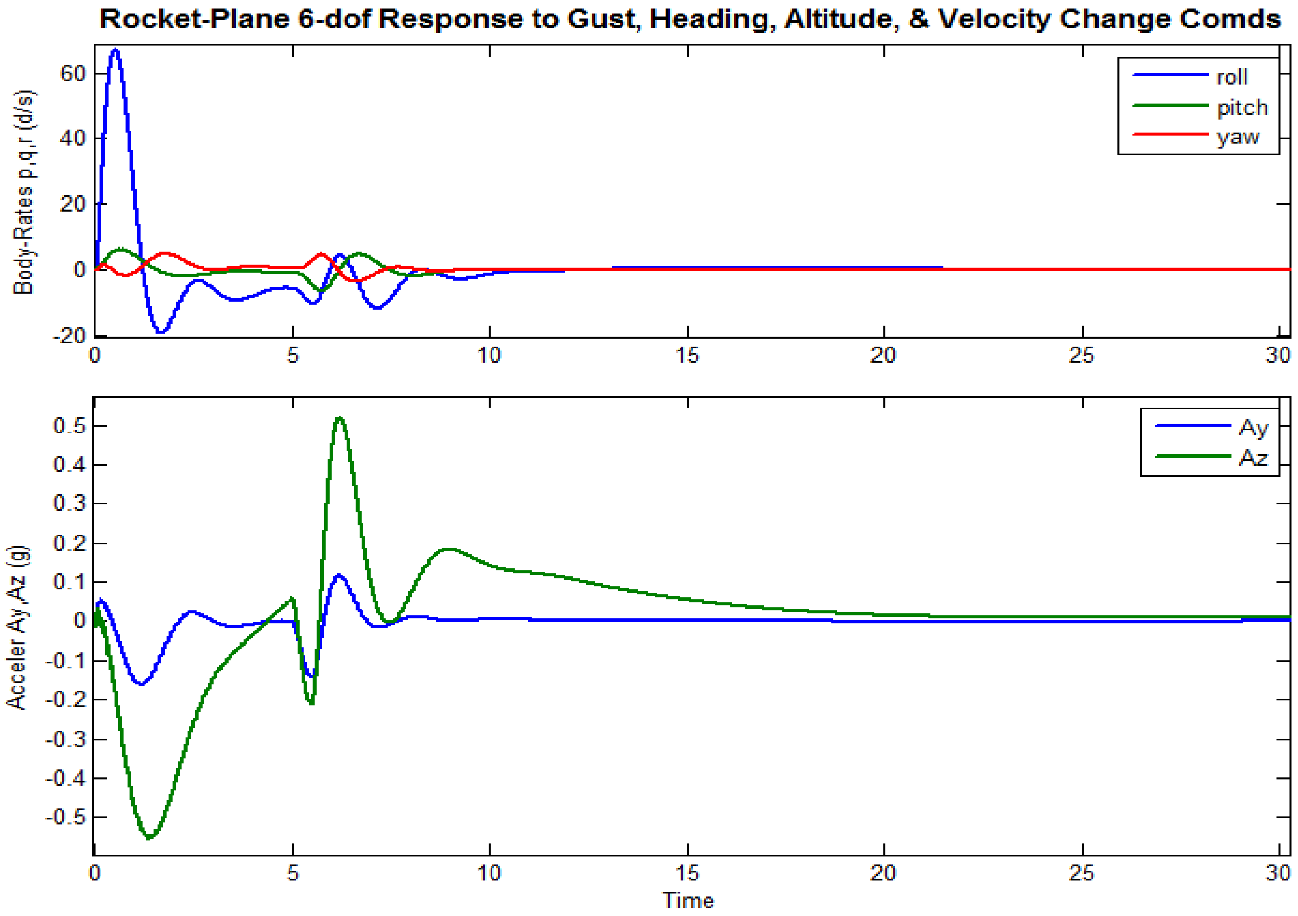


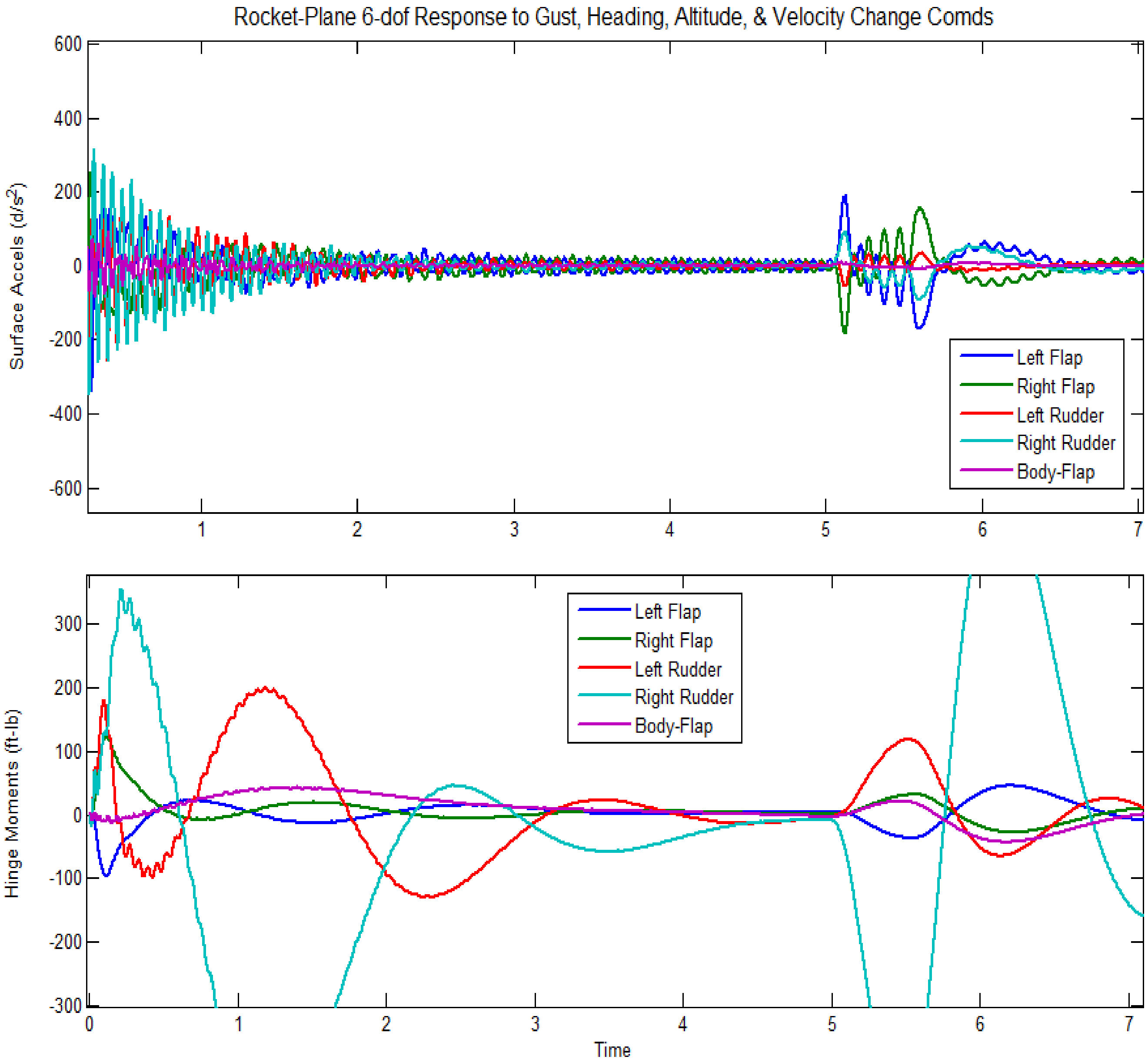



Rocket-Plane 6-dof Response to Gust, Heading, Altitude, & Velocity Change Comds


Rocket-Plane 6-dof Response to Gust, Heading, Altitude, & Velocity Change Comds


Rocket-Plane 6-dof Response to Gust, Heading, Altitude, & Velocity Change Comds
Alpha
Beta
Gamma
Alpha, Beta, Gamma (deg)
4
3
2
1
0
-1
-2
-3
-4
0
5
10
15
20
25
30
Time


Rocket-Plane 6-dof Response to Gust, Heading, Altitude, & Velocity Change Comds
roll
pitch
yaw
Body-Rates p,q,r (d/s)
60
40
20
0
-20
0
5
10
15
20
25
30
Ay
Az
Acceler Ay,Az (g)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
5
10
15
20
25
30
Time



The plots show the effects of structural flexibility in the normal acceleration, the surface accelerations and in the hinge moments. Initially, high frequency resonances are excited at the surface hinges mainly due to the flap deflections. Lower frequency resonances are also excited at 5 (sec) when the gust is applied.

Stability Analysis

The Simulink model “*Open_Loop_Flx.Mdl*” in subdirectory “Mat Anal” is used for stability analysis in the frequency domain. It is a single-input-single-output system for classical control analysis similar to the model that was used for rigid-body analysis. It contains the same elements as the closed-loop flex system but the loop to be analyzed is opened, while the other three loops are closed. The Matlab file "frequ.m" calculates the frequency response across the open-loop and plots the Bode and Nichols charts which are used for evaluating the system stability. To check, for example, the yaw axis stability we must open the yaw loop (as shown below) with the roll, pitch, and throttle loops closed, and calculate the frequency response across the opened yaw loop. The (-1) gain in the open loop is included in order to set the phase correctly for classical control analysis. The system has acceptable phase and gain margins in all four loops with a control system bandwidth of approximately 3.5 (rad/sec). The throttle control loop is slower with only 0.5 (rad/sec) bandwidth.

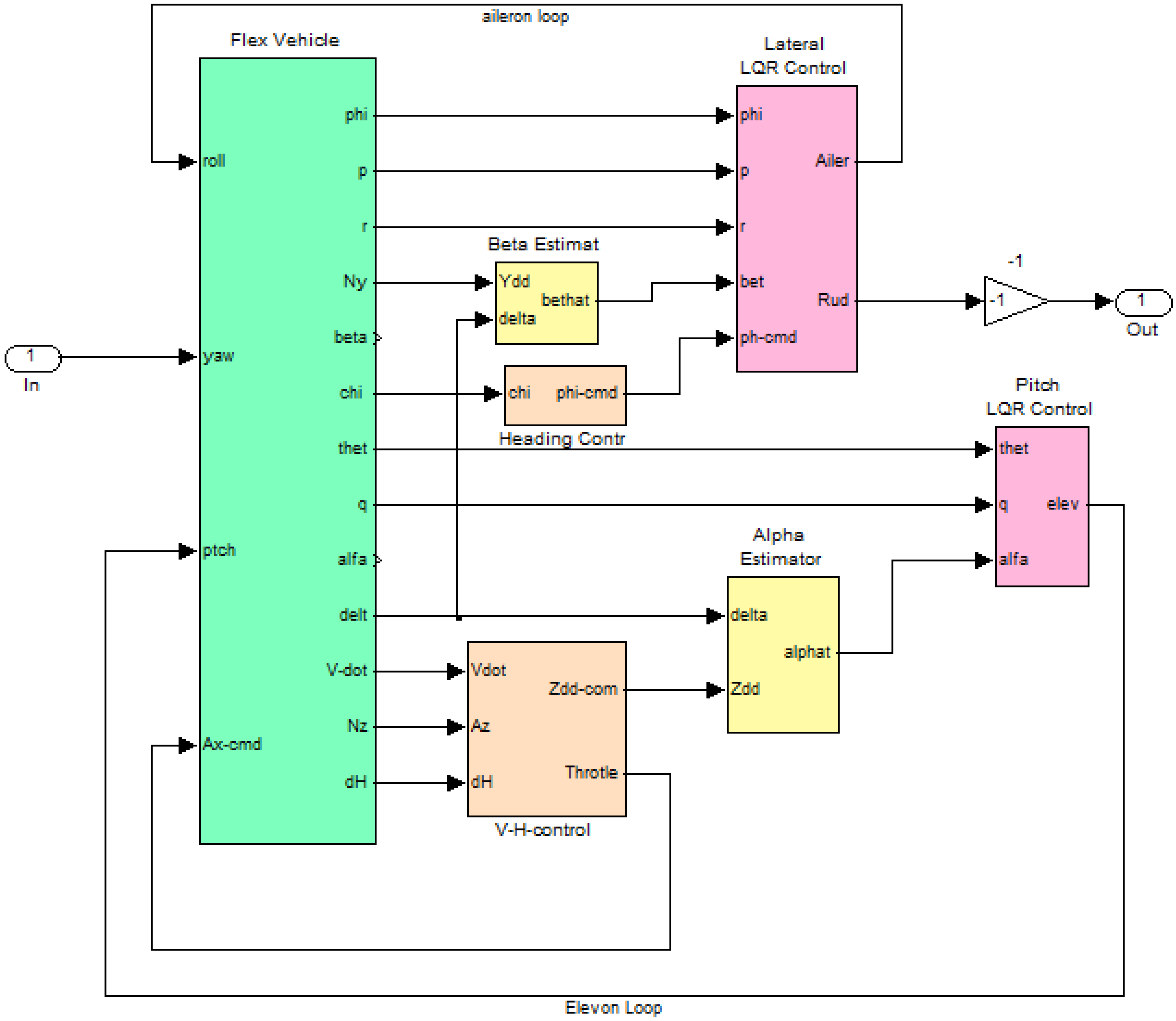
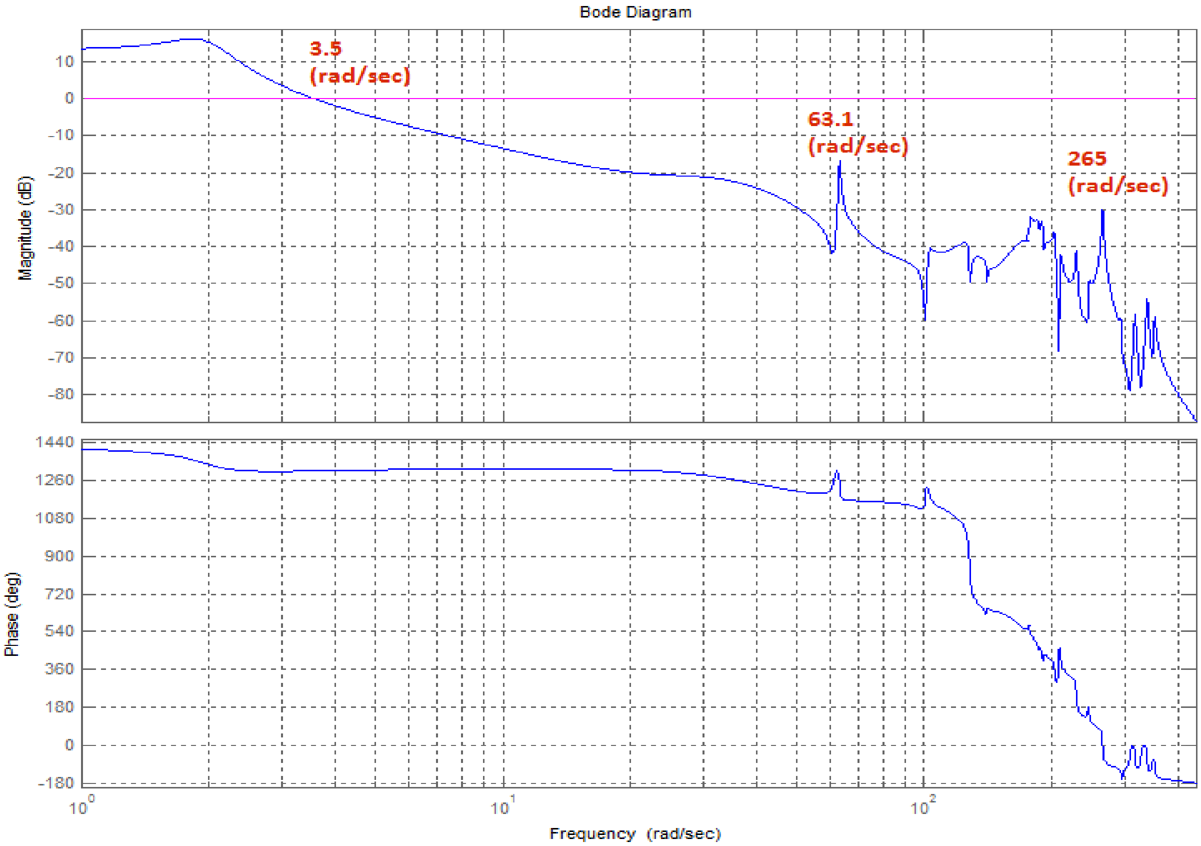
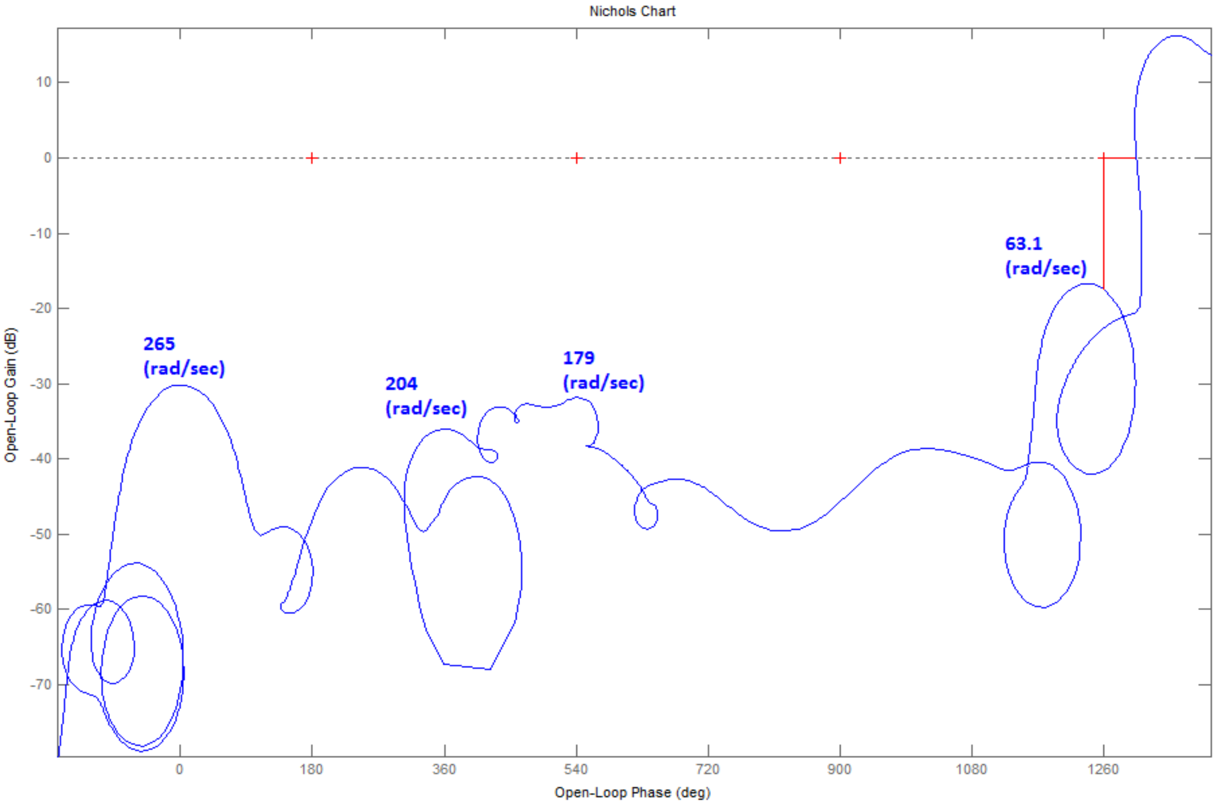


Figure (2.7) Simulink Model "Open_Loop_Flx.Mdl" Used for Stability Analysis

Roll Axis Open-Loop Frequency Response (other loops closed)

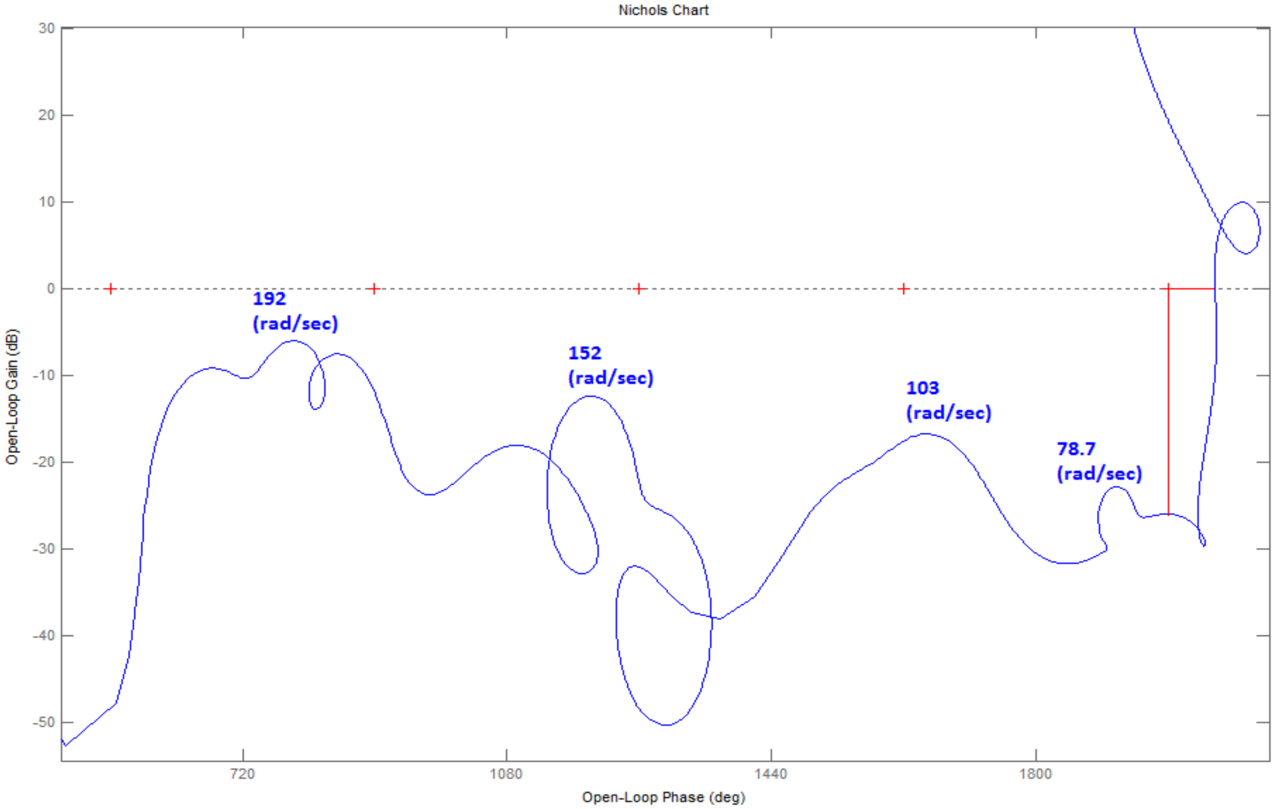


Roll Axis Stability (all other loops closed)

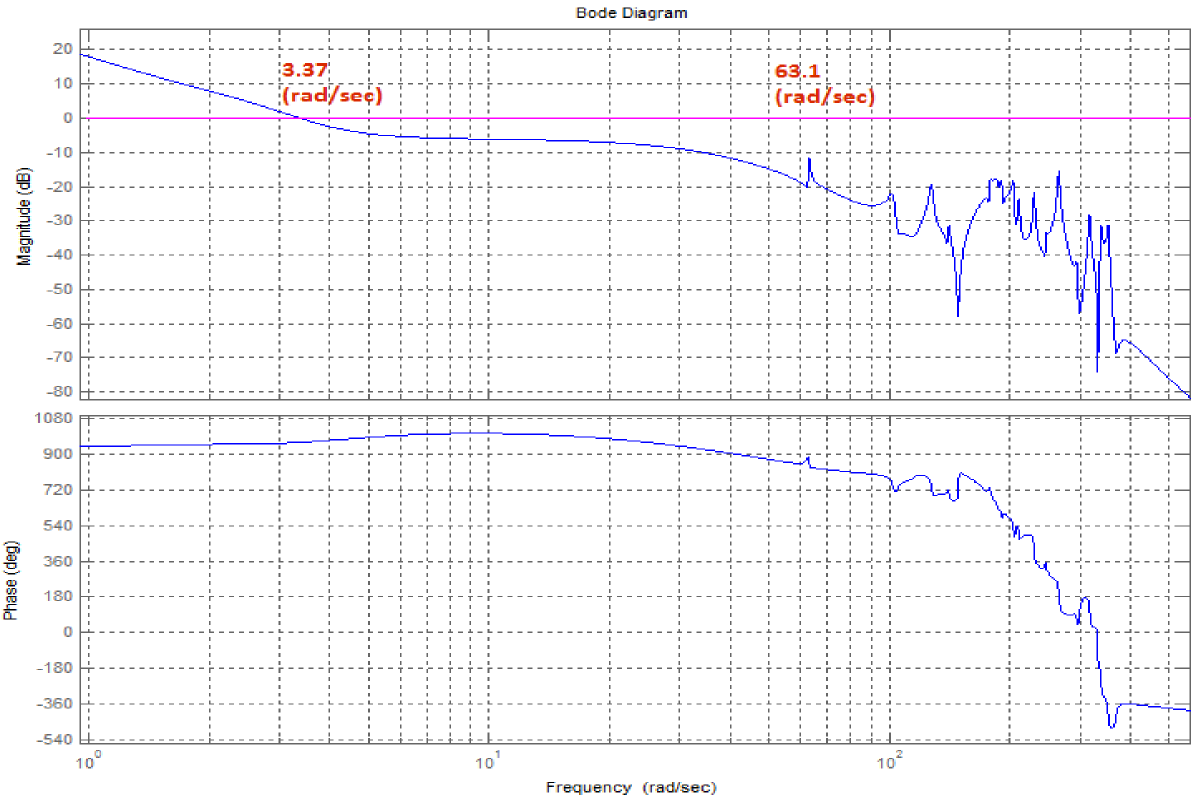


Pitch Axis Open-Loop Frequency Response (other loops closed)

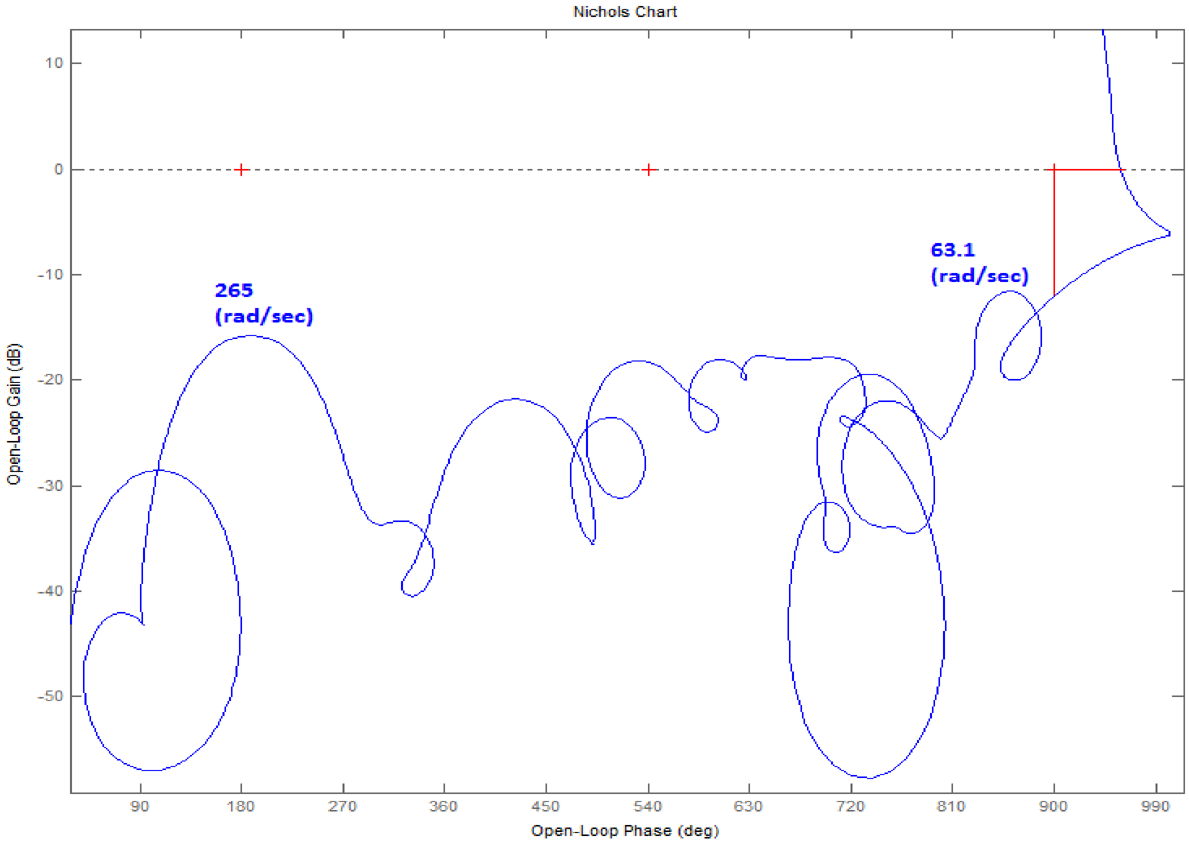
Pitch Axis Stability (all other loops closed)



Yaw Axis Open-Loop Frequency Response (other loops closed)



Yaw Axis Stability (all other loops closed)



3.0 Flexible Vehicle Analysis Using Aero-Elasticity Coefficients

We will now go one step further in our flex vehicle analysis and introduce aero-elasticity coefficients that couple aerodynamics with the flex structure, also known as Generalized Aerodynamic Force Derivatives (GAFD). We will also introduce inertial coupling coefficients that couple surface accelerations with flexibility. We will combine these coefficients with the vehicle model and produce a more accurate representation of structural flexibility.

The GAFD coefficients define how the aerodynamic forces and moments vary as a function of the modal excitation (η_j), surface deflection (δ_{as}), and their rates. Also how the bending modes are excited by the variations in α, β, and the vehicle rates, and how they couple with other modes. There are also coefficients that define how the moment at the hinge of a control surface varies as a function of angles of attack, sideslip, body rates, accelerations, modal displacements, modal rates, and also by the interactions with other control surface deflections and rates.

The mode selection presented in Section 2 will not be repeated, because the previously selected set of flex modes will be used again. In addition to the mode shapes and frequencies, the program will read the GAFD data file, combine it with the vehicle data, and create a flexible vehicle model, the same size as the previous one, but it includes the aero-elastic coupling represented more accurately than the previous model in Section 2. The hinge moment calculations are also more accurate because they are based on a better CFD model that produced those coefficients. We will repeat the analysis that was described in Section 2 using this aero-elastic model, and compare the results between the two flex models.

3.1 Generalized Aerodynamic Force Derivatives (GAFD)

The Generalized Aerodynamic Force Derivatives (GAFD) are used to model the dynamic coupling between structure and aerodynamics on a flight vehicle. They are based on detailed FEM and CFD models and consist of the following sets of coefficients.

- The first set of coefficients define how the vehicle basic aerodynamic forces and moments, such as C_Z, C_m, C_n, etc. are affected by the modal displacements (η_j), and the modal rates (η-dot).

- The second set coefficients describe how the modal displacement (η_j) of a flex mode (j), is excited by the vehicle motion, such as changes in angle of attack, sideslip, body rates, accelerations, control surface deflections, surface rates, and also by modal displacements, rates, and accelerations of other modes.

- The third set of coefficients are hinge moment coefficients. They define how the moment at the hinge of a control surface is affected by changes in the vehicle angles of attack, sideslip, body rates, accelerations, modal displacements, modal rates, and also by the interactions with other control surface deflections and rates.

- The original GAFD data output contains also the rigid-body aerodynamic force and moment derivatives caused by changes in angle of attack, sideslip, body rates, accelerations, control surface deflections, and surface rates. These coefficients, however, are not used by the modeling program. They are not included in the GAFD file and instead more accurate wind tunnel aero derivatives are used which are included in the vehicle input data.

Even though the aero-elastic data preparation is beyond the scope of this document, in the next paragraph we will present a brief description on the GAFD derivation in order to point the user in the right direction and understand what kind of information is needed. The GAFD data are obtained by post processing the generalized aerodynamic forces [Qij] which is a mass matrix obtained from the "Doublet Lattice" program, which is included in the finite elements modeling tools. The Qij terms can be used for flutter analysis, loads, and for control system analysis. The generalized aerodynamic forces are complex and are a function of Mach number and a reduced frequency. For each Mach number and reduced frequency a complex generalized force matrix is generated. In the flutter analysis, a mach number and reduced frequency is assumed and the flutter solution is calculated. If the solution indicates that the flutter frequency is at a different frequency than the assumed reduced frequency, then that solution is invalid. The flutter analysis is then repeated until the flutter frequency equals the assumed reduced frequency. This iterative process is not used in the control system analysis. A complex generalized aerodynamic force matrix is constructed which is independent of reduced frequency. The real part of this complex matrix consists of displacement coefficients and the imaginary part consists of the velocity coefficients. The inputs to the Doublet Lattice program are the Modal Data (mode shapes and mode frequencies) obtained from the finite elements model. The aerodynamic shape of the vehicle is modeled by means of flat plates in the Doublet Lattice program, and it computes the aerodynamic coefficients at different mach numbers. It program finally combines the NASTRAN model with the aerodynamic model and produces the Generalized Aerodynamic Forces, the Qij matrix. The finite-elements-model includes the aero-surfaces with the hinges locked. The hinges are released by the introduction of the inertial coupling coefficients (h-parameters).

The GAFD data at a given mach number consists of matrix pairs, a displacements matrix and a velocities matrix. The displacements matrix describes how the coefficients are affected by changes in $\{\beta, \alpha, p, q, r, \eta_i, \delta_j\}$. The rates matrix describes how the coefficients are affected by the derivatives of: $\{\beta, \alpha, p, q, r, \eta_i, \delta_j\}$. The matrix pairs are calculated at different frequencies: at the 6 rigid body frequencies and at each bending mode. The Displacements matrix for (i) number of modes and for (j) control surfaces is:

$$\begin{pmatrix} C_Y \\ C_Z \\ C_l \\ C_m \\ C_n \\ C_{\eta 1} \\ C_{\eta 2} \\ C_{\eta i} \\ C_{h1} \\ C_{hj} \end{pmatrix} = \begin{bmatrix} C_{Y\beta} & C_{Y\alpha} & C_{Yp} & C_{Yq} & C_{Yr} & C_{Y\eta_1} & C_{Y\eta_2} & C_{Y\eta_i} & C_{Y\delta_1} & C_{Y\delta_j} \\ C_{Z\beta} & C_{Z\alpha} & C_{Zp} & C_{Zq} & C_{Zr} & C_{Z\eta_1} & C_{Z\eta_2} & C_{Z\eta_i} & C_{Z\delta_1} & C_{Z\delta_j} \\ C_{l\beta} & C_{l\alpha} & C_{lp} & C_{lq} & C_{lr} & C_{l\eta_1} & C_{l\eta_2} & C_{l\eta_i} & C_{l\delta_1} & C_{l\delta_j} \\ C_{m\beta} & C_{m\alpha} & C_{mp} & C_{mq} & C_{mr} & C_{m\eta_1} & C_{m\eta_2} & C_{m\eta_i} & C_{m\delta_1} & C_{m\delta_j} \\ C_{n\beta} & C_{n\alpha} & C_{np} & C_{nq} & C_{nr} & C_{n\eta_1} & C_{n\eta_2} & C_{n\eta_i} & C_{n\delta_1} & C_{n\delta_j} \\ C_{\eta 1\beta} & C_{\eta 1\alpha} & C_{\eta 1p} & C_{\eta 1q} & C_{\eta 1r} & C_{\eta 1\eta_1} & C_{\eta 1\eta_2} & C_{\eta 1\eta_i} & C_{\eta 1\delta_1} & C_{\eta 1\delta_j} \\ C_{\eta 2\beta} & C_{\eta 2\alpha} & C_{\eta 2p} & C_{\eta 2q} & C_{\eta 2r} & C_{\eta 2\eta_1} & C_{\eta 2\eta_2} & C_{\eta 2\eta_i} & C_{\eta 2\delta_1} & C_{\eta 2\delta_j} \\ C_{\eta_i\beta} & C_{\eta_i\alpha} & C_{\eta_i p} & C_{\eta_i q} & C_{\eta_i r} & C_{\eta_i\eta_1} & C_{\eta_i\eta_2} & C_{\eta_i\eta_i} & C_{\eta_i\delta_1} & C_{\eta_i\delta_j} \\ C_{h_1\beta} & C_{h_1\alpha} & C_{h_1 p} & C_{h_1 q} & C_{h_1 r} & C_{h_1\eta_1} & C_{h_1\eta_2} & C_{h_1\eta_i} & C_{h_1\delta_1} & C_{h_1\delta_j} \\ C_{h_j\beta} & C_{h_j\alpha} & C_{h_j p} & C_{h_j q} & C_{h_j r} & C_{h_j\eta_1} & C_{h_j\eta_2} & C_{h_j\eta_i} & C_{h_j\delta_1} & C_{h_j\delta_j} \end{bmatrix} \begin{Bmatrix} \beta \\ \alpha \\ p \\ q \\ r \\ \eta_1 \\ \eta_2 \\ \eta_i \\ \delta_1 \\ \delta_j \end{Bmatrix}$$

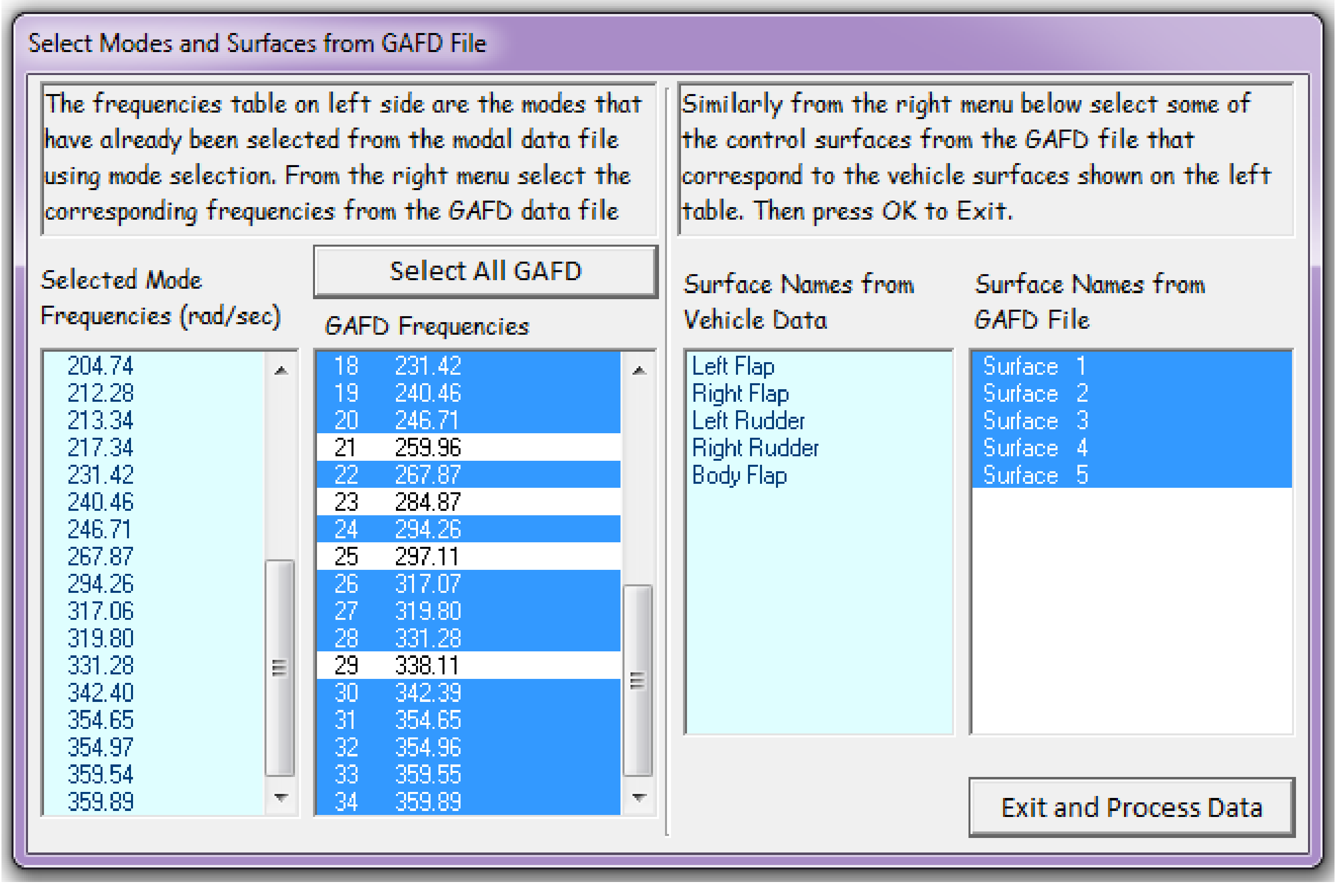
The Velocities matrix for (i) number of modes and for (j) control surfaces is:

$$\begin{pmatrix} C_Y \\ C_Z \\ C_l \\ C_m \\ C_n \\ C_{\eta 1} \\ C_{\eta 2} \\ C_{\eta i} \\ C_{h1} \\ C_{hj} \end{pmatrix} = \begin{bmatrix} C_{Y\dot\beta} & C_{Y\dot\alpha} & C_{Y\dot p} & C_{Y\dot q} & C_{Y\dot r} & C_{Y\dot\eta_1} & C_{Y\dot\eta_2} & C_{Y\dot\eta_i} & C_{Y\dot\delta_1} & C_{Y\dot\delta_j} \\ C_{Z\dot\beta} & C_{Z\dot\alpha} & C_{Z\dot p} & C_{Z\dot q} & C_{Z\dot r} & C_{Z\eta_1} & C_{Z\eta_2} & C_{Z\eta_i} & C_{Z\delta_1} & C_{Z\delta_j} \\ C_{l\dot\beta} & C_{l\dot\alpha} & C_{l\dot p} & C_{l\dot q} & C_{l\dot r} & C_{l\eta_1} & C_{l\eta_2} & C_{l\dot\eta_i} & C_{l\dot\delta_1} & C_{l\delta_j} \\ C_{m\dot\beta} & C_{m\alpha} & C_{m\dot p} & C_{m\dot q} & C_{mr} & C_{m\dot\eta_1} & C_{m\dot\eta_2} & C_{m\dot\eta_i} & C_{m\delta_1} & C_{m\dot\delta_j} \\ C_{n\beta} & C_{n\dot\alpha} & C_{n\dot p} & C_{n\dot q} & C_{n\dot r} & C_{n\eta_1} & C_{n\eta_2} & C_{n\eta_i} & C_{n\delta_1} & C_{n\dot\delta_j} \\ C_{\eta 1\dot\beta} & C_{\eta 1\dot\alpha} & C_{\eta 1 p} & C_{\eta 1 q} & C_{\eta 1\dot r} & C_{\eta 1\dot\eta_1} & C_{\eta 1\dot\eta_2} & C_{\eta 1\eta_i} & C_{\eta 1\delta_1} & C_{\eta 1\delta_j} \\ C_{\eta 2\beta} & C_{\eta 2\alpha} & C_{\eta 2\dot p} & C_{\eta 2\dot q} & C_{\eta 2\dot r} & C_{\eta 2\dot\eta_1} & C_{\eta 2\dot\eta_2} & C_{\eta 2\dot\eta_i} & C_{\eta 2\dot\delta_1} & C_{\eta 2\dot\delta_j} \\ C_{\eta_i\dot\beta} & C_{\eta_i\dot\alpha} & C_{\eta_i p} & C_{\eta_i q} & C_{\eta_i r} & C_{\eta_i\dot\eta_1} & C_{\eta_i\dot\eta_2} & C_{\eta_i\dot\eta_i} & C_{\eta_i\dot\delta_1} & C_{\eta_i\dot\delta_j} \\ C_{h_1\dot\beta} & C_{h_1\dot\alpha} & C_{h_1\dot p} & C_{h_1\dot q} & C_{h_1\dot r} & C_{h_1\dot\eta_1} & C_{h_1\dot\eta_2} & C_{h_1\dot\eta_i} & C_{h_1\dot\delta_1} & C_{h_1\dot\delta_j} \\ C_{h_j\dot\beta} & C_{h_j\dot\alpha} & C_{h_j\dot p} & C_{h_j\dot q} & C_{h_j\dot r} & C_{h_j\dot\eta_1} & C_{h_j\dot\eta_2} & C_{h_j\eta_i} & C_{h_j\dot\delta_1} & C_{h_j\dot\delta_j} \end{bmatrix} \begin{pmatrix} \dot\beta \\ \dot\alpha \\ \dot p \\ \dot q \\ \dot r \\ \dot\eta_1 \\ \dot\eta_2 \\ \dot\eta_i \\ \dot\delta_1 \\ \dot\delta_j \end{pmatrix}$$

The data files used in this vehicle aero-elastic model are in folder in directory *"C:\Flixan\Examples\ Rocket Plane\ Flex With Gafd"*. The GAFD data file containing the aero-elastic coefficients and h-parameters is "*RocketPlane.Gaf*". It was prepared earlier (not shown here) by post-processing the double-lattice output, a much bigger data file, after re-scaling it to be compatible with the vehicle data, and reformatted to be accessible by the vehicle modeling program. It contains the coefficients for 34 frequencies and the 5 control surfaces. The frequencies correspond to the 34 modes in the modal data file "*RocketPlane.Mod*". The sequence of the aero-surfaces is the same as in the vehicle data, which is: left and right flaps, left and right rudders, and body-flap. It also contains the control surface moments of inertia about the hinges, the modal frequencies in (rad/sec), the vehicle reference length and reference area, the reference chords and the reference areas for each control surface, the inertial coupling coefficients which couple the modal equations with the control surface accelerations, and the GAFD coefficients. The inertial coupling coefficients, or h-parameters $h_S(k,j)$, define the excitation of a mode (j) due to the control surface (k) accelerations. They also define the hinge moment variation at a surface (k) due to a mode (j) displacement. They are included in the same GAFD data file.

The vehicle input data-set is "*Rocket Plane at Mach=0.85, Q=150, Aero-Elastic Model with GAFD*", in file *"RocketPlane_Gafd.Inp"*. It is almost identical to the flex vehicle model used in Section 2, except that in this case the "*Include GAFD*" flag in the flags line is set to include the additional GAFD data file. The flags line is below the vehicle title and comments. When the flag is set the program attempts to read and use the GAFD file. Otherwise, if it cannot find the file "*RocketPlane.Gaf*" or if the flag is set to "*Without GAFD*", the vehicle modeling program uses only the modal data by approximating the aero-elastic effects with aerodynamic forces and moments applied at the hinge nodes, which are produced by the control surface deflections and accelerations as it was done in Section 2. There is an identical vehicle data-set in the input file to be used for batch processing. The number of modes to be included is also set to 30 in the vehicle data-set, and the selected modes title is included in the next line. The input file *"RocketPlane-Gafd.Inp"* also includes the selected set of 30 mixed modes that was described and used in Section 2. Actually, the input data file contains two sets of selected modes. A set of 22 pitch modes and a set of 30 mixed modes. The selected set of modes that will be used in this model are located under the title "SELECTED MODAL DATA AND LOCATIONS FOR MIXED AXES MODES", followed by the title of the modes set: "*Rocket Plane at*

Using the mouse, highlight the frequencies on the right menu that correspond to the mode frequencies on the left. Most of them are selected, except of four. The second pair of menus in the right side of the dialog is similar to the pair on the left side and it is used in the same manner for selecting GAFD data for the five control surfaces which are defined in the vehicle data. The GAFD data for the five control surfaces are defined in the same sequence as in the vehicle data. Select all five aero-surfaces from the white menu on the right, that is: left and right flaps, left and right rudders, and body-flap. Finally, click on "*Exit and Process Data*" button and the program will create the state-space model for this vehicle with 30 flex modes and aero-elasticity, and save it in the systems file "*RocketPlane_Gafd.Qdr*" under the same title "*Rocket Plane at Mach=0.85, Q=150, Aero-Elastic Model with GAFD*".



The frequencies table on left side are the modes that have already been selected from the modal data file using mode selection. From the right menu select the corresponding frequencies from the GAFD data file

Similarly from the right menu below select some of the control surfaces from the GAFD file that correspond to the vehicle surfaces shown on the left table. Then press OK to Exit.

Selected Mode Frequencies (rad/sec)

204.74
212.28
213.34
217.34
231.42
240.46
246.71
267.87
294.26
317.06
319.80
331.28
342.40
354.65
354.97
359.54
359.89

GAFD Frequencies

18 231.42
19 240.46
20 246.71
21 259.96
22 267.87
23 284.87
24 294.26
25 297.11
26 317.07
27 319.80
28 331.28
29 338.11
30 342.39
31 354.65
32 354.96
33 359.55
34 359.89

Surface Names from Vehicle Data

Left Flap
Right Flap
Left Rudder
Right Rudder
Body Flap

Surface Names from GAFD File

Surface 1
Surface 2
Surface 3
Surface 4
Surface 5

The vehicle modeling program will compute the flexible vehicle state-space model and save it in file "*RocketPlane_Gafd.Qdr*", under the same title. This flex vehicle system must also be converted to a Matlab function m-file using the "*Export to Matlab*" utility, as demonstrated in Section 1. The system filename is "*vehi_gafd_30flx.m*", and saved in subdirectory "*\Rocket Plane\ Flex With Gafd\ Mat Anal*" where the stability analysis will be performed, using Matlab. Repeat the exporting to Matlab conversion also for the actuator system "*ema_actuator.m*" and the mixing logic matrix "*K4AXMIX.Mat*".

However, there is a much faster way of processing the data and for producing the systems required in the analysis. The input file "*RocketPlane_Gafd.Inp*" includes a batch set on the top "*Batch for the Flexible Rocket Plane with GAFD*". This batch will process the data-sets and create the systems described instantly, in batch mode.

3.3 Flexible System with Aero-Elasticity, Closed-Loop Simulation Model

The Simulink model used in this analysis is “*Closed-Loop-Gafd.mdl*” in directory “*\Rocket Plane\ Flex With Gafd\Mat Anal*”, shown in Figure (3.1). Its structure is almost identical to the simulation model used in Section 2. It consists of the same four control loops. The vehicle system, however, contains the flexible vehicle system with GAFD, title: " *Rocket Plane at Mach=0.85, Q=150, Aero-Elastic Model with GAFD* ", that is loaded into Matlab from file “*vehi_gafd_30flx.m*”.

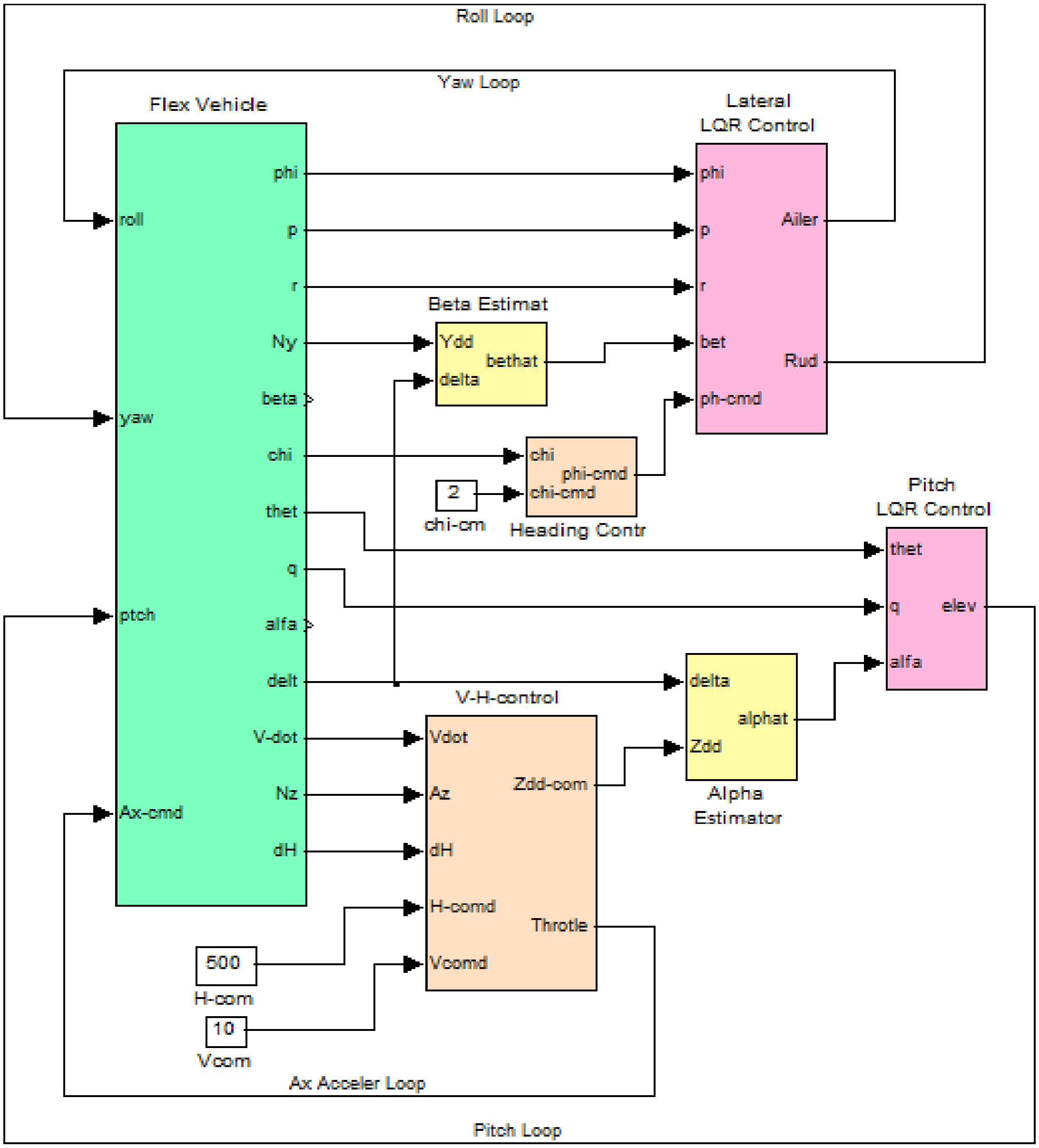
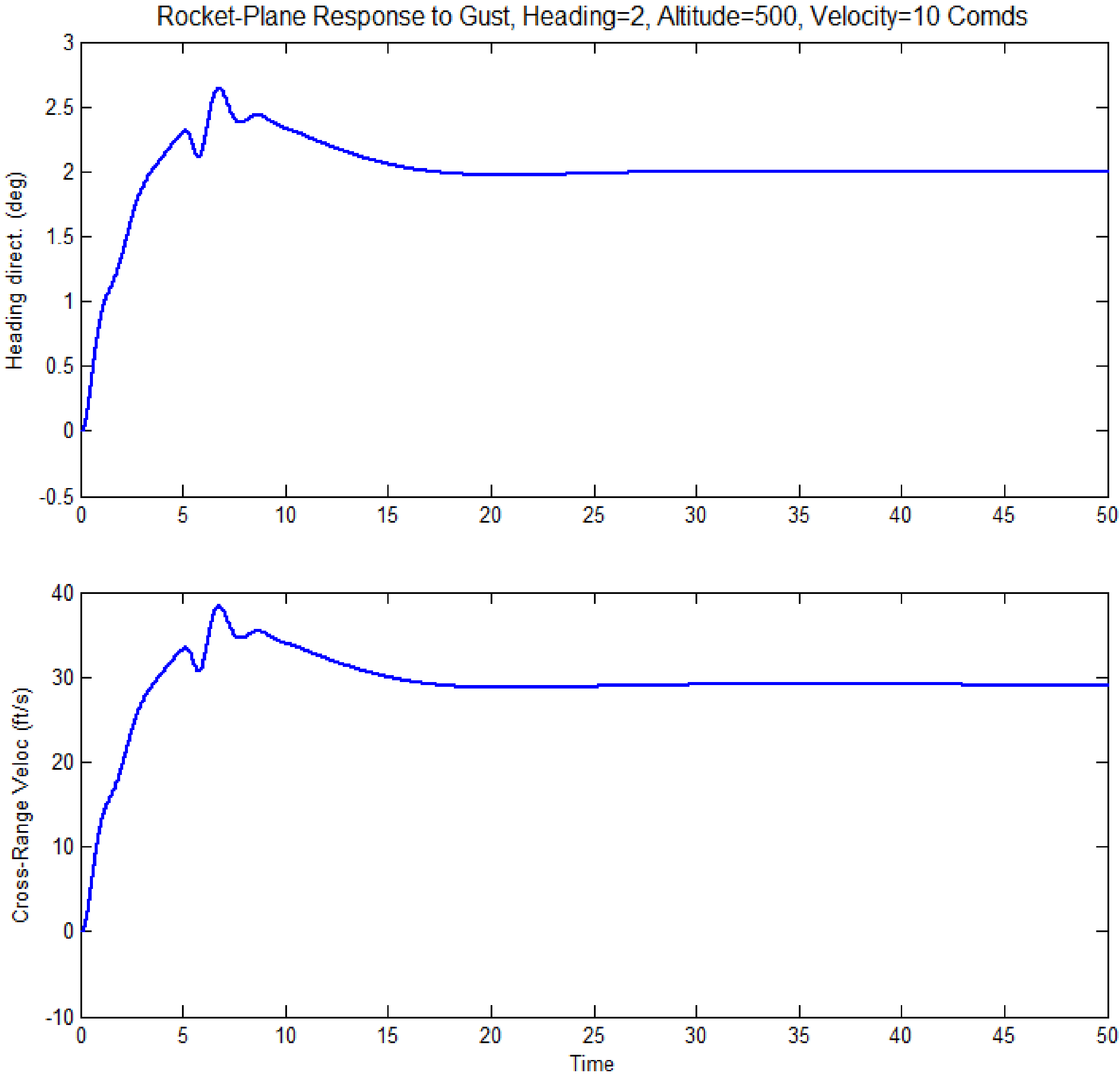
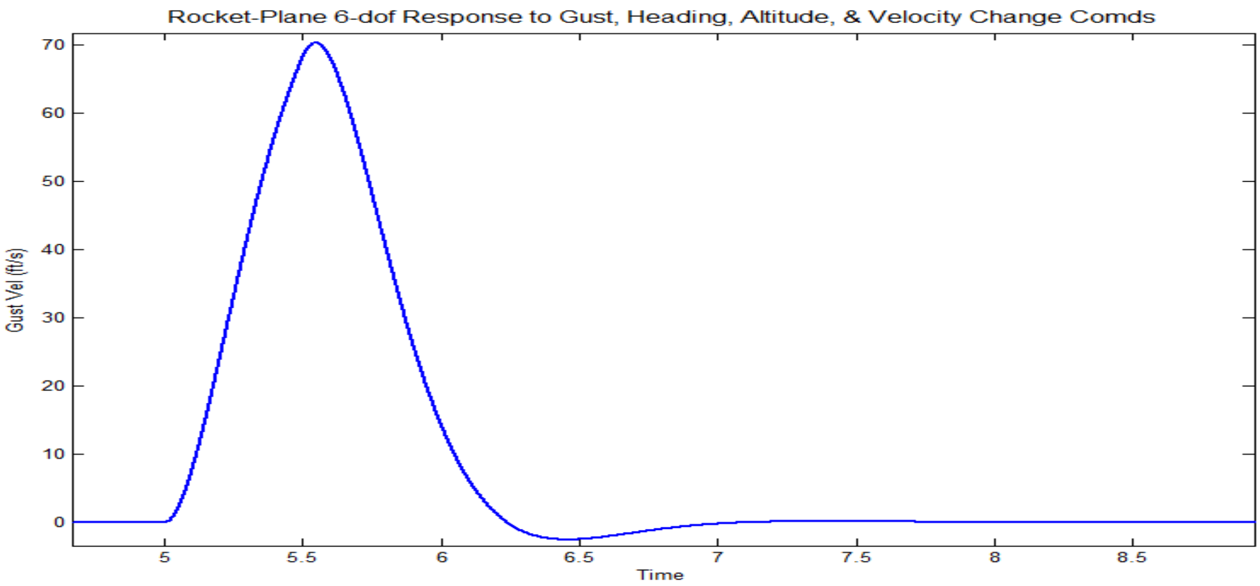


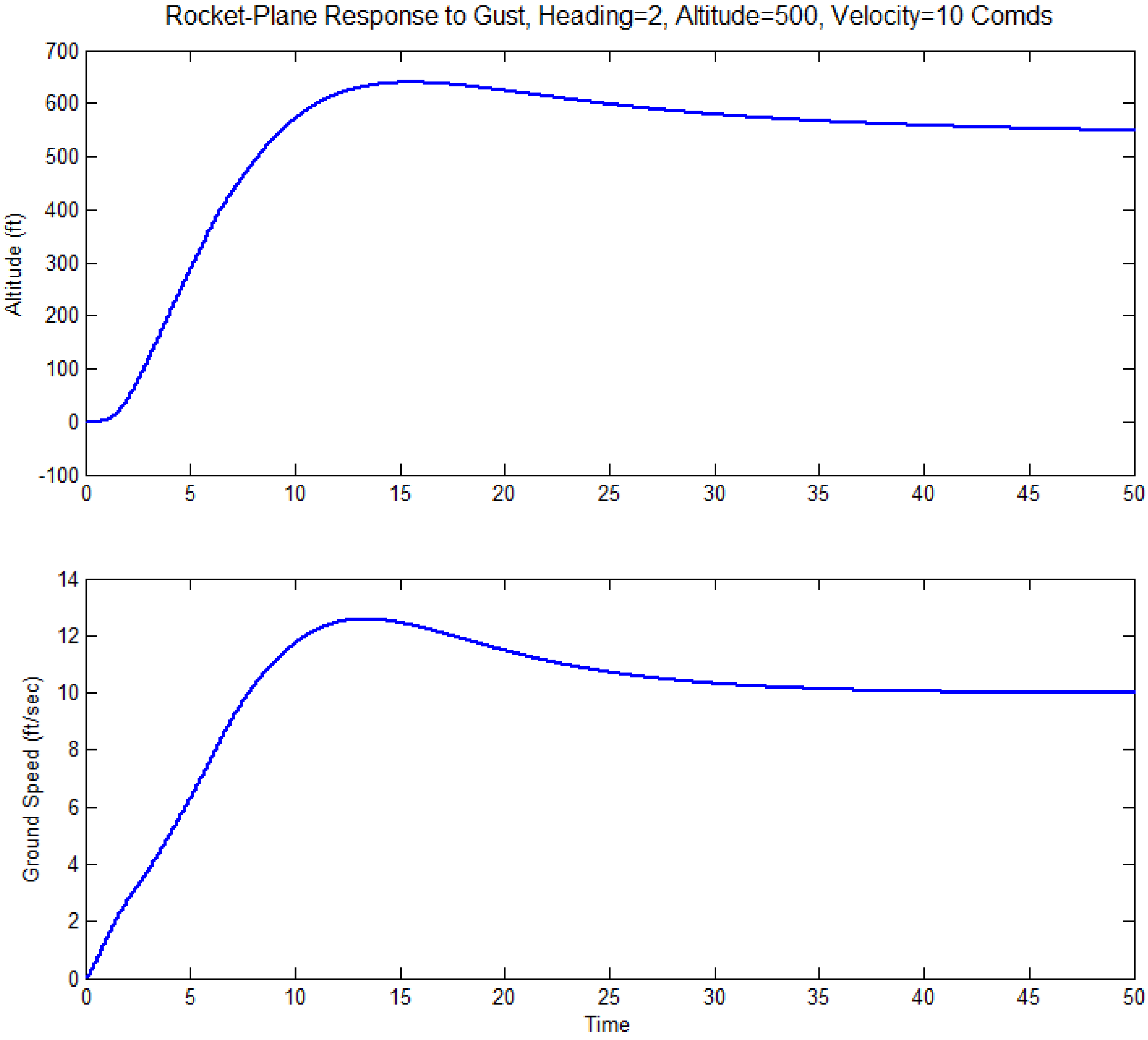
Figure (3.1) Closed-Loop Simulation Model "Closed_Loop_Gafd.Mdl"

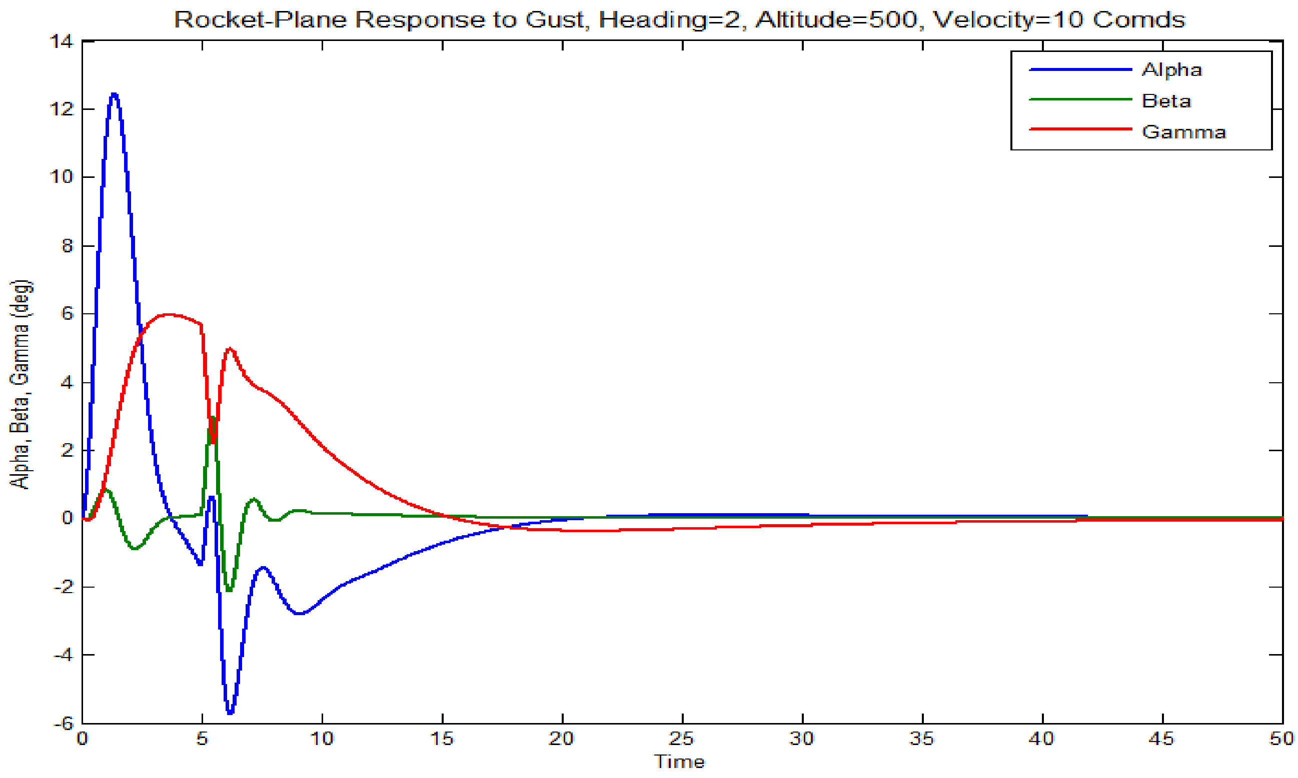
The green vehicle block is very similar to the one described in Section 2. It contains the flexible vehicle, the (6x4) mixing logic matrix “K4AXMIX.mat” which transforms the roll, pitch, yaw, and axial acceleration demands into control surface and throttle commands, and the control surface actuators. The LQR state-feedback gain matrices Kgp and Kgl and the notch filters are the same as before. A notch at 62.3 (rad/sec) was included because the mode at that frequency appears to be a lot stronger with the aero-elastic coupling coefficients than it was in Section 2. The low-pass filter cut-off frequencies were also reduced to accommodate the aero-elastic data.

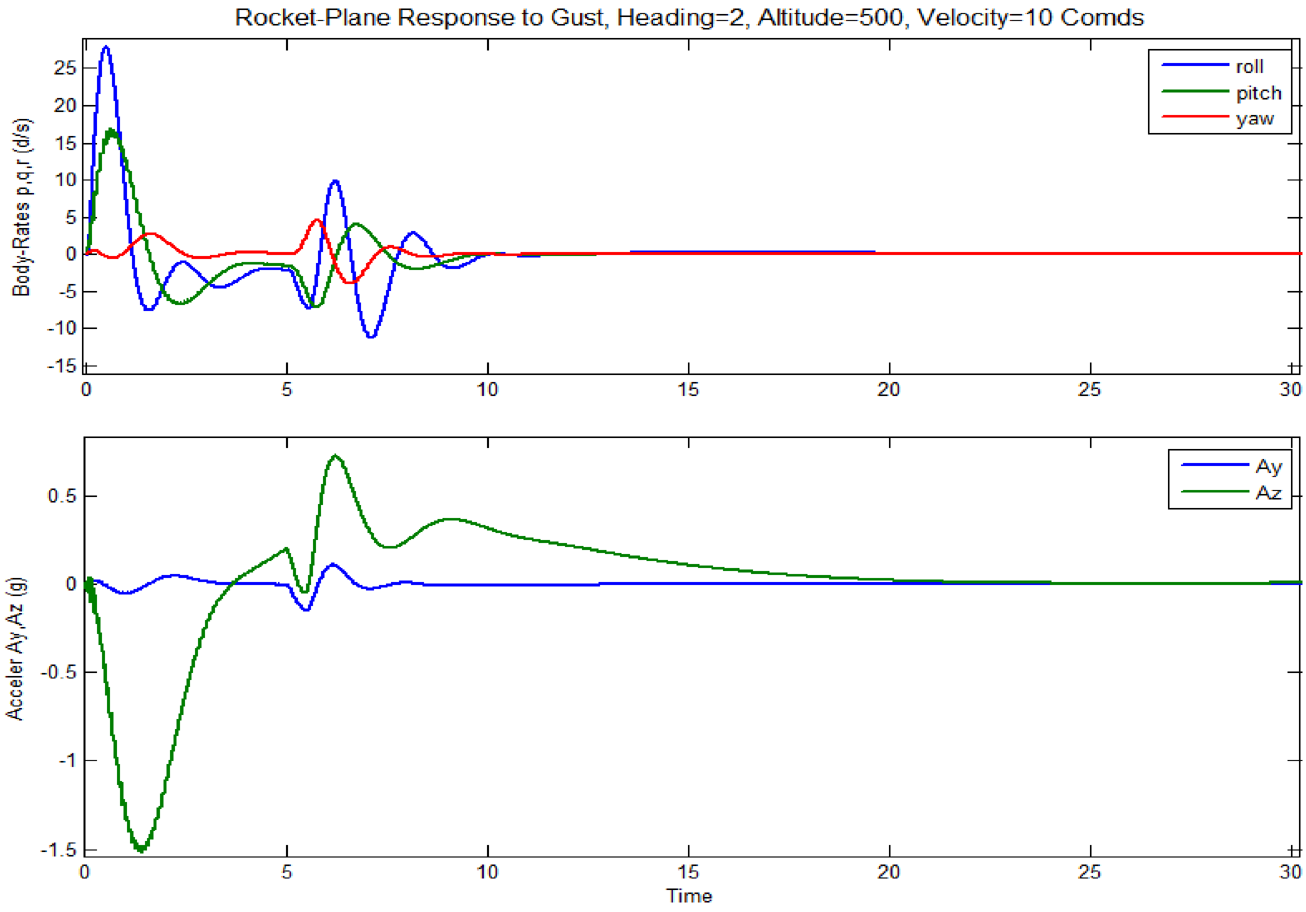
The initialization m-file “run.m” loads the flex vehicle, actuator, mixing logic, and the control system into Matlab. It loads also the alpha & beta estimator parameters, because the estimated alpha and beta replace the real alpha and beta angles required by the LQR controller. A similar 6-dof simulation is performed using the flexible vehicle model that was obtained from the GAFD data. A short duration gust is applied at 5 (sec). The altitude is commanded to increase 500 (feet) this time, the heading direction to increase 2°, and the ground speed to increase 10 (ft/sec). The Matlab script file “Pl.m” is used to plot the simulation data. In general the GAFD model predicts twice as much hinge moment as the ordinary flex model predicted.

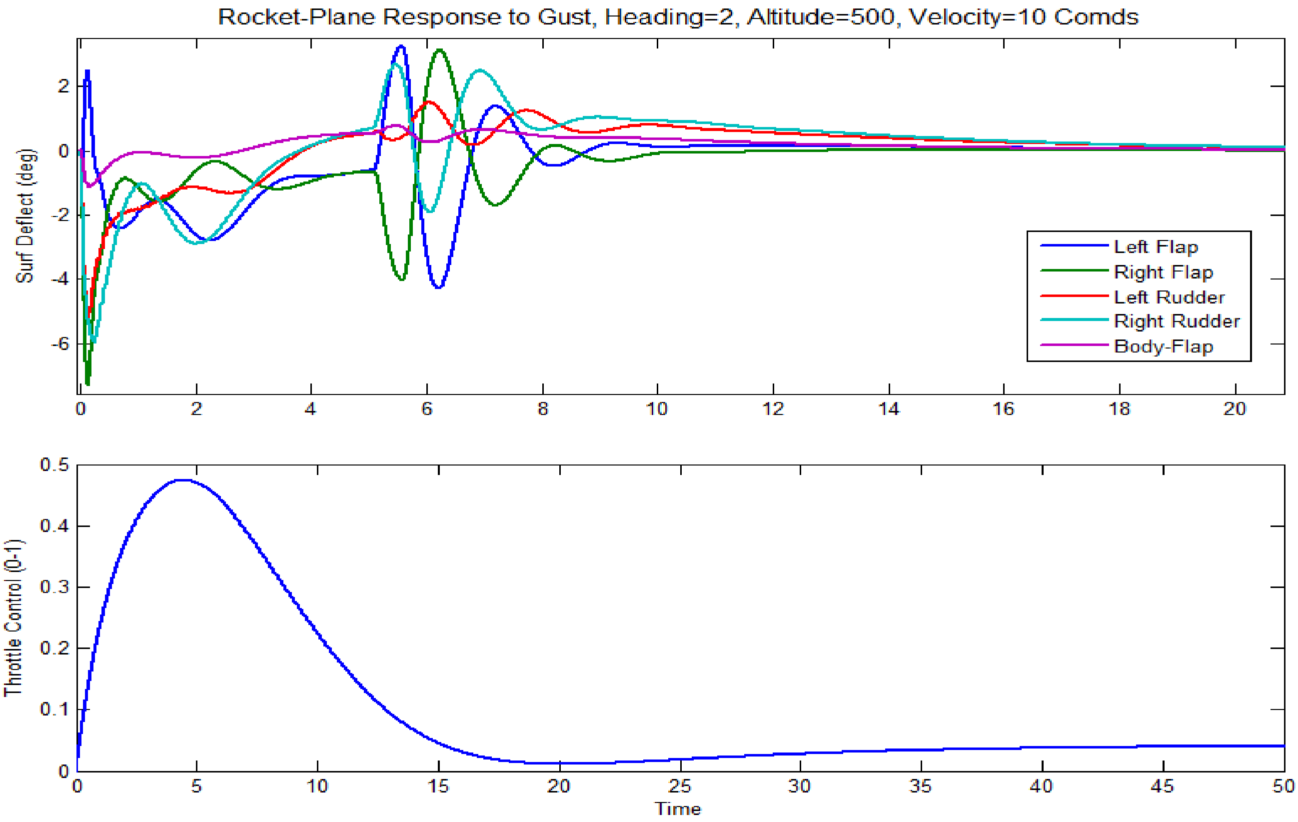


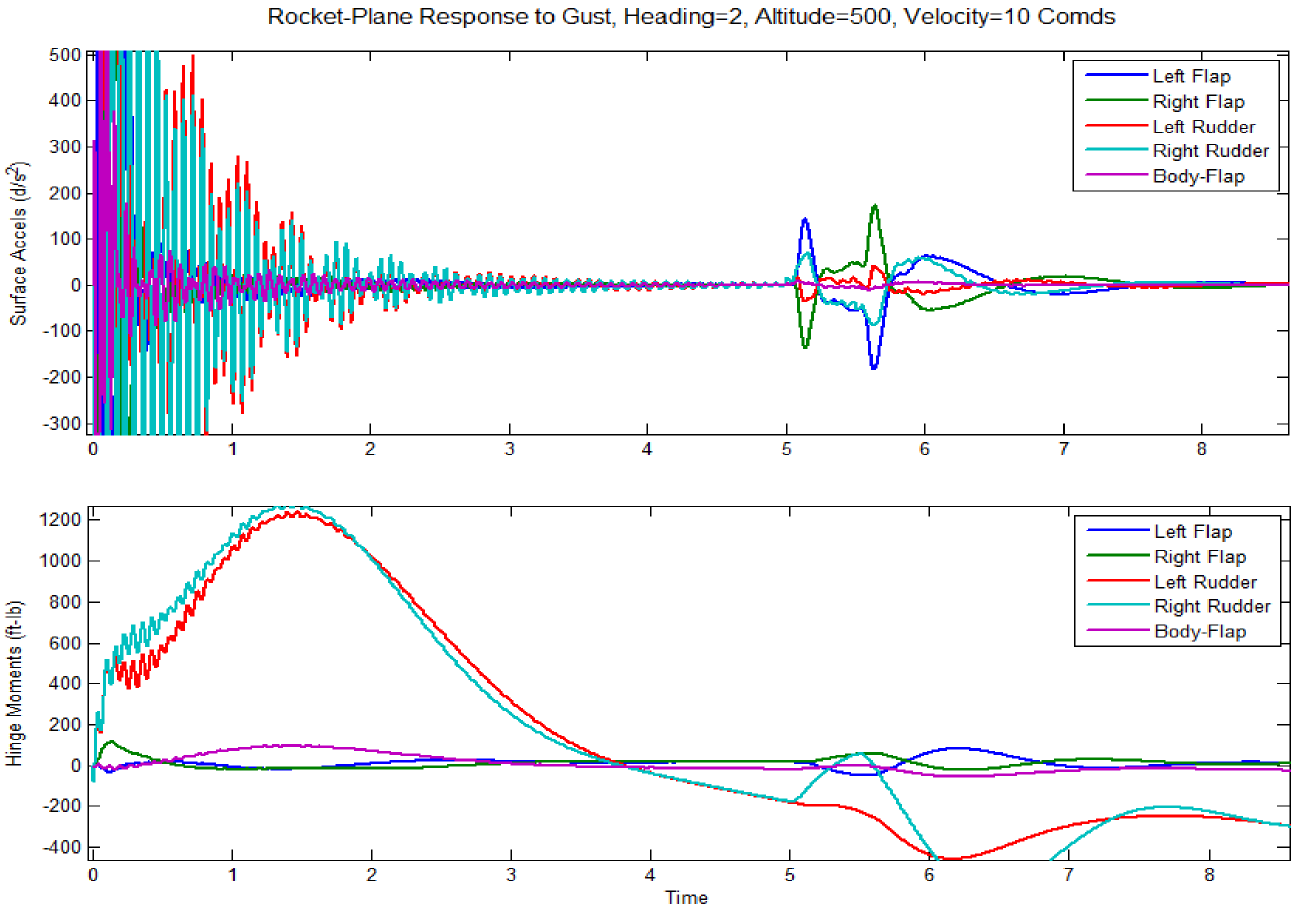

Rocket-Plane 6-dof Response to Gust, Heading, Altitude, & Velocity Change Comds
Gust Vel (ft/s)
70
60
50
40
30
20
10
0
5
5.5
6
6.5
7
7.5
8
8.5
Time


Rocket-Plane Response to Gust, Heading=2, Altitude=500, Velocity=10 Comds
Altitude (ft)
700
600
500
400
300
200
100
0
-100
0
5
10
15
20
25
30
35
40
45
50
Ground Speed (ft/sec)
14
12
10
8
6
4
2
0
0
5
10
15
20
25
30
35
40
45
50
Time


Rocket-Plane Response to Gust, Heading=2, Altitude=500, Velocity=10 Comds
Alpha
Beta
Gamma
Alpha, Beta, Gamma (deg)
14
12
10
8
6
4
2
0
-2
-4
-6
0
5
10
15
20
25
30
35
40
45
50
Time


Rocket-Plane Response to Gust, Heading=2, Altitude=500, Velocity=10 Comds
roll
pitch
yaw
Body-Rates p,q,r (d/s)
25
20
15
10
5
0
-5
-10
-15
0
5
10
15
20
25
30
Ay
Az
Acceler Ay,Az (g)
0.5
0
-0.5
-1
-1.5
0
5
10
15
20
25
30
Time


Rocket-Plane Response to Gust, Heading=2, Altitude=500, Velocity=10 Comds
Surf Deflect (deg)
2
0
-2
-4
-6
0
2
4
6
8
10
12
14
16
18
20
Left Flap
Right Flap
Left Rudder
Right Rudder
Body-Flap
Throttle Control (0-1)
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
25
30
35
40
45
50
Time


Rocket-Plane Response to Gust, Heading=2, Altitude=500, Velocity=10 Comds
Surface Accels (d/s²)
500
400
300
200
100
0
-100
-200
-300
0
1
2
3
4
5
6
7
8
Left Flap
Right Flap
Left Rudder
Right Rudder
Body-Flap
Hinge Moments (ft-lb)
1200
1000
800
600
400
200
0
-200
-400
0
1
2
3
4
5
6
7
8
Left Flap
Right Flap
Left Rudder
Right Rudder
Body-Flap
Time

3.4 Stability Analysis

The Simulink model "*Open_Loop_Gafd.Mdl*" in Figure (3.2) is used for frequency domain open-loop stability analysis in subdirectory "*\Rocket Plane\ Flex With Gafd\ Mat Anal*". It is a single-input-single-output system for classical control analysis similar to the model presented in Section 2. It contains the same elements as the closed-loop flex system with aero-elasticity but the loop to be analyzed is opened, while the other three loops are closed. The Matlab file "*frequ.m*" calculates the frequency response of the open-loop system and plots the Bode and Nichols charts which are used to analyze the system stability margins. In Figure (3.2) it is shown configured for the pitch axis analysis with the pitch loop opened, and the roll, pitch, and throttle loops closed. The results show less conservative flex responses from those predicted in Section 2, although the system still has acceptable phase and gain margins in all four loops with a control system bandwidth of approximately 3.5 (rad/sec).

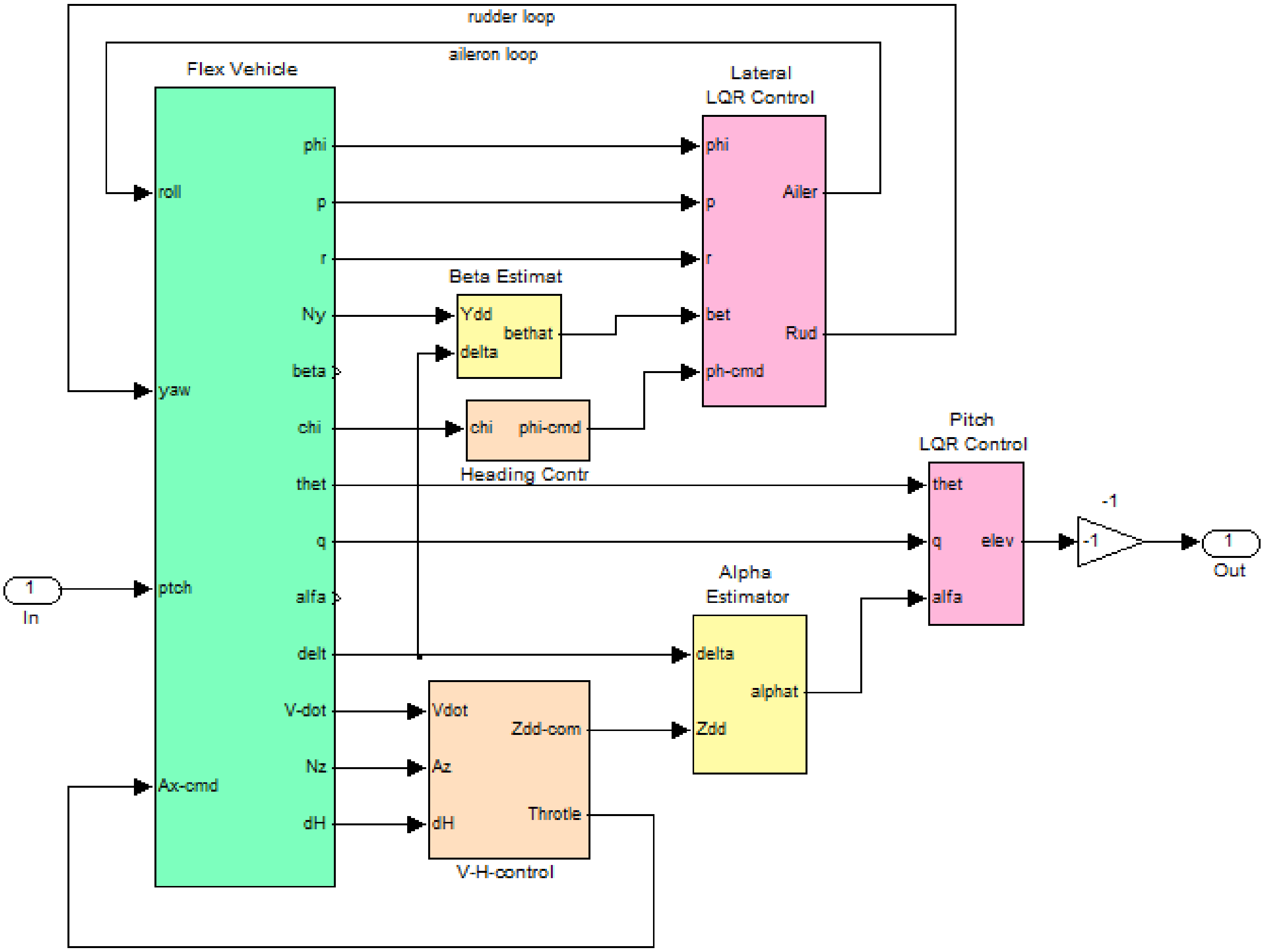
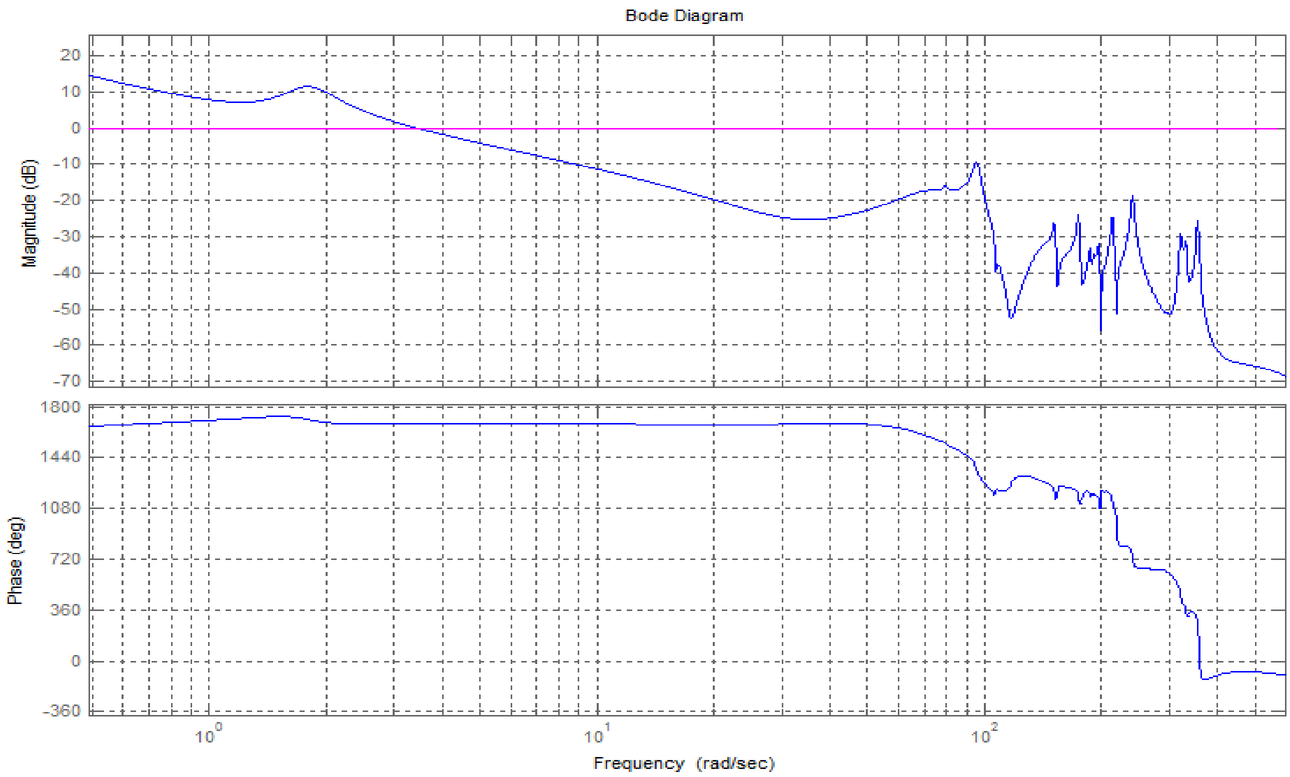
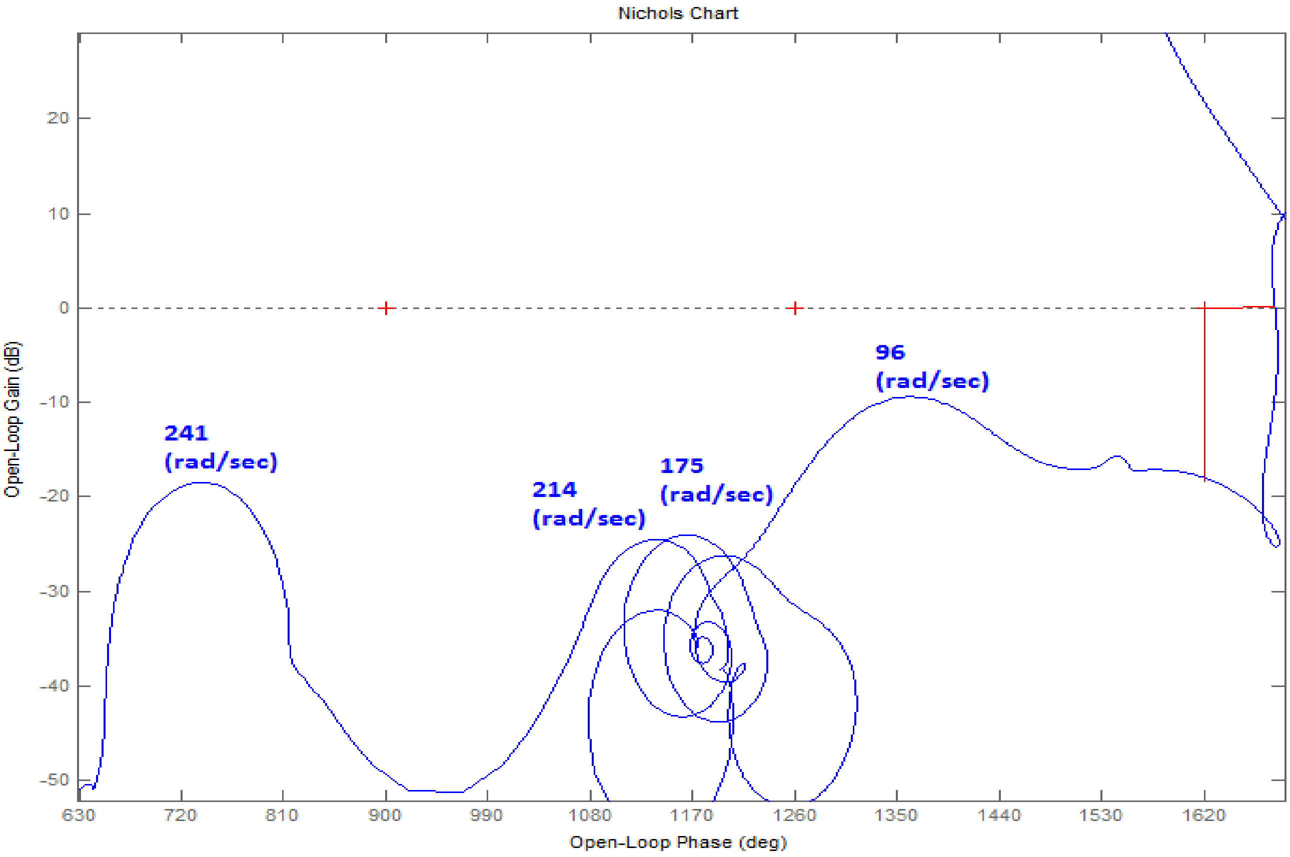


Figure (3.2) Simulink Model "Open_Loop_Gafd.Mdl" Configured for Pitch Axis Open-Loop Frequency Response Analysis

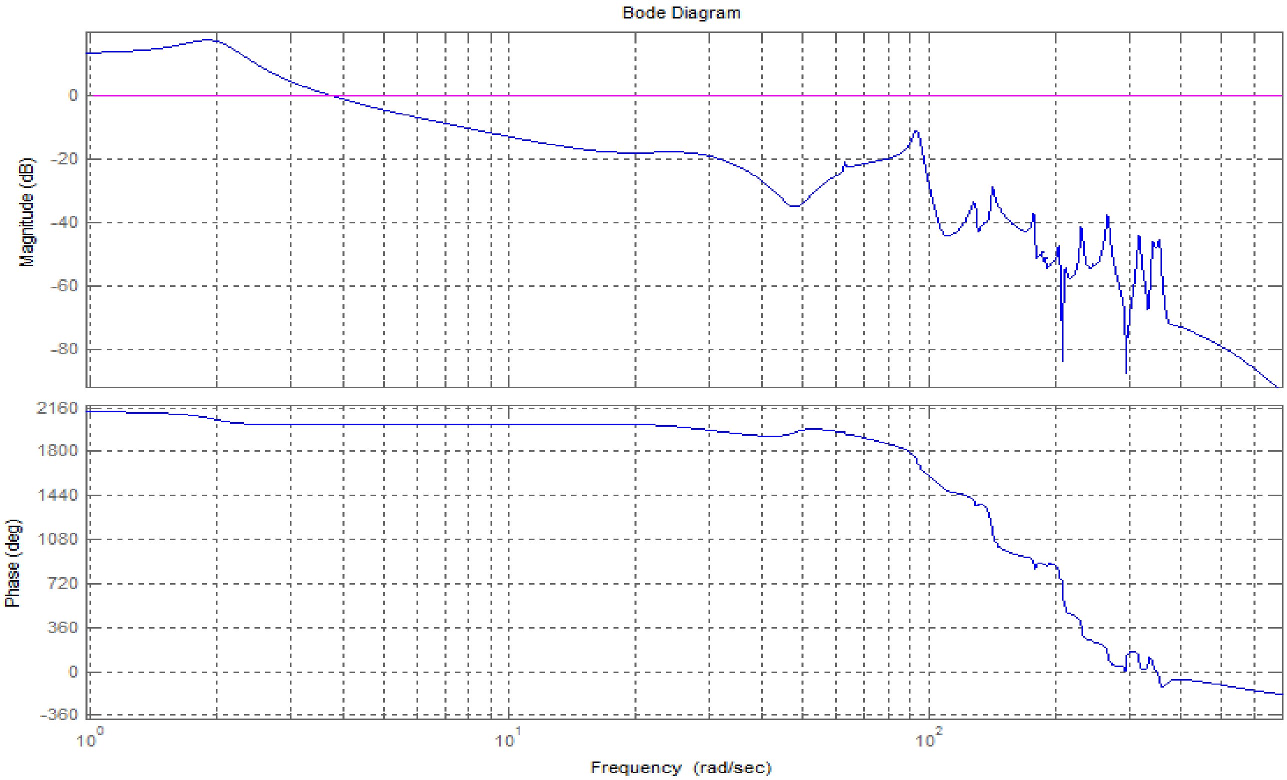
Pitch Axis Open-Loop Frequency Response



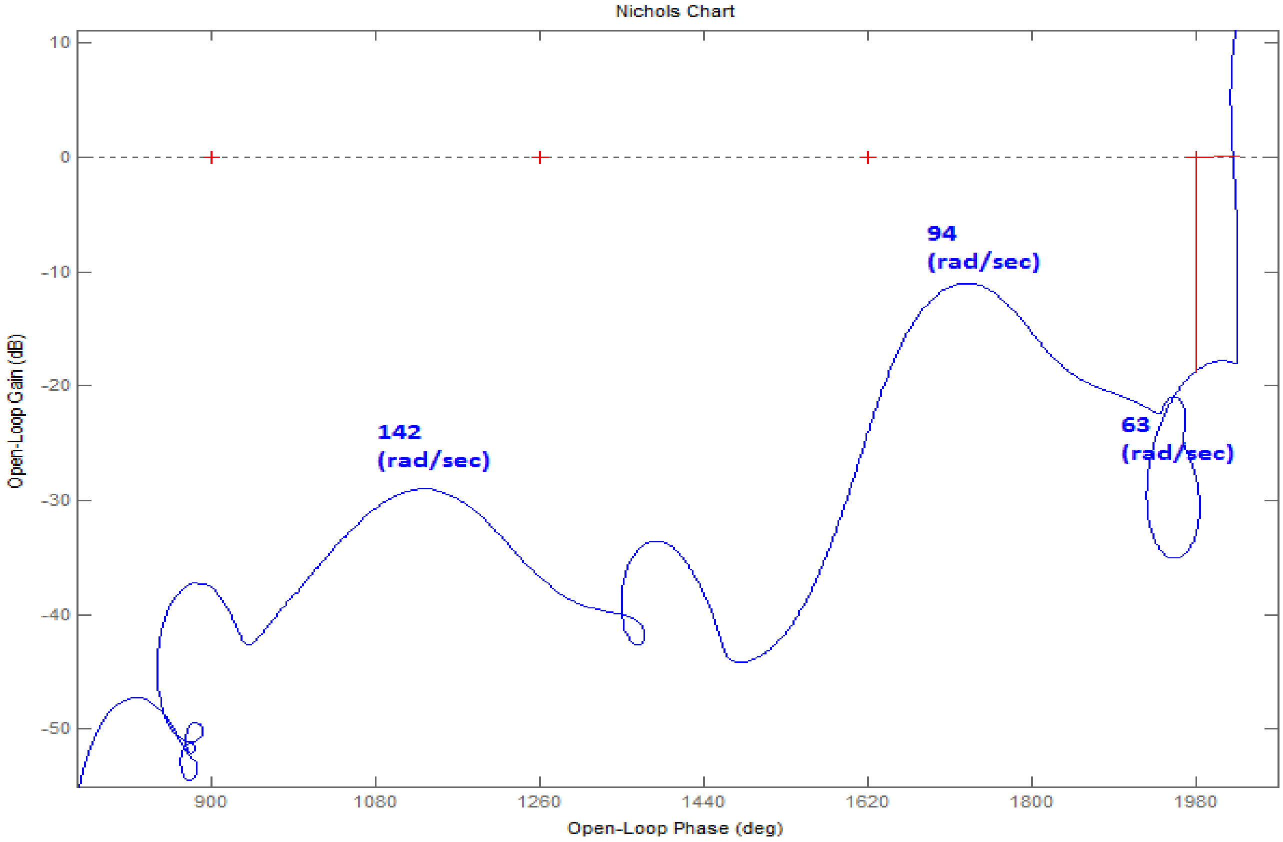
Pitch Axis Stability Analysis



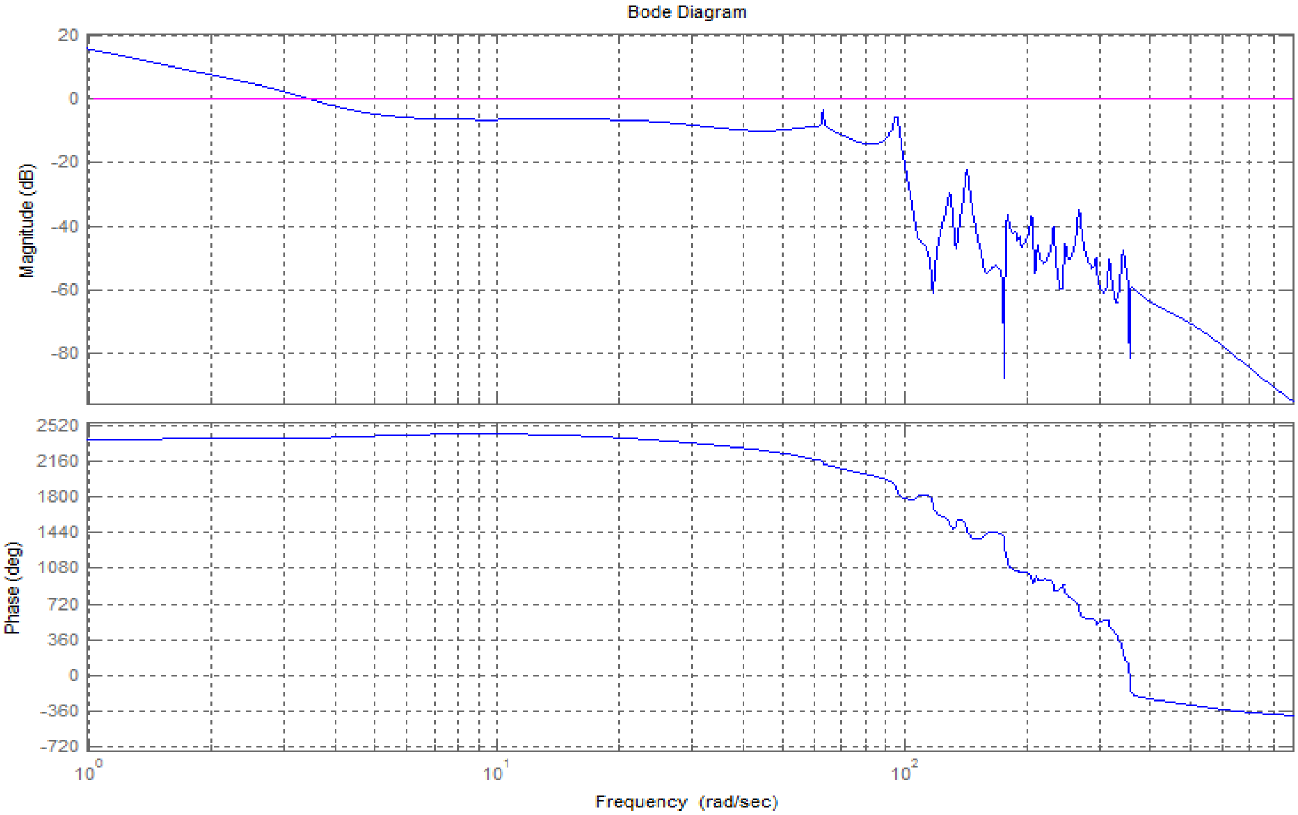
Roll Axis Open-Loop Frequency Response



Roll Axis Stability Analysis



Yaw Axis Open-Loop Frequency Response



Yaw Axis Stability Analysis