# Robustness Analysis of a Lifting Body Aircraft

In this example we have a lifting body glider vehicle at 0.8 Mach and 20 (deg) of angle of attack. The vehicle has four aero-surfaces, an Elevon for pitch control, and a combination of Aileron and two Rudders for lateral control. It also has RCS jets which are not being used in this example. The dynamic model is disturbed by a wind gust input that excites both pitch and lateral dynamics. The plant model has 33 parameters which are not defined accurately. We are going to model the vehicle with its uncertainties extracted in a Δ block, simulate its response to a wind gust, and analyze robustness with respect to uncertainties using μ-analysis.

Modeling

The analysis files for this example are in folder *"C:Flixan\Examples\Robust Analysis Param Uncertainties\ Lifting-Body"*. The vehicle data is in file *"Lifting_Body.Inp"*. This file contains two sets of flight vehicle input data for implementing essentially the same dynamic model using two different approaches: a standard flight vehicle model whose title is *"Lifting-Body at Mach 0.8, Alpha 20, Qbar 400, h=3000 (ft)"*, and a set of vehicle data that references the parameter uncertainties model. The title of the second vehicle data set is *"Lifting-Body at Mach 0.8, Alpha 20, Qbar 400, h=3000 (ft) with 35 Uncertainties"*. This vehicle data references the title of the uncertainties data-set. The uncertainties are bounded in magnitude and the data-set includes magnitudes of maximum parameter variations from their nominal values. They are used to generate the additional input/ output pairs that connect to the "pulled out" uncertainties block Δ. The vehicle parameters may vary from their nominal values, plus or minus the variations. The Flixan program recognizes the uncertainties data-sets by the label *"UNCERTAIN PARAMETER VARIATIONS FROM NOMINAL …"* which is located on the top of the data set. There is also a title for the uncertainties set below that label: *"Uncertainties at Mach=0.8, Alpha=20"* which is used to reference the uncertainties data from the vehicle input data. The uncertainties title must be included in the vehicle data-set that below the line: *"Parameter Uncertainties Data"*, otherwise, the dynamic model will not include the IFL input/ output connections. The variations of the parameters which are known are set to zero in the data-set and they are not extracted in the Δ block. In this example the uncertainties data contains 33 parameter variations, but

μ−Analysis

The Matlab script file "run.m" performs the analysis in Matlab. It loads the coupled system "*Vehicle_35Unc.m*", and decouples it into a pitch and lateral subsystems using the Matlab file "*Split.m*". It separates also the uncertainties input/output pairs into pitch and lateral uncertainties. It synthesizes also an LQR state-feedback control law for the pitch and the lateral systems independently using the Matlab m-file "*K_des.m*". The folder also contains two Simulink models for simulation and μ-analysis, "*Closed_Loop_Pitch.Mdl*" for pitch, and "*Closed_Loop-Lateral.Mdl*" for lateral analysis. They are shown in figures (1 & 2) below.
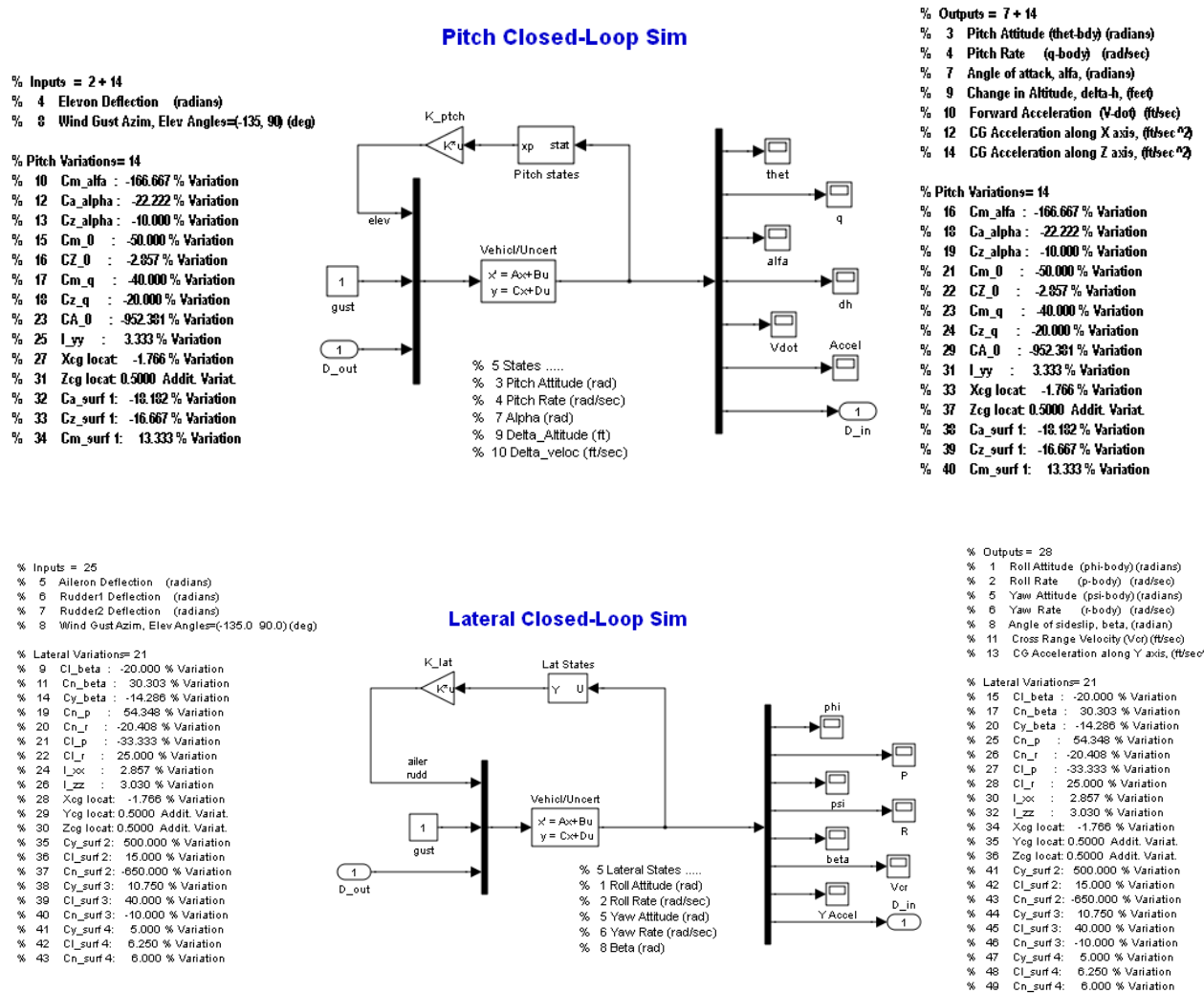
Figures (1 and 2) Closed-Loop Models Used for μ-Analysis

The two systems have their control loops closed via the state-feedback gains K_p and K_l. The 35 uncertainties were separated into 14 pitch and 21 lateral uncertainties shown above. The uncertainty inputs and outputs are combined into vectors which are supposedly connected to the diagonal uncertainty block Δ (not shown). Robustness analysis is performed by calculating the μ frequency response between the input and the output uncertainty vectors for the pitch and the lateral systems separately. The Matlab file "run.m" calculates the μ frequency response, assuming real and not complex parameter variations (complex are easier to calculate but they are too conservative), and plots μ versus frequency, as shown in figure (3). Robustness is guaranteed when μ is less than 1 at all

frequencies for both the pitch and the lateral systems. It means that the pitch and lateral systems will be stable despite all the parameter uncertainties which have bounded and known magnitudes.
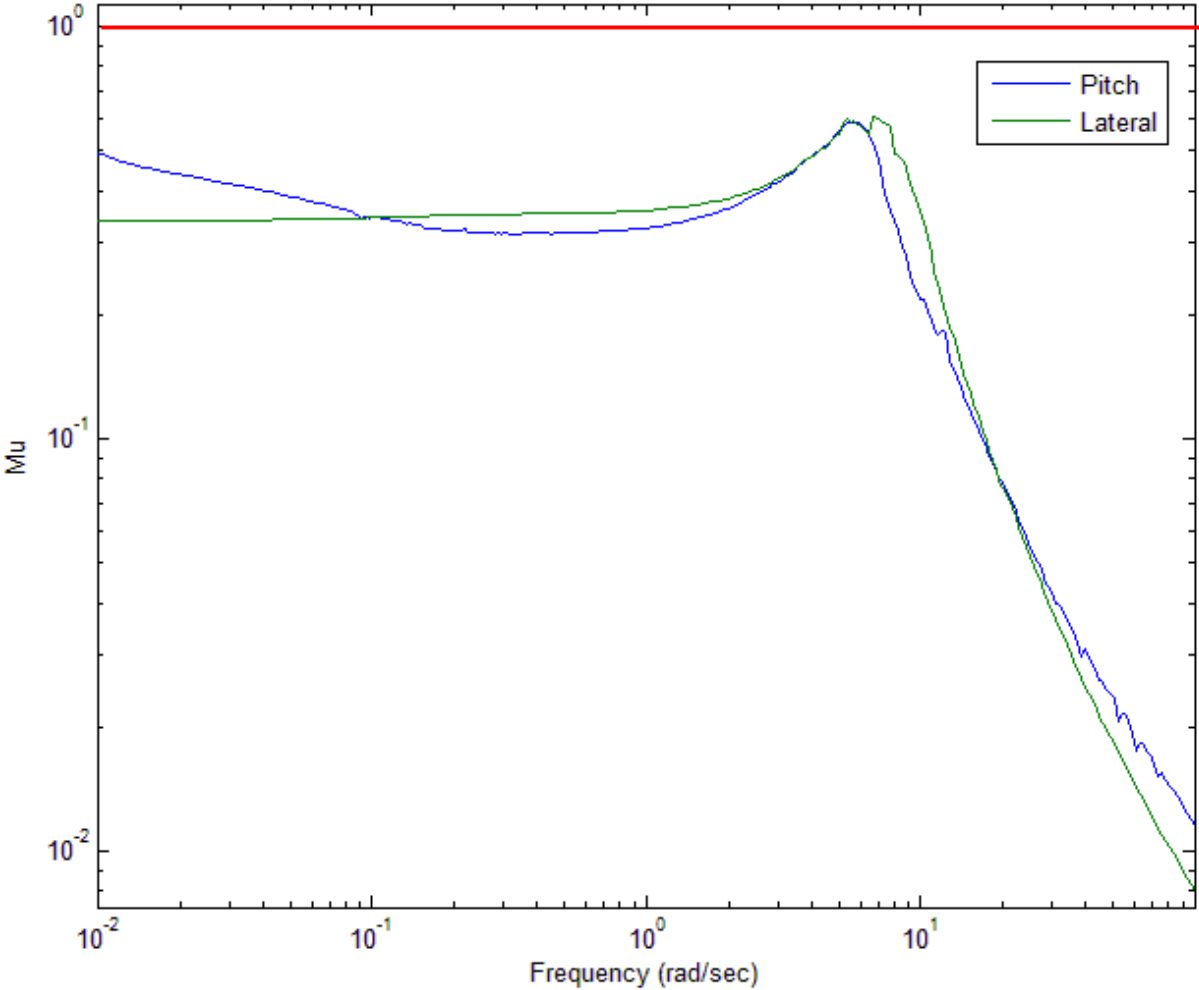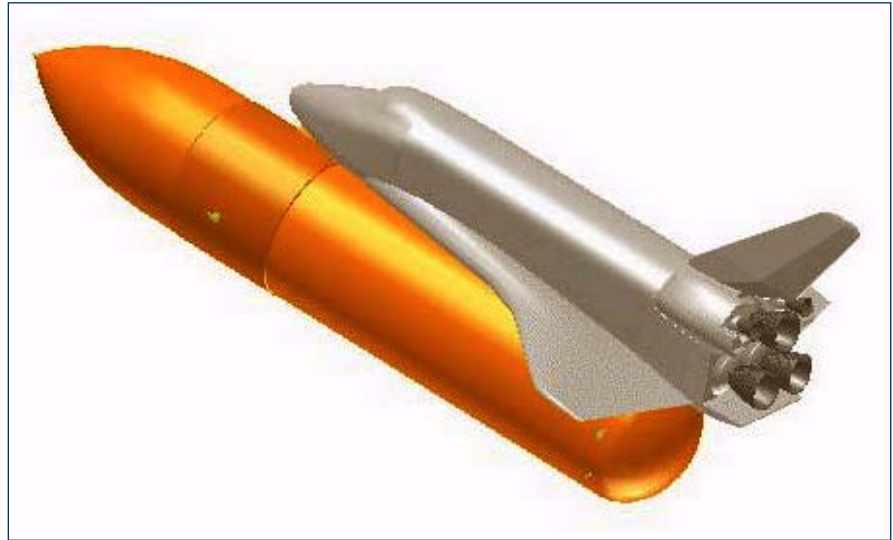

Figure (3) Mu frequency response for 14 pitch and 21 lateral uncertainties, shows that the control system is robust to the parameter variations.

Space Shuttle Robustness Analysis during Second Stage

In this example we will analyze the Space Shuttle vehicle during second stage, immediately after solid rocket booster separation. Although at this time the Shuttle is almost outside the atmosphere, there is still a considerable amount of dynamic pressure for not ignoring the aerodynamics. We will develop a dynamic vehicle model that includes structural flexibility and fuel sloshing effects. The model is defined as uncertain with 45 parameter variations. It will include additional inputs and outputs that attach to the normalized uncertainty block Δ and it will be used to perform robustness analysis.

The analysis data files are in folder "*\Flixan\Examples\Robust Analysis Param Uncertainties\ Shuttle_Ascent*", and the vehicle input data is in file: "*Shuttle_Stg2.Inp*". The vehicle is controlled by the 3 Shuttle main engines which are gimbaling in pitch and yaw. The engine tail-wag-dog dynamics is included in the dynamic model by setting the flag "WITH TWD", and a linear main engine actuator model is also included in the input data file. A gust disturbance is applied in the model perpendicular to the vehicle x axis, skewed and exciting both pitch and lateral directions. The input data file "*Shuttle_Stg2.Inp*" contains two sets of vehicle data, a set that generates a standard flight vehicle model with title "*Space Shuttle Second Stage at T=123 sec*", and a set of vehicle data that generates a similar vehicle model with additional inputs and outputs for the uncertainty model. The title of the second set is "*Space Shuttle Second Stage at T=123 sec (With Uncertain Param)*", and it requires also a set of parameter uncertainties data. The dynamic models include also 20 flexible structure modes. These modes have already been preselected from a big finite elements model and a smaller number of selected modes are included in file "*Shuttle_Stg2.Inp*". The title of the selected modes set is "*Shuttle Second Stage at T=123 sec, Mixed Modes*". The mode selection process is not shown in this example.

The uncertainties data-set is also included in the input file "*Shuttle_Stg2.Inp*". Its title is "*Uncertainties for Shuttle Ascent Second Stage*". It contains maximum variations in vehicle parameters, such as: moments of inertia, angles of attack, sideslip, aero coefficients, engine thrusts, slosh uncertainties, and flex mode frequency and damping variations. The Flight Vehicle Modeling program is processing the uncertainties data set together with the vehicle data to create the augmented state-space model. The title of the uncertainties data is included in the vehicle data, below the line "*Parameter Uncertainties Data*". In this example case both the vehicle and the uncertainties data-sets already exist and they are saved in the input file "*Shuttle_Stg2.Inp*". The Flixan program will process the input data and it will create a state-space model of the vehicle augmented with the additional inputs and outputs that connect to the uncertainty block Δ. The augmented system will be saved in systems file "*Shuttle_Stg2.Qdr*". Notice, that the input/ output pairs that connect to the uncertainty block Δ is greater than the number of parameter variations. This is because some of the uncertainties, such as: engine thrusts, slosh masses,

Control Analysis

The folder "*Examples\Robust Analysis Param Uncertainties\Shuttle_Ascent*" includes several Simulink models used in this robustness analysis. The file "run.m" below loads the systems, and performs linear stability and robustness μ-analysis. One of the simulation models is "*Closed_Loop_Plant.mdl*", shown in figure (1a) below. It uses the combined plant model "*Plant.m*", and the flight control system "*Controller.m*" in closed-loop form. The plant subsystem and the controller are shown in figures (1b and 1c). Figure (2) shows its transient responses to a wind gust step.

```matlab
% Computes the Open-Loop Frequency Response for Flight Control Analysis
% Loop is opened at the Flight Control System Output
d2r=pi/180; r2d=180/pi;
Npv=45;                                        % Number of Param Variations
% Load the sontrol loop subsystems
[Av, Bv, Cv, Dv]= vehicle;                     % Vehicle Nominal Model
[Au, Bu, Cu, Du]= vehicle_unc;                 % Vehicle Model with Uncertainties
[At, Bt, Ct, Dt]= actor_tvc;                   % Actuator + TVC Model
[As, Bs, Cs, Ds]= sensors;                     % Sensors Model
[Ac, Bc, Cc, Dc]= controller;                  % Controller
[Ap, Bp, Cp, Dp]= plant;                       % Combined Plant Model

label='Stage-2 Yaw Axis Stability, (Pitch & Roll Closed)';
[Ao,Bo,Co,Do]= linmod('Open_Loop');           % Frequ Response Model (z-domain)
sys=ss(Ao,Bo,Co,Do);                          % Perform Linearization
w=logspace(-2,2,12000);                       % and Frequ domain analysis
figure(1); Nichols(sys,w); title(label)
figure(2); Bode(sys,w); title(label)

w=logspace(-2,2,500);
[Acp,Bcp,Ccp,Dcp]=linmod('Closed_Loop_Unc');
sys=ss(Acp,Bcp,Ccp,Dcp);
sysf= FRD(sys,w);
blk=[-ones(Npv,1), zeros(Npv,1)];
[bnd,muinfo]= mussv(sysf,blk);

ff= get(muinfo.bnds, 'frequency');
muu=get(muinfo.bnds, 'responsedata');
muu=squeeze(muu);
muu=muu(1,:);
loglog(ff,muu)
xlabel('Frequency (rad/sec)')
ylabel('ssv')
```
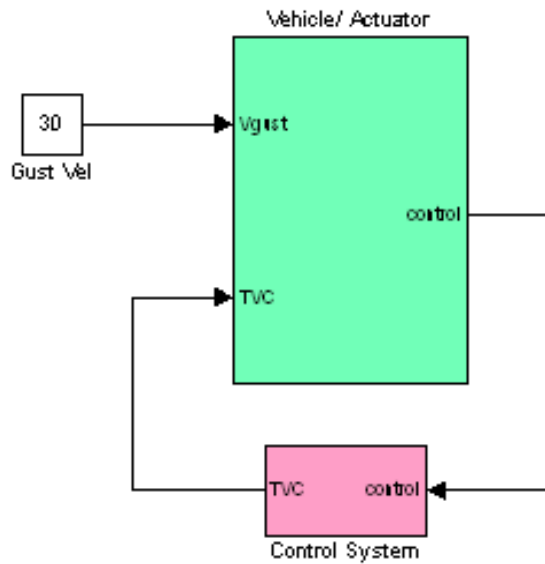
Figure (1a) Plant Model and Flight Control System in Closed-Loop Form

Plant Model
Vehicle/ Actuators/ TVC/ Sensors

x' = Ax+Bu
y = Cx+Du

Plant-Flex-
Continuous

TVC

2

1

Vgust

control

1

K-

r2d1

pqr

r2d

albet

K-

1/32

Nyz

% Inputs = 4
% 1 Roll FCS Command (DP-TVC)
% 2 Pitch FCS Command (DQ-TVC)
% 3 Yaw FCS Command (DR-TVC)
% 4 Wind-Gust Velocity (ft/sec)

% Outputs = 12
% 1 Roll Attitude to FCS
% 2 Roll Rate to FCS
% 3 Pitch Attitude to FCS
% 4 Pitch Rate to FCS
% 5 Yaw Attitude to FCS
% 6 Yaw Rate to FCS
% 7 Angle of Attack, alpha (rad)
% 8 Angle of Sideslip, beta (rad)
% 9 Normal Accelerometer, Nz (ft/sec^2)
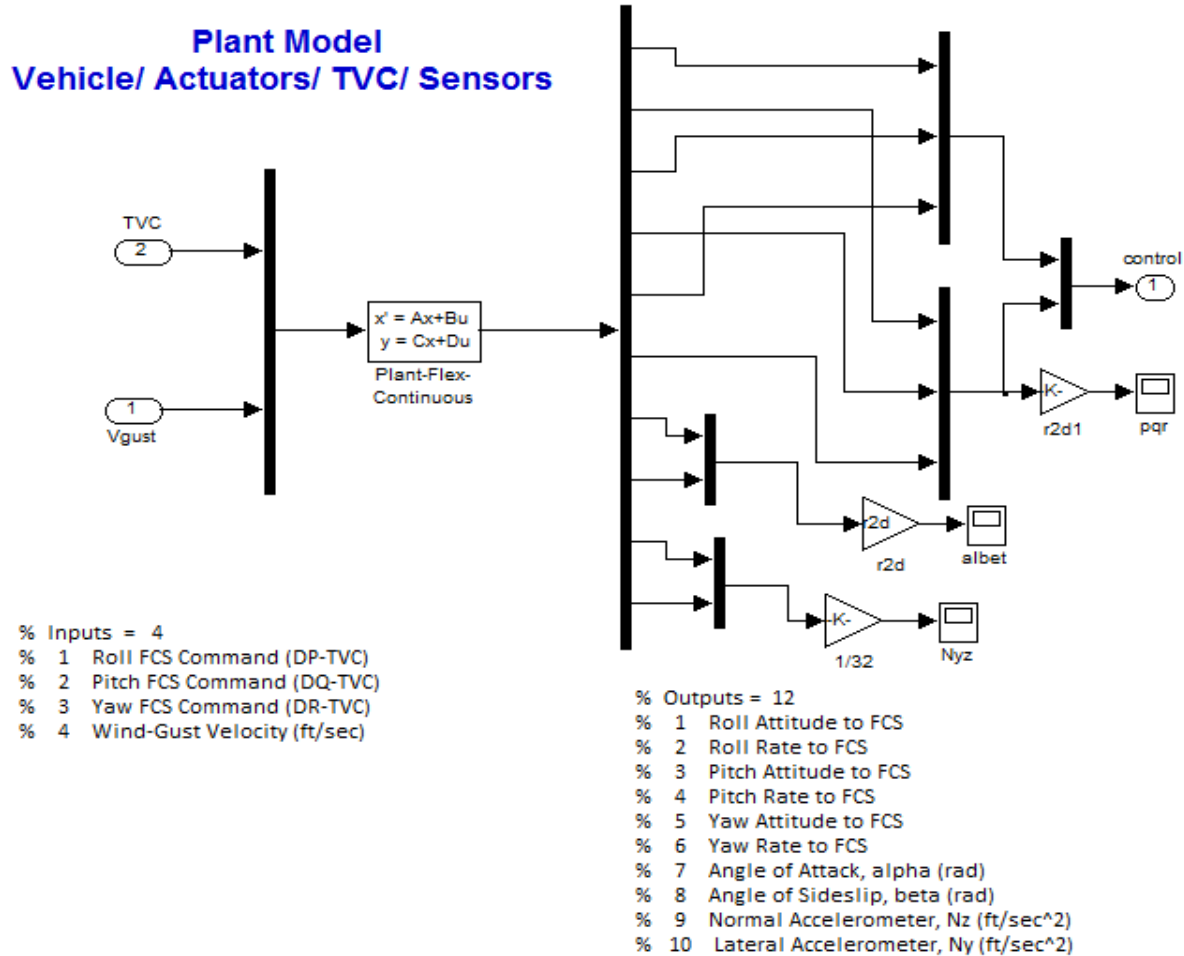% 10 Lateral Accelerometer, Ny (ft/sec^2)

Figure (1b) Plant model from file "plant.m" consisting of Vehicle, Actuators, TVC, and Sensor dynamics

The dynamic model includes the input and output attachments that connect to the uncertainty block Δ. The system's uncertainty inputs and outputs are scaled so that the elements (δ$_i$) of the Δ block vary between -1 and +1. This model is used in robustness analysis by calculating the (μ) structural singular value (SSV) frequency response between the uncertainty inputs and outputs. According to small gain theory, the system is robust to the uncertainties (defined by the parameter variations) when the SSV of the closed-loop system, as seen across Δ, is less than one at all frequencies.

```
% Inputs = 58
%   1   Engine No 1 Pitch Deflect. (rad), Dymax= 10.0000 deg
%   2   Engine No 2 Pitch Deflect. (rad), Dymax= 10.0000 deg
%   3   Engine No 3 Pitch Deflect. (rad), Dymax= 10.0000 deg
%   4   Engine No 1 Pitch Acceleration (rad/sec^2)
%   5   Engine No 2 Pitch Acceleration (rad/sec^2)
%   6   Engine No 3 Pitch Acceleration (rad/sec^2)
%   7   Engine No 1 Yaw Deflect. (rad), Dzmax= 10.0000 deg
%   8   Engine No 2 Yaw Deflect. (rad), Dzmax= 10.0000 deg
%   9   Engine No 3 Yaw Deflect. (rad), Dzmax= 10.0000 deg
%  10   Engine No 1 Yaw Acceleration (rad/sec^2)
%  11   Engine No 2 Yaw Acceleration (rad/sec^2)
%  12   Engine No 3 Yaw Acceleration (rad/sec^2)
%  13   Wind Gust Azim, Elev Angles=( 45.0  90.0) (deg)

Variations = 45
%  14   Cl_beta  :  -44.321 % Variation
%  15   Cm_alpha :  -13.472 % Variation
%  16   Cn_beta  :   23.053 % Variation
%  17   Ca_alpha :   31.831 % Variation
%  18   Cz_alpha :   -6.829 % Variation
%  19   Cy_beta  :  -25.185 % Variation
%  20   Cm_0   :   35.804 % Variation
%  21   CZ_0   :  230.769 % Variation
%  22   CA_0   :   13.937 % Variation
%  23   I_xx   :    6.615 % Variation
%  24   I_yy   :    3.435 % Variation
%  25   I_zz   :    3.616 % Variation
%  26   Xcg locat:  -1.050 % Variation
%  27   Xcg locat:  -1.050 % Variation
%  28   Thr_eng 1:   2.046 % Variation
%  29   Thr_eng 1:   2.046 % Variation
%  30   Thr_eng 2:   2.046 % Variation
%  31   Thr_eng 2:   2.046 % Variation
%  32   Thr_eng 3:   2.046 % Variation
%  33   Thr_eng 3:   2.046 % Variation
%  34   M_slosh 1:   1.660 % Variation
%  35   M_slosh 1:   1.660 % Variation
%  36   X_slosh 1:  -3.093 % Variation
%  37   X_slosh 1:  -3.093 % Variation
%  38   Z_slosh 1:  -1.500 % Variation
%  39   Z_slosh 1:  -1.500 % Variation
%  40   Wslsh_Y 1:   2.257 % Variation
%  41   Wslsh_Z 1:   2.257 % Variation
%  42   M_slosh 2:  10.762 % Variation
%  43   M_slosh 2:  10.762 % Variation
%  44   M_slosh 2:  10.762 % Variation
%  45   X_slosh 2:  -1.596 % Variation
%  46   X_slosh 2:  -1.596 % Variation
%  47   X_slosh 2:  -1.596 % Variation
%  48   Z_slosh 2:  -1.500 % Variation
%  49   Z_slosh 2:  -1.500 % Variation
%  50   Z_slosh 2:  -1.500 % Variation
%  51   Wslsh_Y 2:   2.379 % Variation
%  52   Wslsh_Z 2:   2.379 % Variation
%  53   W_flex 1: 0.5000   Addit. Variat.
%  54   W_flex 2: 0.7000   Addit. Variat.
%  55   W_flex 3: 0.7000   Addit. Variat.
%  56   W_flex 4: 0.8000   Addit. Variat.
%  57   W_flex 5: 0.9000   Addit. Variat.
%  58   W_flex 6: 1.200    Addit. Variat.
```

Vehicle Dynamics with Uncertainties Inputs & Outputs

DY

DY_dd

DZ

DZ_dd

Vgust

Dout

x' = Ax+Bu
y = Cx+Du
Vehicle Model with Uncertainties

r2d   albet

K-

1/32   Nyz

TL

meas

Din

```
% Outputs = 67
%   1   Roll Attitude  (phi-body) (radians)
%   2   Roll Rate    (p-body) (rad/sec)
%   3   Pitch Attitude (thet-bdy) (radians)
%   4   Pitch Rate   (q-body) (rad/sec)
%   5   Yaw Attitude (psi-body) (radians)
%   6   Yaw Rate    (r-body) (rad/sec)
%   7   Angle of attack, alfa, (radians)
%   8   Angle of sideslip, beta, (radian)
%   9   Change in Altitude, delta-h, (feet)
%  10   Forward Acceleration (V-dot) (ft/sec)
%  11   Cross Range Velocity (Vcr) (ft/sec)
%  12   Rate-Gyro # 1, Roll Rate (Body) (rad/sec)
%  13   Rate-Gyro # 2, Pitch Rate (Body) (rad/sec)
%  14   Rate-Gyro # 3, Yaw Rate (Body) (rad/sec)
%  15   Accelerom # 1, (along Z), (ft/sec^2) Translat. Acceler
%  16   Accelerom # 2, (along Y), (ft/sec^2) Translat. Acceler
%  17   Ptch Load-Torque Tly for Engine: 1,(ft-lb)
%  18   Ptch Load-Torque Tly for Engine: 2,(ft-lb)
%  19   Ptch Load-Torque Tly for Engine: 3,(ft-lb)
%  20   Yaw Load-Torque Tlz for Engine: 1,(ft-lb)
%  21   Yaw Load-Torque Tlz for Engine: 2,(ft-lb)
%  22   Yaw Load-Torque Tlz for Engine: 3,(ft-lb)

Variations
%  23   Cl_beta  :  -44.321 % Variation
%  24   Cm_alpha :  -13.472 % Variation
%  25   Cn_beta  :   23.053 % Variation
%  26   Ca_alpha :   31.831 % Variation
%  27   Cz_alpha :   -6.829 % Variation
%  28   Cy_beta  :  -25.185 % Variation
%  29   Cm_0   :   35.804 % Variation
%  30   CZ_0   :  230.769 % Variation
%  31   CA_0   :   13.937 % Variation
%  32   I_xx   :    6.615 % Variation
%  33   I_yy   :    3.435 % Variation
%  34   I_zz   :    3.616 % Variation
%  35   Xcg locat:  -1.050 % Variation
%  36   Xcg locat:  -1.050 % Variation
%  37   Thr_eng 1:   2.046 % Variation
%  38   Thr_eng 1:   2.046 % Variation
%  39   Thr_eng 2:   2.046 % Variation
%  40   Thr_eng 2:   2.046 % Variation
%  41   Thr_eng 3:   2.046 % Variation
%  42   Thr_eng 3:   2.046 % Variation
%  43   M_slosh 1:   1.660 % Variation
%  44   M_slosh 1:   1.660 % Variation
%  45   X_slosh 1:  -3.093 % Variation
%  46   X_slosh 1:  -3.093 % Variation
%  47   Z_slosh 1:  -1.500 % Variation
%  48   Z_slosh 1:  -1.500 % Variation
%  49   Wslsh_Y 1:   2.257 % Variation
%  50   Wslsh_Z 1:   2.257 % Variation
%  51   M_slosh 2:  10.762 % Variation
%  52   M_slosh 2:  10.762 % Variation
%  53   M_slosh 2:  10.762 % Variation
%  54   X_slosh 2:  -1.596 % Variation
%  55   X_slosh 2:  -1.596 % Variation
%  56   X_slosh 2:  -1.596 % Variation
%  57   Z_slosh 2:  -1.500 % Variation
%  58   Z_slosh 2:  -1.500 % Variation
%  59   Z_slosh 2:  -1.500 % Variation
%  60   Wslsh_Y 2:   2.379 % Variation
%  61   Wslsh_Z 2:   2.379 % Variation
%  62   W_flex 1: 0.5000   Addit. Variat.
%  63   W_flex 2: 0.7000   Addit. Variat.
%  64   W_flex 3: 0.7000   Addit. Variat.
%  65   W_flex 4: 0.8000   Addit. Variat.
%  66   W_flex 5: 0.9000   Addit. Variat.
%  67   W_flex 6: 1.200    Addit. Variat.
```

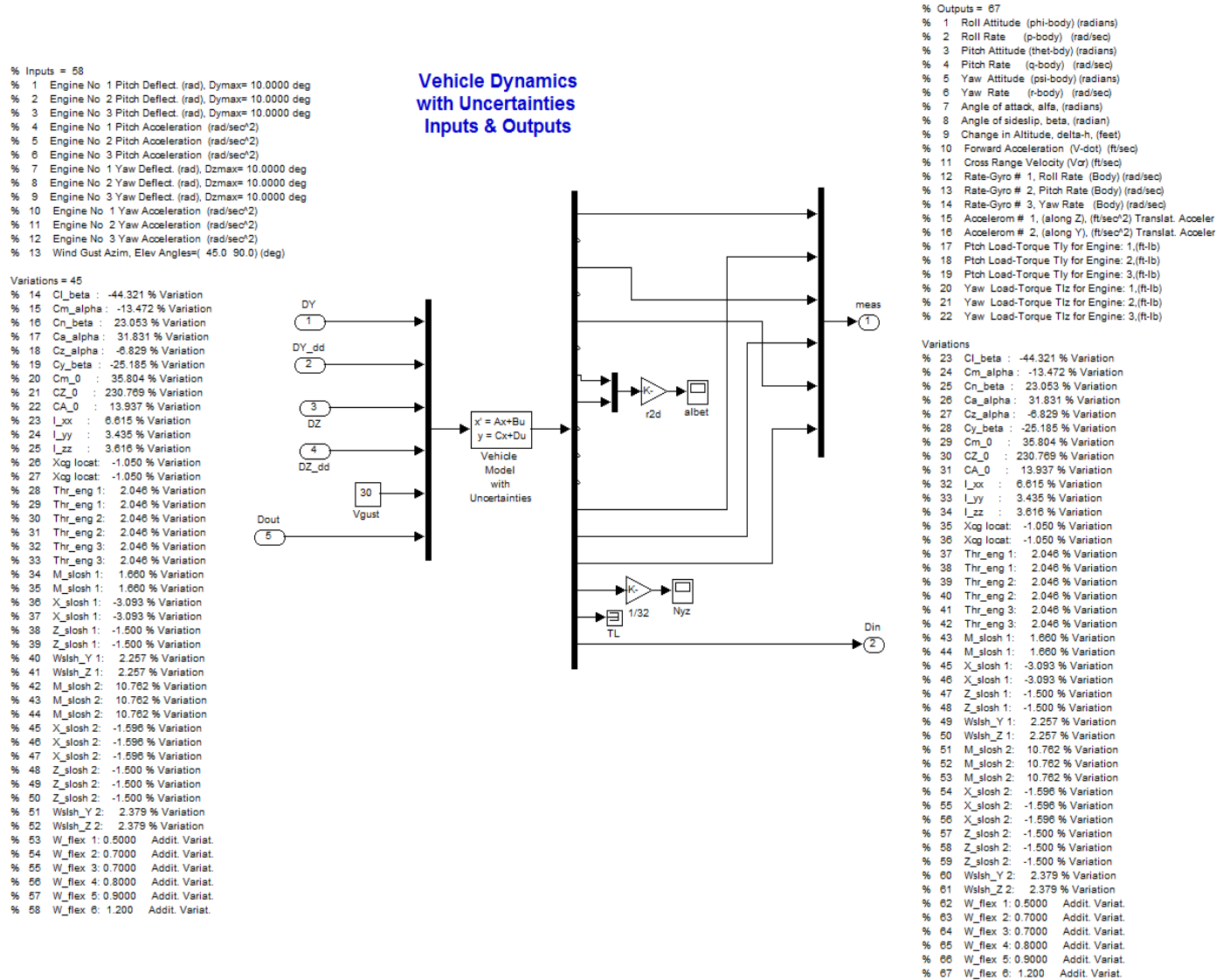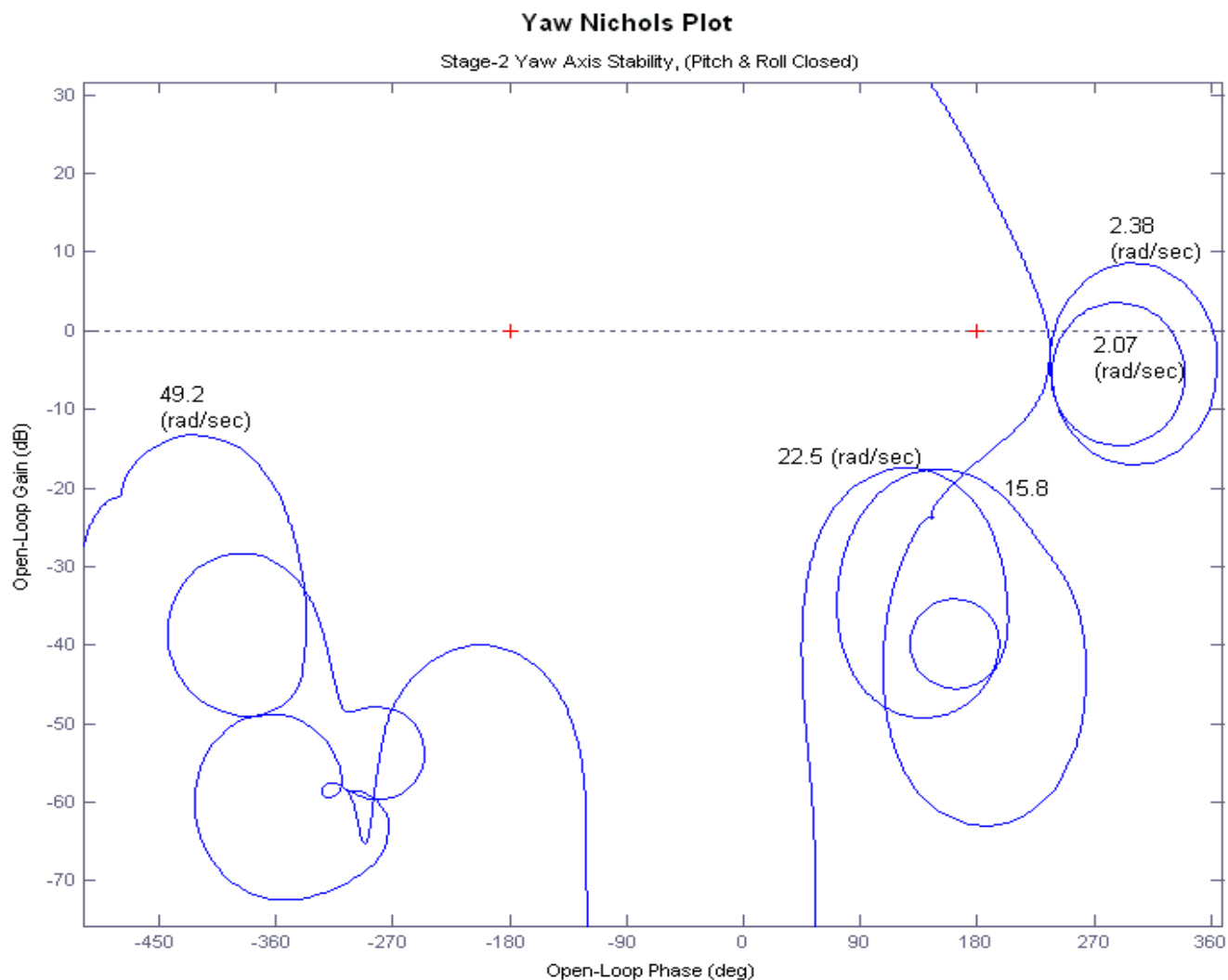**Figure (3b) State-space model consisting of vehicle dynamics plus 45 additional input/ output pairs that connect across the uncertainty block Δ.**

The Matlab file "run.m" uses the open-loop model "*Open_Loop.mdl*" described in Figure (4) to calculate the frequency response and to perform stability analysis using a Nichols chart, as shown in figure (5).

Yaw Nichols Plot

Figure (5) Yaw axis Nichols plot shows slosh and flex resonances. It also shows that the flight control system has sufficient phase and gain margins

Mu Analysis

The Matlab script file "run.m" also uses the closed-loop model with uncertainties "*Closed_Loop-Unc.mdl*" to calculate the μ frequency response across the scaled Δ block, as shown in figure (6). The structural singular value plot shows that the closed-loop system barely meets the μ[M(ω)]<1 robustness requirement. This means that the control system is capable to tolerate the uncertainties without becoming unstable. The Matlab μ analysis algorithm assumes real parameter variations (not complex ones because this is too conservative and it would violate it for low damped resonances).
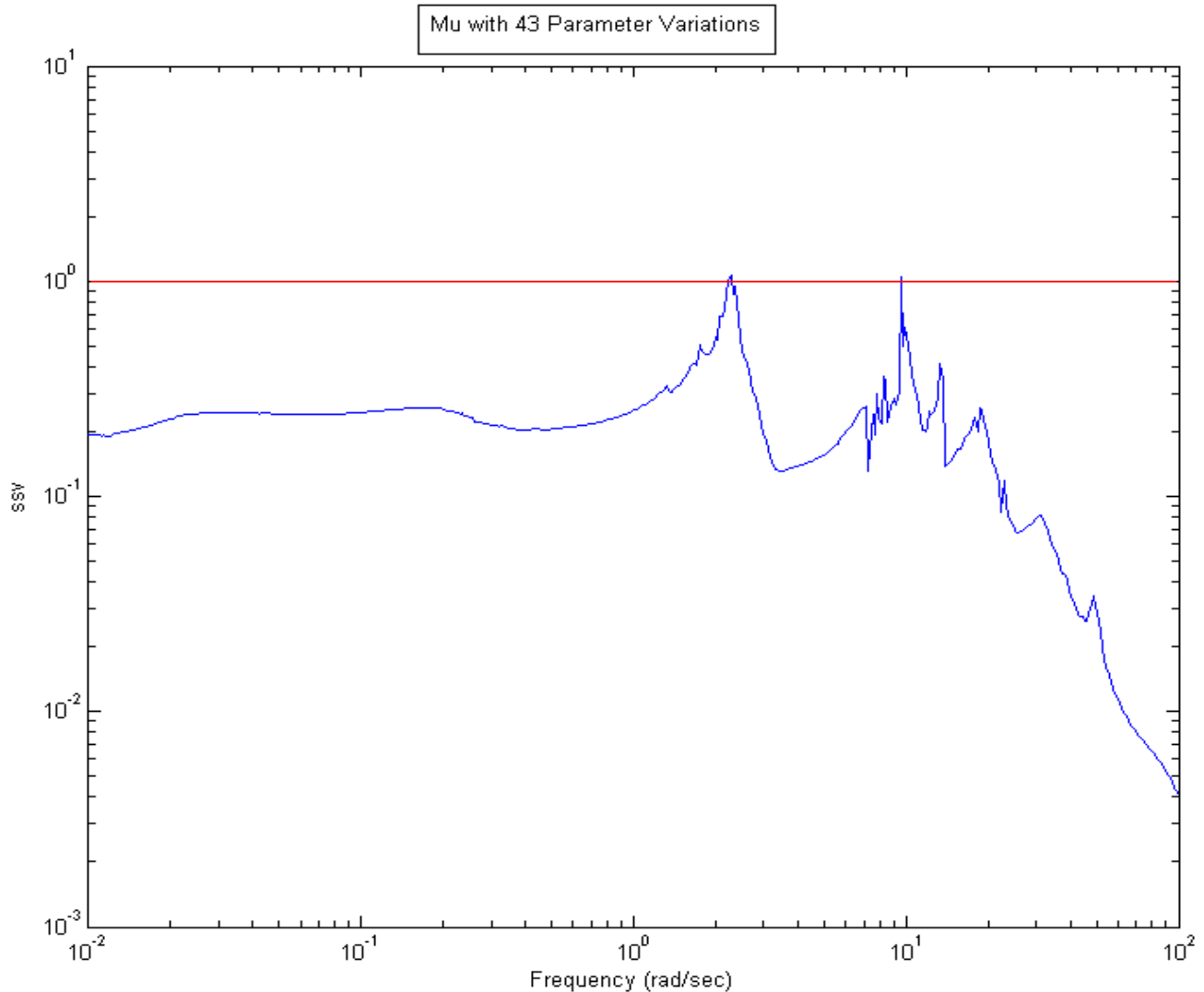
Figure 6 μ-analysis shows that the control system meets the robustness requirements

Robustness Analysis of a Flexible Satellite with Reaction Wheels

In this example we will analyze robustness of a flexible satellite that is controlled by 3 reaction wheels. The project files are in folder "*\Flixan\Examples\Robust Analysis Param Uncertainties\ Satellite_RW*". The input data file that contains the satellite parameters is "*Satellite_RW.Inp*". Its title is "*Flex Satellite with Reaction Wheels*". This data set in addition to vehicle mass properties it contains also: the spin axis direction of the 3 reaction wheels, their initial rotational speeds in (rpm) which are zero in this case, and the moment of inertia for each wheel about its spin axis, which is 0.1 (slug-ft^2). It contains also 3 rate gyros, 3 attitude sensors, and 2 accelerometer sensors. The satellite model also contains 60 structural flexibility modes. The modes are already pre-processed and selected from a finite elements model and the selected modes are included in the input data file "*Satellite_RW.Inp*". The flex modes title is: "*Flex Satellite with Reaction Wheels (60 Flex Modes)*". The block of data for each mode contains the mode frequency and the mode shapes at the locations of the 3 reaction wheels, the 6 gyros, and the 2 accelerometers. The modal data title is also included in the satellite input data-set, under the line "*Number of Flex Modes: 60*".

The input file "*Satellite_RW.Inp*" also includes the uncertainties data-set located below the satellite input data. The Flixan program recognizes the parameter variations data-set by the label "*UNCERTAIN PARAMETER VARIATIONS FROM NOMINAL …*". The uncertainties title is "*Uncertainties for the Flex Satellite with Reaction Wheels*" and it is located below the label. It is also included in the satellite data below the line "*Parameter Uncertainties Data*". The uncertainties data consists of variations in the moments of inertia, in the RW spin axis directions, the RW momentum bias, and the rotor moment of inertia. Variations in some of the flex mode frequencies are also included. The flex mode numbers that have uncertain frequencies appear in line "*Flex Mode Uncertainties (Mode Number): 1 3 4 5 6 42 43 50*". The corresponding max frequency variations are included in the next line "*Flex Mode Frequency Variation (additive): 0.3 0.4 0.6 2.0 2.0 10.0 10.0 10.0*".

Flex Satellite with Reaction Wheels
! This is a simple Satellite model that uses 3 Reaction Wheels, one wheel per axis.
! It contains also 60 flex modes. The modal data are included in bottom of file.
! Title of Parameter Uncertainties data-set is also included for IFL modeling.
! The augmented spacecraft model is used to analyze robustness to structural uncertainties
!
Body Axes Output, Euler Angles

```
Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)   :   50.0   0.0001 0.20896E+08
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)         :   1045.3, 641.02,  505.05,
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :  -0.05,  0.1,  22.625
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)      :   0.0  29000.0   0.0001
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)        :   0.0    0.0  0.0  0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                        :   0.0    0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)  :   0.0 0.0 0.0 0.0, -0.0647,
External Torque and direction about (x,y,z)                                             :   Torque  0.0 1.0   0.0
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)       :   0.0    1.0 1.0
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :   0.0  0.0   0.0 0.0
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:   0.0  0.0   0.0 0.0
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:   0.0 -0.0   0.0 0.0 0.0
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}:   0.0  0.0   0.0 0.0
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:   0.0  0.0   0.0 0.0 0.0
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:   0.0  0.0   0.0 0.0 0.0
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:   0.0  0.0   0.0 0.0 0.0


Number of Thruster Engines, RCS Jets, No Engine Gimbaling                                :   0


Number of Reaction Wheels                                                               :   3
RW No:  1, Spin Direct. Unit Vect, Init Rot Speed (rpm), Rotor Mom of Inertia (slg-ft^2) :   1.0  0.0  0.0  0.0  0.1
RW No:  2, Spin Direct. Unit Vect, Init Rot Speed (rpm), Rotor Mom of Inertia (slg-ft^2) :   0.0  1.0  0.0  0.0  0.1
RW No:  3, Spin Direct. Unit Vect, Init Rot Speed (rpm), Rotor Mom of Inertia (slg-ft^2) :   0.0  0.0  1.0  0.0  0.1


Number of Gyros, (Attitude and Rate)                                                    :   6
Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 6   :  Roll    Rate -0.747 0.114
Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 6   :  Pitch   Rate -0.747 0.114
Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 6   :  Yaw     Rate -0.747 0.114
Gyro No  4 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 6   :  Roll    Attitu -0.747 0.114
Gyro No  5 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 6   :  Pitch   Attitu -0.747 0.114
Gyro No  6 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 6   :  Yaw     Attitu -0.747 0.114


Number of Accelerometers, Along Axes: (x,y,z)                                           :   2
Acceleromet No  1 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 7  :  X-axis Accel 0.338 0.64 25
Acceleromet No  2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 7  :  Y-axis Accel 0.338 0.64 25

Parameter Uncertainties Data
```
Uncertainties for the Flex Satellite with Reaction Wheels

```
Number of Bending Modes                                                                 : 60
```
Flex Satellite with Reaction Wheels (60 Flex Modes)
```
-----------------------------------------------------------------------------------------------------
```

New Uncertainties for Satellite
! This is a new set of parameter variations for a Satellite with three Reaction Wheels. The wheel
! moment of inertia about the rotor, and the spin axis direction are not known very accurately.
! Also the wheel speed is unknown. There are also uncertainties in the spacecraft moments of
! inertia, and also in the flex mode frequencies and damping coefficient (zeta).

```
Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)   :   0.00    0.0    0.0
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)         :   50.0  30.0  25.0 0.0
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)       :   0.0   0.0    0.0
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)      :   0.00   0.0    0.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)        :   0.0    0.0    0.0  0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                        :   0.0    0.0    0.0  0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)  :   0.0    0.0    0.0  0.0
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:   0.0    0.0    0.0  0.0
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:   0.0    0.0    0.0  0.0
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}:   0.0    0.0    0.0  0.0
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:   0.0    0.0    0.0  0.0
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:   0.0    0.0    0.0  0.0
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:   0.0    0.0    0.0  0.0


Number of Reaction Wheels, (Variations from Nominal Wheel Parameters)                    :   3
RW No:  1, Spin Direct. Unit Vect, Init Rot Speed (rpm), Rotor Mom of Inertia (slg-ft^2) :   0.0  0.1  0.1  10.0   0.02
RW No:  2, Spin Direct. Unit Vect, Init Rot Speed (rpm), Rotor Mom of Inertia (slg-ft^2) :   0.1  0.0  0.1  10.0   0.02
RW No:  3, Spin Direct. Unit Vect, Init Rot Speed (rpm), Rotor Mom of Inertia (slg-ft^2) :   0.1  0.1  0.0  10.0   0.02

Flex Mode Uncertainties (Mode Number)                                                   :  1    3     4     5     6    42
43    50
Flex Mode Frequency (omega) Variation (additive)  (rad/sec)                             :  0.3  0.4   0.6   2.0   2.5 25
Flex Mode Damping Coefficient (zeta) Variation (additive)                              : 0.001 0. 0.001 0.0 0.001 0.
-----------------------------------------------------------------------------------------------------
```

```
SELECTED MODAL DATA AND LOCATIONS FOR : Pre-Selected
Flex Satellite with Reaction Wheels (60 Flex Modes)
! The flex modes used in this analysis are the same modes that were selected and used in a
! previous flex spacecraft analysis. We start from Mode #7 because the first 6 rigid-body modes
! are not included in the modal data because the rigid-body dynamics are included in the vehicle.
! The Nodes correspond to vehicle locations defined in the vehicle data above.

MODE#  1/  7, Frequency (rad/sec), Damping (zeta), Generalized Mass=    3.0998       0.50000E-02   12.000
DEFINITION OF LOCATIONS (NODES)           phi along X   phi along Y   phi along Z   sigm about X  sigm about Y  sigm

                        Node ID#     Modal Data at the  3 Reaction Wheels...
Reaction Wheel #1 CG      58041     -0.20700D+00  -0.43105D-03   0.41763D-01  -0.11013D-02   0.64943D-01  -
Reaction Wheel #2 CG      58042     -0.20783D+00  -0.16859D-02  -0.31758D-01  -0.11002D-02   0.64942D-01  -
Reaction Wheel #3 CG      58043     -0.20632D+00  -0.17810D-02  -0.38780D-01  -0.11033D-02   0.64942D-01  -


                        Node ID#     Modal Data at the  6 Gyros ...
Inertial Attitude Sensors 31001      0.51466D-01   0.39036D-02   0.38773D-01  -0.11017D-02   0.64943D-01  -
Inertial Attitude Sensors 31001      0.51466D-01   0.39036D-02   0.38773D-01  -0.11017D-02   0.64943D-01  -
Inertial Attitude Sensors 31001      0.51466D-01   0.39036D-02   0.38773D-01  -0.11017D-02   0.64943D-01  -
Inertial Attitude Sensors 31001      0.51466D-01   0.39036D-02   0.38773D-01  -0.11017D-02   0.64943D-01  -
Inertial Attitude Sensors 31001      0.51466D-01   0.39036D-02   0.38773D-01  -0.11017D-02   0.64943D-01  -
Inertial Attitude Sensors 31001      0.51466D-01   0.39036D-02   0.38773D-01  -0.11017D-02   0.64943D-01  -


                        Node ID#     Modal Data at the  2 Accelerometers, along (x,y,z)...
Accelerometers            31002      0.51889D-01   0.27108D-02  -0.32279D-01
Accelerometers            31002      0.51889D-01   0.27108D-02  -0.32279D-01


                        Node ID#     Modal Data at the Disturbance Point
RCS Jet #8    (+X)        98008     -0.19474D+00   0.14764D-02   0.14237D+00  -0.11013D-02   0.64937D-01

-----------------------------------------------------------------------------------------------------------
```
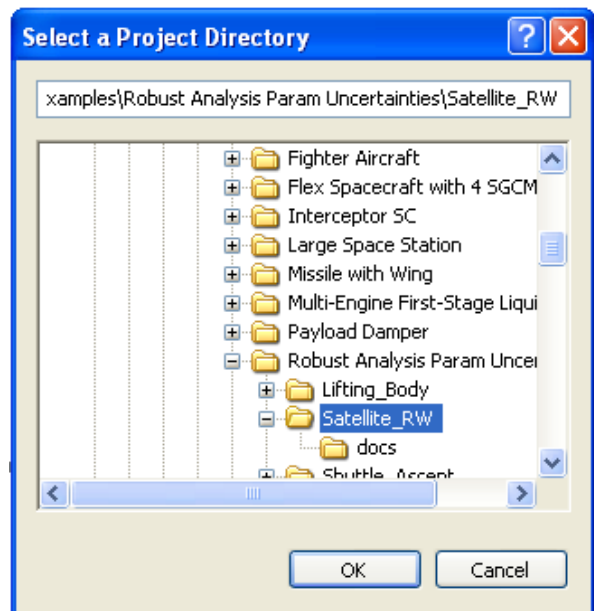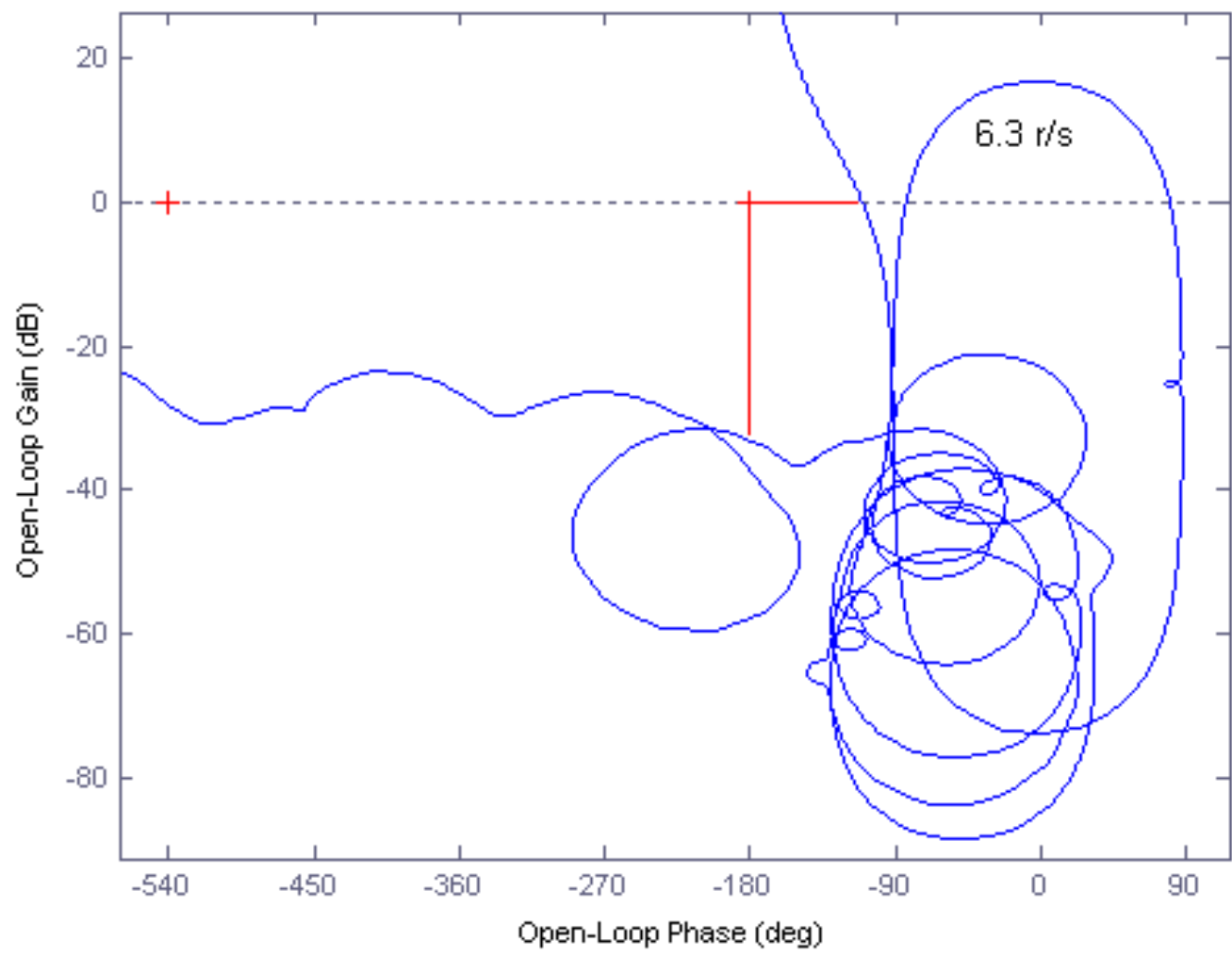
Generating the Uncertain Satellite State-Space Model

There are several ways of processing this input data file in Flixan and to create the satellite state-space model with the additional IFL uncertainty I/Os. It can either be processed by running the batch set which is located on the top of file "*Satellite_RW.Inp*" similar to the previous two examples, or it can be processed by running the flight vehicle modeling program just like we would normally run a typical flight vehicle input data. The uncertainties data will also be processed because they are referenced by their title in the vehicle data. Notice, that if the uncertainties title is not included in the vehicle data they will be ignored and the vehicle model will not include the additional input/ output pairs that connect to the Δ block. In this example we will use the third option which is to run the parameter uncertainties modeling program. The uncertainties are already in the input file.

Start the Flixan program and select the current project folder: "*\Flixan\Examples\Robust Analysis Param Uncertainties\Satellite_RW*". Then from the main menu select "*Program Functions*", "*Robust Control Synthesis Tools*", and "*Modeling Vehicle Parameter Uncertainties*", as shown below.

Yaw Axis Stability, (Pitch & Roll Closed)

6.3 r/s

The Simulink model in Figure (3) is "*Closed_Loop.mdl*" and the green satellite block includes the Flixan generated flex vehicle with uncertainties system "satellite_60flx.m". The satellite dynamics model (green block) is shown in detail in figure (4), and the controller is a simple PD control law. The file "run.m" calculates also the structural singular value (μ) frequency response of the closed-loop system with the loop opened across the parameter variations block Δ, see Figure (3). It calculates the (μ) using two different methods, (a) assuming that the variations in block Δ are complex with magnitudes less than 1 (this method is usually too conservative for low damped resonances), and (b) assuming that the variations in block Δ are real numbers with magnitudes varying between ±1 (slower to compute).

Figure (3) Closed-Loop Simulation Model "*Closed_Loop.mdl*" uses a simple PD control law.

Flex Satellite with 3 Reaction Wheels

Trw1

Trw

Trw

2
Trw

Clock   Time

x' = Ax+Bu
y = Cx+Du

Flex Satellite

1
Unco

Filter

In Ou

pqr

pqr

rate1

atti

atti

accel

acc

rpm

rpm

rate

3

attit

2

att

1
Unci

% Inputs = 32
% 1   Reaction Wheel No 1 Input Torque (ft-lb)
% 2   Reaction Wheel No 2 Input Torque (ft-lb)
% 3   Reaction Wheel No 3 Input Torque (ft-lb)
% 4   Ext Torque Direct.=(0.0000 1.0000 0.0000)

Param Variations= 28
% 5   I_xx   :   4.783 % Variation
% 6   I_xx   :   4.783 % Variation
% 7   I_yy   :   4.680 % Variation
% 8   I_yy   :   4.680 % Variation
% 9   I_yy   :   4.680 % Variation
% 10   I_zz   :   4.950 % Variation
% 11   I_zz   :   4.950 % Variation
% 12   RWinert 1:   20.000 % Variation
% 13   RW_H0 1: 0.2094E-01 Addit. Variat.
% 14   RW_H0 1: 0.2094E-01 Addit. Variat.
% 15   HDirect 1:   14.003 % Variation
% 16   HDirect 1:   14.003 % Variation
% 17   RWinert 2:   20.000 % Variation
% 18   RW_H0 2: 0.2094E-01 Addit. Variat.
% 19   RW_H0 2: 0.2094E-01 Addit. Variat.
% 20   HDirect 2:   14.003 % Variation
% 21   RWinert 3:   20.000 % Variation
% 22   RW_H0 3: 0.2094E-01 Addit. Variat.
% 23   RW_H0 3: 0.2094E-01 Addit. Variat.
% 24   HDirect 3:   14.003 % Variation
% 25   W_flex 1: 0.3000   Addit. Variat.
% 26   W_flex 2: 0.4000   Addit. Variat.
% 27   W_flex 3: 0.6000   Addit. Variat.
% 28   W_flex 4: 2.000   Addit. Variat.
% 29   W_flex 5: 2.000   Addit. Variat.
% 30   W_flex 6: 10.00   Addit. Variat.
% 31   W_flex 7: 10.00   Addit. Variat.
% 32   W_flex 8: 10.00   Addit. Variat.

% Outputs = 50
% 1   Roll Attitude (phi-body) (radians)
% 2   Roll Rate   (p-body) (rad/sec)
% 3   Pitch Attitude (thet-bdy) (radians)
% 4   Pitch Rate   (q-body) (rad/sec)
% 5   Yaw Attitude (psi-body) (radians)
% 6   Yaw Rate   (r-body) (rad/sec)
% 7   Angle of attack, alfa, (radians)
% 8   Angle of sideslip, beta, (radian)
% 9   Change in Altitude, delta1-h, (feet)
% 10   Forward Acceleration (V-dot) (ft/sec)
% 11   Cross Range Velocity (Vcr) (ft/sec)
% 12   Rate-Gyro # 1, Roll Rate (Body) (rad/sec)
% 13   Rate-Gyro # 2, Pitch Rate (Body) (rad/sec)
% 14   Rate-Gyro # 3, Yaw Rate   (Body) (rad/sec)
% 15   Gyro # 4, Roll Attitude   (Body) (radians)
% 16   Gyro # 5, Pitch Attitude (Body) (radians)
% 17   Gyro # 6, Yaw Attitude   (Body) (radians)
% 18   Accelerom # 1, (along X), (ft/sec^2) Translat. Acceler
% 19   Accelerom # 2, (along Y), (ft/sec^2) Translat. Acceler
% 20   Reaction Wheel # 1 Rate (rpm) relative to vehicle
% 21   Reaction Wheel # 2 Rate (rpm) relative to vehicle
% 22   Reaction Wheel # 3 Rate (rpm) relative to vehicle

Param Variations= 28
% 23   I_xx   :   4.783 % Variation
% 24   I_xx   :   4.783 % Variation
% 25   I_yy   :   4.680 % Variation
% 26   I_yy   :   4.680 % Variation
% 27   I_yy   :   4.680 % Variation
% 28   I_zz   :   4.950 % Variation
% 29   I_zz   :   4.950 % Variation
% 30   RWinert 1:   20.000 % Variation
% 31   RW_H0 1: 0.2094E-01 Addit. Variat.
% 32   RW_H0 1: 0.2094E-01 Addit. Variat.
% 33   HDirect 1:   14.003 % Variation
% 34   HDirect 1:   14.003 % Variation
% 35   RWinert 2:   20.000 % Variation
% 36   RW_H0 2: 0.2094E-01 Addit. Variat.
% 37   RW_H0 2: 0.2094E-01 Addit. Variat.
% 38   HDirect 2:   14.003 % Variation
% 39   RWinert 3:   20.000 % Variation
% 40   RW_H0 3: 0.2094E-01 Addit. Variat.
% 41   RW_H0 3: 0.2094E-01 Addit. Variat.
% 42   HDirect 3:   14.003 % Variation
% 43   W_flex 1: 0.3000   Addit. Variat.
% 44   W_flex 2: 0.4000   Addit. Variat.
% 45   W_flex 3: 0.6000   Addit. Variat.
% 46   W_flex 4: 2.000   Addit. Variat.
% 47   W_flex 5: 2.000   Addit. Variat.
% 48   W_flex 6: 10.00   Addit. Variat.
% 49   W_flex 7: 10.00   Addit. Variat.
% 50   W_flex 8: 10.00   Addit. Variat.

Figure (4) Spacecraft dynamics subsystem uses the uncertain state-space model "*Satellite_60flx.m*" which provides the input/ output pairs that couple with the variations block Δ

Figure (5) shows the μ-analysis results of the closed-loop system between the uncertainty inputs and the uncertainty outputs. The results show comparison between the two methods used to calculate the SSV, (a) using complex parameter variations, which is easier to calculate but too conservative, and (b) using real parameter variations which is slower to compute, but not as conservative as the complex variations. The complex variations being conservative violate our robustness requirement, but the (μ) with real variations satisfy the requirement of being less than one at all frequencies, with plenty of margin. It means that the system maintains stability in presence of all possible parameter variations as long as the variations lie between -1 and +1. Remember, that the individual max parameter variations were used during system creation to scale the uncertainty inputs and outputs, and the uncertainty block is, therefore, normalized assuming that its individual elements vary between -1 and +1.
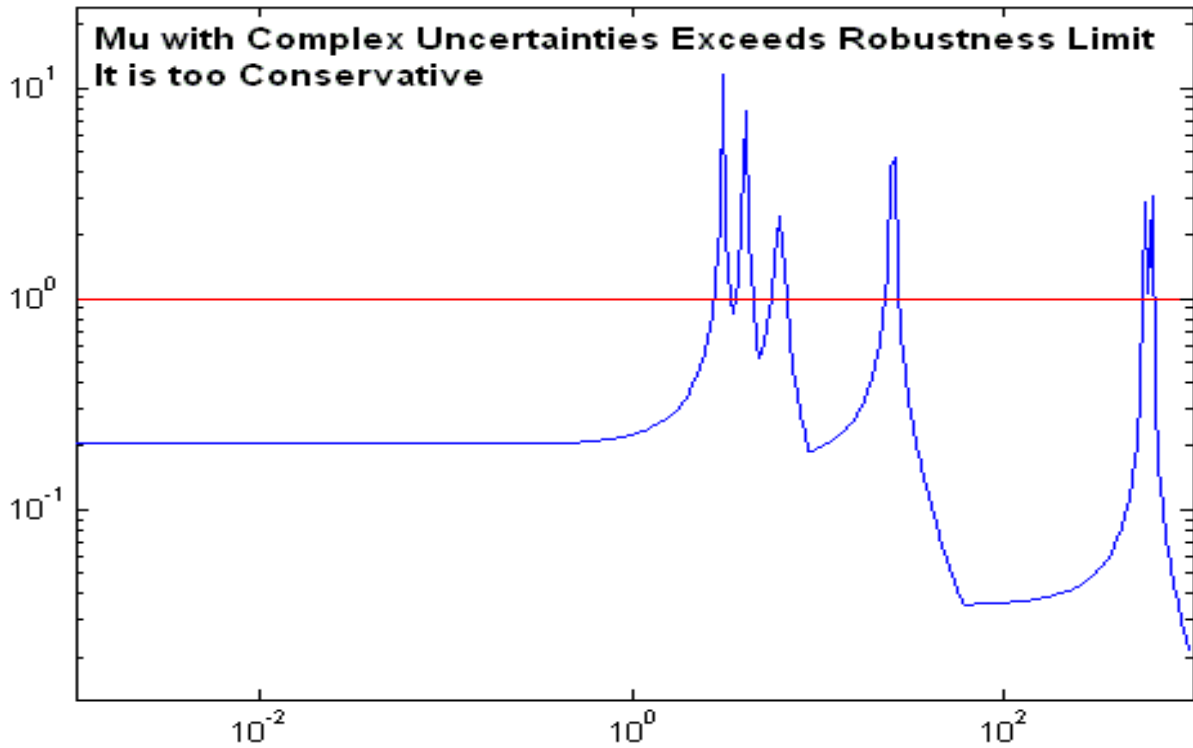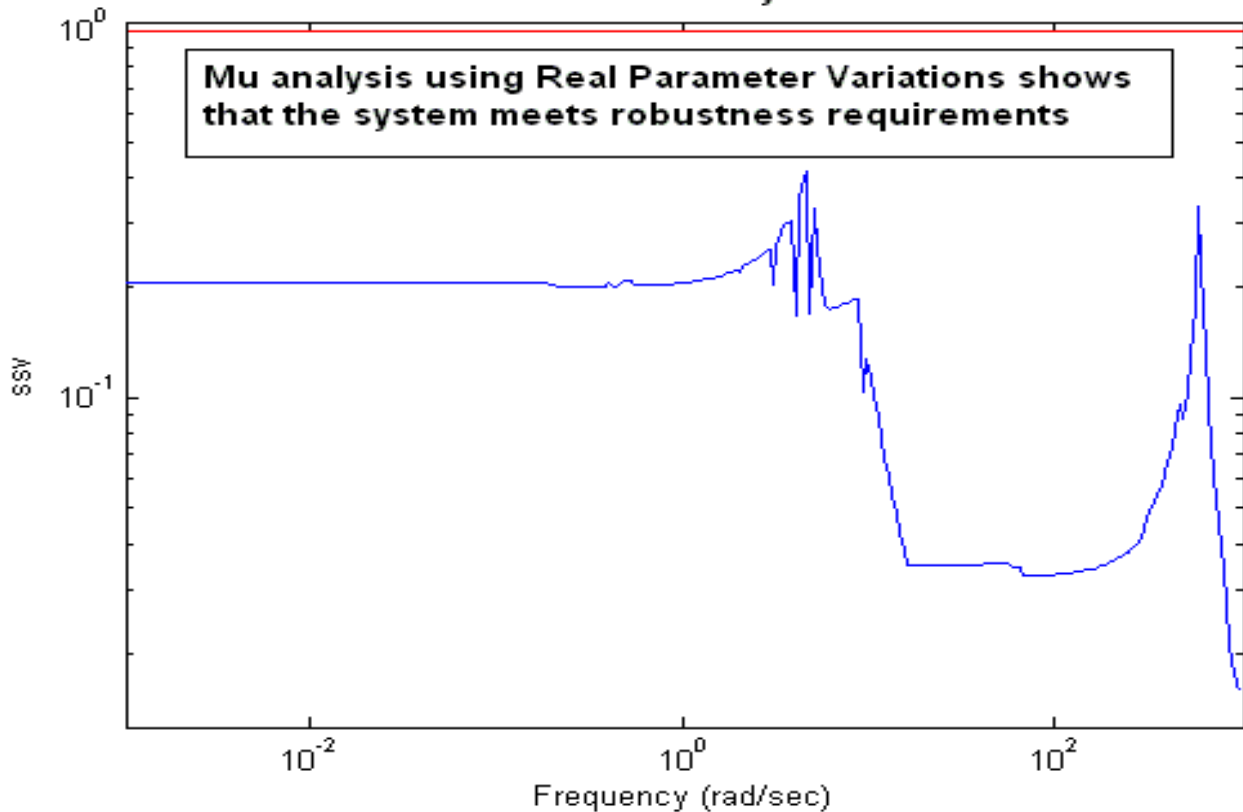
Figure (5) SSV of the Closed-Loop system as seen across the perturbation block Δ. Analysis using complex variations is too conservative and violates robustness requirement, but real variations meets robustness requirement

# Robustness Analysis of a Flexible Satellite with Single Gimbal Control Moment Gyros

In this example we have a flexible spacecraft that is controlled by a cluster of four Single-Gimbal CMGs. The dynamics of a spacecraft with CMGs is highly non-linear and it is a function of the CMG gimbal angles. In this analysis, however, we will use a linearized model generated by the Flixan flight vehicle modeling program and also use linear control laws to analyze stability and robustness at a fixed gimbal angle positions. The analysis is performed at the initialization gimbal positions where the total system momentum is zero. The four SGCMGs have the same momentum 1200 (ft-lb-sec) and they are mounted in a four-sided pyramid configuration, as shown in figure (2). The pyramid angle (β) is 68°. The four gimbaling directions are perpendicular to the surfaces of the pyramid and the initial gimbal angles are at zero where the four momentum vectors (h_i) are parallel to the base of the pyramid, producing a total CMG momentum equal to zero. The positions in Figure (2) is the CMG initialization state and we will analyze the system stability and robustness in this position to parameter uncertainties by creating dynamic models in this orientation using the Flight Vehicle Modeling program (FVMP) and Matlab. The gimbal angles, gimbal directions, and momentum reference directions are inputs to the program. It is also possible to create linear models in any other CMG orientations and to perform similar analysis.

Figure (1) shows a cluster of four SG-CMG mounted on a rigid structure which is isolated from the spacecraft structure by means of vibration isolation struts. Notice that, the pyramid mounting structure of Figure (2) is a standard model used for visualization purposes. The actual mounting of the CMGs are not exactly as shown on the pyramid but they can be translated anywhere on the spacecraft, as long as, the gimbal direction vectors and the momentum reference directions are parallel to those in the pyramid model. The momentum reference directions for each CMG are defined to be the momentum directions when the gimbal angles are zero.
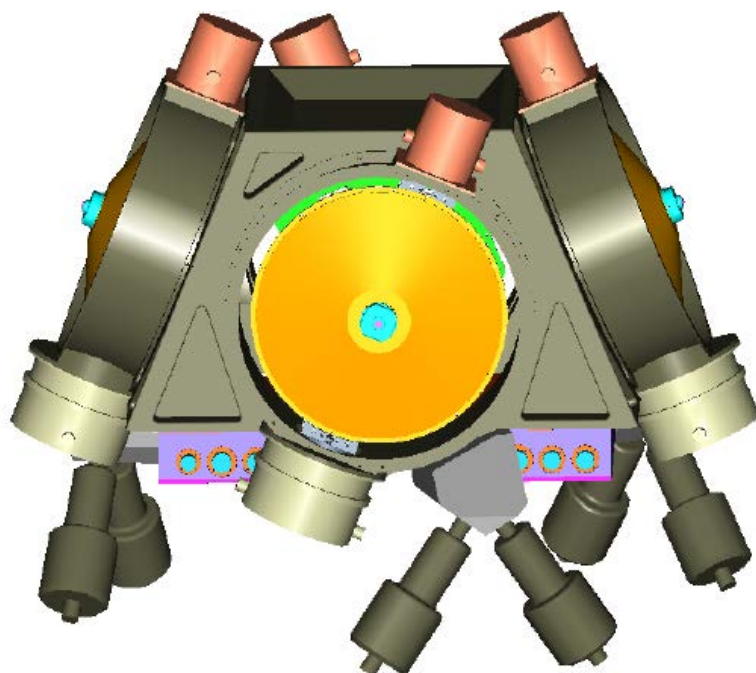
The FVMP creates two spacecraft models, a nominal model, and a similar model that has 61 additional inputs and outputs for the 61 parameter uncertainties. Dynamically both models are the same, but the second model has 61 internal parameter uncertainties "pulled out" as a Δ block by the fictitious inputs and outputs. We will use the nominal model to analyze stability and to prove that the nominal system is stable and then we will use the model with the uncertainties to analyze robustness. We must close the control loop (without commands) and, as long as the system is stable, perform μ-analysis across the uncertainty inputs and outputs to check if there is any combination of uncertainties that will drive it unstable.

The analysis is performed in folder "*C:\Flixan\Examples\Robust Analysis Param Uncertainties\ Satellite_SGCMG*". The spacecraft parameters are in the input file "*FlexSc_4CMG.Inp*". The file includes also a set of modal data consisting of 40 selected modes. The mode selection process is not shown here because it is fully described in another example. The input file also includes the parameter uncertainties in a separate set of data, title: "*Uncertainties for Flexible Agile Spacecraft with 4 SG-CMG*". The parameter uncertainties are additive variations to the nominal spacecraft parameters. Notice that, not all parameters should be varied and the user must use caution in selecting which parameters to vary, because perturbing some parameters does not create a plant variation, and this causes an error in the program. In this example we vary the spacecraft moments and products of inertia, we vary the frequencies in 8 flex modes, that is, the strongest 8 modes. We also add uncertainty in the CMG momentum ± 50 (ft-lb-sec), the momentum direction, the initial gimbal angle (δ_0), the CMG moment of inertia about the gimbal axis (J_g), and the pyramid surface orientation angles (β, γ). We did not include variations in the gimbal directions and the CMG inertias (J_s, J_o) because they do not create plant variations, that is, only in this example.

σ is the CMG gimbal axis direction
h is the initial CMG momentum direction
h-dot is the initial CMG torque direction
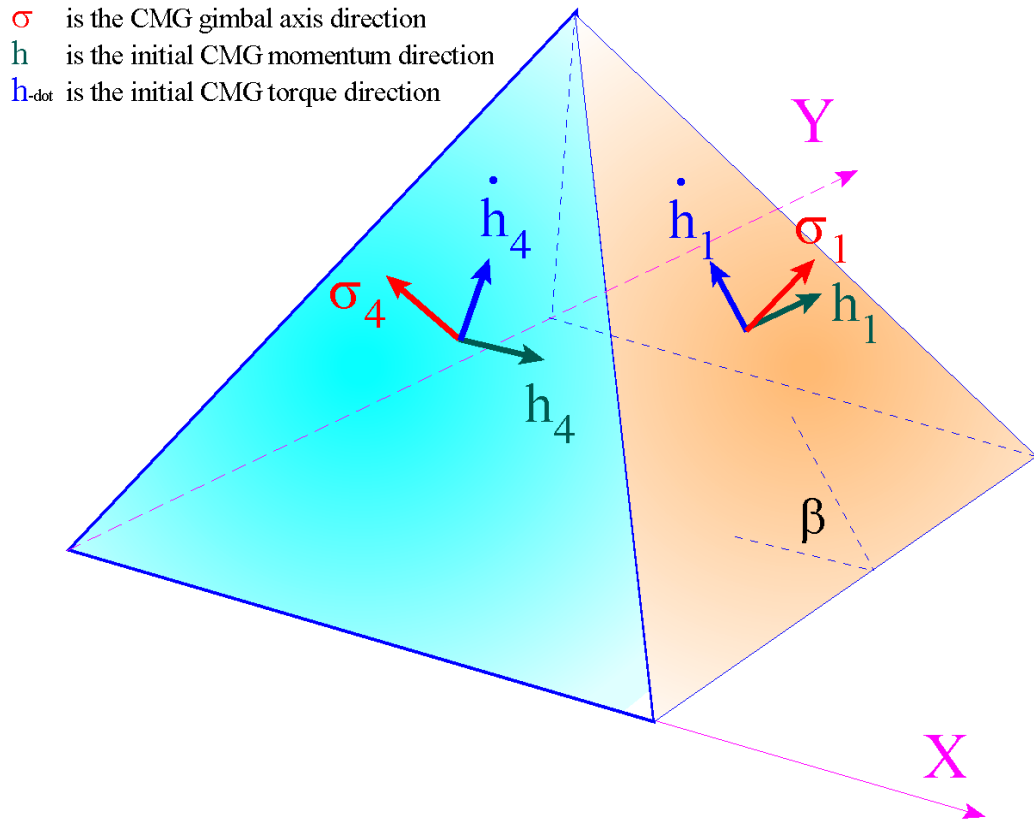
Figure 2 Array of four CMGs in a Pyramid Configuration

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Spacecraft Model Creation

We will now process the input file "FlexSc-4CMG.Inp" that already contains the spacecraft data, the uncertainties, and the modal data.

```
FLIGHT VEHICLE INPUT DATA ......
Flexible Agile Spacecraft with 4 SG-CMG (Uncertainties)
! This is a Flex Spacecraft model that includes four 1200 (ft-lb-sec) SGCMGs
! Uncertainties in the mass properties and in the CMGs are also included
Body Axes Output, Attitude=Euler Angles

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)   :   202.05  32.17 0.20896E+08
Moments and products of Inertias Ixx, Iyy, Izz, -Ixy,-Ixz,-Iyz, in (lb-sec^2-ft)        :   1.41E+4  1.3E+4 0.1759E+4
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :   0.0  -0.20975  0.62
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :   0.0 25500.0 0.0 700000.
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)         :   0.0  0.0  0.0  0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                         :   0.0  0.0  0.0  0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   :   0.0  0.0  0.0  0.0
External Force direction unit vect: (x,y,z), Force application point: (x,y,z)   (feet)   :   Force  1.0 0.0 0.0  -12.1
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)        :   1.0    1.0       1.0
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :   0.0  -0.209  0.6201   0.0
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  0.0  0.0  0.0  0.0   0.0
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:  0.0  0.0  0.0  0.0   0.0
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.0  0.0  0.0  0.0   0.0
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:  0.0  0.0  0.0  0.0   0.0
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:  0.0  0.0  0.0  0.0   0.0

Number of Single Gimbal Control Moment Gyros (SG CMG) ?                                   :   4
CMG: 1 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg)     :   1200.0     0.0
       Gimbal Direction Vector (m)                                                        :   0.927183   0.0      0.3746
       Momentum Reference Direction Vector (r)                                            :   0.0        1.0      0.0
       SGCMG Surface Orientation angles in Pyramid (beta & gamma), (Figure 2.2)           :   68.0       90.0
       SGCMG Moments of Inertia (Js, Jg, Jo) about: spin, gimbal and output axes          :   1.2        0.6      0.8
CMG: 2 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg)     :   1200.0     0.0
       Gimbal Direction Vector (m)                                                        :   0.0        0.927 0.3746
       Momentum Reference Direction Vector (r)                                            :   -1.0       0.0      0.0
       SGCMG Surface Orientation angles in Pyramid (beta & gamma), (Figure 2.2)           :   68.0       180.0
       SGCMG Moments of Inertia (Js, Jg, Jo) about: spin, gimbal and output axes          :   1.2        0.6      0.8
CMG: 3 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg)     :   1200.0     0.0
       Gimbal Direction Vector (m)                                                        :   -0.927183  0.0  0.3746
       Momentum Reference Direction Vector (r)                                            :   0.0        -1.0     0.0
       SGCMG Surface Orientation angles in Pyramid (beta & gamma), (Figure 2.2)           :   68.0       270.0
       SGCMG Moments of Inertia (Js, Jg, Jo) about: spin, gimbal and output axes          :   1.2        0.6      0.8
CMG: 4 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg)     :   1200.0     0.0
       Gimbal Direction Vector (m)                                                        :   0.0        -0.9271 0.3746
       Momentum Reference Direction Vector (r)                                            :   1.0        0.0      0.0
       SGCMG Surface Orientation angles in Pyramid (beta & gamma), (Figure 2.2)           :   68.0       0.0
       SGCMG Moments of Inertia (Js, Jg, Jo) about: spin, gimbal and output axes          :   1.2        0.6      0.8

Number of Gyros, (Attitude and Rate)                                                     :   9
Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2    :   Roll   Attit 6.53 0.0 -1.1
Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2    :   Pitch  Attit 6.53 0.0 -1.1
Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2    :   Yaw    Attit 6.53 0.0 -1.1
Gyro No  4 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2    :   Roll   Rate  6.53 0.0 -1.1
Gyro No  5 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2    :   Pitch  Rate  6.53 0.0 -1.1
Gyro No  6 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2    :   Yaw    Rate  6.53 0.0 -1.1
Gyro No  7 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 1    :   Roll   Rate -2.93 0.2 -0.2
Gyro No  8 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 1    :   Pitch  Rate -2.93 0.2 -0.2
Gyro No  9 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 1    :   Yaw    Rate -2.93 0.2 -0.2

Number of Accelerometers, Along Axes: (x,y,z)                                            :   6
Acceleromet No  1 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2   :   X-axis Accel  6.5 0.0 -1.1
Acceleromet No  2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2   :   Y-axis Accel  6.5 0.0 -1.1
Acceleromet No  2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2   :   Z-axis Accel  6.5 0.0 -1.1
Acceleromet No  4 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4   :   X-axis Accel  5.2 0.0 -2.0
Acceleromet No  5 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4   :   Y-axis Accel  5.2 0.0 -2.0
Acceleromet No  6 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4   :   Z-axis Accel  5.2 0.0 -2.0

Parameter Uncertainties Data
Uncertainties for Flexible Agile Spacecraft with 4 SG-CMG


Number of Bending Modes                                                                  : 40
Flexible Agile Spacecraft with 4 SG-CMG, All Flex Modes
-----------------------------------------------------------------------------------------------------------
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Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)  :  0.0    0.0    0.0
Moments and products of Inertias Ixx, Iyy, Izz, -Ixy,-Ixz,-Iyz, in (lb-sec^2-ft)       : 160.0  150.0  25.0, 0.4,
0.2, 1.8
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)       :  0.0    0.0    0.0
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)      :  0.0    0.0    0.0    0.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)        :  0.0    0.0    0.0    0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                        :  0.0    0.0    0.0    0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)  :  0.0    0.0    0.0    0.0
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}: 0.0,  0.0,  0.0,   0.0,
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}: 0.0,  0.0,  0.0,   0.0,
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}: 0.0,  0.0,  0.0,   0.0,
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}: 0.0,  0.0,  0.0,   0.0,
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}: 0.0,  0.0,  0.0,   0.0,


Number of Single Gimbal Control Moment Gyros (SG CMG) Variations ?                      :  4
CMG: 1 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg)    :  50.0   1.0
       Gimbal Direction Vector (m) variation                                            :  0.0    0.0    0.0
       Momentum Reference Direction Vector (r) variation                                :  0.1    0.0    0.1
       SGCMG Surface Orientation angle in Pyramid (beta & gamma) variations,(Figure 2.2) :  1.5    1.5
       SGCMG Moments of Inertia (Js, Jg, Jo) variat. about: spin, gimbal and output axes :  0.0    0.06   0.0
CMG: 2 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg)    :  50.0   1.0
       Gimbal Direction Vector (m) variation                                            :  0.0    0.0    0.0
       Momentum Reference Direction Vector (r) variation                                :  0.0    0.1    0.1
       SGCMG Surface Orientation angle in Pyramid (beta & gamma) variations,(Figure 2.2) :  1.5    1.5
       SGCMG Moments of Inertia (Js, Jg, Jo) variat. about: spin, gimbal and output axes :  0.0    0.06   0.0
CMG: 3 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg)    :  50.0   1.0
       Gimbal Direction Vector (m) variation                                            :  0.0    0.0    0.0
       Momentum Reference Direction Vector (r) variation                                :  0.1    0.0    0.1
       SGCMG Surface Orientation angle in Pyramid (beta & gamma) variations,(Figure 2.2) :  1.5    1.5
       SGCMG Moments of Inertia (Js, Jg, Jo) variat. about: spin, gimbal and output axes :  0.0    0.06   0.0
CMG: 4 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg)    :  50.0   1.0
       Gimbal Direction Vector (m) variation                                            :  0.0    0.0    0.0
       Momentum Reference Direction Vector (r) variation                                :  0.0    0.1    0.1
       SGCMG Surface Orientation angle in Pyramid (beta & gamma) variations,(Figure 2.2) :  1.5    1.5
       SGCMG Moments of Inertia (Js, Jg, Jo) variat. about: spin, gimbal and output axes :  0.0    0.06   0.0

Flex Mode Uncertainties (Mode Number)                                                   :  1   4   6  24  26  33  34
Flex Mode Frequency Variation (additive)                                                : 0.4 0.7 1.2 3.0 3.2 3.4 4.0
-------------------------------------------------------------------------------------------------------------------
```

```
MODE#  1/  7, Frequency (rad/sec), Damping (zeta), Generalized Mass=    3.1905        0.30000E-02    12.000
DEFINITION OF LOCATIONS (NODES)           phi along X   phi along Y   phi along Z   sigm about X  sigm about Y  sigm

                          Node ID#     Modal Data at the  4 Single Gimbal GMGs...
Control Moment Gyros (CMG) Loc     6     0.10717D-02  -0.16639D-01  -0.31137D-01  -0.42835D-01  -0.29132D-02  -
Control Moment Gyros (CMG) Loc     6     0.10717D-02  -0.16639D-01  -0.31137D-01  -0.42835D-01  -0.29132D-02  -
Control Moment Gyros (CMG) Loc     6     0.10717D-02  -0.16639D-01  -0.31137D-01  -0.42835D-01  -0.29132D-02  -
Control Moment Gyros (CMG) Loc     6     0.10717D-02  -0.16639D-01  -0.31137D-01  -0.42835D-01  -0.29132D-02  -
                          Node ID#     Modal Data at the  9 Gyros ...
Attitude, Rate, Accelerom Sens     2     0.45900D-02  -0.76587D-01   0.46744D-01  -0.42835D-01  -0.29132D-02  -
Attitude, Rate, Accelerom Sens     2     0.45900D-02  -0.76587D-01   0.46744D-01  -0.42835D-01  -0.29132D-02  -
Attitude, Rate, Accelerom Sens     2     0.45900D-02  -0.76587D-01   0.46744D-01  -0.42835D-01  -0.29132D-02  -
Attitude, Rate, Accelerom Sens     2     0.45900D-02  -0.76587D-01   0.46744D-01  -0.42835D-01  -0.29132D-02  -
Attitude, Rate, Accelerom Sens     2     0.45900D-02  -0.76587D-01   0.46744D-01  -0.42835D-01  -0.29132D-02  -
Attitude, Rate, Accelerom Sens     2     0.45900D-02  -0.76587D-01   0.46744D-01  -0.42835D-01  -0.29132D-02  -
Pointing Antena                    1     0.19735D-02  -0.34853D-01   0.29482D-01  -0.42912D-01  -0.26934D-02  -
Pointing Antena                    1     0.19735D-02  -0.34853D-01   0.29482D-01  -0.42912D-01  -0.26934D-02  -
Pointing Antena                    1     0.19735D-02  -0.34853D-01   0.29482D-01  -0.42912D-01  -0.26934D-02  -
```

Simulation Model

Now, let us take a look at the linear simulation model in file "*Lin_Flex_Sim.Mdl*", shown in Figure (2), that uses the nominal spacecraft system from file "*sc_4cmg_flex.m*", without the uncertainties. This simulation model is initialized using file "start.m", which also loads the two spacecraft systems into the Matlab workspace.
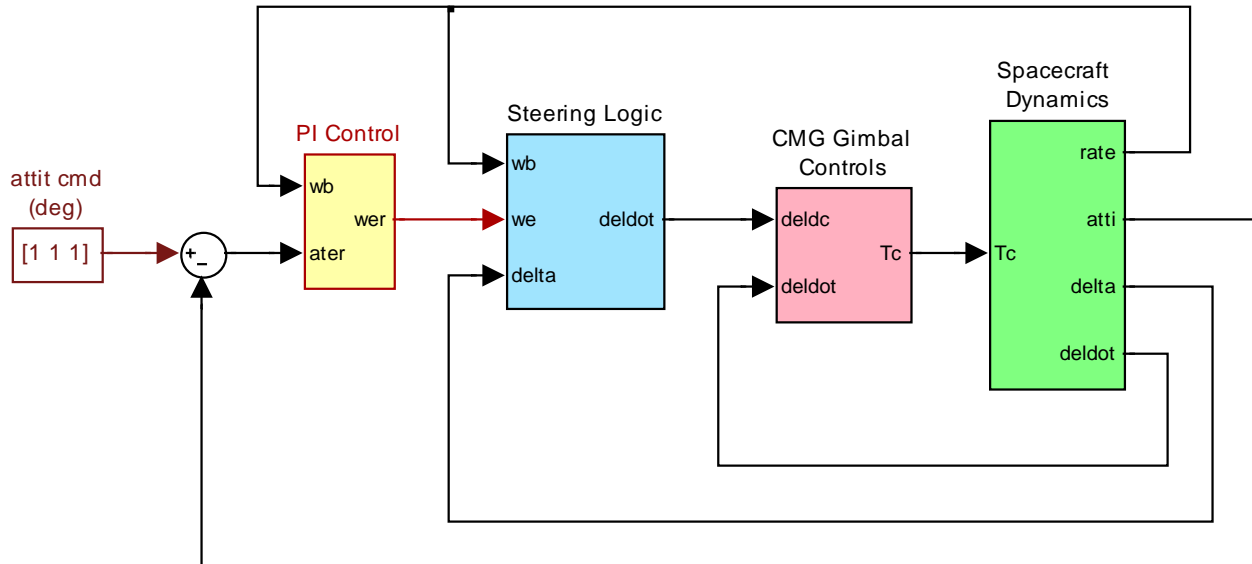
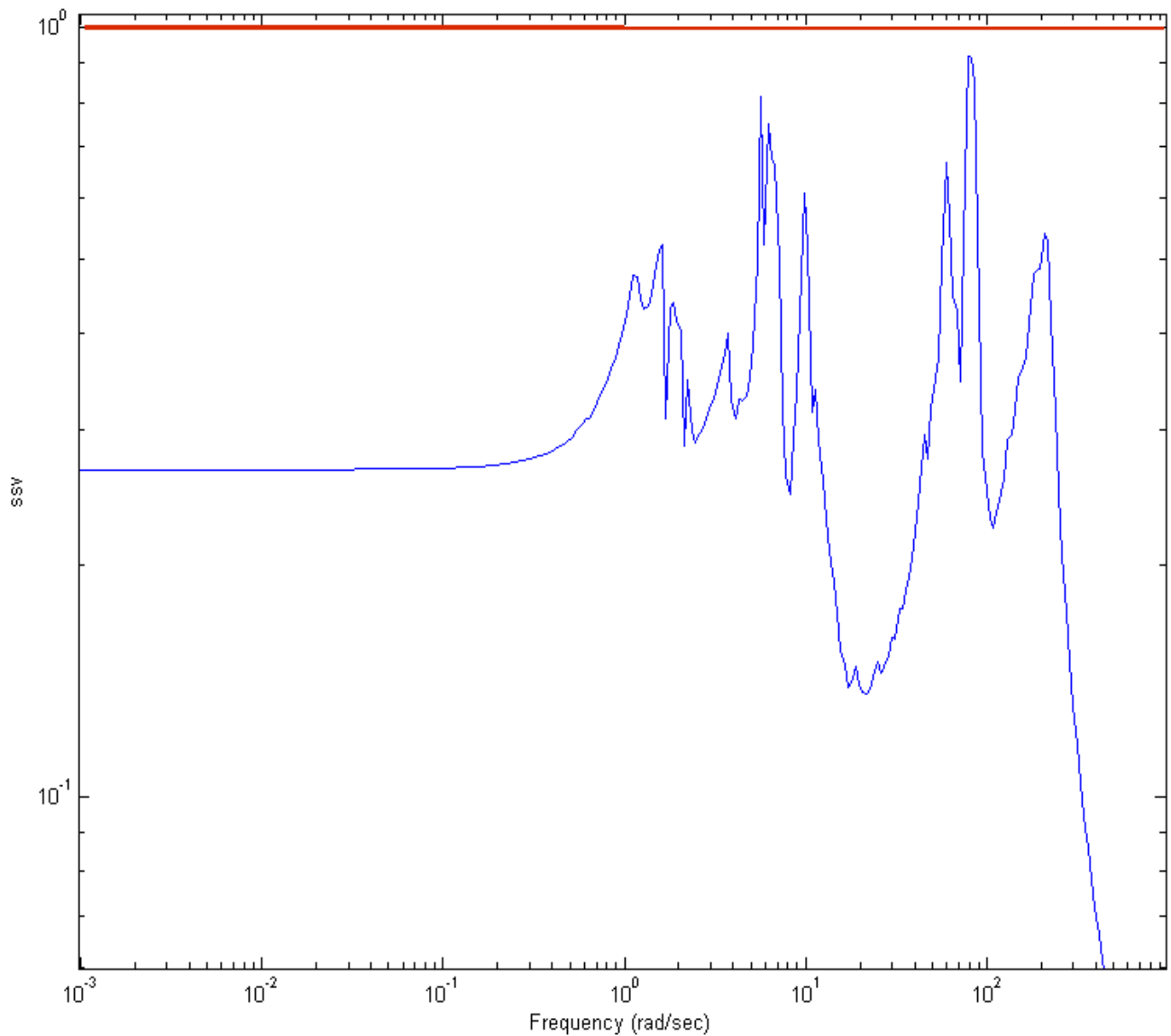**Figure (2) Simulink model for the Flex Agile Spacecraft with 4 SG-CMGs, in file "*Lin_Flex_Sim.mdl*"**

The spacecraft dynamics (green) block consists of the state-space system "*sc_4cmg-flex.m*" (without uncertainties). The input (Tc) is a vector of four CMG gimbal torques which control the gimbal rates. The outputs are: spacecraft attitude, rates, CMG gimbal angles, and gimbal rates. The gimbal rate commands come from the CMG steering logic and the gimbal rate control system provides the gimbal torques required to control the rates. The purpose of the steering logic is to control the spacecraft rate by creating gimbal rate commands at the 4 CMG gimbals. The inputs to the steering logic are: spacecraft rates, gimbal angles, and spacecraft rate error. The attitude control system is a simple PI. The (D) part of the PID is included in the steering. In the simulation the spacecraft is commanded to perform a one degree rotation in all 3 directions. The purpose of the simulation is to demonstrate nominal system stability.
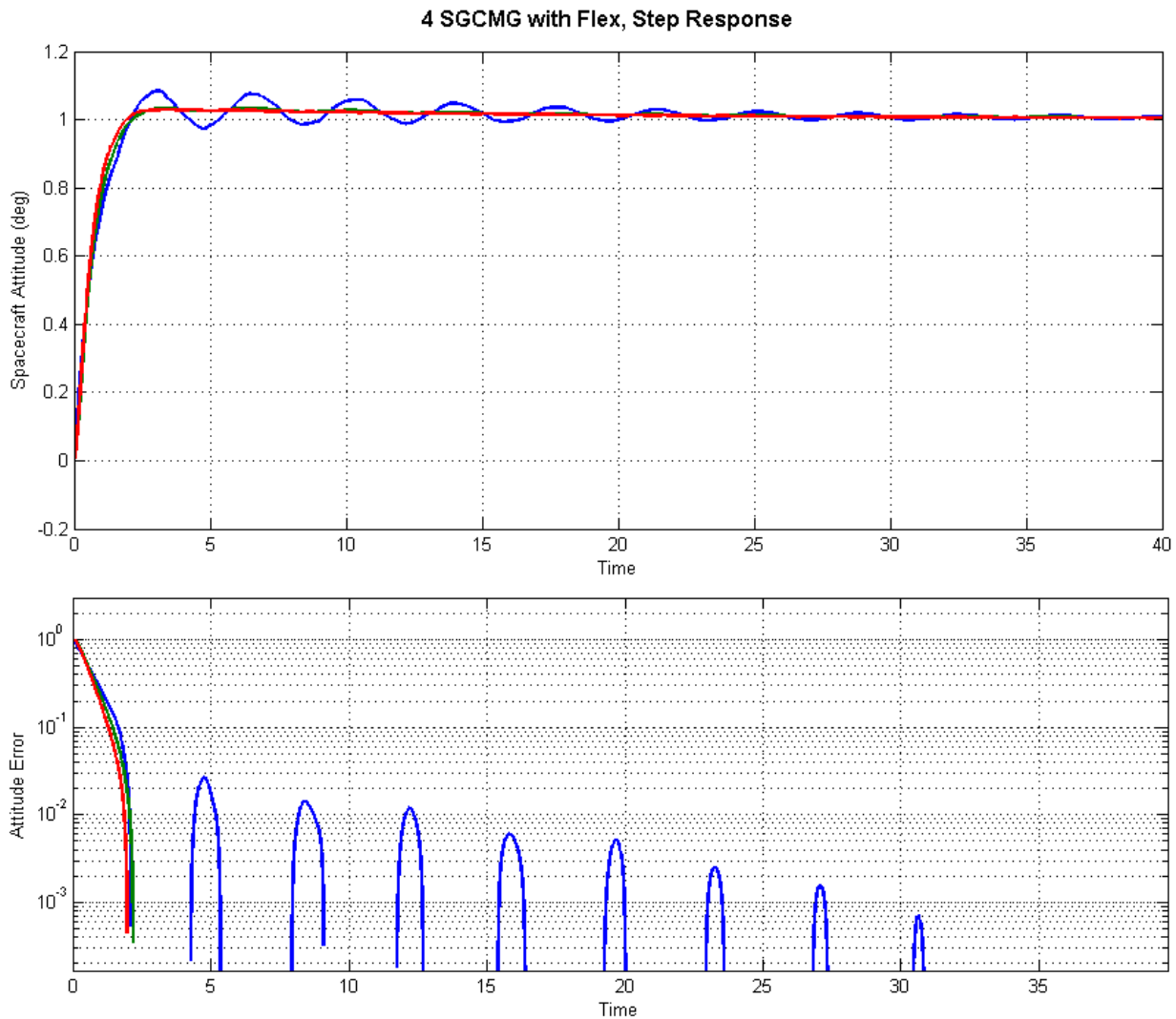
Figure (3a) Spacecraft attitude response to one degree command in all 3 directions

Figure (3a) shows a stable attitude response to 1° command in all 3 directions. The roll axis (blue) takes longer to settle because flexibility is stronger in roll. Figure (3b) shows the spacecraft rate and acceleration at two separate locations with different flex mode sensitivity. Figure (3c) shows the gimbal angles and gimbal rates. It also shows the CMG momentum. The roll momentum oscillates as the CMGs respond to the roll structural oscillations.

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Figure (8) Structured Singular Value frequency response across the perturbations block Δ is less than one at all frequencies. The system, therefore, is robust to the parameter variations.