Creating Plant Models for Analyzing System Robustness

Robustness is the ability of the control system to tolerate uncertainties and variations, either internal or external. The ability of a system to handle external disturbances is evaluated by the effect of the disturbances on some sensitive outputs, like an optical sensor, or a structural load sensor. Well known sensitivity analysis tools are used to evaluate the system sensitivity between a certain inputs and outputs. The question is how do we analyze a system's robustness to internal parameter variations? How much parameter variations can a system tolerate before it becomes unstable, or stops performing properly? Parameter uncertainties can be seen as imprecise knowledge of the plant model parameters, such as: the mass properties, moments of inertia, aerodynamic coefficients, vehicle altitude, dynamic pressure, center of gravity, etc. The uncertainties of a model are specified in terms of variations in the actual plant parameters, above or below their nominal values. These uncertainties are called *"Structured"*, in contrast with the *"Unstructured"* uncertainties which are described in the frequency domain in terms of maximum amplitude error in the transfer function model.

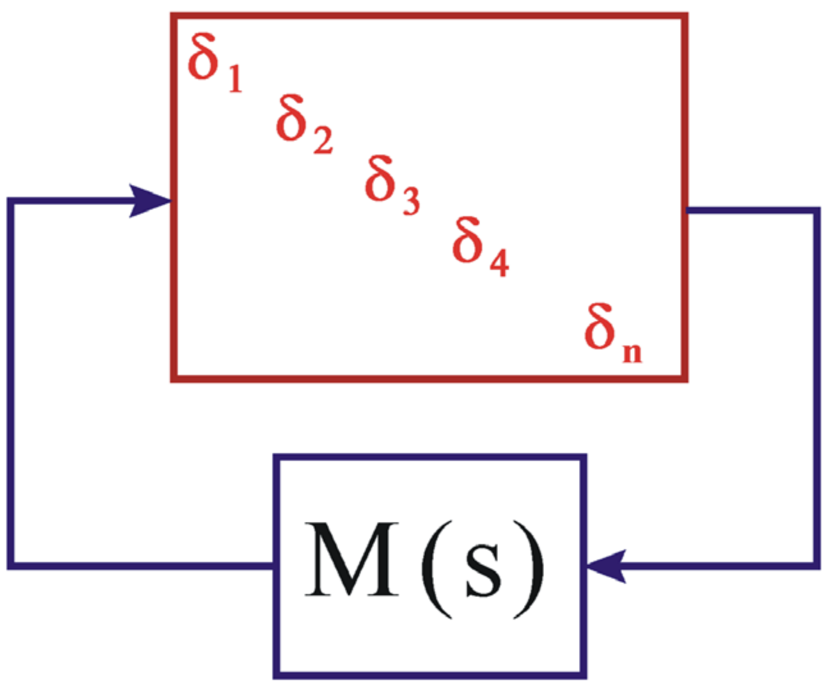

Figure 1 Uncertainties are extracted from the plant M(s) and placed in a diagonal Δ block

In this section we present a method for modeling real parameter uncertainties that have known and bounded max variation magnitudes. Each parameter variation is "pulled out" of the uncertain plant model and it is placed inside a diagonal block Δ that contains only the uncertainties, while the remaining plant is assumed to be known (best guess). The Δ block is attached to the known plant

M(s) by means of (n) input/ output "wires", where (n) is the number of plant uncertainties, as shown in Figure (1). In essence we are creating (n) additional inputs and outputs to the plant M(s) that connect to the uncertainties block Δ, which is a block diagonal matrix Δ= diag($\delta_1,\delta_2,\delta_3,...\delta_n$). The individual elements of Δ may be scalars or matrices and each element represents a real uncertainty in the plant. They may be aerodynamic coefficient variations from nominal values, moment of inertia variations, thrust variations, etc. The magnitude of each element represents the maximum possible variation of the corresponding parameter above or below its nominal value. M(s) represents the known dynamics consisting of the plant model with the control system in closed-loop form.

The internal uncertainties are "pulled out" of the plant M(s) and are connected to M(s) by fictitious inputs and outputs. The method used to extract them as is called the Internal Feedback Loop (IFL) method and it will be described in the next section. The augmented state-space model is then used to analyze robustness using μ-methods, similar to sensitivity analysis. The system in Figure (1) configuration is defined to be robust if it remains stable despite all possible variations in the Δ block, as long as the magnitude of each individual variation is below the uncertainty $\delta_{(i).}$ The structured singular value (μ) is the perfect tool for analyzing this type of robustness problems in the frequency domain. To make the analysis easier, the plant M(s) inputs and outputs are scaled so that the individual elements of the diagonal uncertainty block Δ can now vary between +1 and -1. The value of 1/ μ(M) represents the magnitude of the smallest perturbation that will destabilize the normalized closed-loop system M(s). According to the small gain theorem, the closed-loop system is robust as long as μ(M) across the normalized block Δ is less than one at all frequencies. But the question is how do we extract the uncertainties out of the model?

In the following sections we will also present criteria for evaluating closed-loop system robustness using μ-methods. The μ-tools are extended to analyze simultaneously robustness and performance in the presence of disturbances. We will present criteria for analyzing nominal performance, which is performance in presence of external disturbances alone, ignoring parameter variations. We will also present criteria for analyzing robust performance, which attempts to satisfy both performance and robustness, this is, stability and performance in presence of external disturbances and also robustness in the presence of internal parameter uncertainties, at the same time. We will also demonstrate how to use the Flixan program to create dynamic models for analyzing robustness. The magnitudes of the uncertainties are defined in the input data and the dynamic models are augmented with additional inputs and outputs which capture the uncertain parameter effects. The augmented models are then used to analyze robustness as it will be demonstrated by four real vehicle analysis examples.

The Internal Feedback Loop (IFL) Structure

The IFL concept allows internal parameter perturbations in a plant to be treated like external disturbances in the system. This representation allows us to use μ-tools for robustness analysis, or to apply H_∞ plus other robust methods to design control systems that can tolerate internal parameter variations. To utilize the IFL concept the system must be expressed by the following equation:

$$\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \left\{ \begin{bmatrix} A & B \\ C & D \end{bmatrix} + \begin{bmatrix} \Delta A & \Delta B \\ \Delta C & \Delta D \end{bmatrix} \right\} \begin{bmatrix} x \\ u \end{bmatrix}$$

Suppose that they are (l) independently perturbed parameters p_1, p_2, ... p_l with bounded parameter variations δp_i, where $|\delta p_i| \leq 1$. The perturbation matrix ΔP= [ΔA, ΔB; ΔC, ΔD] can be decomposed with respect to each parameter variation as follows:

$$\Delta_i = -\sum_{i=1}^{L} \delta p_i \begin{pmatrix} \alpha_x^{(i)} \\ \alpha_y^{(i)} \end{pmatrix} \begin{pmatrix} \beta_x^{(i)} & \beta_u^{(i)} \end{pmatrix}$$

The perturbation matrix ΔP is assumed to have a rank-1 dependency with respect to each parameter (p_i). For each parameter p_i

$\alpha_x^{(i)}$ and $\alpha_y^{(i)}$ are column vectors

$\beta_x^{(i)}$, and $\beta_u^{(i)}$ are row vectors

The plant uncertainty ΔP due to all perturbations can be written in the following form

$$\Delta P = -\begin{pmatrix} M_x \\ M_y \end{pmatrix} \Delta \begin{pmatrix} N_x & N_u \end{pmatrix} = -M \Delta N$$

Where M_x and M_y are stacks of column vectors and N_x and N_u are stacks of row vectors as shown below

$$M_x = \begin{bmatrix} \alpha_x^{(1)} & \alpha_x^{(2)} & \ldots & \alpha_x^{(L)} \end{bmatrix}; \quad M_y = \begin{bmatrix} \alpha_y^{(1)} & \alpha_y^{(2)} & \ldots & \alpha_y^{(L)} \end{bmatrix}$$

and

$$N_x = \begin{bmatrix} \beta_x^{(1)} \\ \vdots \\ \beta_x^{(L)} \end{bmatrix}; \quad N_u = \begin{bmatrix} \beta_u^{(1)} \\ \vdots \\ \beta_u^{(L)} \end{bmatrix}$$

$\Delta = \text{diag} [\delta p_1, \delta p_2, \delta p_3, \ldots \delta p_l]$

Notice, that in order to simplify the implementation, the columns of matrices M_x and M_y and the rows of matrices N_x and N_u are scaled, so that the elements of the diagonal block Δ have unity upper bound. Now let us introduce two new variables (z_p and w_p) and rewrite the equations in the following system form in order to express it as a block diagram.

$$z_p = N_x x + N_u u \quad and \quad w_p = -\Delta z_p$$

The perturbed state-space system can be expressed by the following augmented representation which is the same as the original system in the upper left side, with some additional input and output vectors, an input and an output for each parameter uncertainty.

$$\begin{pmatrix} \dot{x} \\ y \\ z_p \end{pmatrix} = \begin{bmatrix} A & B & M_x \\ C & D & M_y \\ N_x & N_u & 0 \end{bmatrix} \begin{pmatrix} x \\ u \\ w_p \end{pmatrix}$$

If we further separate the plant inputs (u) into disturbances (w) and controls (u_c). That is: u=[w, u_c], and if we also separate the plant outputs (y) into performance criteria (z) and control measurements (y_m), the above system is augmented as shown below.

$$\begin{pmatrix} \dot{x} \\ z \\ y_m \\ z_p \end{pmatrix} = \begin{bmatrix} A & B_1 & B_2 & M_x \\ C_1 & D_{11} & D_{12} & M_w \\ C_2 & D_{21} & D_{22} & M_{ym} \\ N_x & N_w & N_{uc} & 0 \end{bmatrix} \begin{pmatrix} x \\ w \\ u_c \\ w_p \end{pmatrix}$$

The above formulation is useful for μ-synthesis or robustness/ performance analysis using μ-methods. It is shown in block diagram form in Figure (2). The uncertainties block Δ is normalized to unity by scaling the columns in the M_x , M_w , and M_{ym} matrices and rows in the N_x , N_w , and N_{uc} matrices after dividing with the square root of the corresponding singular value. The normalized parameter variations block Δ is connected to the plant by means of the inputs w_p and the outputs z_p. When the controller feedback loop between (y_m) and (u_c) is closed, the controller K(s) which is designed based on the nominal plant P(s), is also expected to keep the plant stable despite all possible variations inside Δ block. This property is defined as Robust Stability. In addition to robust stability the controller must also satisfy performance requirements between the disturbances (w) and the criteria (z) not only for the nominal plant (Nominal Performance), but also for the perturbed plant that has the uncertainty loop closed via the Δ block. This property is known as Robust Performance. It means, that the system must be stable and it must satisfy the performance criteria (z) when excited by (w), despite all possible internal plant variations captured in Δ, which are normalized and their individual magnitudes δ_i can vary between -1 and +1.

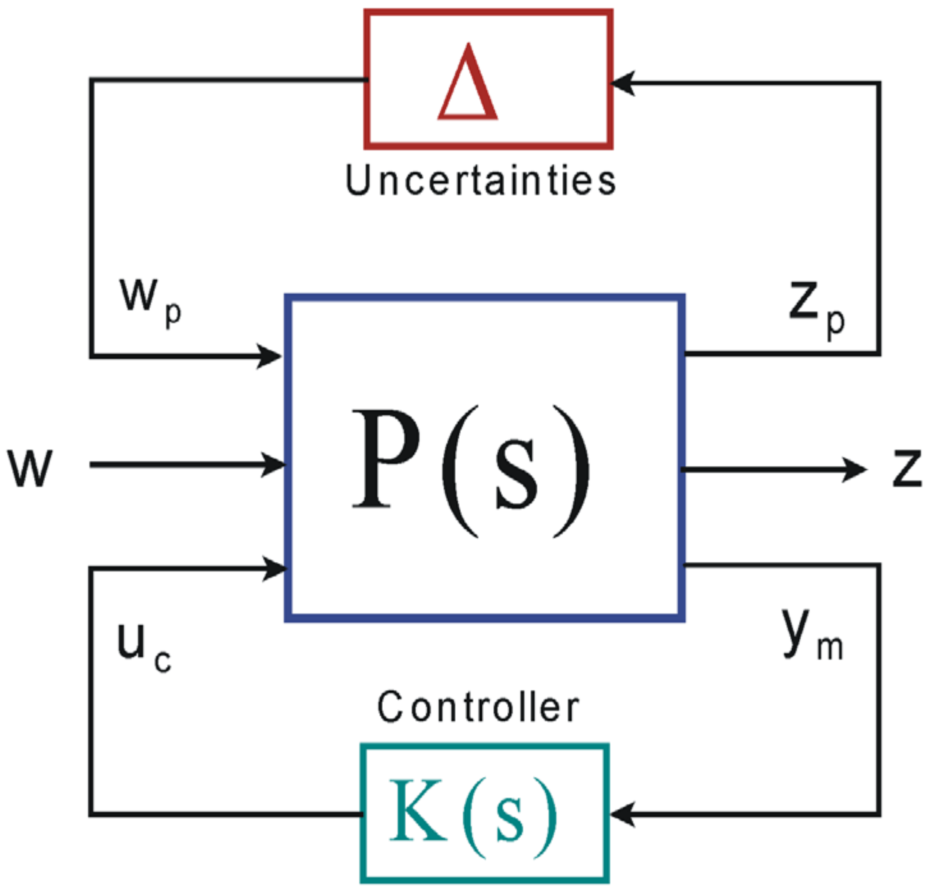

Figure 2 Robustness analysis block showing the Uncertainties IFL loop, the control feedback loop, the disturbances (w), and performance outputs (z)

This system can also be represented in matrix transfer function form as follows

$$\begin{pmatrix} z_p \\ z \\ y_m \end{pmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \begin{pmatrix} w_p \\ w \\ u_c \end{pmatrix}$$ where:

$$w_p = -\Delta z_p \quad and \quad u_c = -K(s)\, y_m$$