

# Hypersonic Rocket Plane



This example analyzes a rocket-plane that can either take off horizontally from the ground or it can be dropped from another aircraft. When it takes off from the ground it climbs up to an altitude of 76,000 feet. Then it turns off its engine and glides back to the ground and lands like an airplane using only aero-surfaces for control. The vehicle is powered by a 63,000 (lb) rocket engine during ascent and uses four aero-surfaces during both ascent and entry: an elevon, a body-flap, an aileron, and a rudder. In this example we will use the “Trim” program to calculate the aero-surface trim angles and evaluate the vehicle performance during the entire flight, from ground take off to landing. The analysis is separated into two parts, the boost phase and the descent to the ground phase.

## 1. Ascent Phase

During ascent the rocket plane uses a variable thrust booster engine to regulate its axial acceleration. The engine thrust is along the vehicle x axis and the engine does not gimbal. The aircraft is controlled by the control surfaces. We are going to use the “Trim” program to perform the following analysis: (a) calculate the trim angles of the control surfaces and the engine thrust as a function of time, (b) evaluate the vehicle performance during the entire boost phase, (c) investigate the effects of XCG variations on the trim angles and performance, (d) analyze stability by means of contour plots, (e) analyze maneuverability against disturbances by using vector diagrams, (f) take advantage of multiple effectors which are available in pitch to perform trimming modifications, and (g) analyze the lateral effects of a beta disturbance and of a YCG shift.

## 1.1 Flight Vehicle Data Files

The data for the hypersonic vehicle example during ascent is in folder "*C:\Flixan\Trim\Examples\Hypersonic Vehicle\Ascent*". This folder contains data files which are inputs to "Trim" and files which are generated by the "Trim" program. We will first describe the input data files. There are two ascent trajectory files in this directory, a high altitude release "*Drop\_Boost.Traj*", and a ground take-off trajectory file "*Hyp\_Ascent.Traj*". In this example we are going to focus on the ground take-off trajectory. The flight lasts for 89 seconds. The initial weight at ground take-off is 30,000 (lb) and its final weight at 76,000 (ft) altitude is 14,000 (lb). The final Mach number at engine cut-off is 4.5. The angle of attack in the beginning of flight is positive and it becomes negative at around Mach 1.

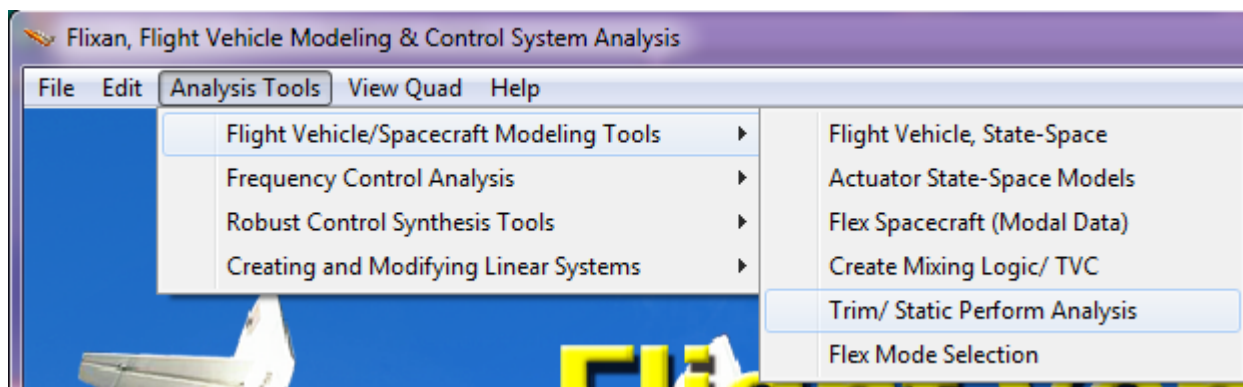
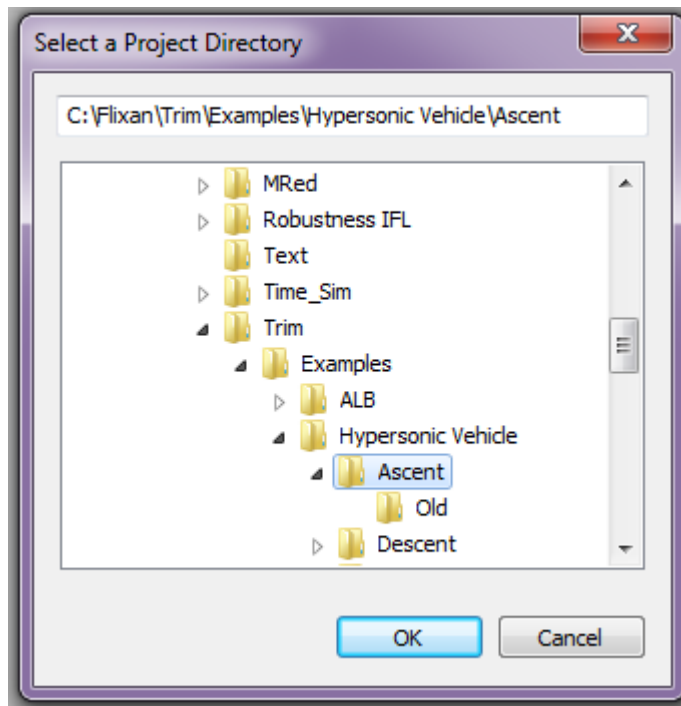
The basic aero-dynamic coefficients during ascent are in file "*Hyp\_Ascent.Aero*". The data is a 3-dimensional array consisting of 12 Mach numbers ranging from Mach: 0.3 to Mach: 10, three angles of sideslip (-5°, 0°, +5°), and 16 angles of attack ranging between -6° and +30°. The aero-surface coefficients are in file "*Hyp-Ascent.Delt*". They represent increments contributed by the four aero-surface deflections in addition to the basic moments and forces. The aero-surfaces file consists of four sets of data for all four aero-surfaces. The aero-surface coefficients are 4-dimensional arrays. The first 3 array elements correspond to the same Mach, alpha and beta as in the basic aero coefficients array. The fourth element in the array corresponds to surface deflections which range between -20° to +20°. The surface increment angles are not the same in the aero-surfaces. The file "*Hyper.HMco*" contains the hinge moment coefficients data for all four aero-surfaces. It consists of a 4-dimensional array of coefficients (as a function of Mach, alpha, beta, and delta) for each surface and it looks very similar to the aero-surface coefficients file. It is used for sizing the control surface actuator torques. The file "*Hyp\_Ascent.Unce*" contains the aerodynamic uncertainties data.

The mass properties are in file "*Hyper.Mass*". The first column contains the vehicle mass in (slugs). The remaining columns contain the corresponding moments and products of inertia, the CG location, and the vehicle length which is constant in this case. The main engine data is in file "*Hyper1.Engn*". It contains the nominal engine thrust, its location and orientation relative to vehicle coordinates, its maximum pitch and yaw deflections which are zero in this case because the engine is not gimbaling, and its maximum throttling capability which in this case is  $\pm 40\%$  relative to its nominal thrust of 62,000 (lb). The file contains additional engine parameters (such as the engine mass, its moment of inertia about the gimbal, and its CG moment arm from the pivot point). These parameters are not used by "Trim" but they are transferred to the Flight Vehicle Modeling Program when the "*State-Space Modeling*" option is selected, and they are used for modeling the dynamic coupling between the engine nozzle and the vehicle (tail-wags-dog and actuator load-torque). This, obviously, does not apply in this case because the engine does not pivot. We have included, however, an alternative engine data file "*Hyper2.Engn*" which contains some engine data modifications as an optional design. When using this file, in addition to its

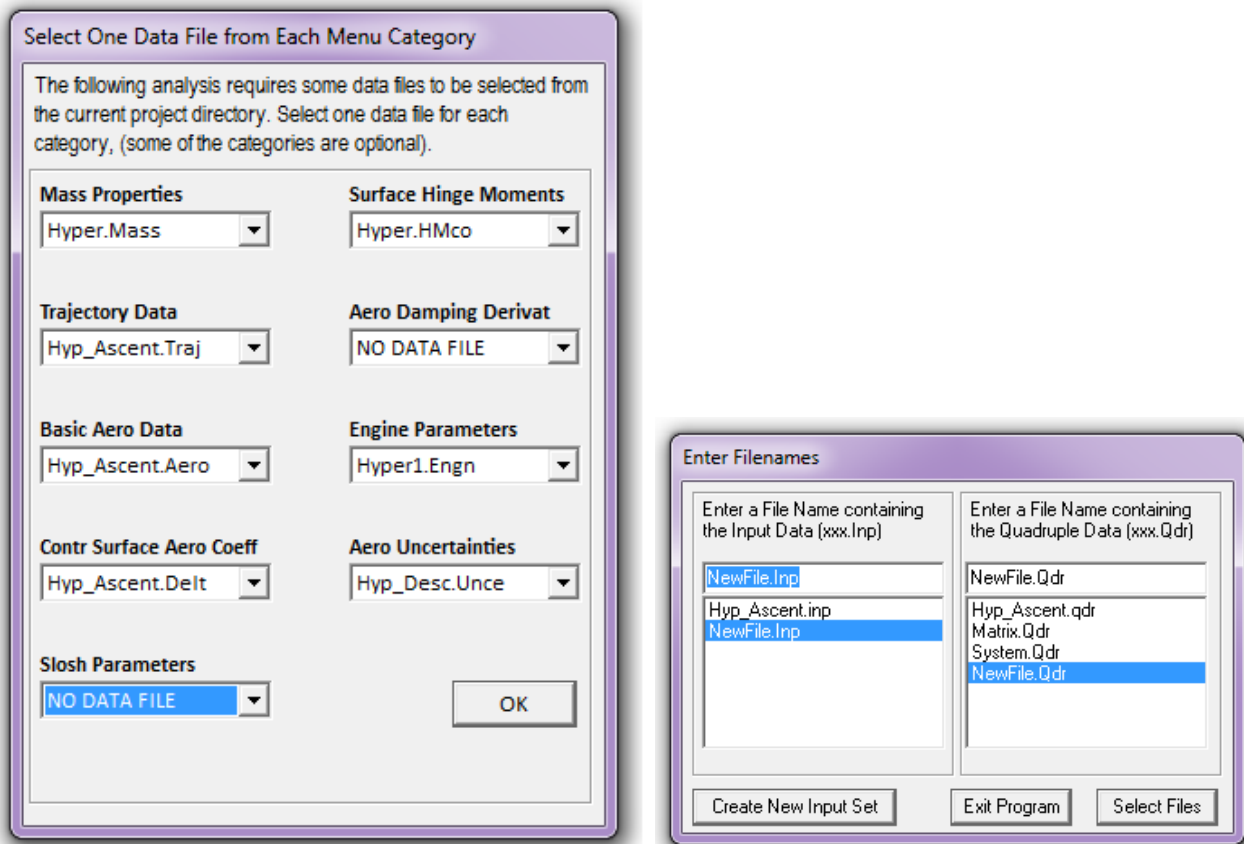
original thrust variation capability, the main engine is also allowed to gimbal  $\pm 5^\circ$  in both pitch and yaw. In addition, the second engine data file includes also two RCS thrusters of  $\pm 2,000$  (lb) max thrust firing in  $\pm z$  and in the  $\pm y$  directions. The second propulsion data file, however, is used mainly for tutorial analysis purposes and it is not the baseline design.

## 1.2 Checking out the Aero Data

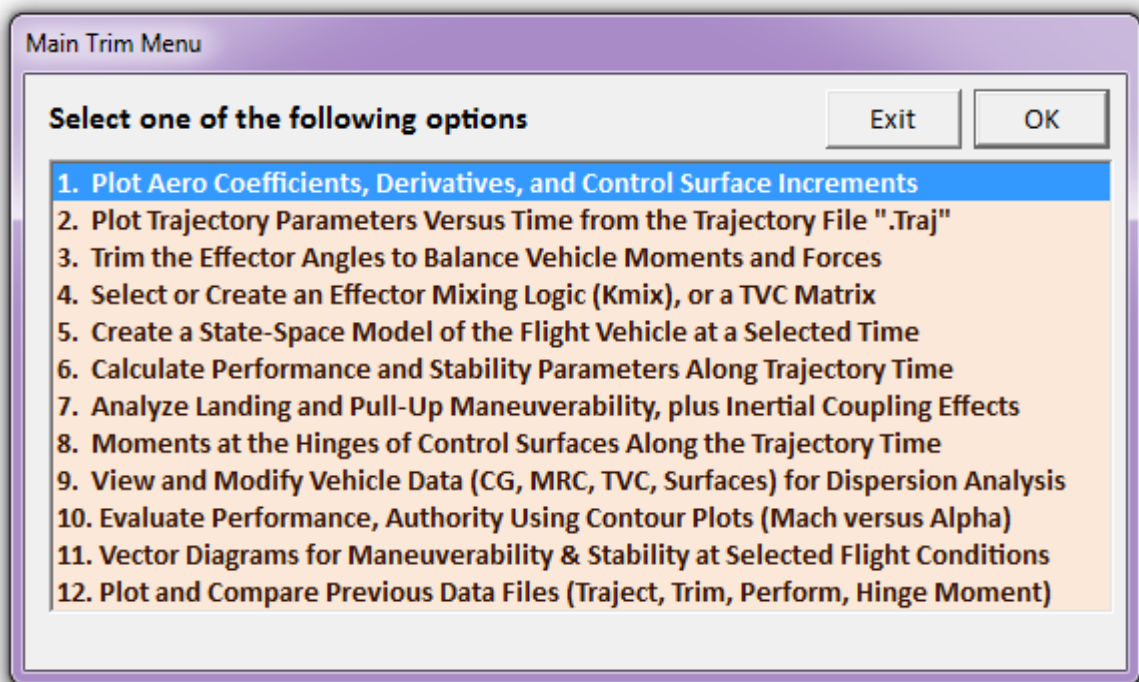
Before beginning the analysis let us first check some of our data files starting with the aero data, using the graphic utilities provided in "Trim". The program includes utilities for plotting the aero coefficients and their derivatives as a function of Mach, alpha, and beta. After starting Flixan you must select the project folder that contains the current analysis files. Then, from the Flixan main menu select "Analysis Tools", "Flight Vehicle/ Spacecraft Modeling Tools", and then "Trim/ Static Performance Analysis", as shown.



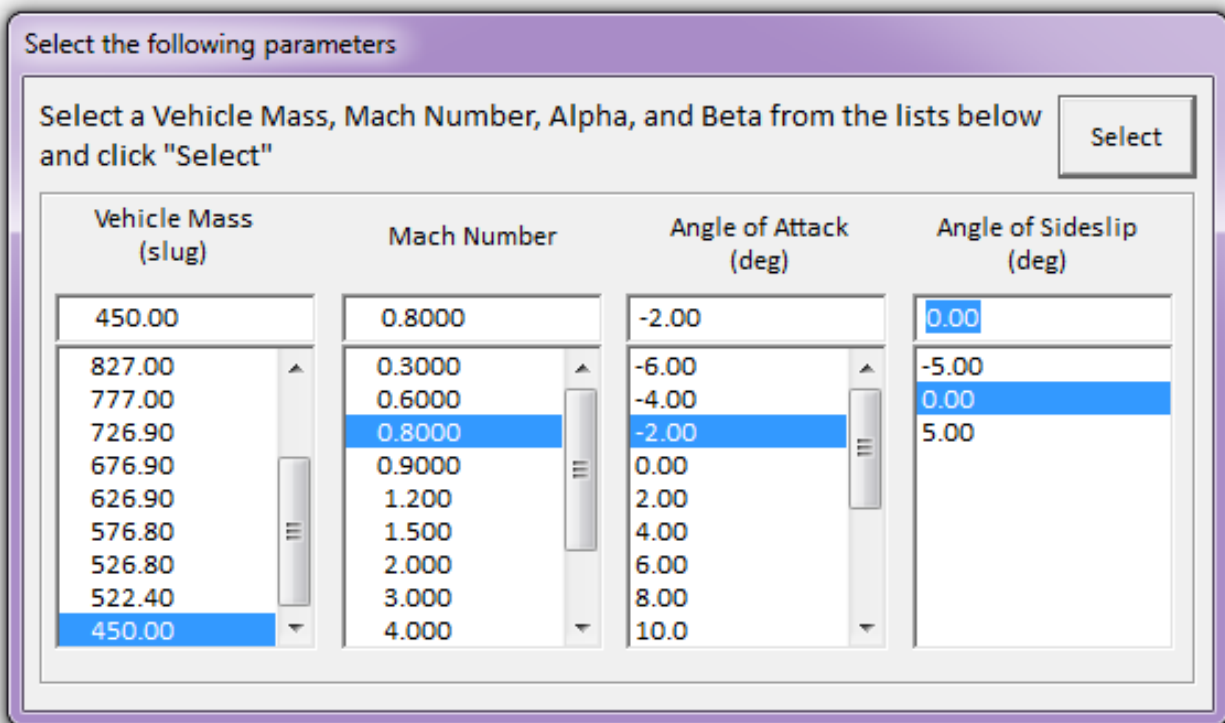
The following dialog is used for selecting the input files that will be used by the Trim program. Some of the files like the slosh parameters, hinge moment coefficients, aero uncertainties, and damping derivatives are optional. If they are missing you will not be able to perform some of the functions, like for example, uncertainties analysis or to calculate the hinge moments. The slosh parameters are not directly used by Trim but they are transferred to the vehicle input data file for generating linear systems using the flight vehicle modeling program (FVMP). In this case select the following files, as shown below. The next filename selection dialog is used for selecting an input data file for the FVMP and a systems file for saving linear systems and matrices. In this case you may select the default file names and click on "Select Files", because we are not going to use any of these types of files yet.



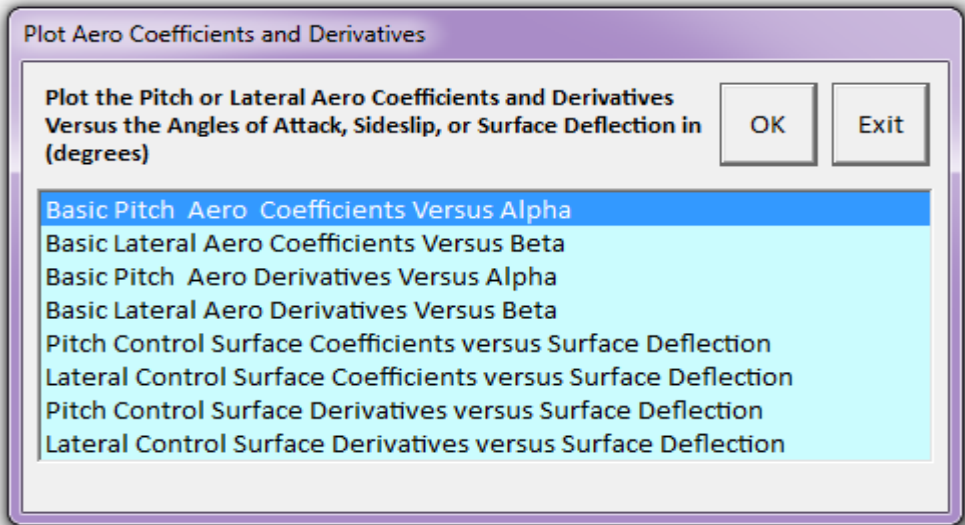
The following is the Trim main menu which selects the functions performed by the Trim program. Select the first option for plotting the aero data, as shown below, and click "OK".



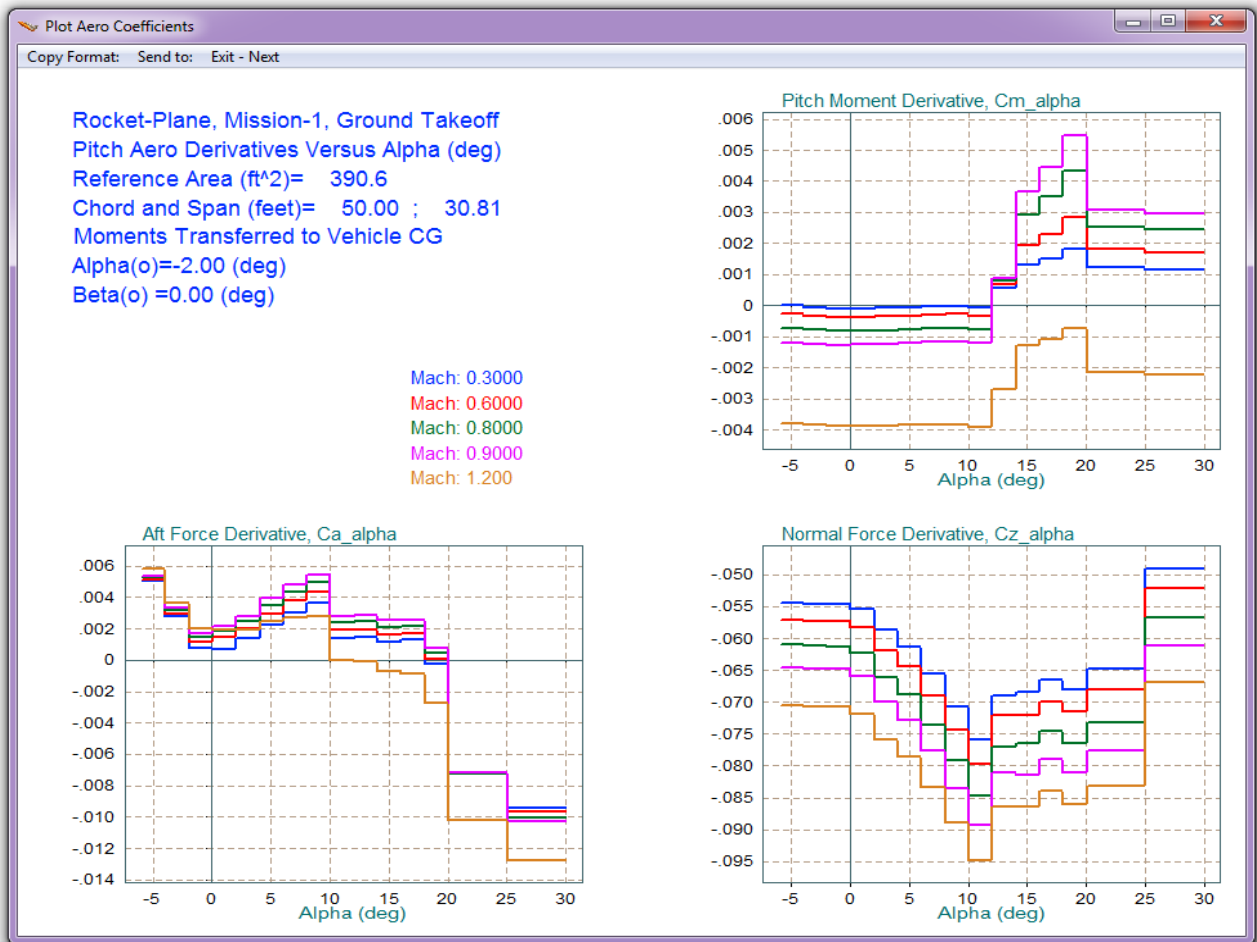
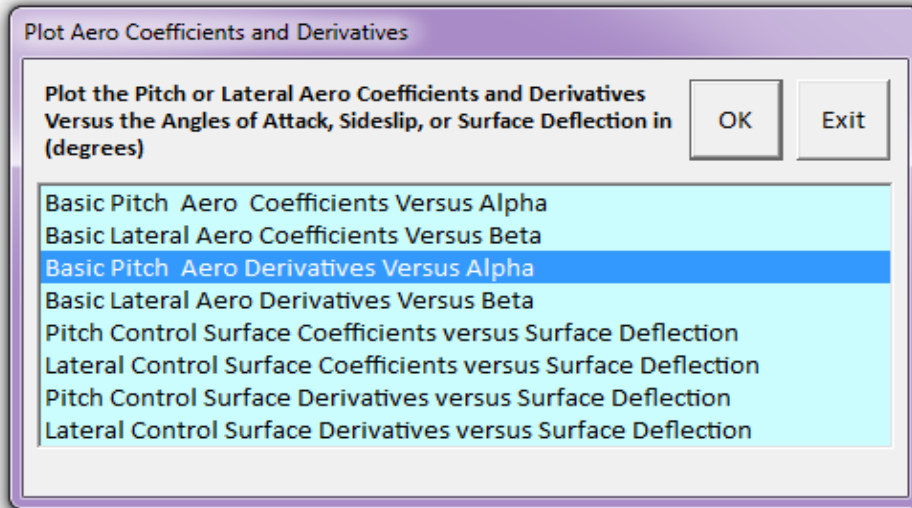
The following dialog contains several menus for choosing a flight condition as a function of Mach number, angles of attack and sideslip, and vehicle mass (slugs). Select a flight condition and click on "Select". The mass is used for transferring the aero moments from the moments reference center (MRC) to the corresponding vehicle CG.



From the following menu you may select the type of aero data that you would like to plot. In the following case we select the basic pitch aero coefficients, and click "OK". In the plots below we see the pitch moment  $C_m$ , the normal force  $C_z$ , and the aft force  $C_A$  aero coefficients, as a function of (Mach and  $\alpha$ ). Five separate Mach curves are shown for comparison.

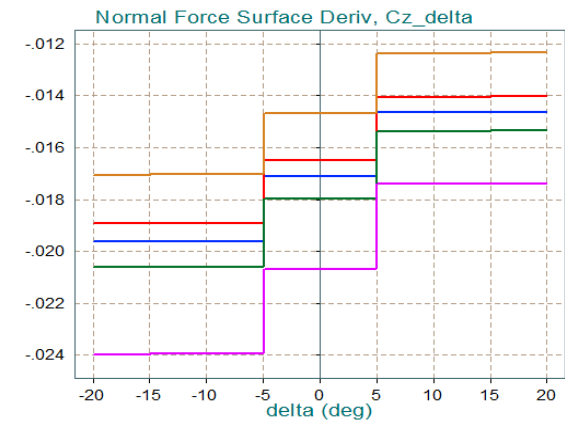
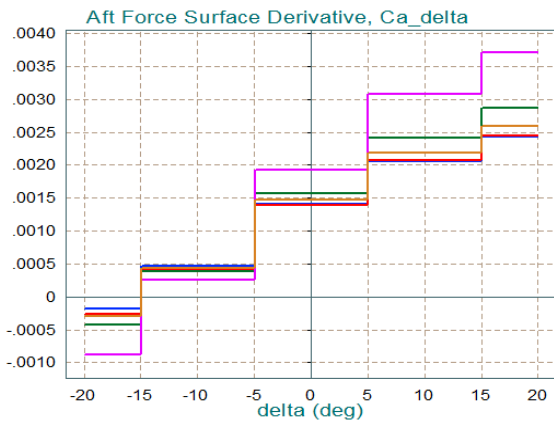
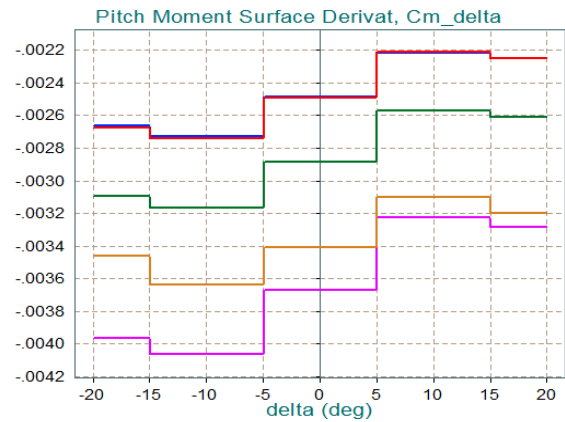


Next, select the pitch aero derivatives, and click "OK". The plot below shows the  $C_{m\alpha}$ ,  $C_{A\alpha}$ , and  $C_{Z\alpha}$  derivatives as a function of (Mach and  $\alpha$ ). From the top menu bar (above the plot) click on "Exit/Next" to return to the aero coefficients plotting menu. From there you may either plot another set of aero data or you may return to the Main Trim Options menu. You may also try other options, such as, plotting the longitudinal and lateral aero-surface derivatives.



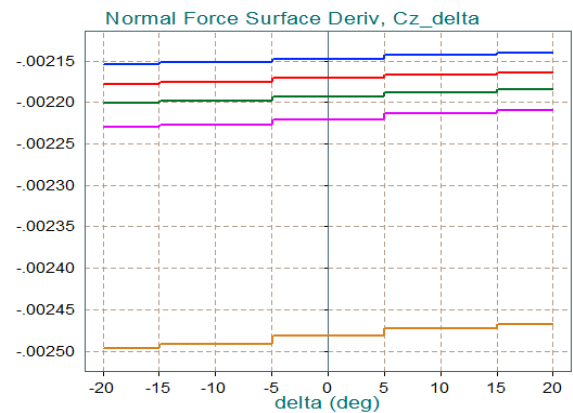
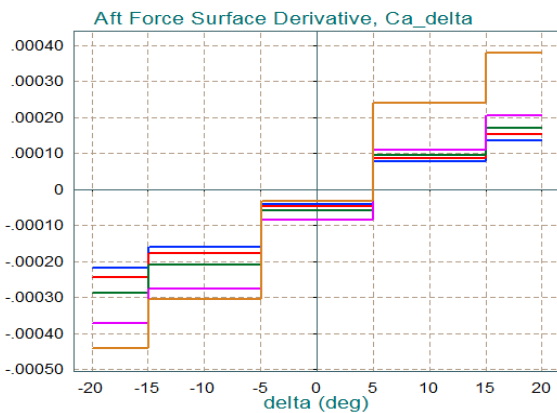
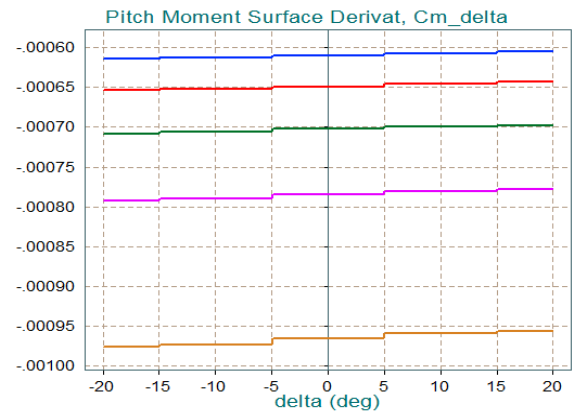
Rocket-Plane, Mission-1, Ground Takeoff  
 Aero-Surface Derivat vers Deflect of Elevon  
 Reference Area (ft<sup>2</sup>)= 390.6  
 Chord and Span (feet)= 50.00 ; 30.81  
 Moments Transferred to Vehicle CG  
 Alpha(o)=-2.00 (deg)  
 Beta(o)=0.00 (deg)

Mach: 0.3000  
 Mach: 0.6000  
 Mach: 0.8000  
 Mach: 0.9000  
 Mach: 1.200



Rocket-Plane, Mission-1, Ground Takeoff  
 Aero-Surface Derivat vers Deflect of Body Flap  
 Reference Area (ft<sup>2</sup>)= 390.6  
 Chord and Span (feet)= 50.00 ; 30.81  
 Moments Transferred to Vehicle CG  
 Alpha(o)=-2.00 (deg)  
 Beta(o)=0.00 (deg)

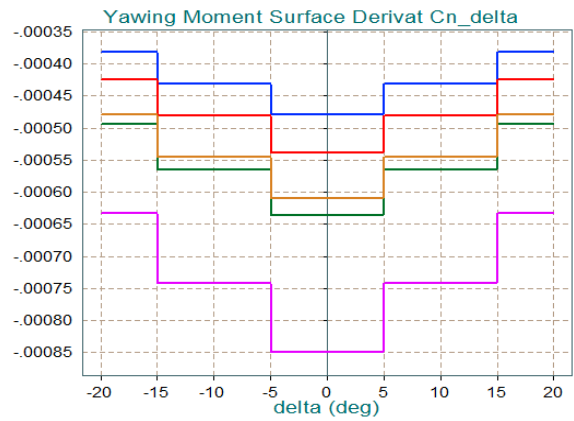
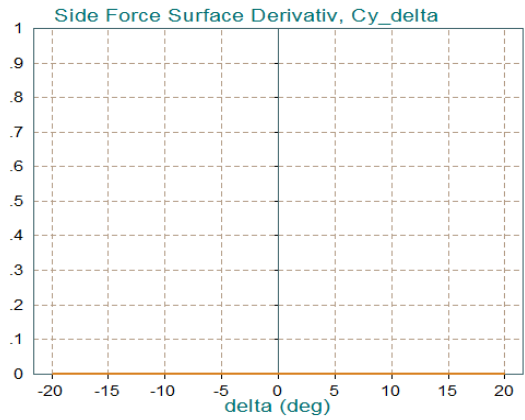
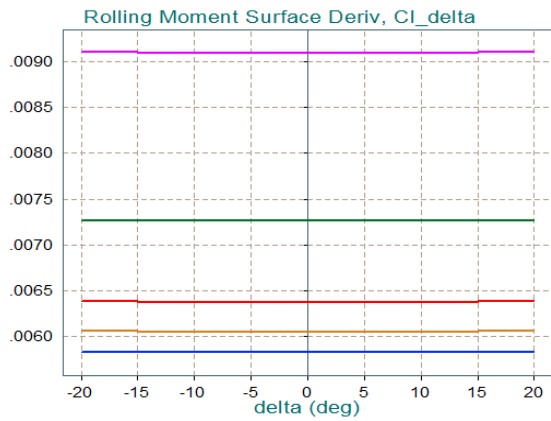
Mach: 0.3000  
 Mach: 0.6000  
 Mach: 0.8000  
 Mach: 0.9000  
 Mach: 1.200





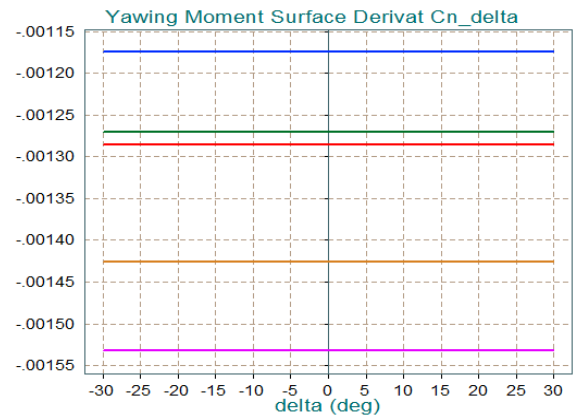
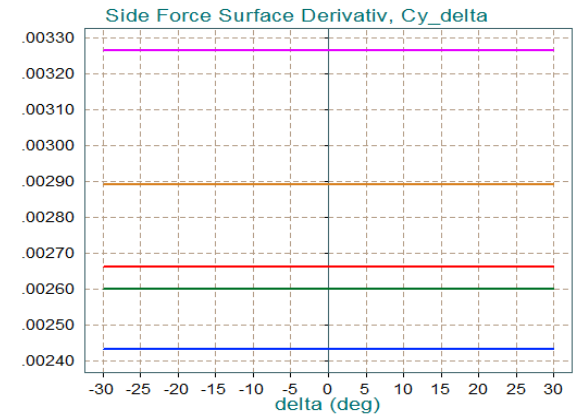
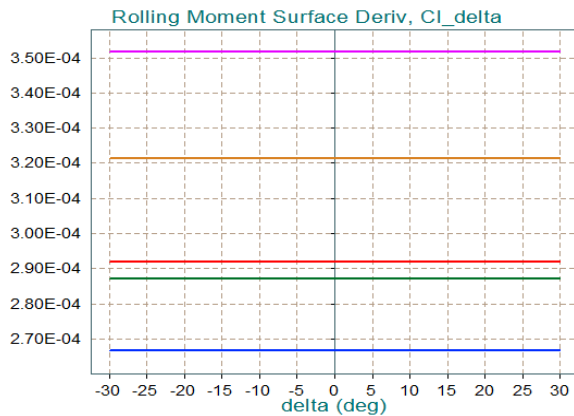
Rocket-Plane, Mission-1, Ground Takeoff  
 Aero-Surface Derivat vers Deflect of Aileron  
 Reference Area (ft<sup>2</sup>)= 390.6  
 Chord and Span (feet)= 50.00 ; 30.81  
 Moments Transferred to Vehicle CG  
 Alpha(o)=-2.00 (deg)  
 Beta(o) =0.00 (deg)

Mach: 0.3000  
 Mach: 0.6000  
 Mach: 0.8000  
 Mach: 0.9000  
 Mach: 1.200



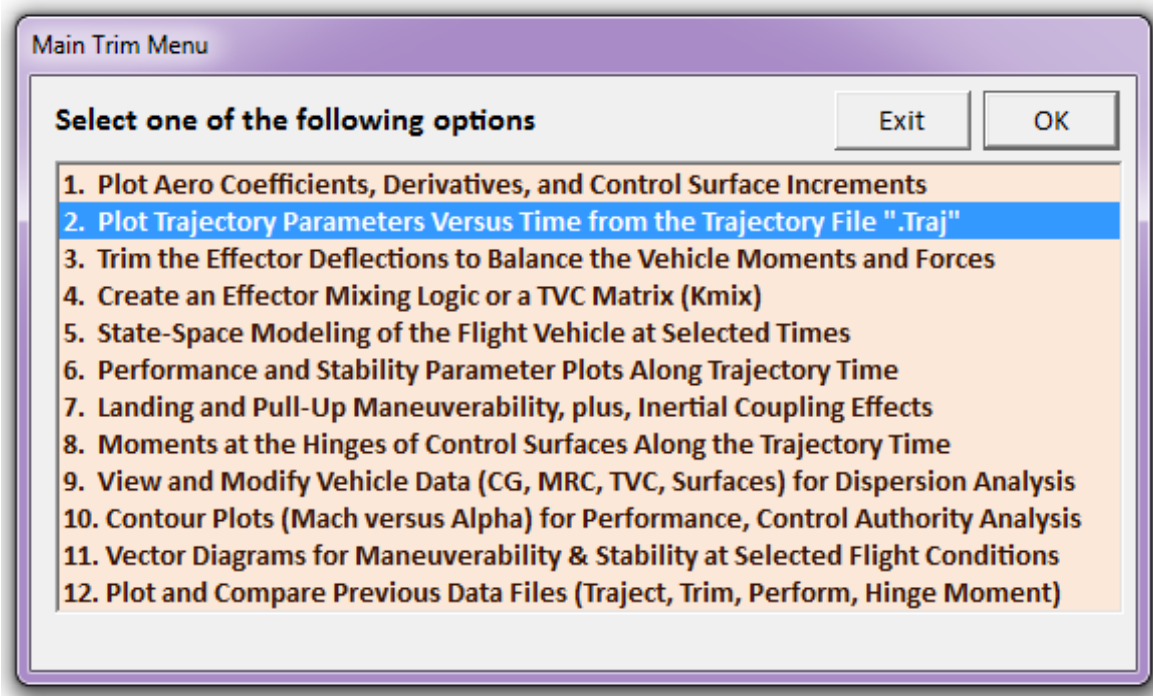
Rocket-Plane, Mission-1, Ground Takeoff  
 Aero-Surface Derivat vers Deflect of Rudder  
 Reference Area (ft<sup>2</sup>)= 390.6  
 Chord and Span (feet)= 50.00 ; 30.81  
 Moments Transferred to Vehicle CG  
 Alpha(o)=-2.00 (deg)  
 Beta(o) =0.00 (deg)

Mach: 0.3000  
 Mach: 0.6000  
 Mach: 0.8000  
 Mach: 0.9000  
 Mach: 1.200



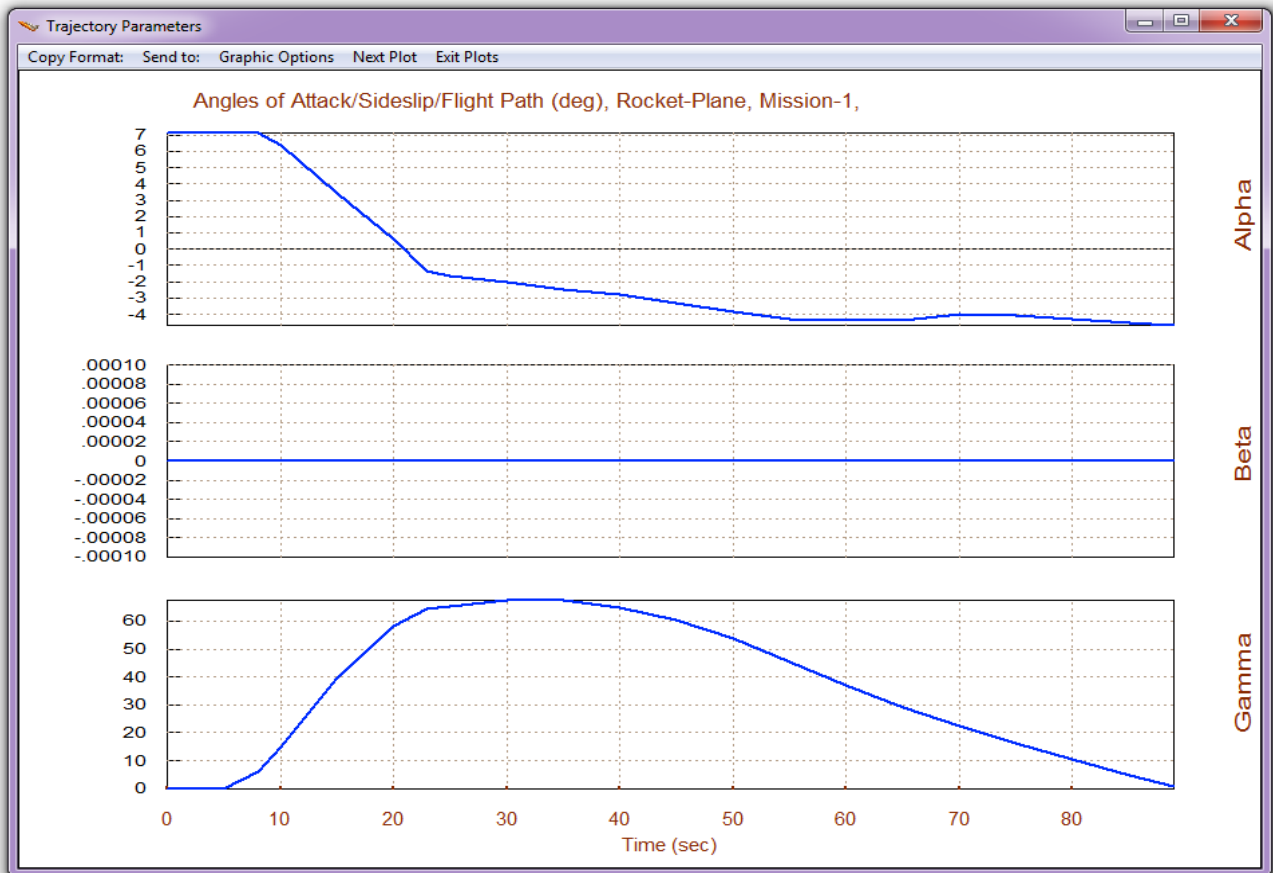
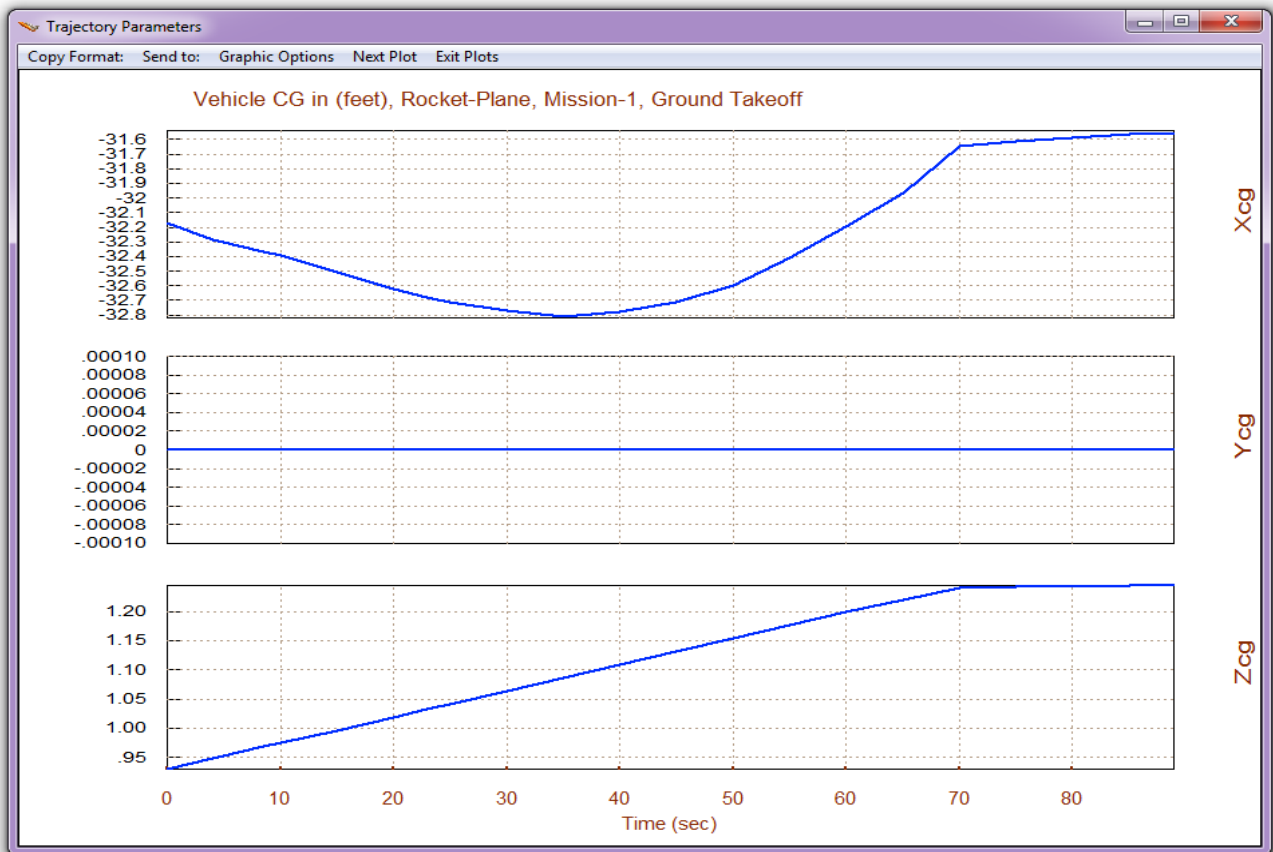
## 1.2 Checking the Trajectory Data

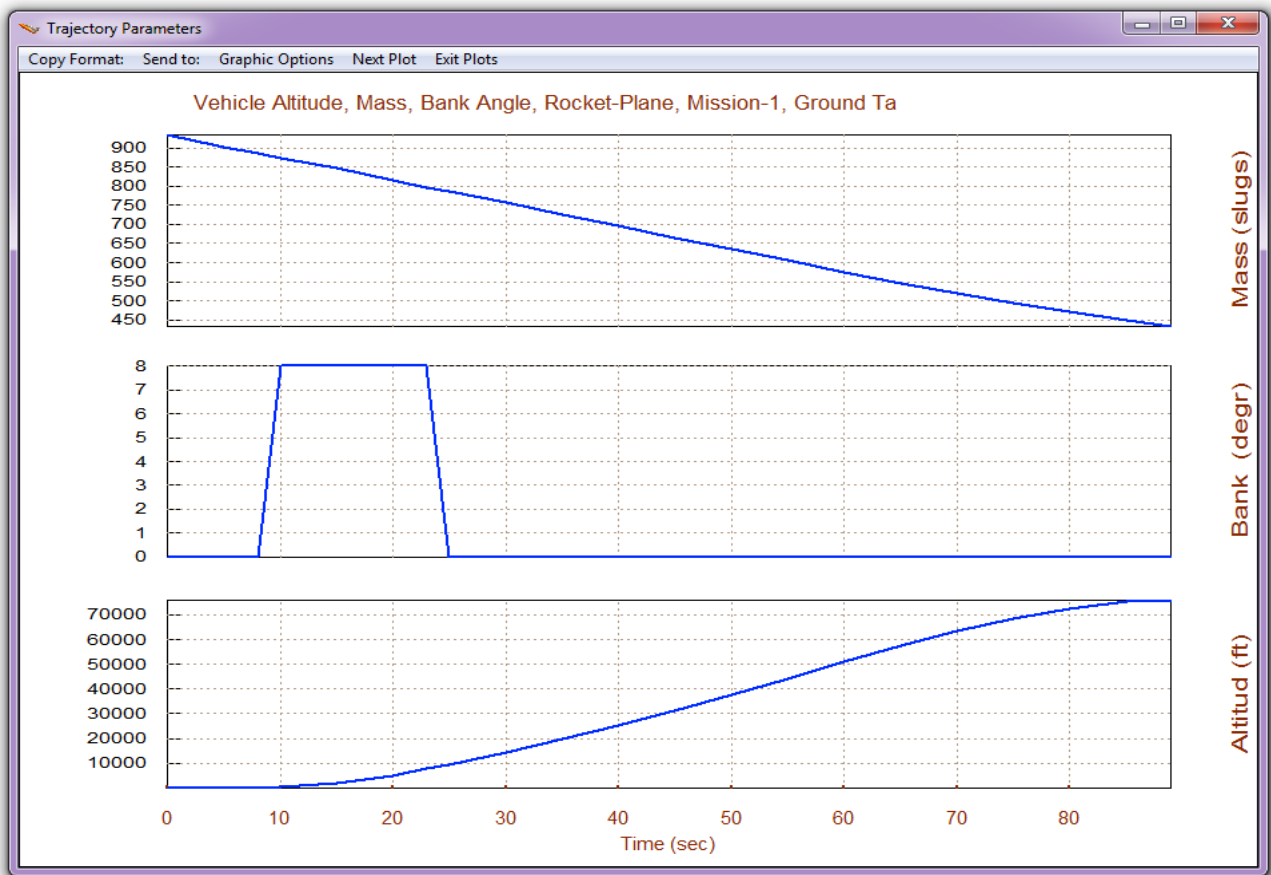
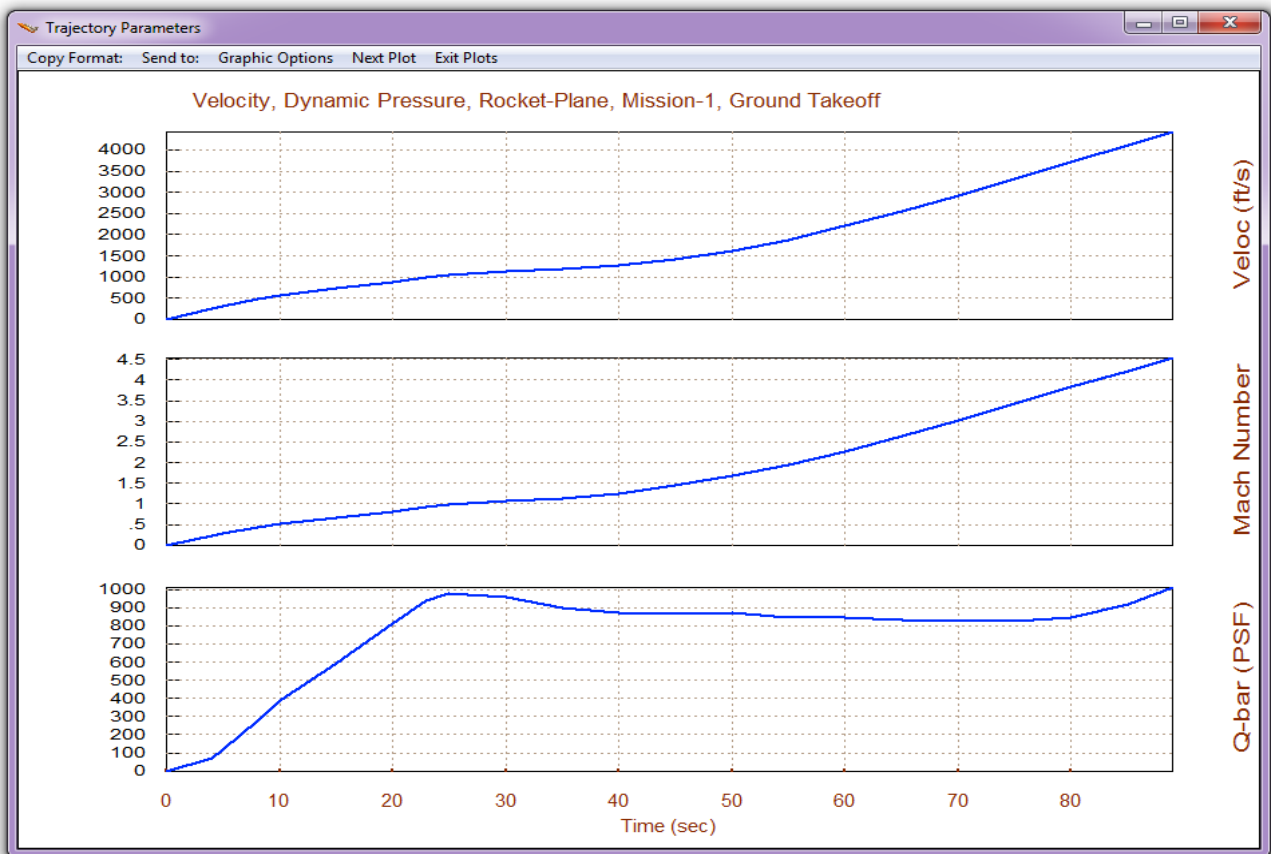
The next important thing to check before beginning the analysis is the trajectory data. Return to the Trim main menu, select the second option which is "Plot the Trajectory Parameters Versus Time" and click "OK".

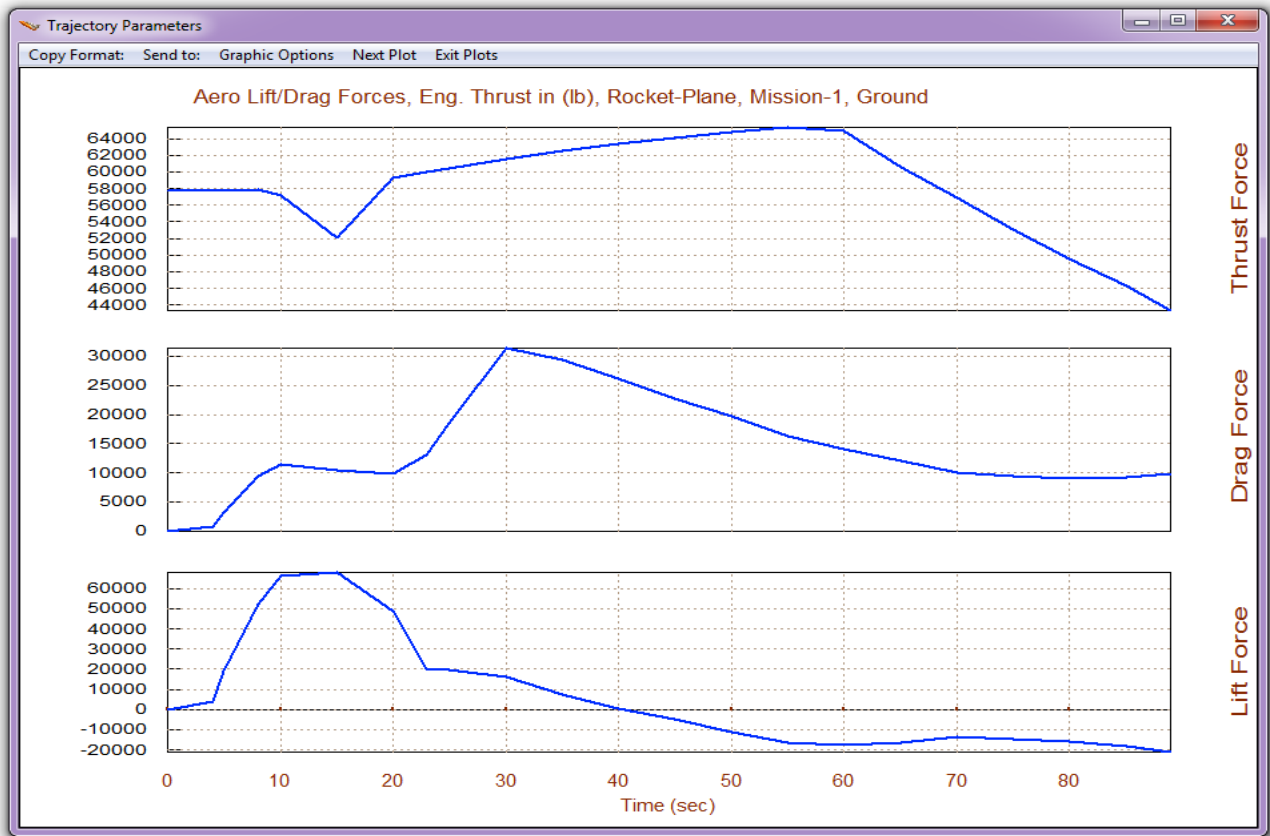
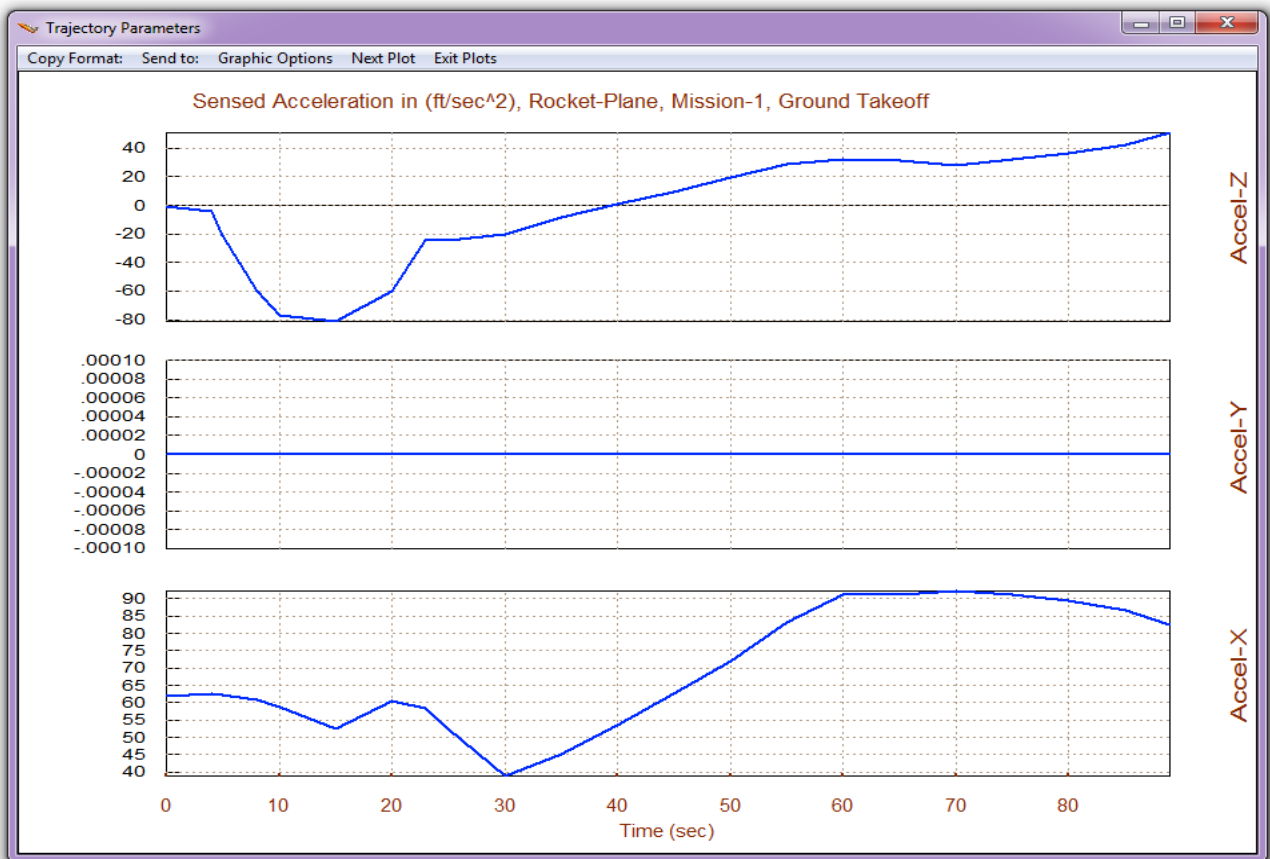


The following plots show the trajectory parameters, which are read by the program from file "Hyp\_Ascent.Traj", as a function of time. The first plot shows the CG travel during the flight. The CG location is calculated from the vehicle mass versus time and also by the mass properties file "Hyper.Mass" that contains the CG coordinates as a function of mass. The next plot shows  $\alpha$  which is initially positive and it changes sign after Mach 1. The sideslip  $\beta$  is zero throughout the flight because there is lateral symmetry. The flight path angle  $\gamma$  is positive during the climb and it decays to zero towards the end of the ascent phase. The dynamic pressure reaches a Max-Q of 970 (psf) at 26 seconds, just before Mach 1. There is an 8° bank angle between 10 and 22 sec for turning its heading direction. The engine thrust varies between 44,000 and 64,000 (lb). The normal and axial accelerations are also shown in (ft/sec<sup>2</sup>). The axial x-acceleration reaches almost 3 g's. The normal z-acceleration towards the end of the flight is positive (down) as the vehicle is flying with a negative ( $\alpha$ ).

On the top of each plotting window there is a horizontal menu bar that includes several options. If you click on "Next Plot" it will move to the next plot containing trajectory data. If you click on "Exit Plots" it will go back to the Trim main menu shown above.

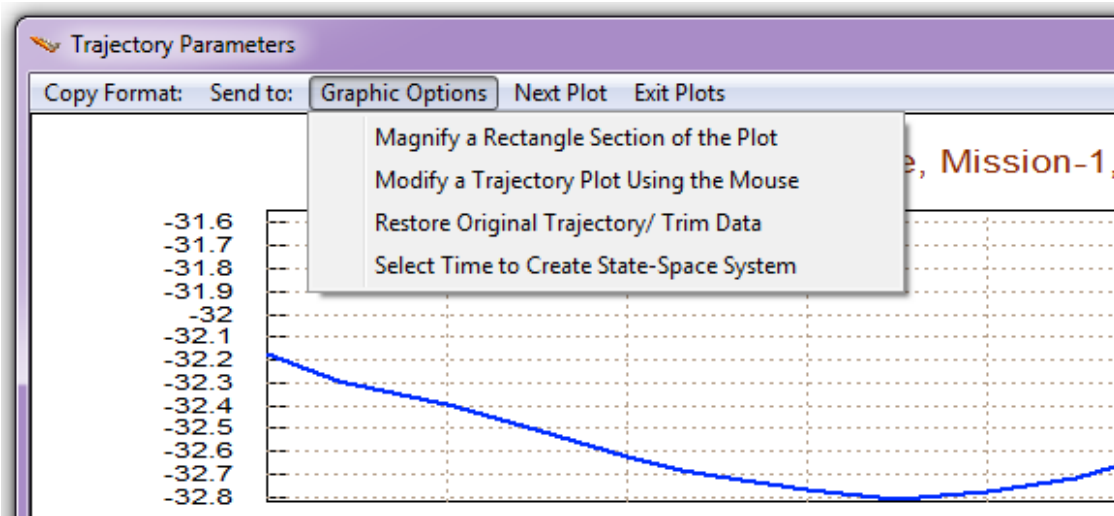






### 1.3 Modifying the $X_{CG}$ in the Trajectory Data

Sometimes it is suitable to modify some of the trajectory parameters in order to evaluate the vehicle's response to parameter variations, such as: CG shift due to an unbalanced payload, angle of attack or sideslip variations from trim, dynamic pressure, accelerations, Mach number, or it may be necessary to introduce an external force or torque that represents disturbances due to engine misalignments, actuator failures, etc. This can be done graphically by manipulating the trajectory parameter curves directly on the screen by using the mouse. From one of the trajectory plots, such as the ones shown below, go to the top horizontal menu and click on "*Graphic Options*", and then from the vertical pop-up menu select the option "*Modify a Trajectory Plot Using the Mouse*".



You may now begin to modify the trajectory variables, one at a time. In this example we will modify only the CG. The following menu shows the trajectory variables that can be modified by the user. Select to modify the CG location along the X axis, and click on "*Select a Variable to Modify*". A plot of the  $X_{CG}$  location versus time appears in the next window-dialog. The original  $X_{CG}$  travel is shown by the green line on the top. You may use the mouse to modify it and shift the  $X_{CG}$  location a couple of feet back towards the engine making it less stable. The modified  $X_{CG}$  travel is shown by the yellow line below the original  $X_{CG}$  curve. You may "*Continue with Another Variable to Modify*", but in this example click on "*Save the Modified Trajectory*" and the modification will remain active in memory for the following analysis. The original trajectory filename is "*Hyp\_Ascent.Traj*". The trajectory modification is automatically saved in file "*Hyp\_Ascent1.Traj*". You may co-plot the two trajectory files together, as shown in the trajectory plot below, by returning to the Trim main menu, selecting the last option (12) and then, the two trajectory files. The modified XCG is the red curve. The  $Y_{CG}$  and the  $Z_{CG}$  travel are the same in both trajectories (shown in red). You may also rename and save the modified trajectory to a different filename "*-XCG.Traj*" to protect it from getting overwritten. After completing the dispersion analysis the original trajectory can be restored by going back to the same menu above and clicking on "*Restore the Original Trajectory*".

Modify Trajectory Data

Trajectory modifications are used for evaluating the vehicle performance under dispersed conditions. Some variables can be modified graphically using the mouse. This does not destroy the original trajectory which can be restored later.

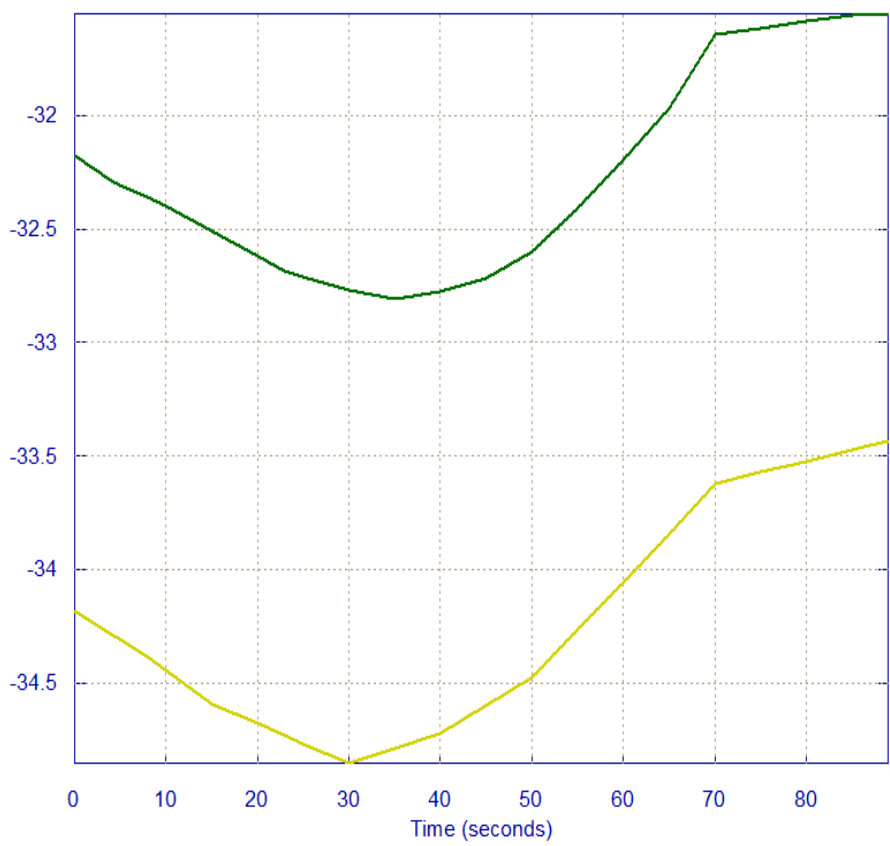
- Angle of Attack, alpha, (deg)
- Angle of Sideslip, beta, (deg)
- Dynamic Pressure,  $Q_{bar}$ , (psf)
- Mach Number, (Mach), (---)
- Roll Rate, (P), (deg/sec)
- Pitch Rate, (Q), (deg/sec)
- Yaw Rate, (R), (deg/sec)
- Accelerat. Along X, (Ax) (ft/sec<sup>2</sup>)
- Accelerat. Along Y, (Ay) (ft/sec<sup>2</sup>)
- Accelerat. Along Z, (Az) (ft/sec<sup>2</sup>)
- CG Location Along X (feet)**
- CG Location Along Y (feet)
- CG Location Along Z (feet)
- Total Engine Thrust, (Te), (lb)
- Disturbance Force along X, Fd-x (lb)
- Disturbance Force along Y, Fd-y (lb)
- Disturbance Force along Z, Fd-z (lb)
- Disturbance Moment about X, Ld-x (ft-lb)
- Disturbance Moment about Y, Md-y (ft-lb)
- Disturbance Moment about Z, Nd-z (ft-lb)

Select a Variable to Modify

Exit Menu

Modify the Trajectory Variables

CG Location Along X, (Xcg), (ft)



This curve shows the time history of the selected trajectory variable. This profile can be modified graphically by repeated adjustments using the mouse.

Specify a range between two points on the curve to be reshaped. Place the cursor at a point on the curve and click the mouse to select point (A), and then select point (B) shown by red dots. The mid-point (purple dot) is found and highlighted.

The mid-point may be shifted vertically to a new location. Click again to define a new shape between A and B. You may repeat several times as needed. When you have finished reshaping the curve you may "Continue With Another Variable to Modify".

Save the Modified Trajectory

Continue with Another Variable to Modify

Main Trim Menu

Select one of the following options

Exit

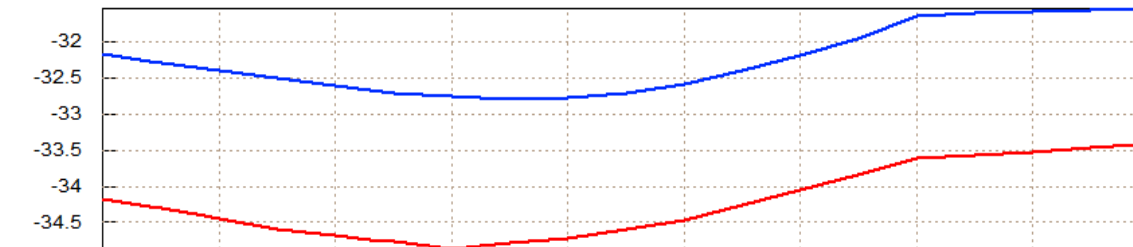
OK

1. Plot Aero Coefficients, Derivatives, and Control Surface Increments
2. Plot Trajectory Parameters Versus Time from the Trajectory File ".Traj"
3. Trim the Effector Deflections to Balance the Vehicle Moments and Forces
4. Create an Effector Mixing Logic or a TVC Matrix (Kmix)
5. State-Space Modeling of the Flight Vehicle at Selected Times
6. Performance and Stability Parameter Plots Along Trajectory Time
7. Landing and Pull-Up Maneuverability, plus, Inertial Coupling Effects
8. Moments at the Hinges of Control Surfaces Along the Trajectory Time
9. View and Modify Vehicle Data (CG, MRC, TVC, Surfaces) for Dispersion Analysis
10. Contour Plots (Mach versus Alpha) for Performance, Control Authority Analysis
11. Vector Diagrams for Maneuverability & Stability at Selected Flight Conditions
12. Plot and Compare Previous Data Files (Traject, Trim, Perform, Hinge Moment)

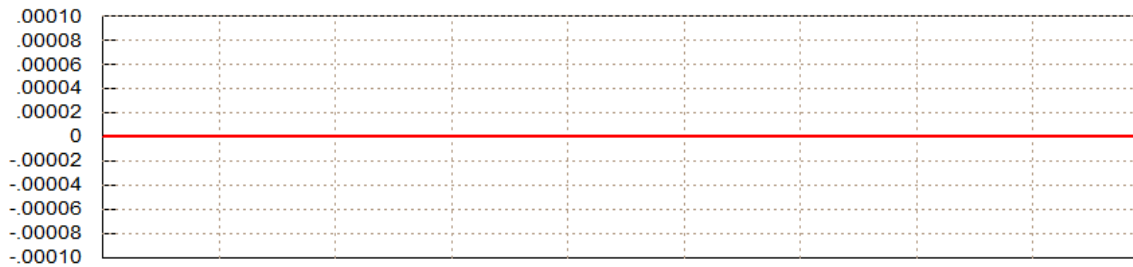
Trajectory Parameters

Copy Format: Send to: Graphic Options Next Plot Exit Plots

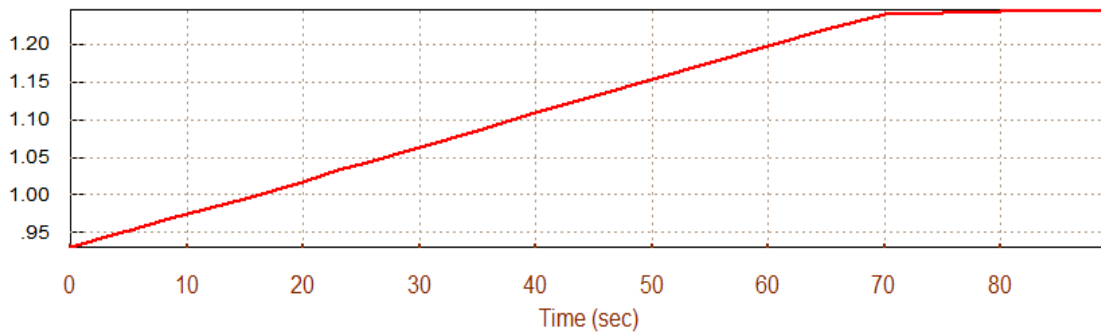
Vehicle CG in (feet), User Modified Trajectory ...



Xcg



Ycg

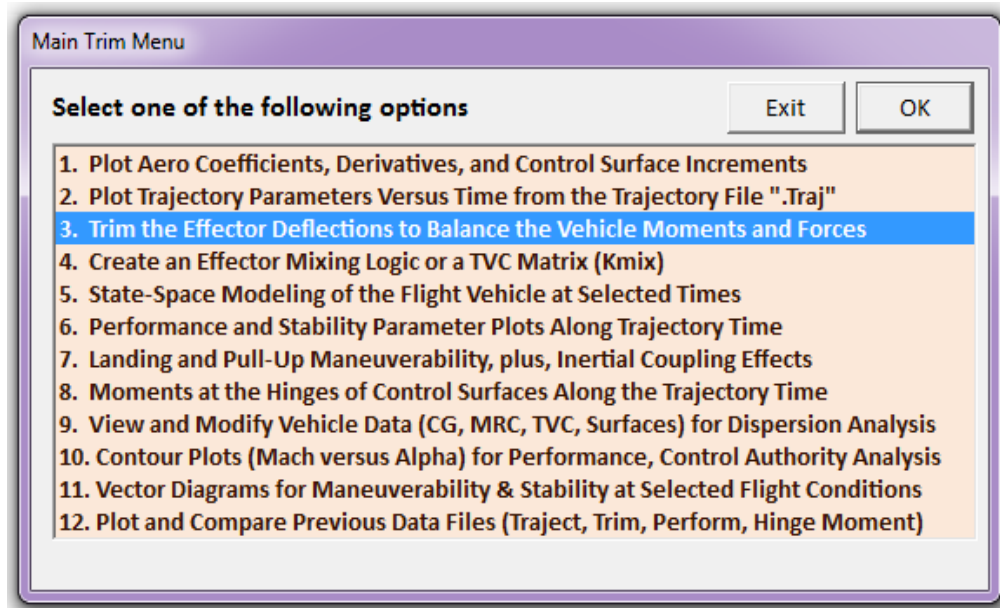


Zcg

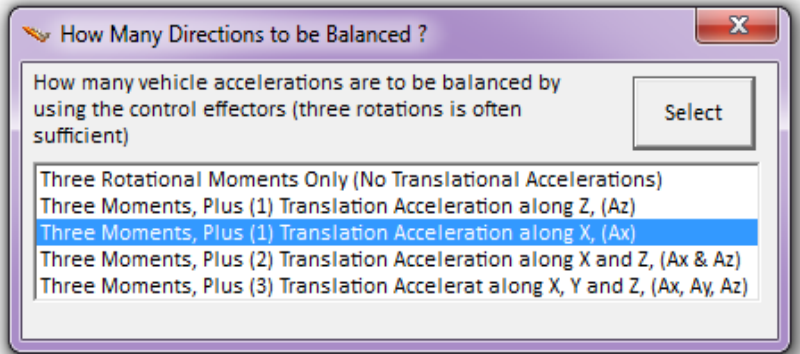
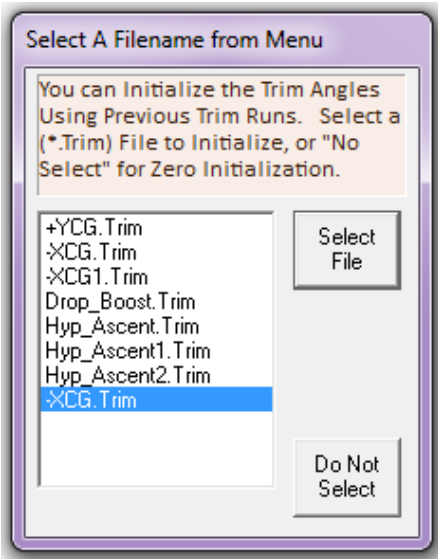


## 1.4 Trimming the Effectors and Comparing the Trim Results

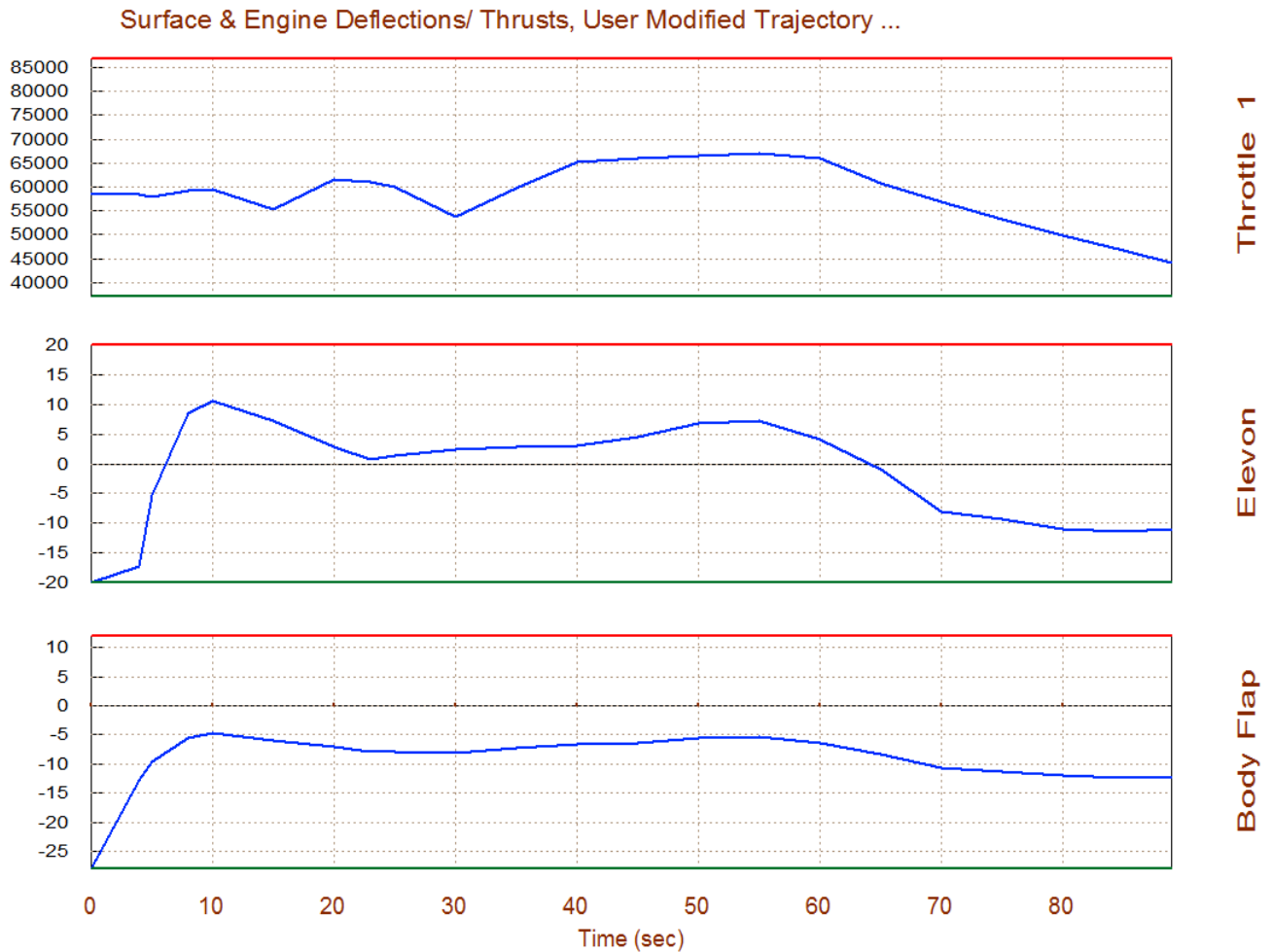
Let us now trim the vehicle effectors along the trajectories. The user has the option to adjust graphically the maximum control capability of each effector versus time from their original maximum deflections defined in the effector data file. This time we will not apply any limitations to the effectors maximum control authority but allow the program to use their full capability during Trim. Further trimming adjustments are possible when the vehicle has multiple effectors manipulating the same directions and in order to trade-off the activity of some effectors against others. In this example we have 5 effectors: a throttling main engine which is defined in file "*Hyper1.Engn*", and the four aero-surfaces which are defined in file "*Hyp-Ascent.Delt*". Later on we will introduce more controls by using a gimbaling engine and thruster jets and examine how this may improve the overall performance. In section 1.3 we recently created a modified trajectory with an aft shifted  $X_{CG}$ . Let us continue using this modified trajectory to trim and then we will restore the original trajectory with the proper CG, re-trim the effectors and compare the trim results between the two cases: nominal and aft  $X_{CG}$  trajectories. Return to the Trim menu and without restoring the original trajectory select option (3) to "*Trim the Effector Deflections*" and click "OK".



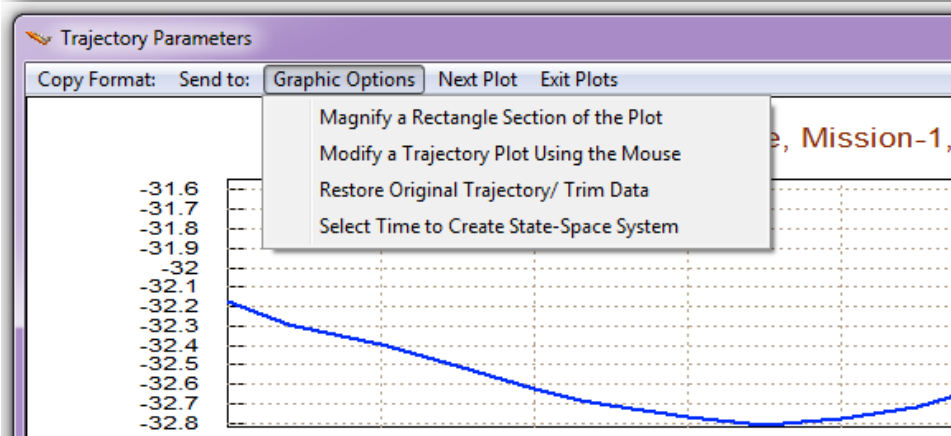
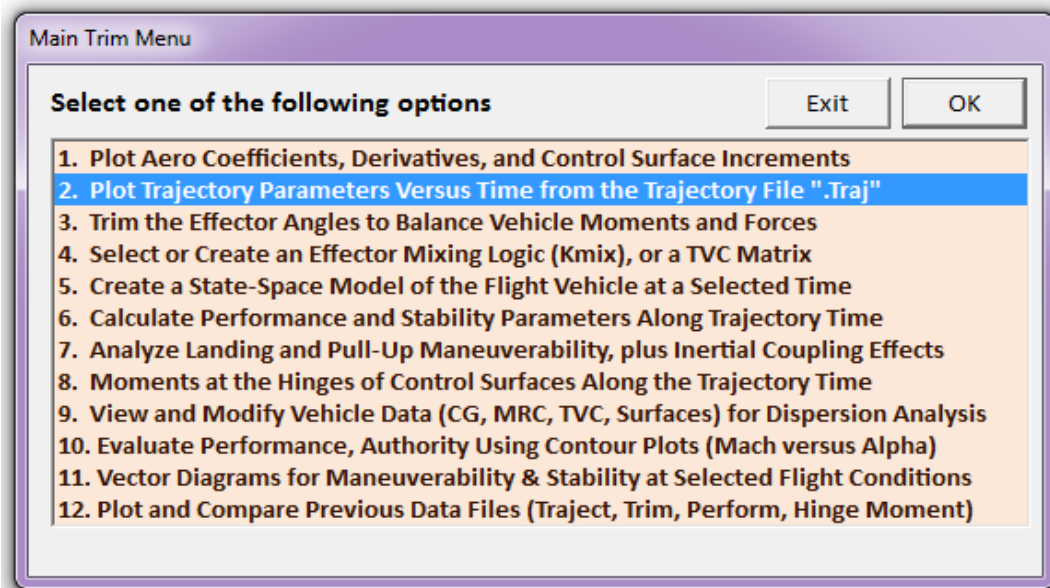
In the next menu the program wants to know how to initialize the trimming process. You may select a previous trim file to initialize the algorithm by selecting the file and clicking on "*Select File*" or you may start from zero deflection angles and throttle control by clicking on "*Do Not Select*" a file. Since we have not yet created a trim file in this configuration we do not select an initialization file. In the next menu the user must chose the directions along which to trim the vehicle. Since we have a throttlable engine, in addition to the 3 rotations, we would also like to trim in the x-direction in order to match the acceleration. In this setup we can eventually design a flight control system that will control 3 rotations and may include velocity control by varying the engine thrust.



The Trim program now calculates the control surface trim angles along the  $X_{CG}$  modified trajectory and also the engine thrust, which varies from its 63,000 (lb) nominal value. The thrust variation is because the trimming program is trying to match the trajectory's axial acceleration.

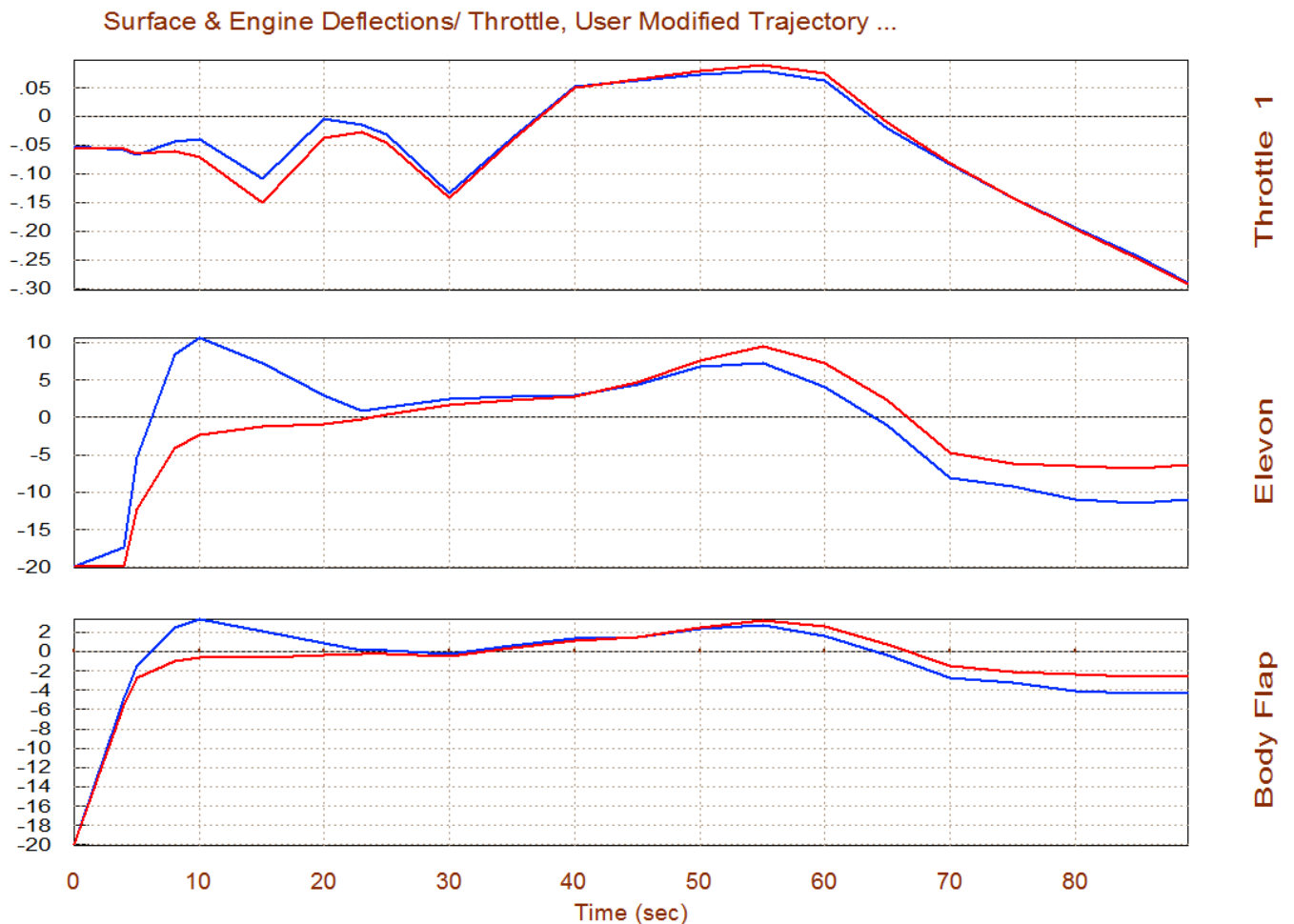
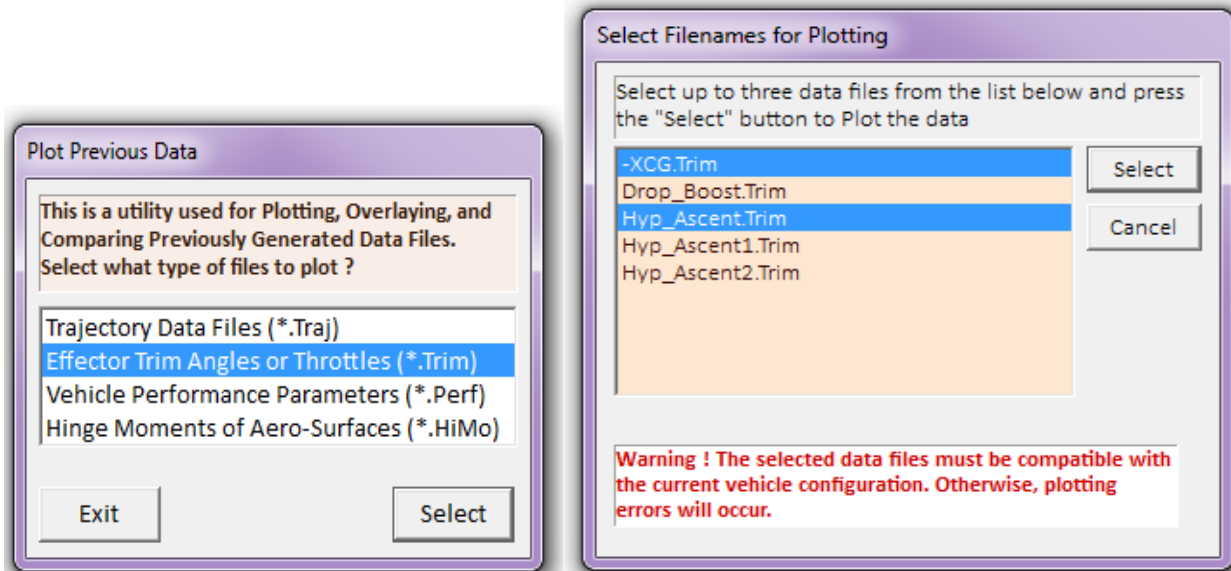


At this point our vehicle has perfect lateral symmetry without any lateral disturbances so the aileron and rudder deflections are zero, as expected. However, there is a sizeable amount of deflections in both, Elevon and in the Body-Flap. Their maximum and minimum deflection limits are:  $\pm 20^\circ$  relative to the bias position. The bias position is defined in the aerosurface data and in this case the Body-Flap is biased at  $-8^\circ$ . The throttle control is limited to  $\pm 40\%$ . The deflection limits are shown by the red and green lines, which for the time being they are constant, but they can be varied when we make trimming adjustments. For example, we might like to trade some Elevon deflection against the Body-Flap. The effector trim histories for the modified trajectory are saved in file "-XCG.Trim" for plotting. The max/min deflection limits are also saved in this file.



Now, let us return to the trajectory plotting option (2), restore the original trajectory from the top menu, and repeat the trimming process using the original "*Hyp\_Ascent.Traj*" and not the trajectory with the modified Xcg. When the trim is completed the trim data from the original trajectory is saved in file "*Hyp\_Ascent.Trim*". We may now plot the two trim files together and discuss their differences. Return to the Trim main menu and select again option (12) for plotting and comparing data. This time we select the second option for plotting and comparing effector trim files. From the menu on the right select the two "Trim" files that were recently created: the first one is from

the modified Xcg trajectory (blue), and the second one is created from the original trajectory (red). Notice, that the aft X<sub>CG</sub> modified trajectory requires in general larger deflections than the original forward X<sub>CG</sub> trajectory. This is because the vehicle is further from neutral stability and towards instability.

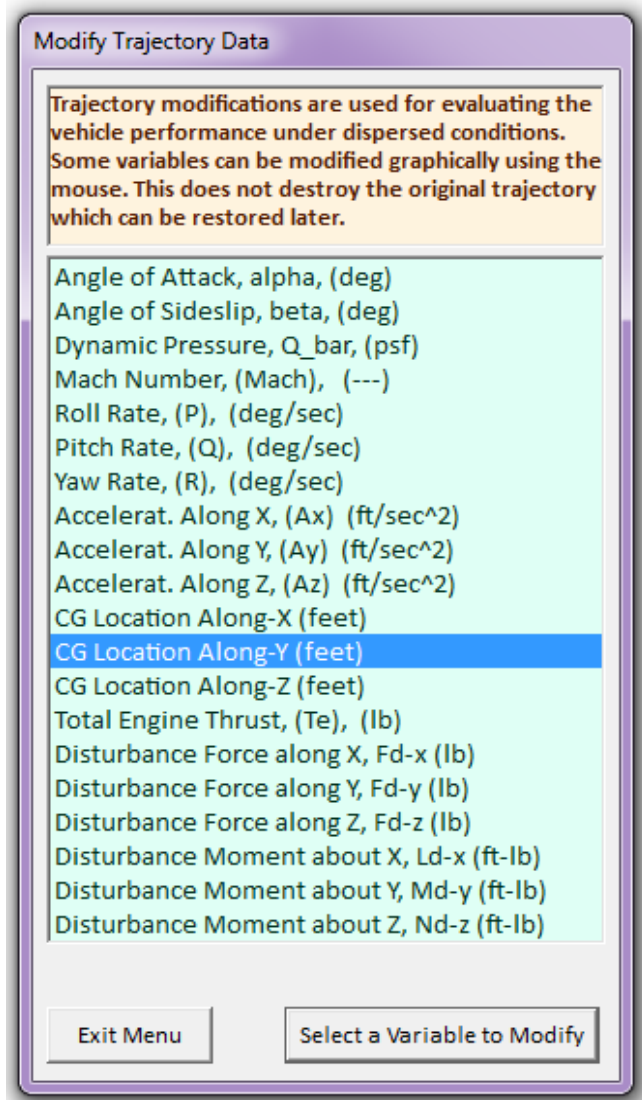


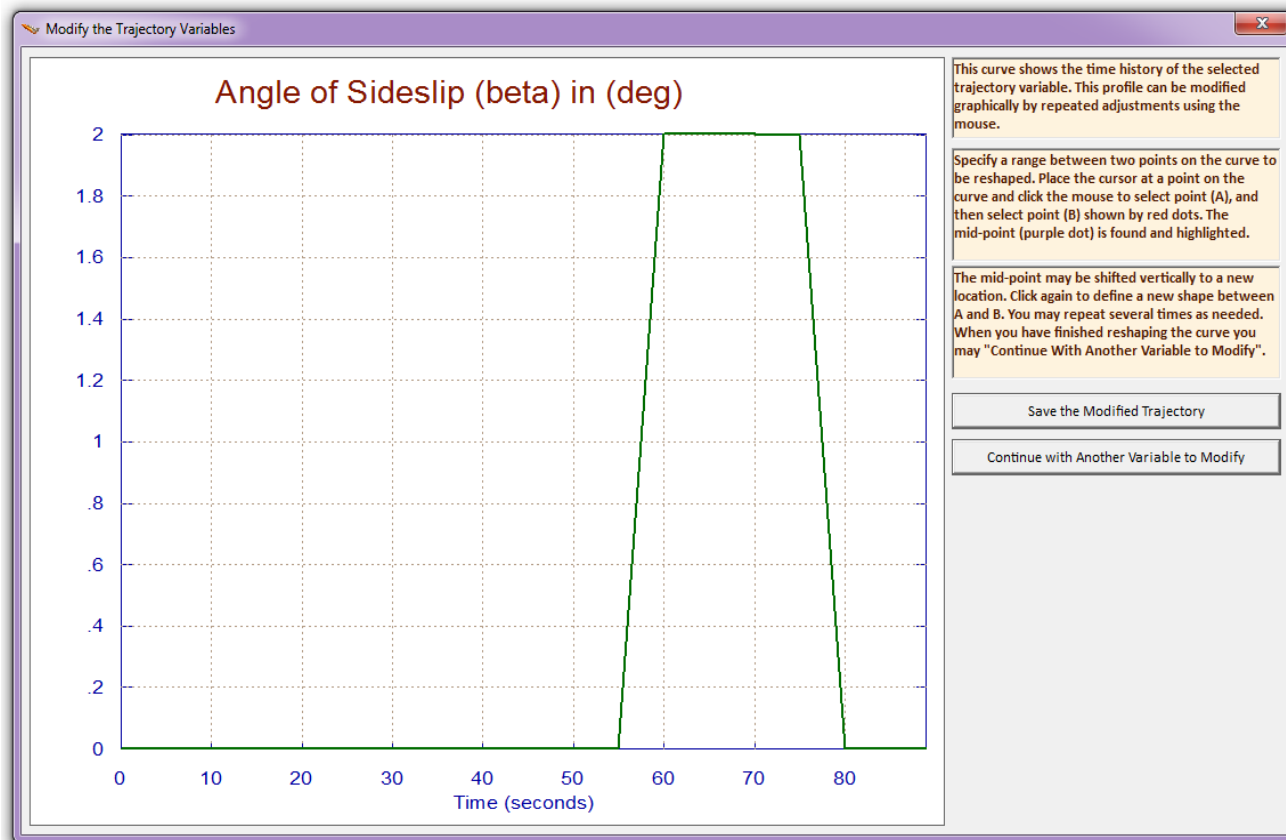
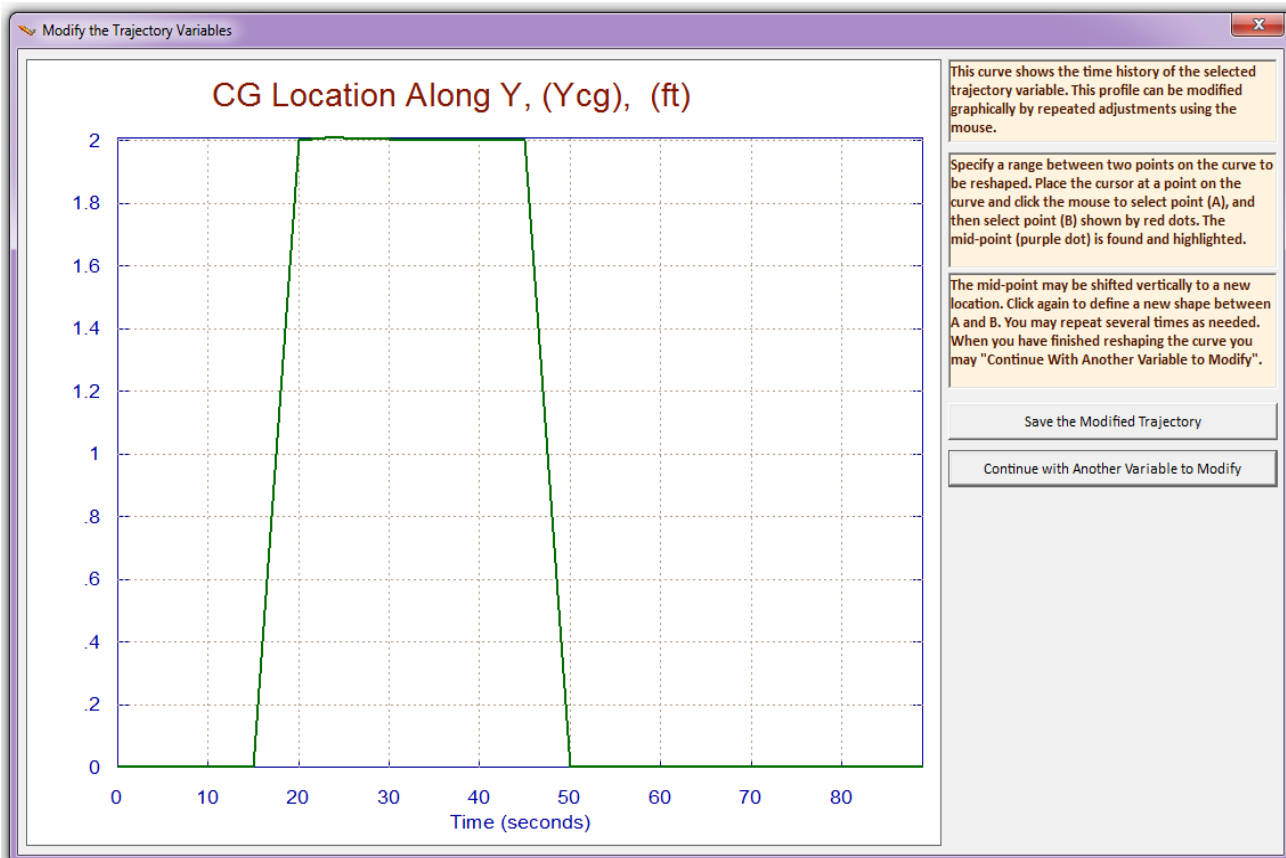
## 1.5 Effects of $\beta$ disturbance and $Y_{CG}$ Variation on the Aileron and Rudder

This time we will analyze dispersion effects in the lateral axes. We will introduce a beta disturbance and a  $Y_{CG}$  offset, re-trim, and evaluate their effects on the aileron and the rudder that control and trim the roll and yaw axes. The disturbance effects will occur at different flight times. We will first shift the  $Y_{CG}$  location, which is originally at the center-line, from zero to 2 (feet) towards the right wing, for a period of 30 seconds, and bring it back to zero. Then at a later time period we will apply a steady  $2^\circ$   $\beta$ -disturbance for a period of 20 sec. Let us begin again by plotting the nominal trajectory, as in Section 1.2. From one of the trajectory plots, go to the horizontal top menu bar and click on "Graphic Options". Then from the vertical pop-up menu select the option "Modify a Trajectory Plot Using the Mouse".

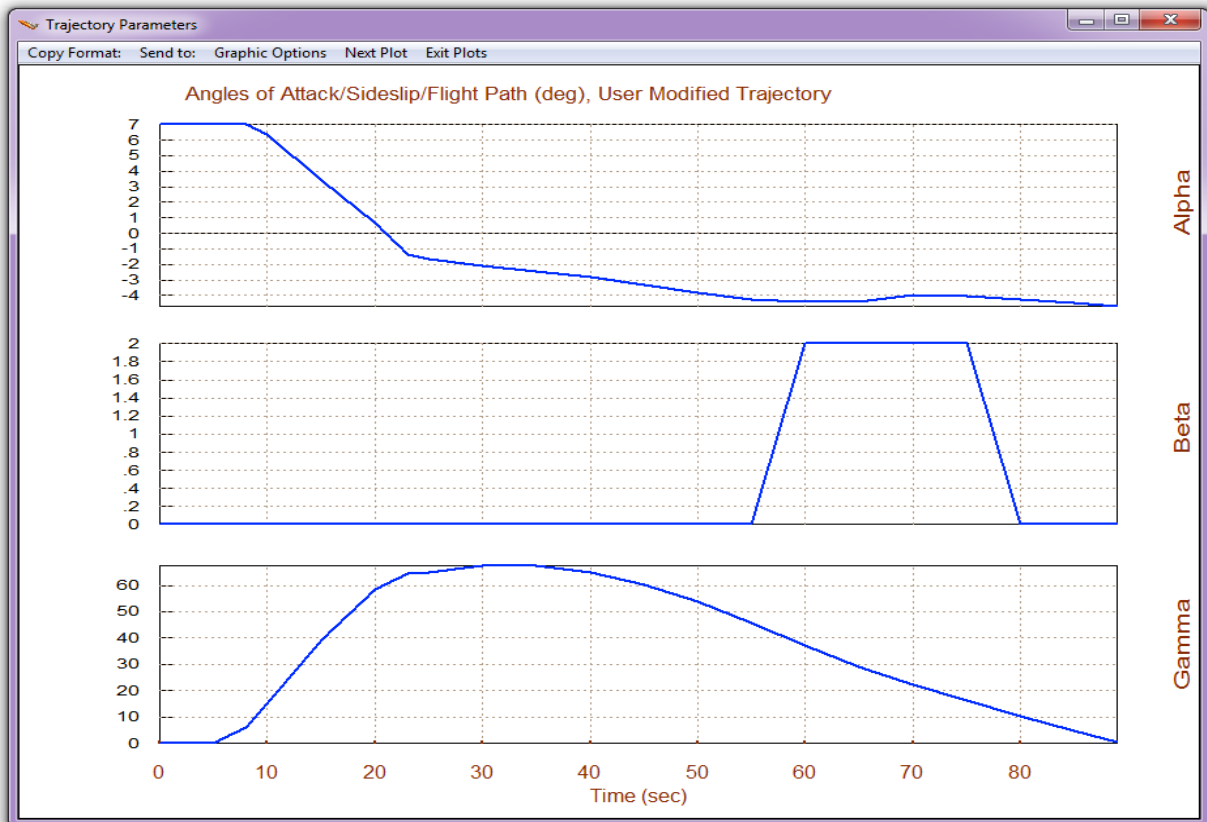
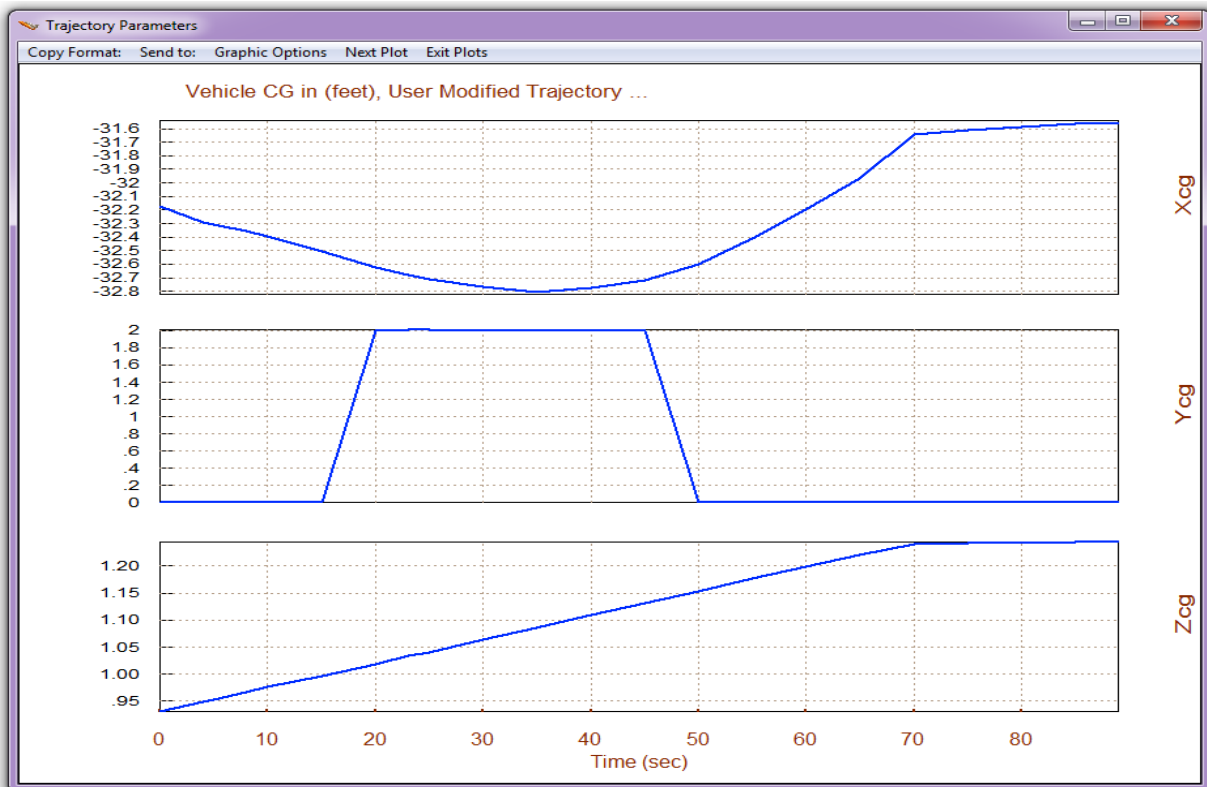
The following menu comes up and it is used for selecting trajectory variables to be modified by the analyst. First choose the CG location along the y-axis and click on "Select Variable to Modify". Using the dialog-plot of  $Y_{CG}$  versus time, modify the plot interactively using the mouse, one small section at a time. It is finally shown shifted from zero to +2 (ft) for the time period between 20 and 50 (sec). When the modification is complete, click on "Continue with Another Variable to Modify" to return to the previous menu.

Now select the angle of sideslip ( $\beta$ ) to modify which was originally zero. With a few mouse clicks this variable can be modified to the rectangular shape, as shown in the following dialog-plot. It has a  $2^\circ$  peak in the time period between 60 and 80 sec. Finally click on "Save the Modified Trajectory" to save it and it will remain active in memory until you remove it or restart the program. This user modified trajectory was also saved in file "+YCG+Beta.Traj" for further analysis. When you click on "Exit Menu" the program returns to the trajectory plots.

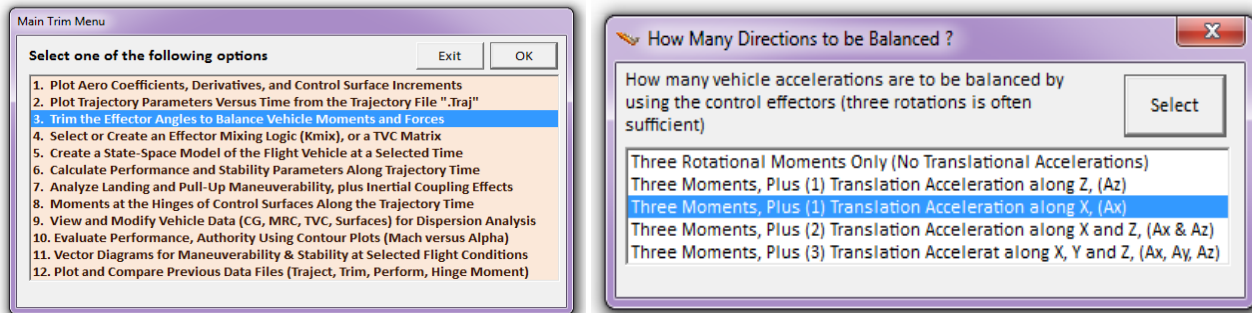




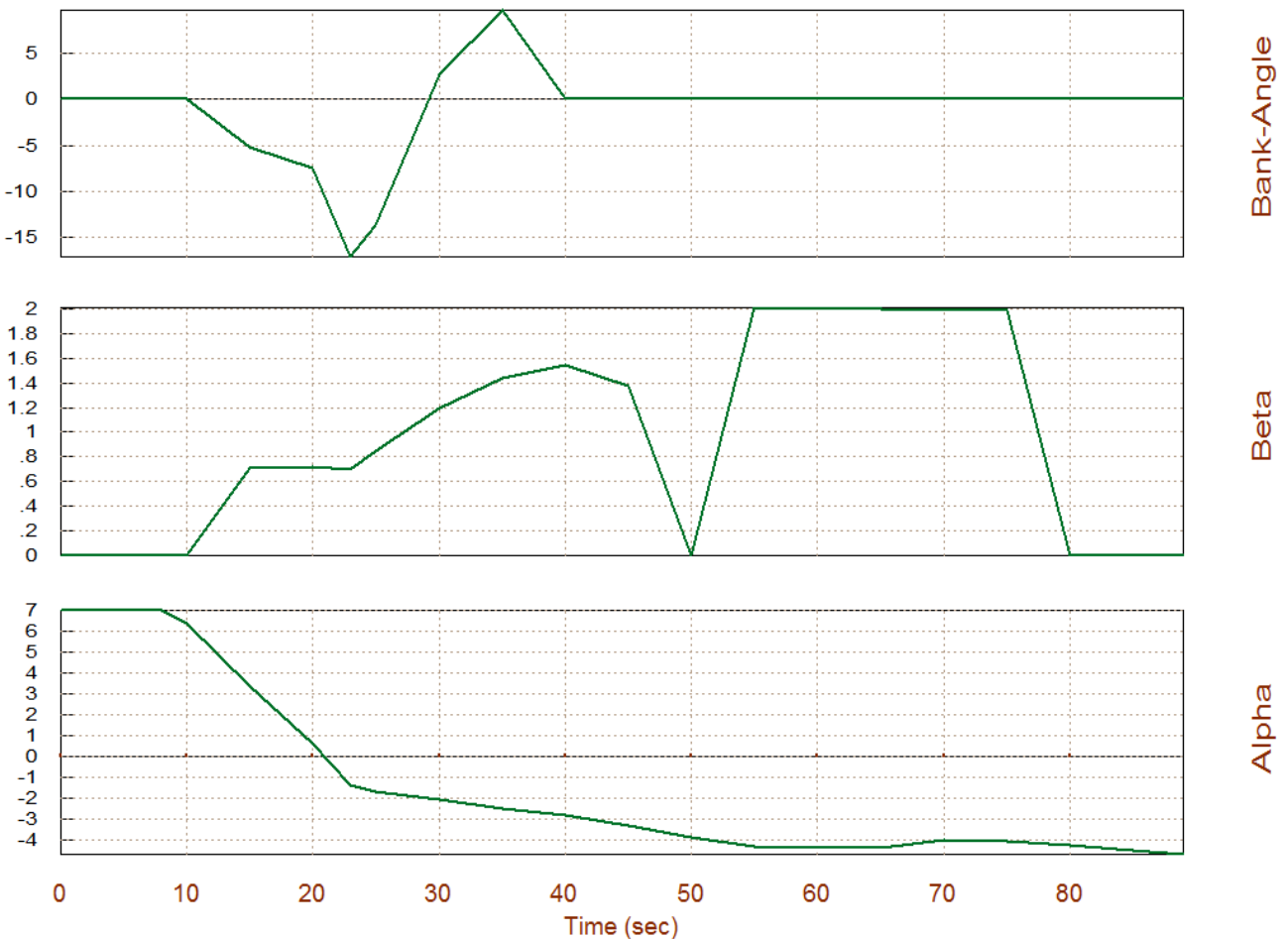
The following plots show that the trajectory modifications were saved as a user modified trajectory which is now active in Trim. It will remain active until it is replaced with the original.



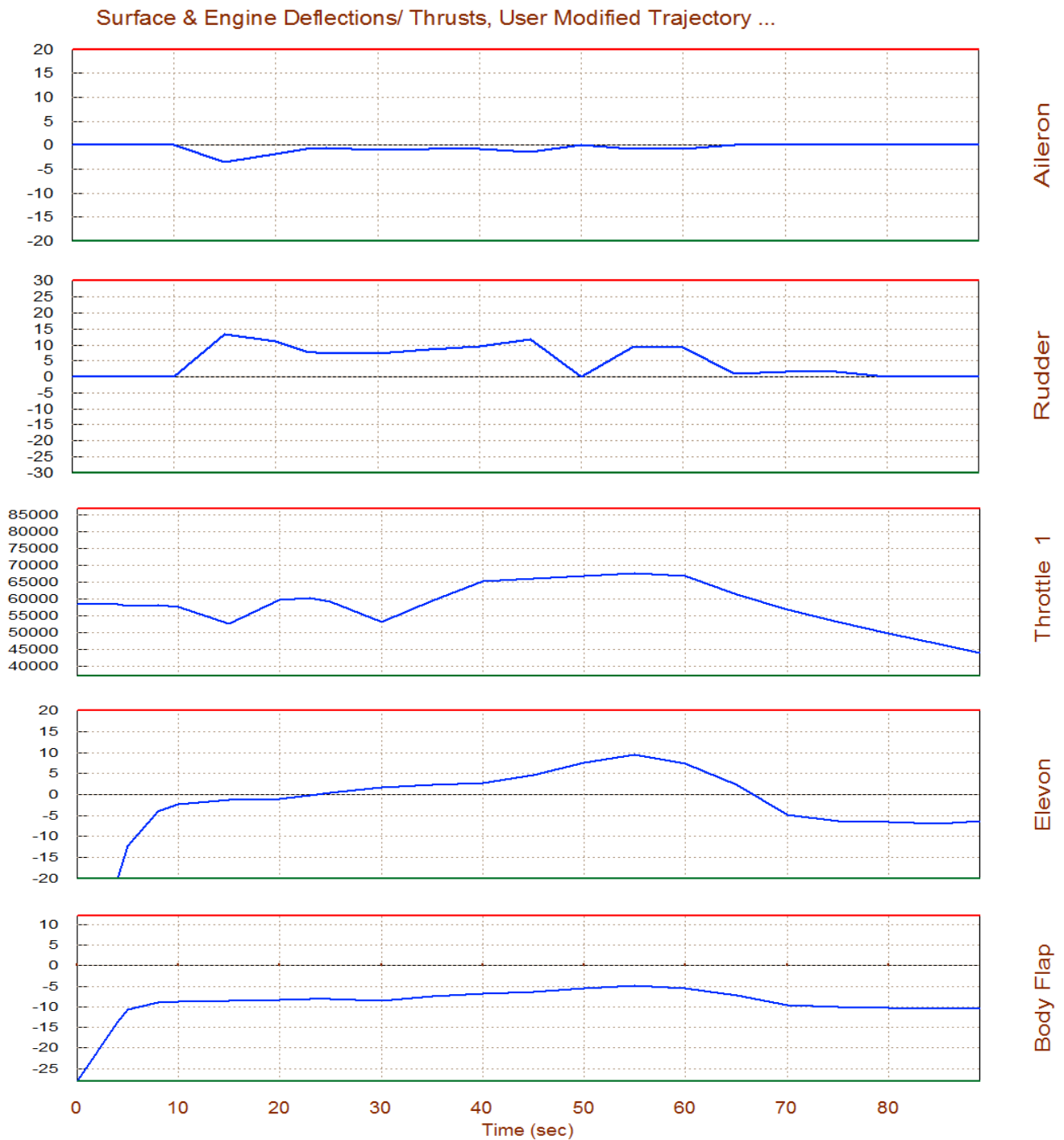
The next step is to return to the Trim menu and repeat the trimming process using the latest modified trajectory. Select again option (3) for "*Trimming the Effector Deflections*". Select to trim 3 rotations plus x-acceleration, as before. The first plot shows the residual moments in roll, pitch, and yaw, which ideally they should be zero when trimming is performed perfectly. They are pretty close to zero except for the first 3 seconds in the pitch axis where the elevon hits its -20 (deg) limit at take-off. It requires a lot of pitching moment at take-off as it is maintaining a high ( $\alpha$ ). The plot below shows the induced sideslip beta, initially due to the +YCG offset, and later due to the beta disturbance.



Alpha, Beta, and Bank Angle ( $\phi$ ), Assuming Effectors are Trimmed (deg)





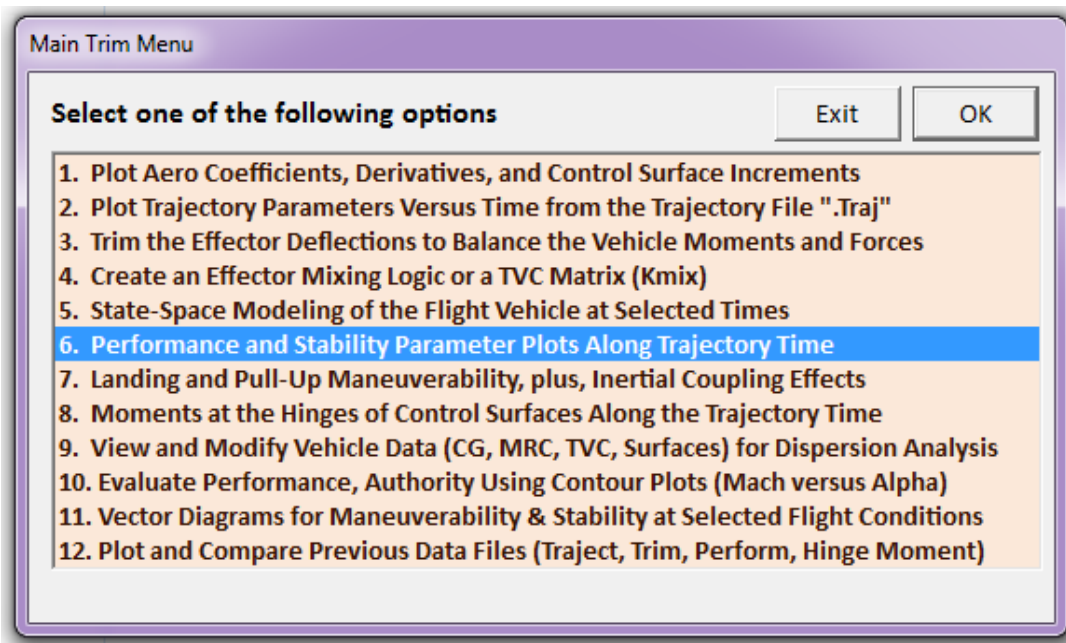


The plot above shows the aileron and rudder deflections required to trim the vehicle with the lateral trajectory dispersions described. The +YCG offset (in the time period between 10 and 50 sec) couples with the main engine thrust and induces a positive yawing moment on the vehicle. The rudder deflection is positive because it is generating a negative yaw torque trying to balance the positive yaw due to thrusting. The rudder also creates a positive roll torque and so the aileron deflection is negative trying to take it out. In the time period between 60 and 80 (sec) where the 2° of beta excitation is applied (the Y<sub>CG</sub> is back in the middle now), this excitation causes a positive yawing moment (because the vehicle is stable in yaw). The rudder deflection is positive again

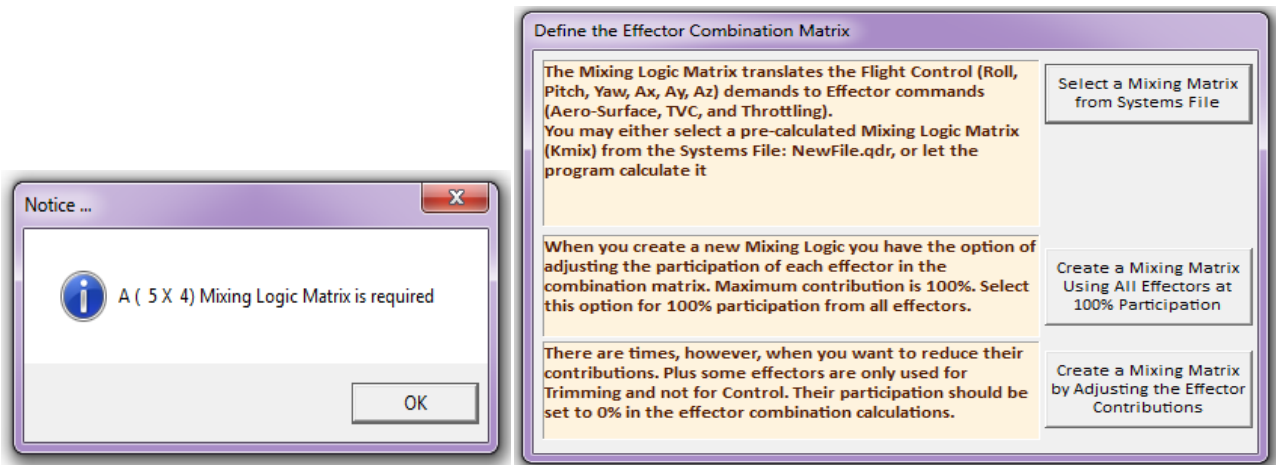
creating a negative moment against it. It also creates a positive roll and the aileron deflection is negative trying to take it out. The Elevon and Body-Flap trim deflections are the same as in the nominal trajectory. We may now click on "Exit Plots" to return to the main Trim menu.

## 1.6 Analyzing Performance Parameters along the Trajectory

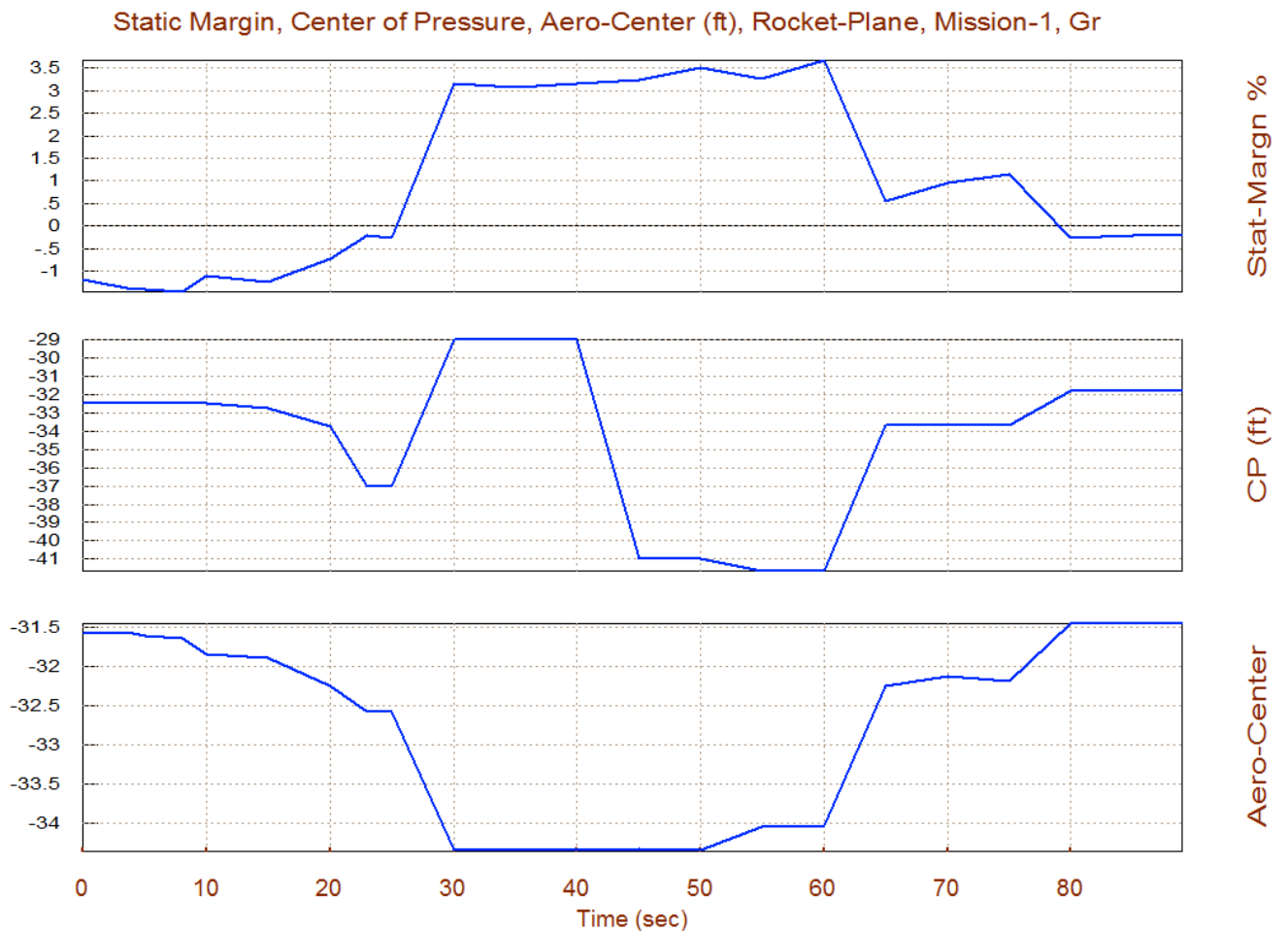
We will now use option (6) to plot the performance and stability parameters along the trajectory time. These parameters were discussed in Section 3. However, in order to calculate the vehicle static performance the program needs to know how the effectors combine together to control the four directions specified. The mixing logic matrix defines the effector control allocation along the four trimming directions. Roll, pitch, yaw, and axial directions will independently be controlled. Although at this point we are not designing a control system an effector combination matrix is required to define the four control loops and to decouple the accelerations between the four axes. This matrix is also needed for the calculation of control authority. It was not required for trimming because in trimming we are only balancing moments and are not distributing control. The analyst may use a constant pre-calculated mixing matrix and the program will select it from a systems file (.Qdr) and use it. In this case we do not have a mixing matrix picked and will let the program calculate it along the trajectory. The matrix must have 4 inputs for the 4 control directions (3 rotations plus x-acceleration), and 5 outputs (4 control surfaces plus the engine throttle control).

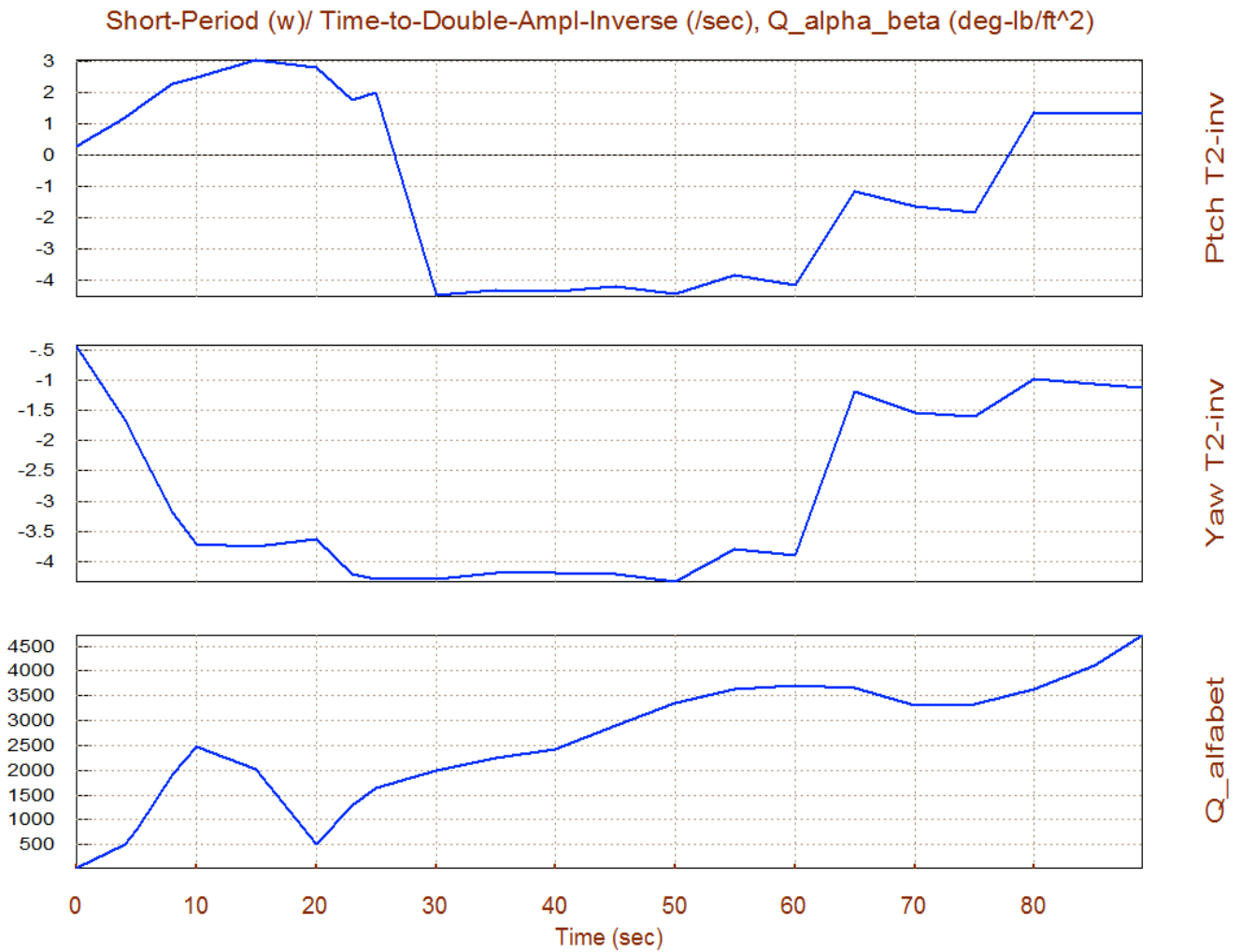


From the effector combination menu below, select the second option to let the program create a mixing matrix using 100% effector participation. Also, as we described in Section (3) we must also specify a worst case wind-shear environment which at steady-state is defined by the maximum dispersion angles ( $\alpha_{\max}$  and  $\beta_{\max}$ ) from trim. Enter  $2^\circ$  for both angles and the program will plot the performance parameters, as shown below.



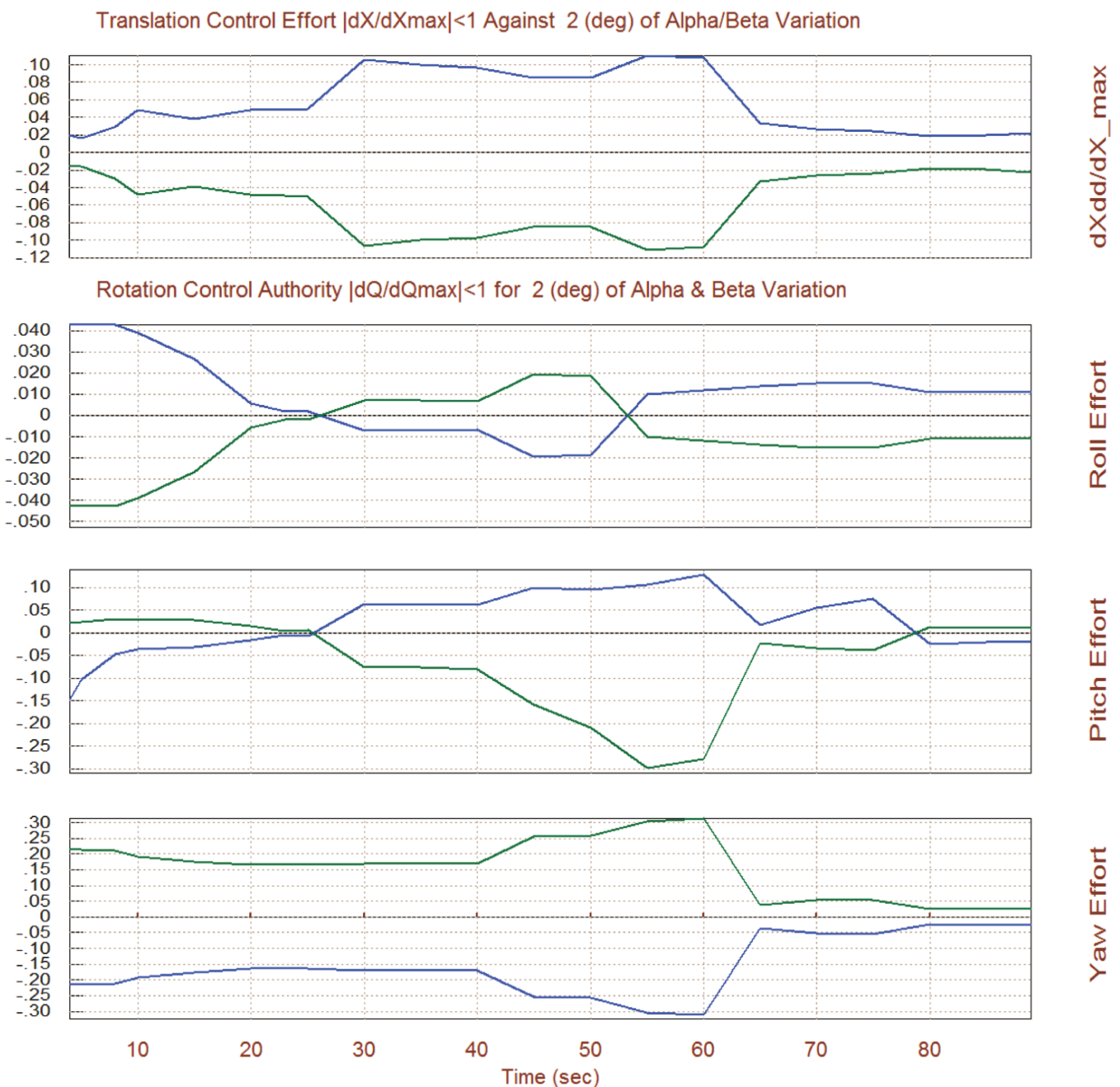
The first plot shows the static margin, Equation (3.12), the center of pressure, and the location of the aerodynamic center along the x axis in vehicle coordinates. The static margin indicates that initially the vehicle is longitudinally unstable, then it becomes stable for a period of 55 seconds, and towards the end of the boost phase it becomes unstable again.



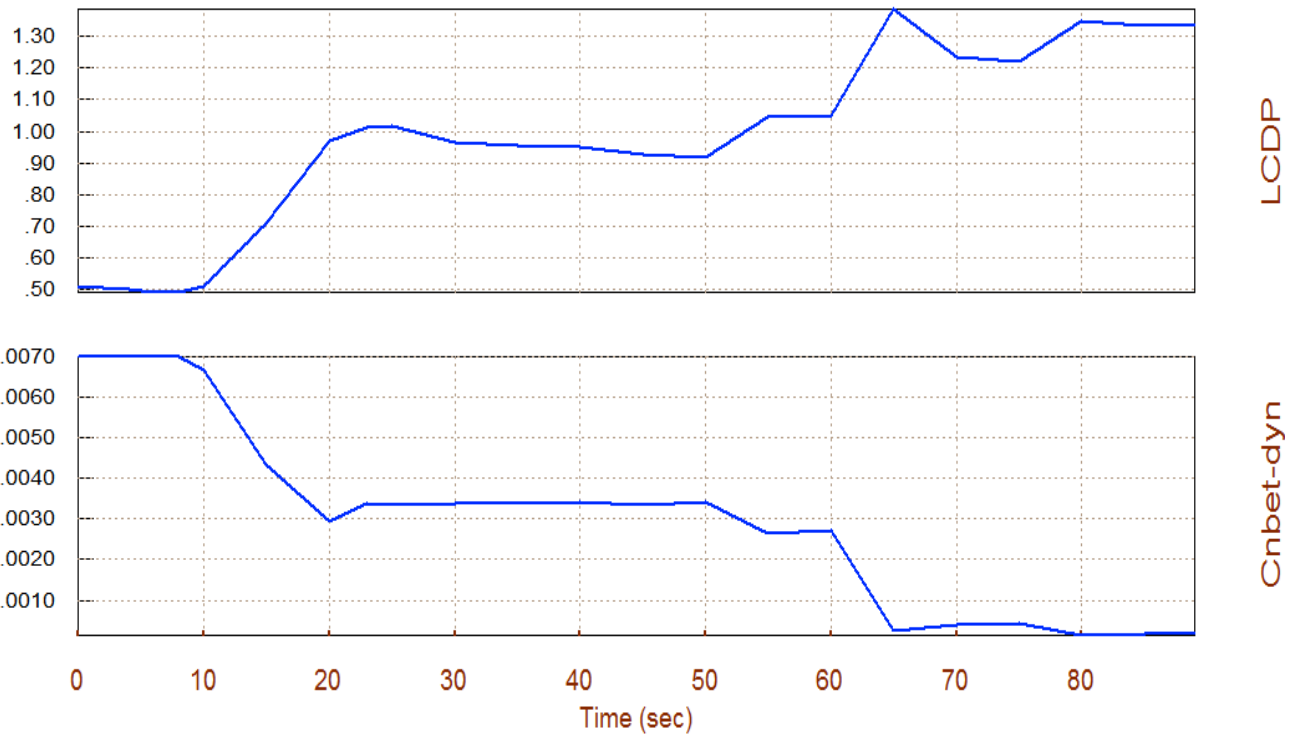


The pitch instability is also confirmed by the T2-inverse parameter which shows that at 15 seconds the time-to-double amplitude is short,  $T_2=0.33$  sec. It implies that the vehicle actuators must be fast enough in order to be able to catch-up with this fast rate of divergence. During the stable region the short period resonance reaches  $\omega_p=4.3$  (rad/sec) at  $T=30$  sec. Towards the end of boost the pitch axis becomes unstable again with a  $T_2=0.59$  sec. In the lateral direction the vehicle is stable during the entire boost phase with the "Dutch-Roll" resonance reaching to  $\omega_d=4.5$  (rad/sec) at 24 sec, and remaining at about the same level for a period of 40 seconds.

The combined ( $Q\alpha$  &  $Q\beta$ ) normal and lateral load indicator parameter is shown in the bottom plot. It is indicative of normal and lateral loading when the vehicle is flying at a constant  $\alpha_{max}=2^\circ$  and  $\beta_{max}=2^\circ$  dispersions. Its value is pretty high 4500 (psf-deg) towards the end of boost where the dynamic pressure is very high. This of course is conservative because the vehicle is not intended to fly at such a high angle of attack and sideslip during high  $Q_{bar}$ .

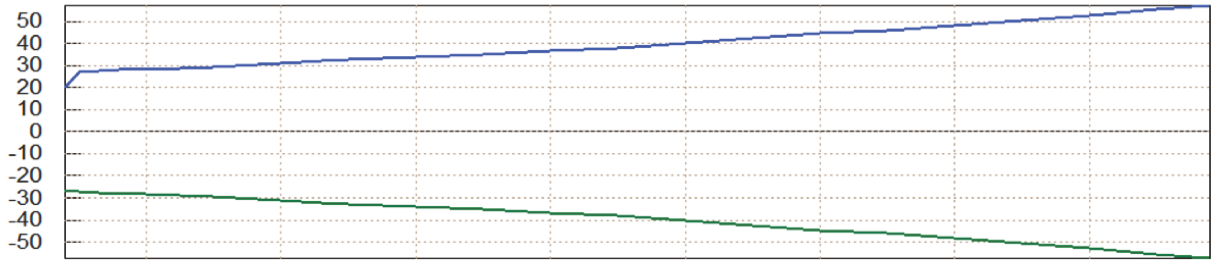


The above plot shows the control effort of the effectors system in the four controlled directions against  $\alpha$  or  $\beta$  dispersions as it was described in equations (3.24) through (3.26). The blue curves correspond to  $+\alpha_{max}$  in pitch or  $+\beta_{max}$  in lateral, and the green curves correspond to  $-\alpha_{max}$  or  $-\beta_{max}$  dispersions. The control efforts are sufficiently below the 0.5 limit in all 3 rotational directions and also in the x translation. It means that the control authority of the FCS is more than sufficient to counteract against moments and forces originating from the 2° of  $\alpha_{max}$  and  $\beta_{max}$  dispersions, plus it has additional control authority left for other functions (gusts, commands).



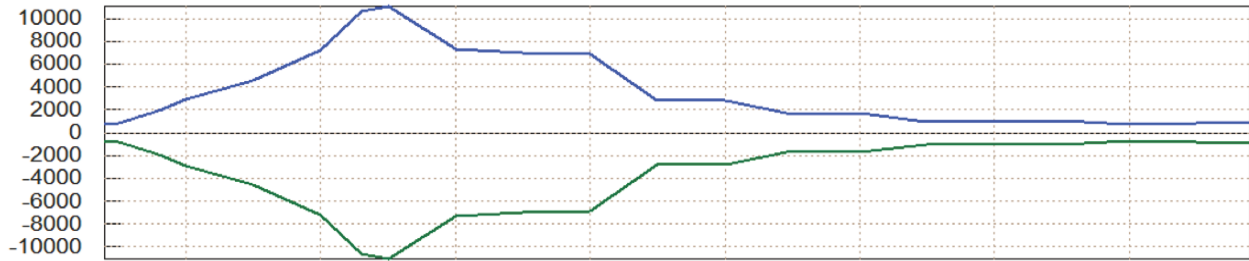
The bank angle with cross-wind is defined in equation 3.24 and it has no significant meaning in this case, it is only useful near landing. The LCDP ratio is described in equation 3.28 and in Figure 3.3. It is an indicator of roll axis dynamic controllability and in this case it looks pretty good because (a) it is positive and its magnitude is reasonably close to one, which is an ideal value, and (b) it does not change sign which causes roll reversals and unreliable situations. In fact it proves that the vehicle has excellent turn coordination. The  $C_n\beta$ -dynamic parameter, defined in equation 3.16, is positive throughout the boost phase which also shows that the vehicle is directionally stable. It drops pretty low in magnitude towards the end of the boost phase near neutral stability. This causes reduction in the yaw control effort.

Max Linear Accelerations in (ft/sec<sup>2</sup>), at Maximum +ve and -ve Control Demands

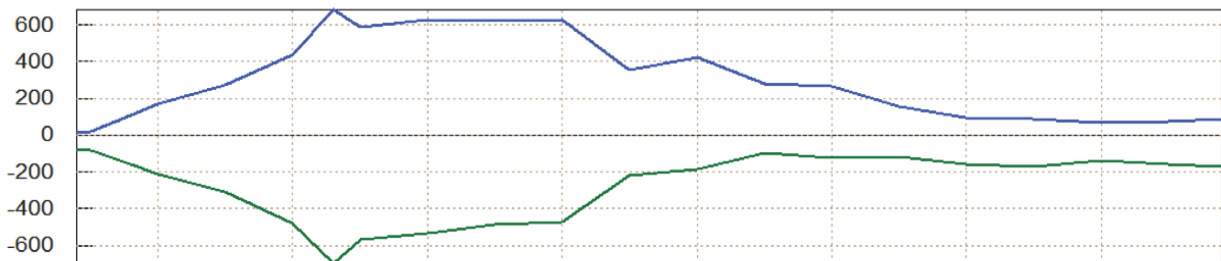


X-accel(Max)

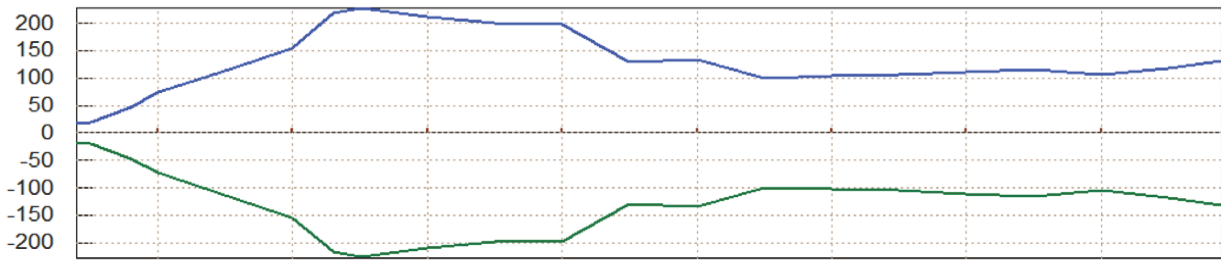
Max Angular Accelerations (rad/sec<sup>2</sup>), at Maximum +ve and -ve Control Demands



P\_dot (Max)



Q\_dot (Max)



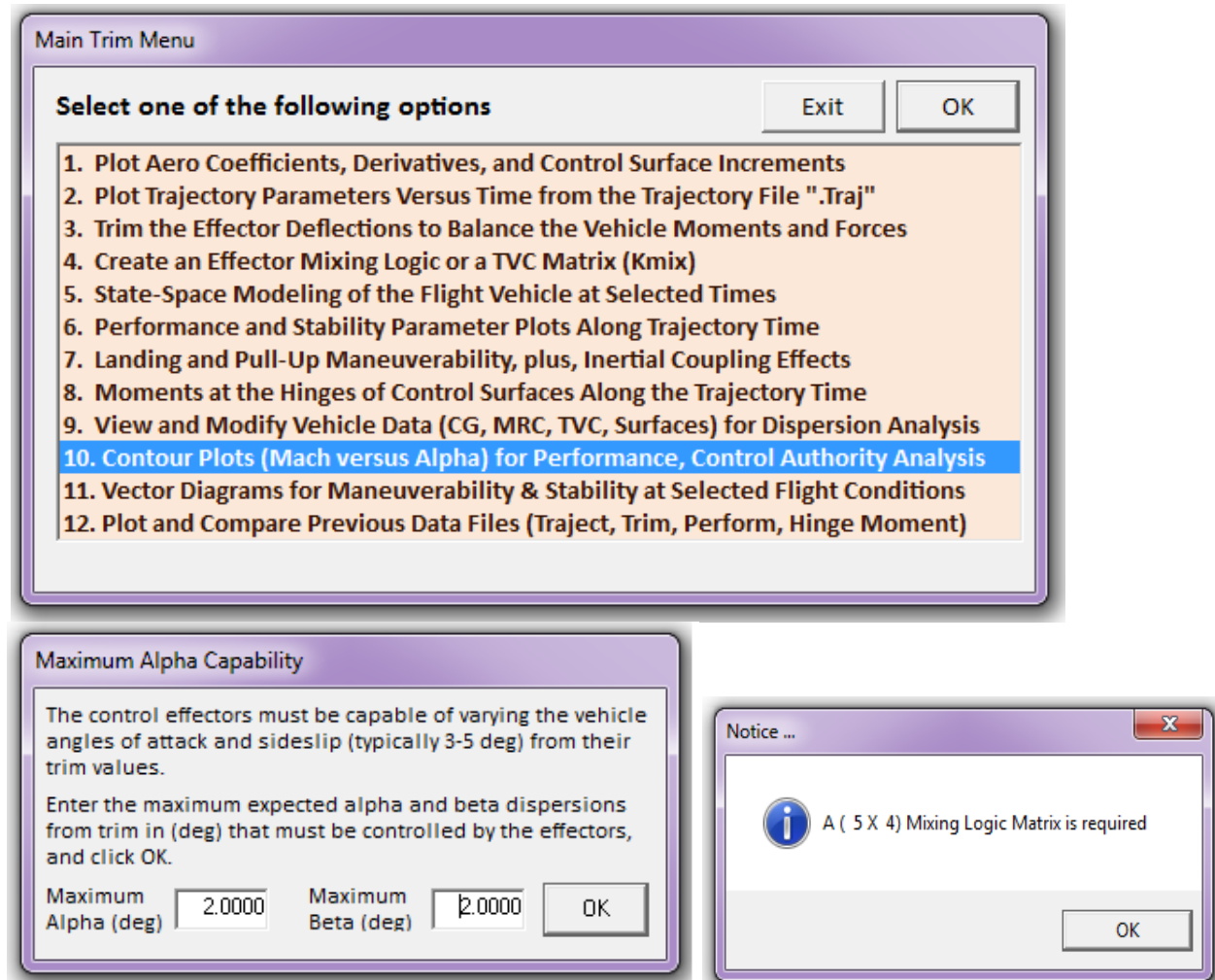
R\_dot (Max)

10 20 30 40 50 60 70 80  
Time (sec)

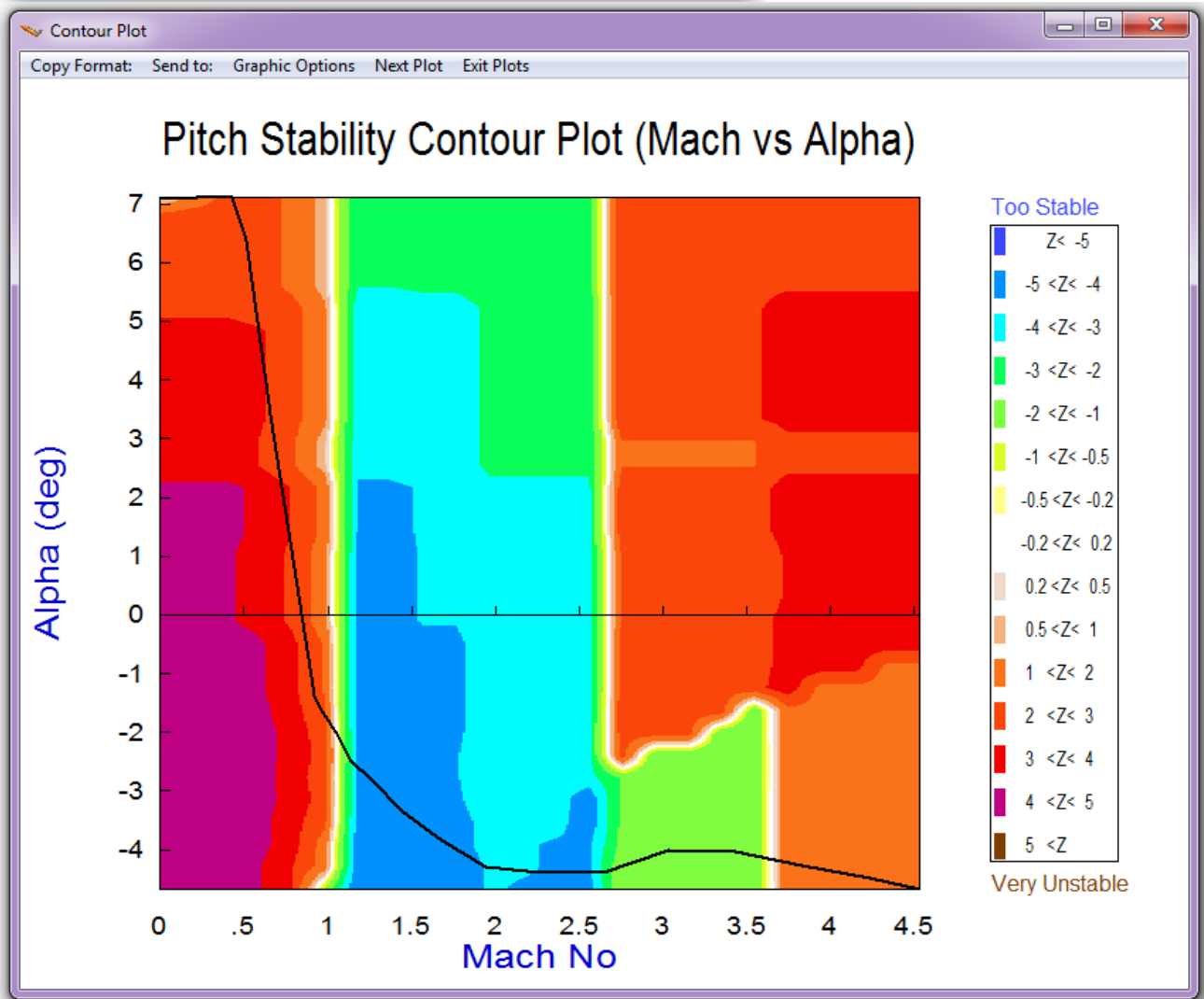
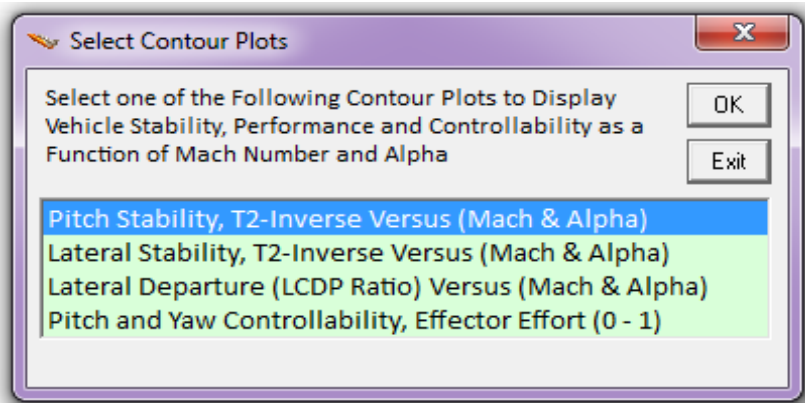
This plot shows the maximum acceleration capabilities from trim in the four controlled directions, at full positive control demand (blue) and full negative demand (green). That is, a translational acceleration along x in (feet/sec<sup>2</sup>), and 3 rotational accelerations in (rad/sec<sup>2</sup>). The maximum accelerations are described in equations (3.31 through 3.34).

## 1.7 Analyzing Stability & Performance during Ascent Using Contour Plots

Contour plots are 3-dimensional plots that allow us to visually inspect vehicle performance over the entire Mach versus Alpha range rather than in the vicinity of the trajectory. Sometimes the vehicle flight path may deviate from its expected alpha versus Mach trajectory and we would like to make sure that the planned trajectory is not close to any regions of unacceptable performance. Contour plots are also useful in understanding how to reshape trajectories and improving performance. From the main menu select option (10) to create contour plots of some important parameters against Mach and  $\alpha$ . Similar to our analysis in section 1.6 we must specify the steady-state aero disturbances which are defined by the maximum dispersion angles  $\alpha_{max}$  and  $\beta_{max}$  from trim, which are both  $2^\circ$ , as shown below. The program also requires the effector combination matrix, and since we do not have one selected we will allow the program to calculate it along the trajectory by selecting the second option in the Mixing Logic selection dialog. The matrix has 4 inputs corresponding to the 4 control directions and 5 outputs corresponding to the effector commands. Use the contour plot selection menu to select one of the performance parameters to be displayed color-coded, in a 2-dimensional array versus Mach and Alpha.

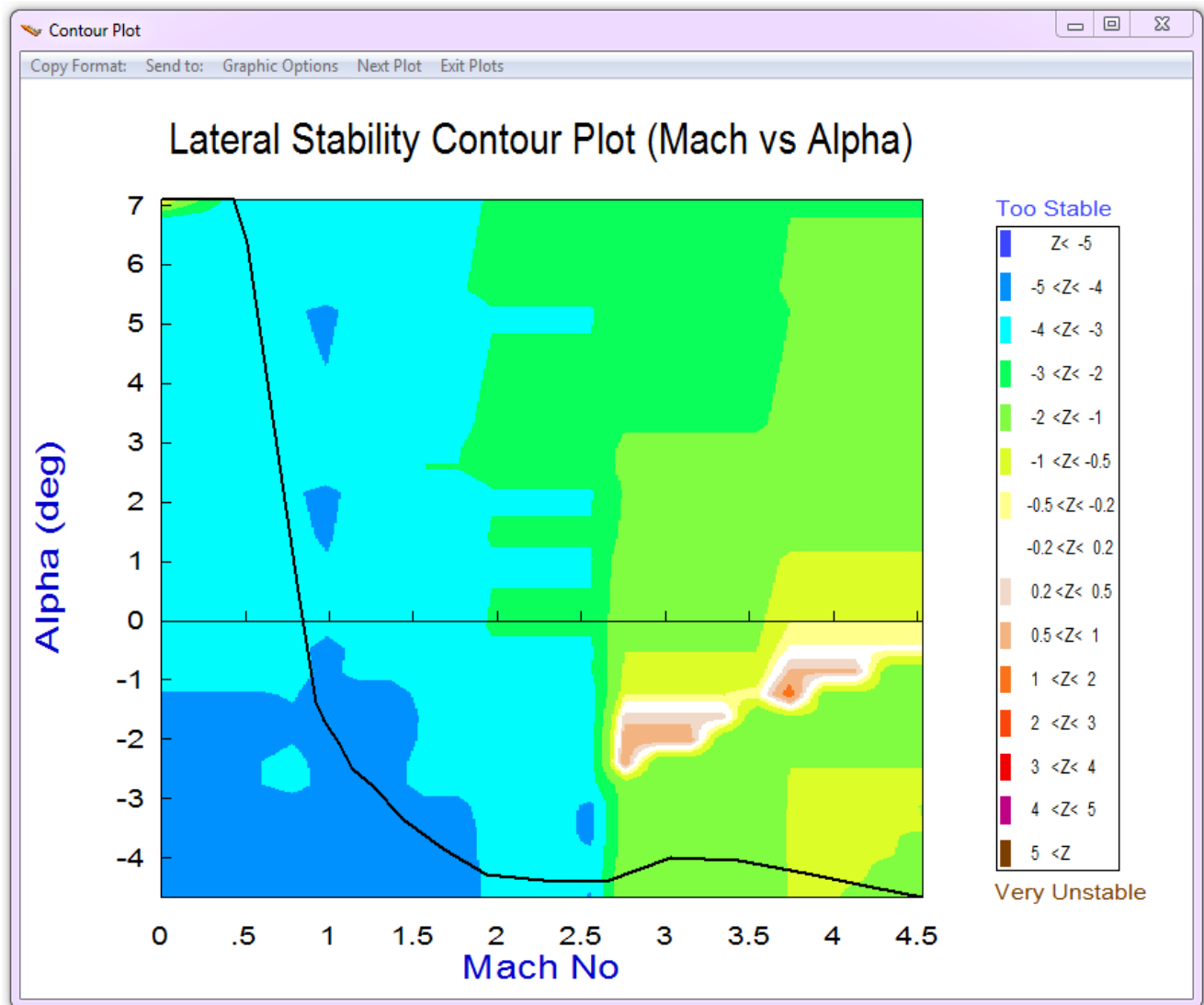




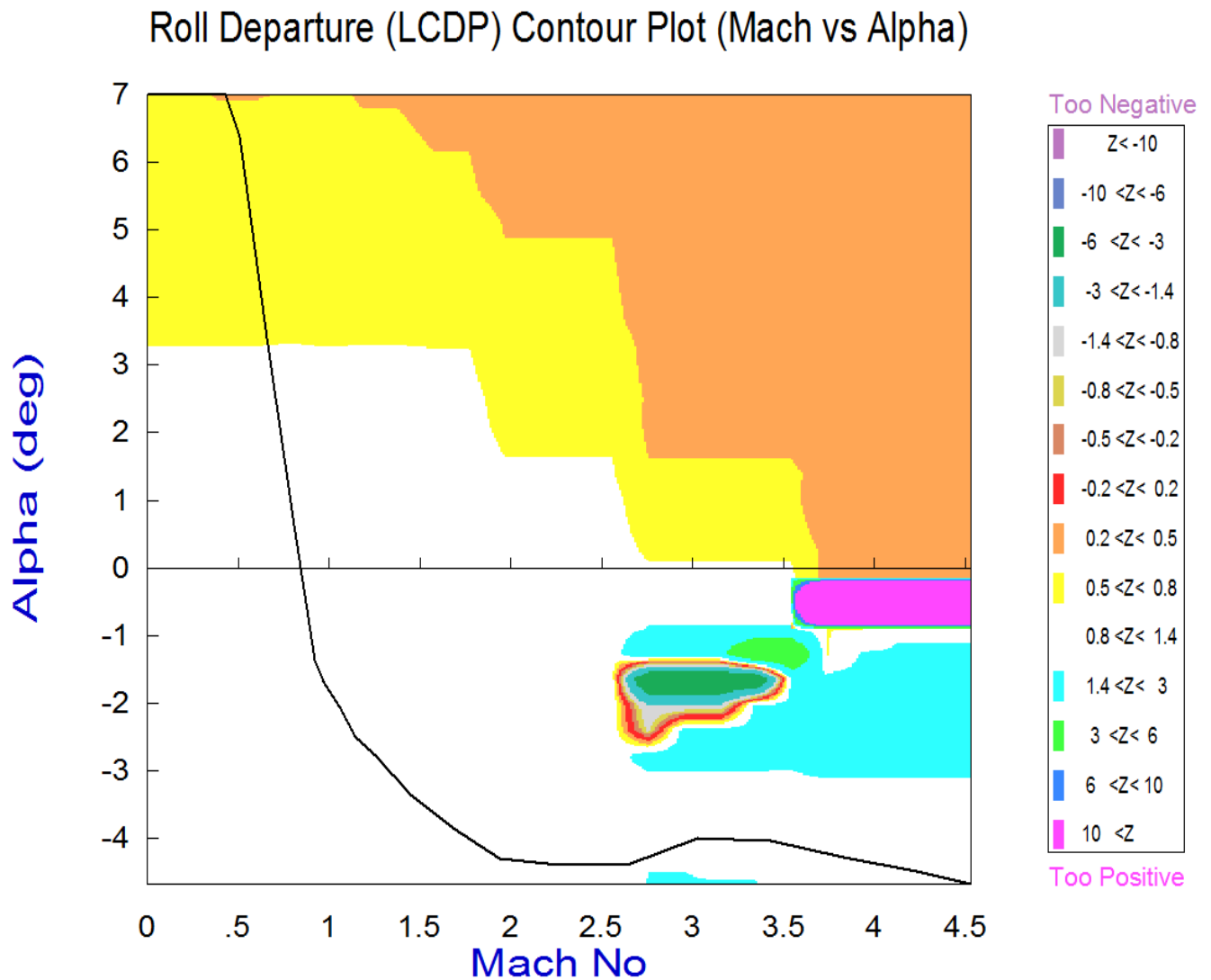


The first parameter to be shown is the pitch stability which was described in equations (3.14 & 3.15). The trajectory is represented by a black line across the Mach versus Alpha array. It starts at  $T=0$  (sec) in the upper left hand corner where ( $\alpha=7^\circ$ , Mach=0) and it ends at  $T=89$  (sec) in the lower right corner where ( $\alpha=-4.5^\circ$ , Mach=4.5). The color symbolizes the value of the performance parameter, which in this case it is T2-inverse for pitch stability. The color variations give us an indication on how pitch stability changes across the entire of Mach versus Alpha range. It also

shows us how the trajectory, illustrated by the black line, travels across the (Mach vs Alpha) field. By comparing our trajectory's performance relative to the overall performance of the array we can decide if there is a need to change trajectory course in order to improve it. In our example, we see that initially the vehicle is statically unstable as it crosses through the red orange region, with a time to double,  $T_2=1/Z$ , reaching 0.33 (sec). Then the trajectory crosses through the neutrally stable (white) region to the stable region (blue, cyan, green) where the short period resonance reaches  $\omega_p= 4.3$  (rad/sec). Then it crosses through the neutral white region again and it ends up in the unstable (orange) region towards the end of the boost phase. However, the stability parameter remains within acceptable bounds in the entire course avoiding the "Too Stable" or "Too Unstable" regions that should be avoided. In lateral directions the vehicle stability parameter is in the stable region through the entire trajectory with the Dutch-Roll resonance reaching  $\omega_d= 4.5$  (rad/sec), see equations (3.17 & 3.18). There are a couple of mildly unstable island regions at ( $\alpha=-2^\circ$ , Mach=3, and  $\alpha=-1^\circ$ , Mach=4) which are not crossed or located near the trajectory.

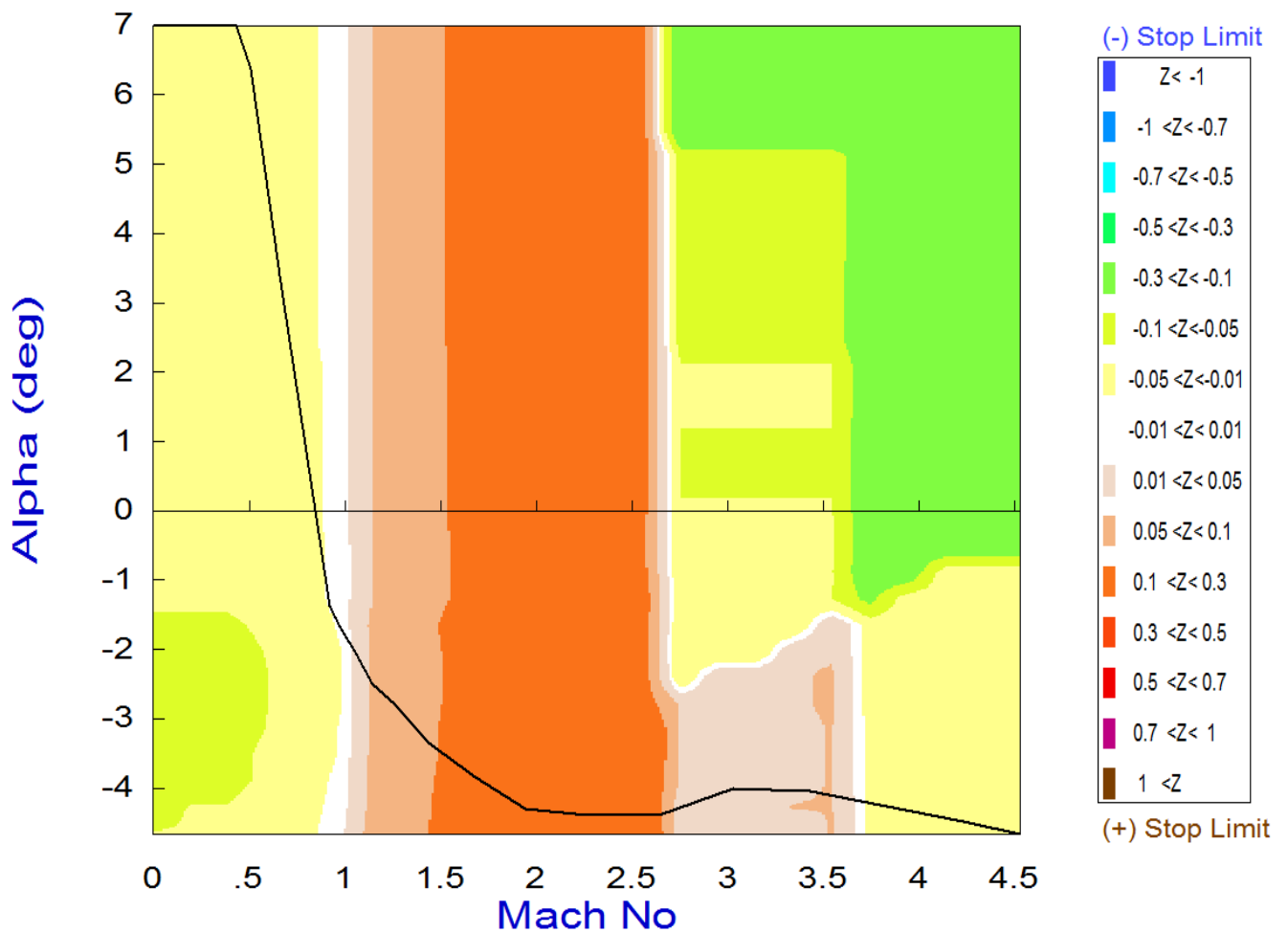


The roll departure LCDP ratio is also very good. It is entirely in the positive region (symbolized by bright colors) and mostly in the white region demonstrating that it has an excellent turn coordination, see Figures (3.3 & 4.2). There are a couple of bad regions, however, to be avoided. One of them is at ( $\alpha=-2^\circ$ , Mach=3) where the LCDP-ratio changes sign (dark colors surrounded by a red band). If the trajectory would pass through that region it would require reversal in the roll control gain twice and it would make it vulnerable to aero uncertainties. There is also a region at around ( $\alpha=-0.6^\circ$ , Mach=4) where the LCDP ratio exceeds 10 meaning that roll maneuvers in that region would induce too much beta transients. Fortunately, these regions are not crossed by the trajectory. In fact, our trajectory is sufficiently far from those regions.

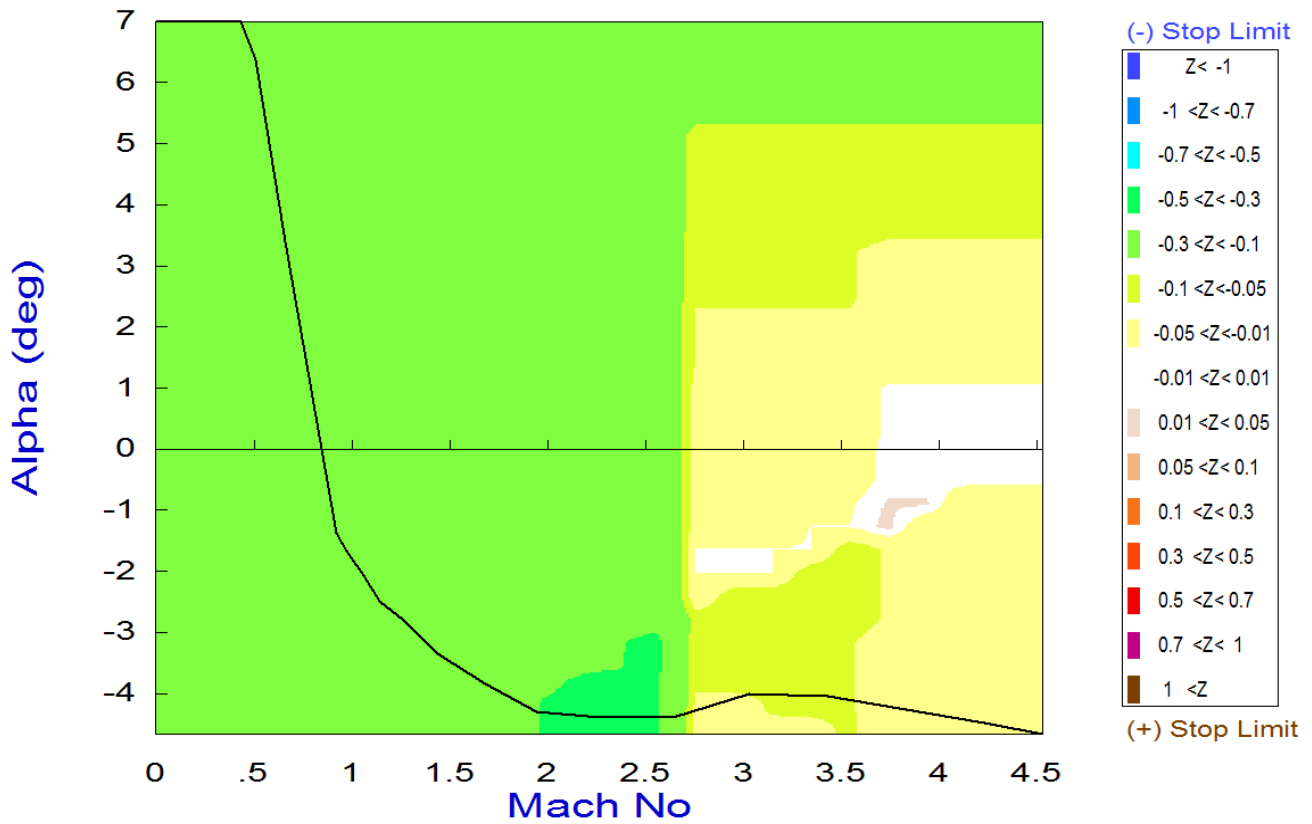


The next 3 plots show how the control authority of the effectors system varies against wind-shear disturbances which are defined by the dispersions  $\alpha_{\max} = 2^\circ$  and  $\beta_{\max} = 2^\circ$ . As it was already described in Section 3, the control effectors will overcome the disturbances when the control effort parameter is less than one, or even better, less than 0.5. In this case, the effector system satisfies the control effort requirement in all 3 axes. The worst control authority happens to be in yaw, where the control effort reaches 0.3. In pitch, the control direction changes sign a couple of times because the stability parameter also changes sign and the control direction reverses.

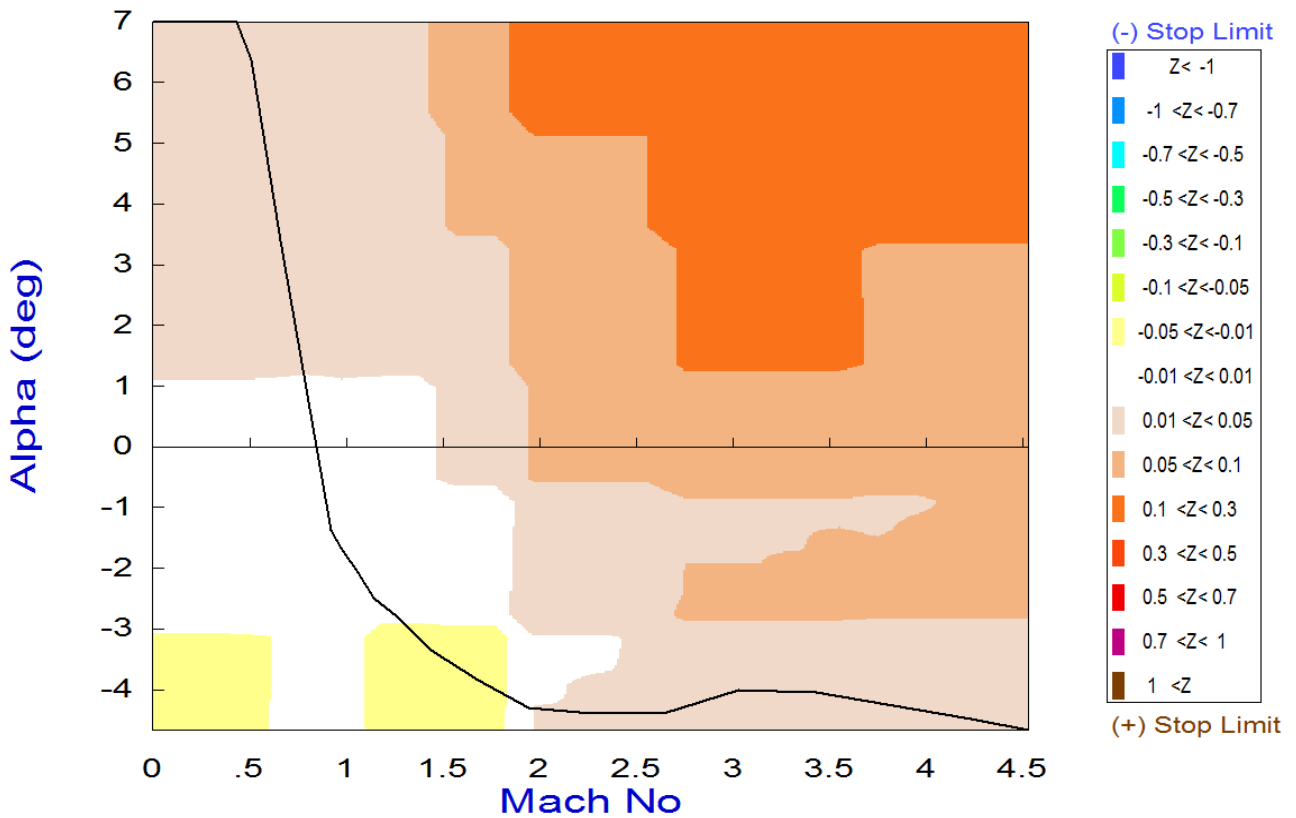
Pitch Control Effort Contour Plot (Mach vs Alpha)



### Yaw Control Effort Contour Plot (Mach vs Alpha)



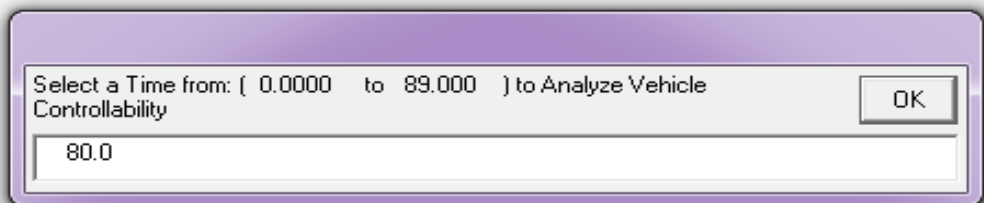
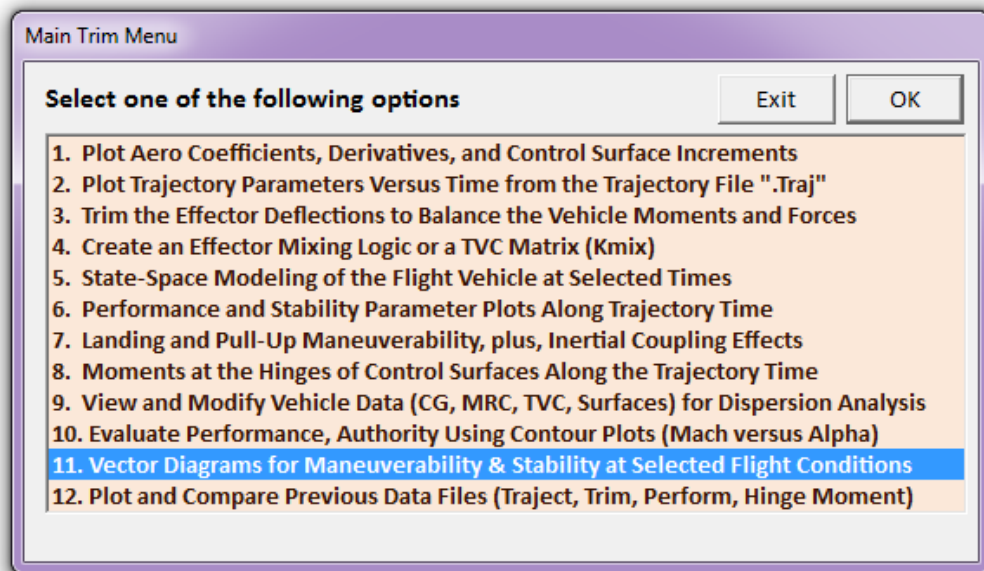
### Roll Control Effort Contour Plot (Mach vs Alpha)



## 1.8 Analyzing Maneuverability Using Vector Diagrams

Vector diagrams are 2-dimensional plots for analyzing vehicle controllability at a specified flight condition. We compare the control capability of the effector system in two directions against the effects of a wind-shear disturbance which is expressed by the dispersions it causes in the angles of attack and sideslip and determine if the effector system provides sufficient control authority to neutralize the disturbance moments and forces. This is not just a magnitude comparison but it also allows us to examine the directions of the controls against the disturbance. It helps us evaluate the orthogonality of the control system, compare the acceleration magnitudes of the controls against the wind disturbance, and determine if the controls are powerful enough and their directions are capable of counteracting the disturbance moments along roll, pitch, yaw, and axial directions in this case.

Assuming that the initialization files and trim conditions are the same as before, from the Trim menu select option-11 for plotting vector diagrams, and pick an arbitrary flight condition at  $t=80$  sec. The following dialog consists of menus used for selecting the vehicle mass, Mach number, and the angles of attack and sideslip. The default values correspond to the selected flight time. You may keep those parameters or select different values from the menus. In this case we select the default values and click "Select". The disturbances are caused by wind-shear defined by the maximum alpha and beta produced. In the following dialog enter the maximum disturbance angles ( $\alpha_{\max}$  and  $\beta_{\max}$ )= $2^\circ$ .



Select the following parameters

Select a Vehicle Mass, Mach Number, Alpha, and Beta from the lists below and click "Select"

Vehicle Mass (slug)	Mach Number	Angle of Attack (deg)	Angle of Sideslip (deg)
440.00	4.000	-4.00	0.00
827.00	0.9000	-4.00	-5.00
777.00	1.200	-2.00	0.00
726.90	1.500	0.00	5.00
676.90	2.000	2.00	
626.90	3.000	4.00	
576.80	4.000	6.00	
526.80	6.000	8.00	
522.40	8.000	10.0	
440.00	10.00	12.0	

Maximum Alpha Capability

The control effectors must be capable of varying the vehicle angles of attack and sideslip (typically 3-5 deg) from their trim values.

Enter the maximum expected alpha and beta dispersions from trim in (deg) that must be controlled by the effectors, and click OK.

Maximum Alpha (deg)  Maximum Beta (deg)

Define the Effector Combination Matrix

The Mixing Logic Matrix translates the Flight Control (Roll, Pitch, Yaw, Ax, Ay, Az) demands to Effector commands (Aero-Surface, TVC, and Throttling). You may either select a pre-calculated Mixing Logic Matrix (Kmix) from the Systems File: NewFile.qdr, or let the program calculate it

Select a Mixing Matrix from Systems File

When you create a new Mixing Logic you have the option of adjusting the participation of each effector in the combination matrix. Maximum contribution is 100%. Select this option for 100% participation from all effectors.

Create a Mixing Matrix Using All Effectors at 100% Participation

There are times, however, when you want to reduce their contributions. Plus some effectors are only used for Trimming and not for Control. Their participation should be set to 0% in the effector combination calculations.

Create a Mixing Matrix by Adjusting the Effector Contributions

From the menu bar at the top, click on "*Select Vector Diagrams*" and then from the vertical pop-up menu select "*Moments per Max Controls, and per Max Alpha*". You may also select "*Accelerations per Max Control and per Max Alpha*", as shown in Figure 1.8.1, which plots the roll and yaw moments and accelerations produced when the roll and yaw FCS demands are maximized to effector saturation values. The solid blue vector corresponds to max positive yaw FCS demand  $\delta R_{+FCS\_Max}$  and the dashed blue vector corresponds to max negative yaw demand  $\delta R_{-FCS\_Max}$ . Similarly, the green vectors correspond to the peak roll FCS demands  $\delta p_{\pm FCS\_Max}$ . The control moments are pointing close to their intended directions but not exactly, there is some roll to yaw cross-coupling, but notice that the control accelerations are pointing exactly in the commanded directions without any cross-coupling. This is because the effector mixing matrix is compensating against the  $I_{xz}$  product of inertia. The two small red vectors show the moments and angular accelerations generated by the disturbance variations in the angles of attack and sideslip  $\pm\alpha_{max}$  and  $\pm\beta_{max}$  from their trim positions. The red rectangles represent uncertainties in  $C_l$  and  $C_n$  due to  $\pm\beta_{max}$  and although the red vectors are small the uncertainty rectangles are large. The yellow and cyan rectangles around the tips of the control vectors represent the uncertainties in roll/yaw control moments and accelerations. The figure shows that roll and yaw controllability exceeds the disturbance moments and accelerations generated by the  $\pm\alpha_{max}$  and  $\pm\beta_{max}$  dispersions, including the uncertainties envelope. The uncertainties are read from file "*Hyp\_Ascent.Unce*".

The vector diagrams in Figure 1.8.2 show the effector system controllability in two longitudinal directions, pitch and axial acceleration. This time the vectors are not symmetric as in the lateral directions. It appears that it is twice easier to torque and rotate the vehicle in the negative pitch direction when you apply a max negative pitch control than it is to rotate it in the positive direction with a maximum positive control. This is because the vehicle is statically unstable in this flight condition and it is flying with a negative  $\alpha = -4^\circ$ , and therefore, its natural tendency would be to rotate in the negative direction and to produce more negative pitch acceleration. There is also unsymmetry in the x acceleration control. It is easier for the vehicle to accelerate when applying a maximum positive throttle when alpha is negative than it is to slow it down with less thrust (negative throttle). Despite the lack of symmetry in the controls the aero disturbance in the pitch and x-acceleration directions is very small which indicates that the vehicle is controllable in both pitch and axial directions.



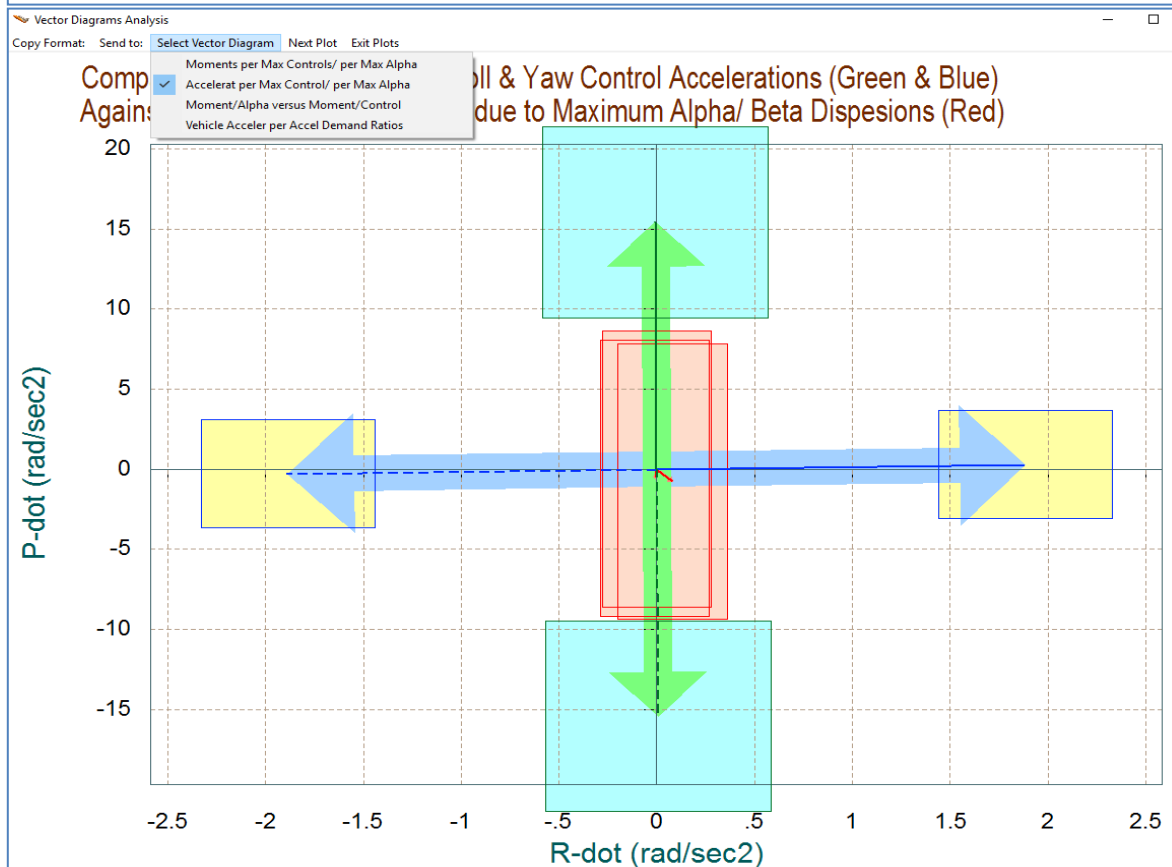
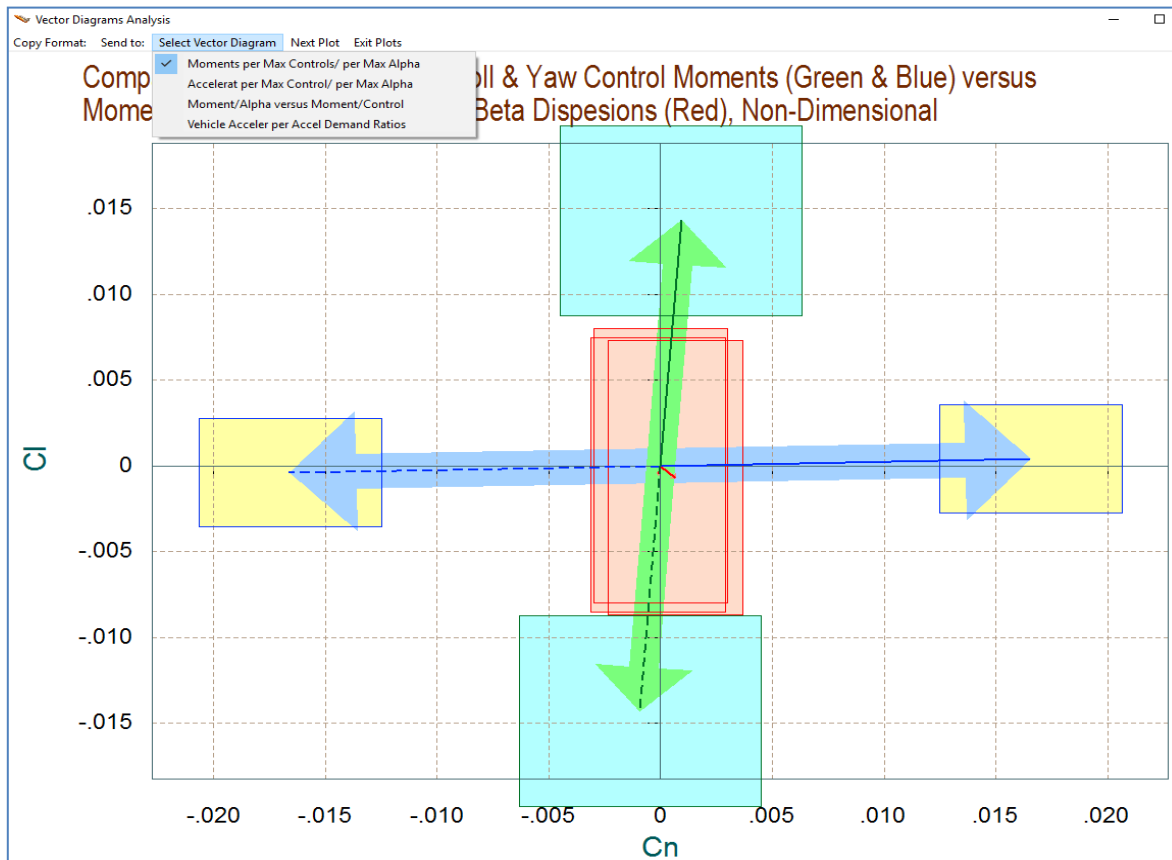
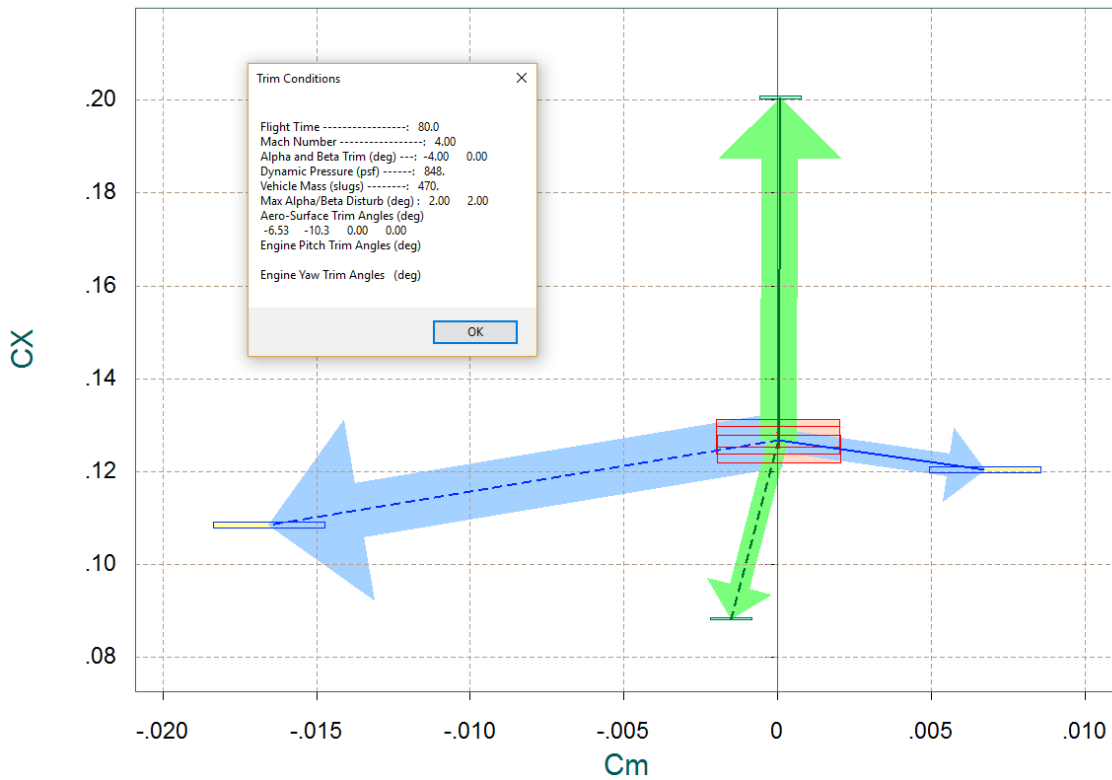


Figure 1.8.1 Maximum Roll & Yaw Moments and Accelerations Vector Diagrams Including Uncertainties

Comparison between Maximum Pitch Control Moment and Axial X-Force (Blue & Green) Against Disturbances due to Maximum Alpha Variation (red), Non-Dimensional



Comparison between Maximum Pitch and Axial X-Force Control Accelerat (Blue & Green) Against Aero Disturbance due to Maximum Alpha Variation (red)

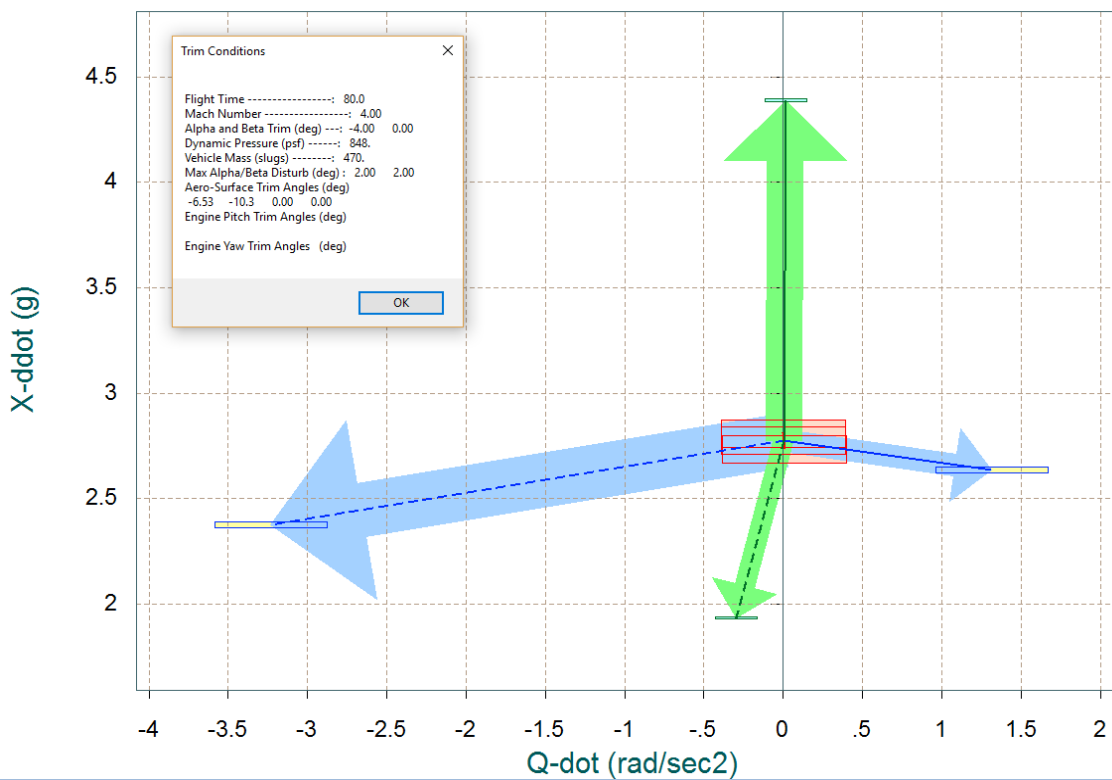


Figure 1.8.2 Maximum Pitch Moment, Axial Force, and Accelerations Vector Diagram Including Uncertainties

You may now go back to the top menu bar and click on "*Select Vector Diagrams*" and then from the vertical pop-up menu select "*Moments per Alpha versus Moments per Control*" to plot vector diagram partials. Click on "Next Plot" until you see the vector diagrams shown in Figure 1.8.3. These are partials vector diagrams in the longitudinal directions. The blue vectors show the pitch moment, normal and axial force partials per pitch acceleration demand  $\{C_m\delta Q_{FCS}, C_x\delta Q_{FCS}, C_z\delta Q_{FCS}\}$ . It shows how a pitch demand produces a positive effect in pitch and z acceleration. The pitch demand has no effect in the x direction. The green vector represents the partials  $\{C_m\delta X_{FCS}, C_x\delta X_{FCS}\}$ . It is the effect of an axial acceleration demand in pitch moment and x-force, and it is entirely in the  $C_x$  direction. The red vectors are the  $\{C_{x\alpha}, C_{z\alpha}, C_{m\alpha}\}$  partials. They are in pairs because they are calculated at the two extreme values of  $\pm\beta_{max}$ . They are scaled to make them comparable with the control partials as described in equations (7.1 through 7.4).

The vector diagram partials in Figure 1.8.4 show the variation in roll and yaw moments per roll and yaw acceleration demands in  $(rad/sec^2)$ . The blue vector is  $\{C_n\delta R, C_l\delta R\}$  and it is entirely in the yaw direction. The green vector is  $\{C_n\delta P, C_l\delta P\}$  and affects mainly roll but it also couples a little into yaw. The bottom figure also shows the sideforce per yaw acceleration demand. The red vectors are the  $\{C_n\beta, C_l\beta, C_y\beta\}$  partials. The red rectangle centered at the tip of the  $\{C_n\beta, C_l\beta\}$  vectors is due to the uncertainties in the two partials obtained from file "*Hyp\_Ascent.Unce*". Also the yellow and the cyan rectangles at the tips of the control vector partials represent the uncertainties in the control partials. The diagrams show that the controls are nicely decoupled from each other and that each control is affecting their corresponding direction. All directions are easily controllable because the disturbance vectors with respect to  $\alpha$  and  $\beta$  are relatively small in comparison with the control vectors, even with uncertainties.

The last set of vector diagrams in Figures 1.8.5 is acceleration partials. Go back to the top menu bar and click on "*Select Vector Diagrams*" and then from the vertical pop-up menu select "*Vehicle Accelerations per Acceleration Demand Ratios*". These are partials of accelerations achieved per accelerations demanded in specific directions. The first figure shows the longitudinal partials of accelerations per acceleration demands in pitch and axial directions. The blue vector is  $\{\dot{Q}/\delta Q_{FCS}, \ddot{X}/\delta Q_{FCS}\}$ , and the green vector is  $\{\dot{Q}/\delta X_{FCS}, \ddot{X}/\delta X_{FCS}\}$ . The second figure shows the partials of accelerations per acceleration demands in roll and yaw. The green vector is  $\{\dot{P}/\delta P_{FCS}, \dot{R}/\delta P_{FCS}\}$ , and the blue vector is  $\{\dot{P}/\delta R_{FCS}, \dot{R}/\delta R_{FCS}\}$ . The axis units are in  $(rad/sec^2)/(rad/sec^2)$  for rotational accelerations or in  $(ft/sec^2)/(ft/sec^2)$  for translational accelerations. The acceleration partials analyze the effectiveness of the effector mixing logic to properly scale and to decouple the accelerations in the controlled directions. The mixing logic matrix in this case was created by the Flixan algorithm specifically for the t=80 sec flight condition. All vectors are unit vectors pointing in their corresponding directions and they are completely decoupled, indicating that controllability is achievable in all 4 directions, and the accelerations achieved are equal to the demanded accelerations.

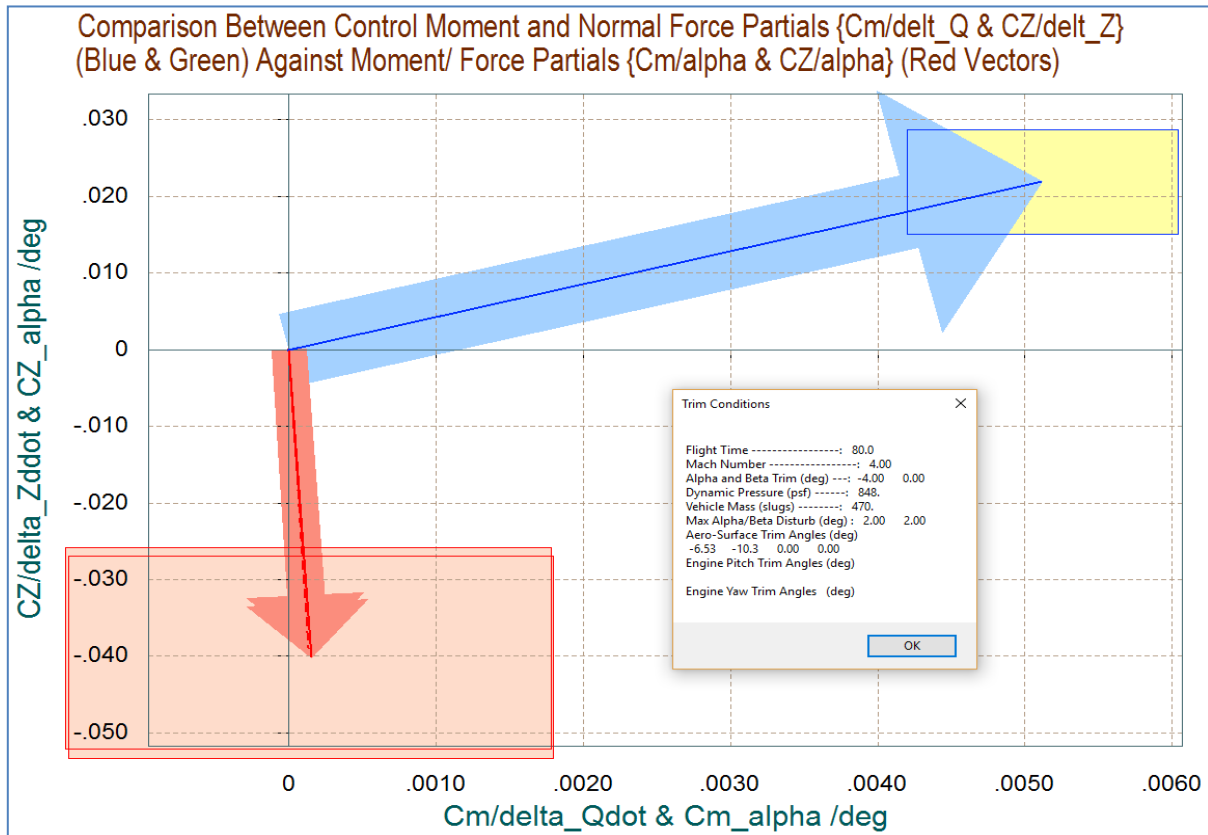
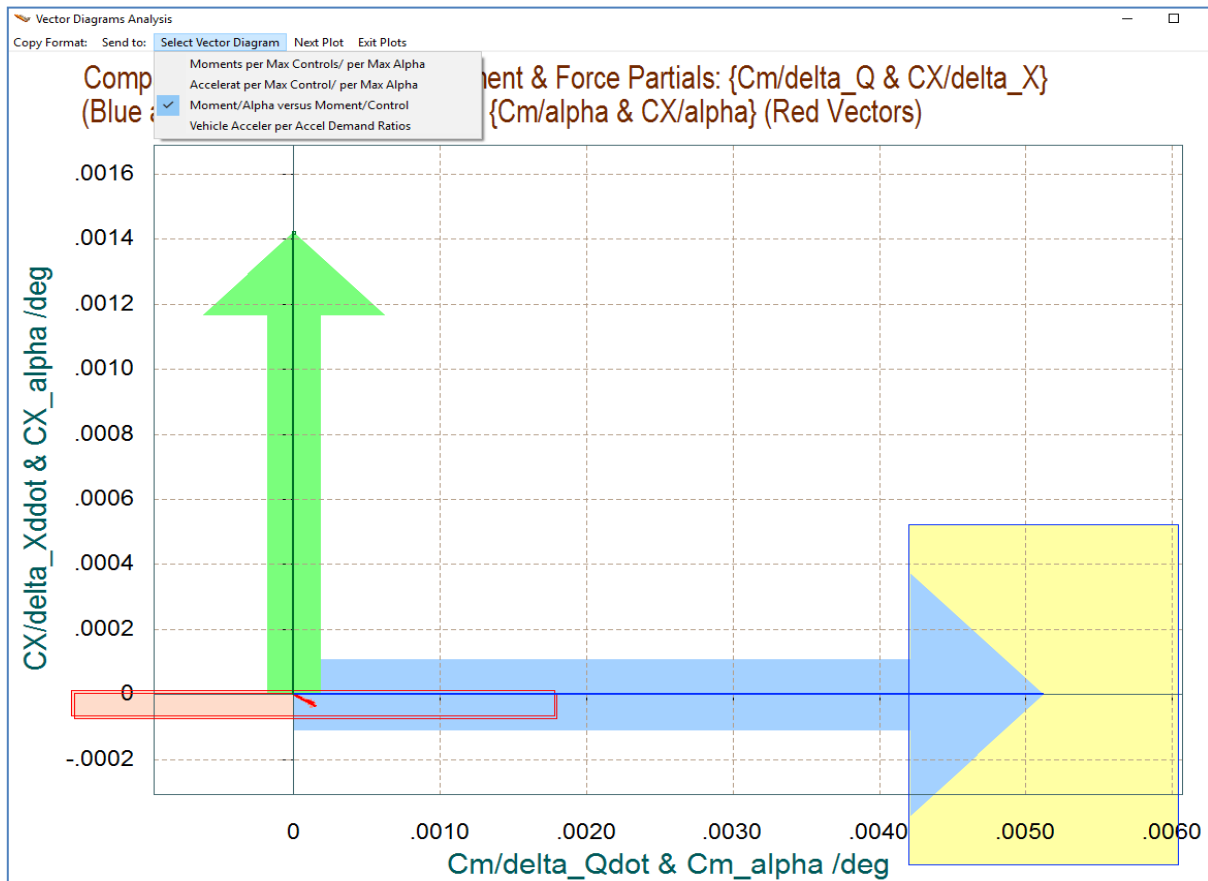


Figure 1.8.3 Moment and Force Partial in the Longitudinal Directions

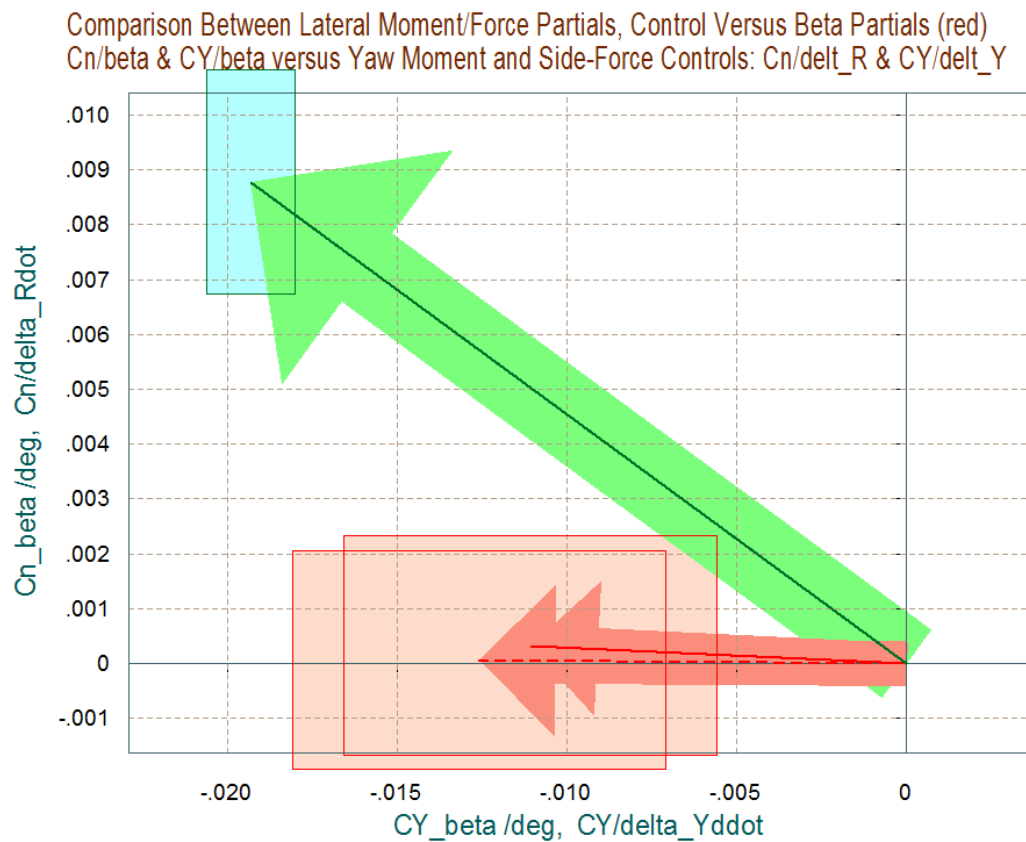
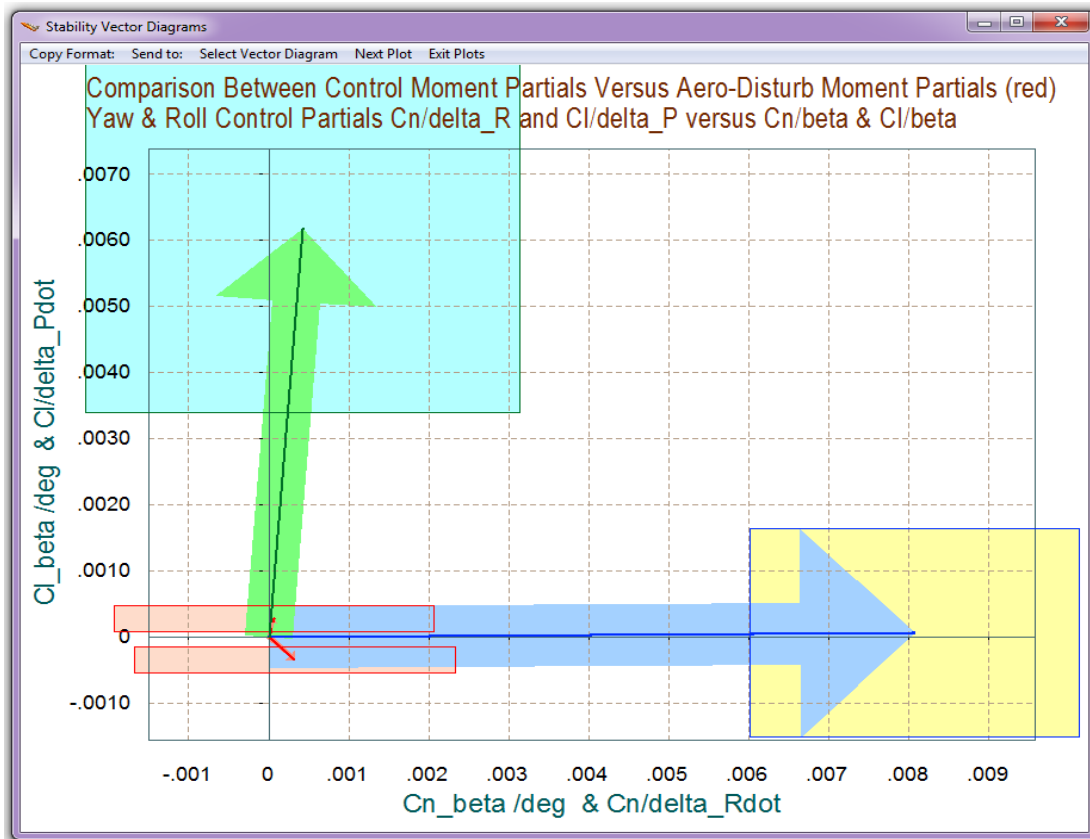


Figure 1.8.4 Roll, Yaw, and Sideforce Partial with respect to Roll & Yaw Control Demands, and Beta

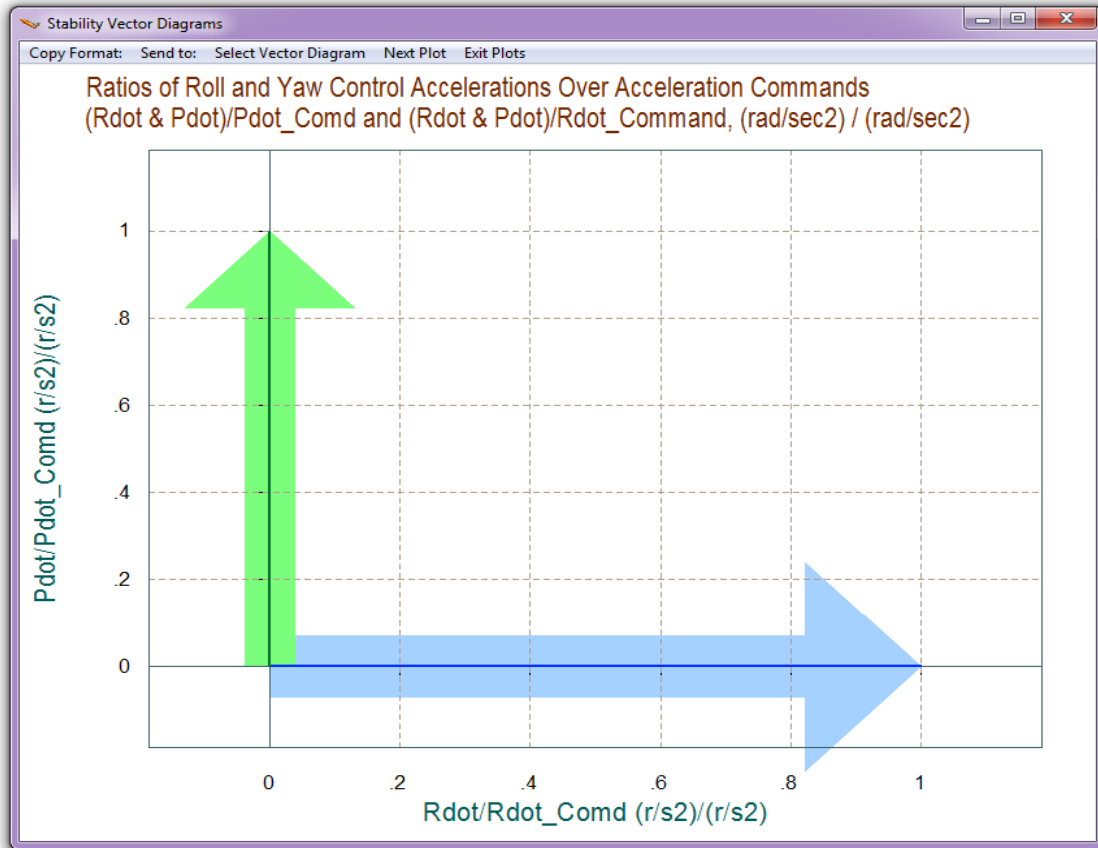
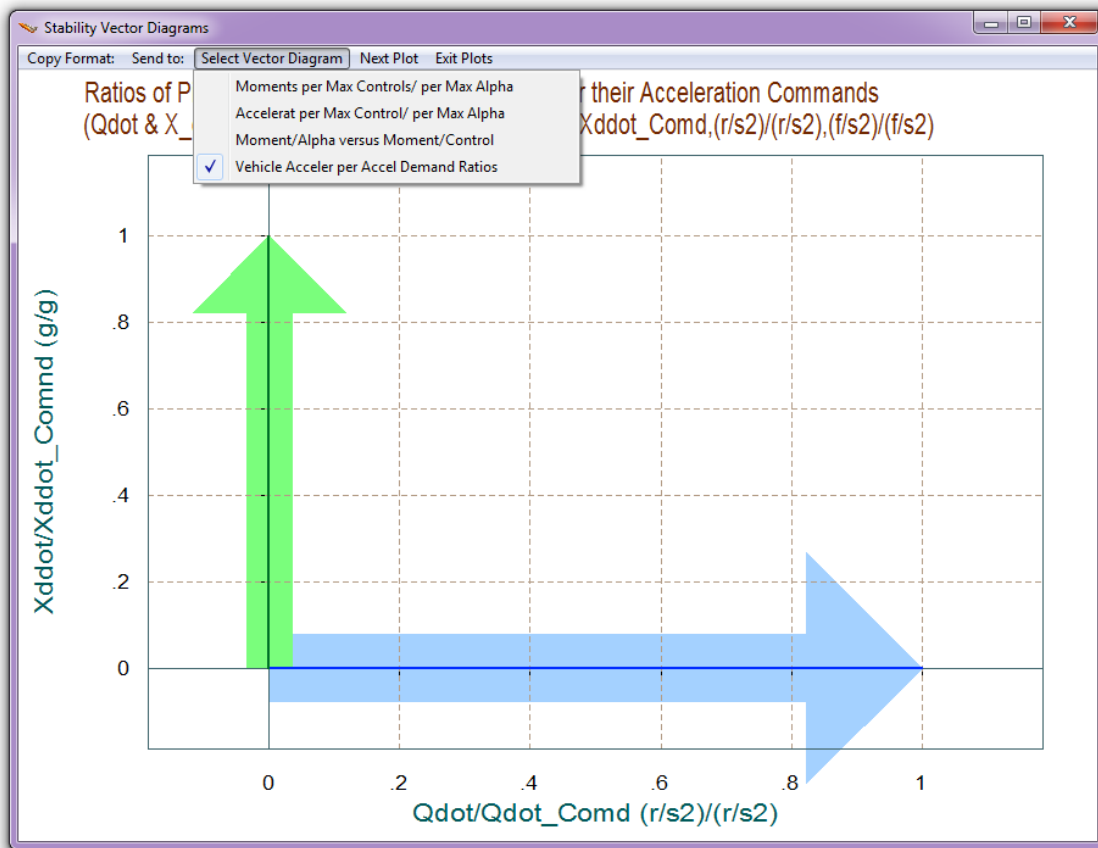


Figure 1.8.5 Acceleration Partial with respect to Acceleration Demands

## Wind-speed Variations Vector Diagrams

In a similar manner we can analyze the effects on the vehicle produced by variations in the air-speed relative to the nominal speed  $V_0$ . From the Trim menu select vector diagrams (11) again, and a different flight time  $t=50$  sec, and an airspeed variation  $v_{max} = \pm 500$  (feet/sec) from nominal  $V_0$ . From the menus below select the default mass, Mach #,  $\alpha$  and  $\beta$  values, and in the next dialog allow the program to generate an effector combination matrix with full 100% participation from all effectors (second option).

Select a Time from: ( 0.0000 to 89.000 ) to Analyze Vehicle Controllability

50.0

**Maximum Aero Disturbances**

The control effectors must be capable of varying the vehicle angles of attack and sideslip (typically 3-5 deg) from their trim values.

Enter the worst expected alpha and beta dispersions in (deg), and also delta-velocity in (ft/sec) from trim that must be controlled by the effectors, and click OK.

Maximum Alpha (deg)  Maximum Beta (deg)

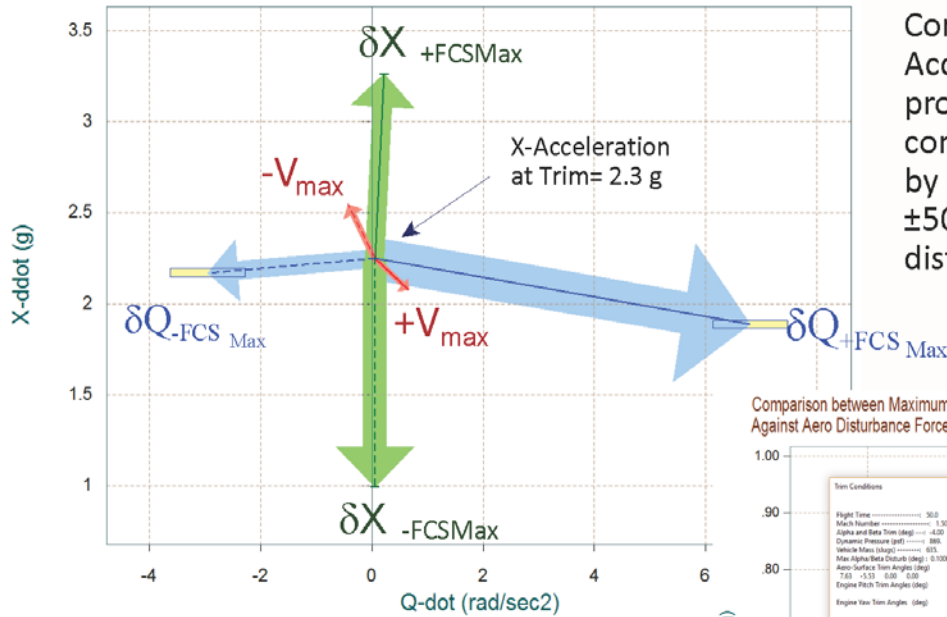
Maximum Change in Velocity due to Wind in (feet/sec)

Select the following parameters

Select a Vehicle Mass, Mach Number, Alpha, and Beta from the lists below and click "Select"

Vehicle Mass (slug)	Mach Number	Angle of Attack (deg)	Angle of Sideslip (deg)
626.90	1.500	-4.00	0.00
827.00	0.9000	-4.00	-5.00
777.00	1.200	-2.00	0.00
726.90	1.500	0.00	5.00
676.90	2.000	2.00	
626.90	3.000	4.00	
576.80	4.000	6.00	
526.80	6.000	8.00	
522.40	8.000	10.0	
440.00	10.00	12.0	

Comparison between Maximum Pitch and Axial X-Force Control Accelerat (Blue & Green)  
Against Aero Disturbance due to Delta-Velocity Dispersion (red)



Comparison of Vehicle Accelerations at t=50 sec, produced by maximizing the controls from Trim, and also by airspeed variations  $V_{max} = \pm 500$  (ft/sec) due to wind disturbances.

Blue and Green Vectors show the variations in Pitch, Axial, and Normal Acceleratios produced by the maximization of the Pitch and Axial controls. The Red Vectors show the acceleration variations generated by the air-speed variation  $\pm V_{max}$  from  $V_0$

Comparison between Maximum X and Z Control Accelerations in g (Blue & Green)  
Against Aero Disturbance Forces due to Delta-Velocity Dispersion (Red)

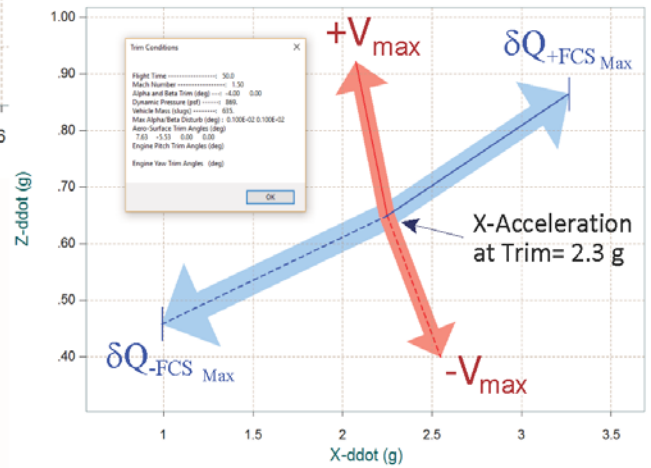
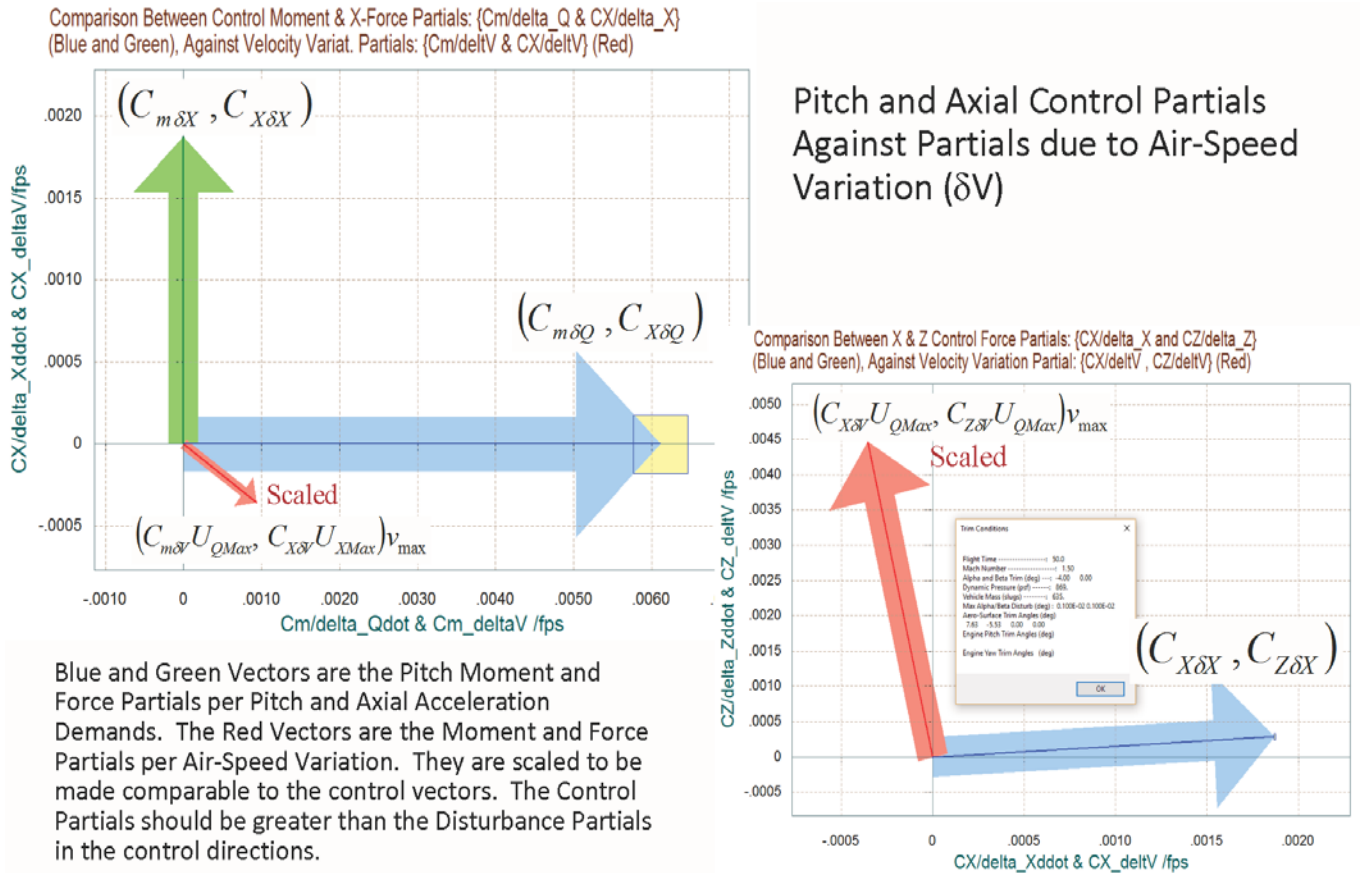


Figure 1.8.6 Maximum Accelerations Produced by the Pitch and Thrust Variation Controls and also by Wind Variations  $\pm v_{max}$

In Figure 1.8.6 the vehicle has an x-acceleration 2.3 g. The blue and green vectors show the maximum pitch and axial accelerations that can be achieved by the pitch and axial acceleration controls. The red vectors show the acceleration variations generated by the  $\pm v_{max}$  wind variations along the velocity vector. Obviously a head-wind  $+v_{max}$  produces negative or rather a less positive acceleration and a tail-wind will produce a more positive acceleration, as shown. Since the vehicle is statically unstable and flying with a negative angle of attack ( $\alpha_0 = -4^\circ$ ) in this flight condition, an increase in airspeed  $+v_{max}$  produces positive pitching moment and a positive (down) z-acceleration, as shown in the lower right diagram of Figure 1.8.6. The control accelerations are obviously greater than the accelerations produced by the airspeed variations.





**Figure 1.8.7 Pitch Moment and Axial Force Control Partial against Scaled Partial per Airspeed Variations; it shows that the control partials dominate the disturbance partials.**

Figure 1.8.7 shows the partials of the pitch moment and axial force per control demands against the partials of the pitch moment and axial force per variation in airspeed ( $\delta V$ ). It shows that the effector combination matrix, designed by the program, perfectly decouples the two control directions because both control partials (blue and green vectors) are pointing in their proper directions. The disturbance partials (per  $\delta V$ ) are the red vectors and they are scaled to be made comparable with the control partials, as already described. It shows that the vehicle is statically unstable and an increase in airspeed produces a significant positive z-force (down) due to the negative  $\alpha_0$ . In this configuration, we do not have a control allocation in the z-direction to counteract the effect of the  $\delta V$  disturbance. In the pitch and x-axis directions, however, the control partials are significantly greater than the scaled disturbance partial.

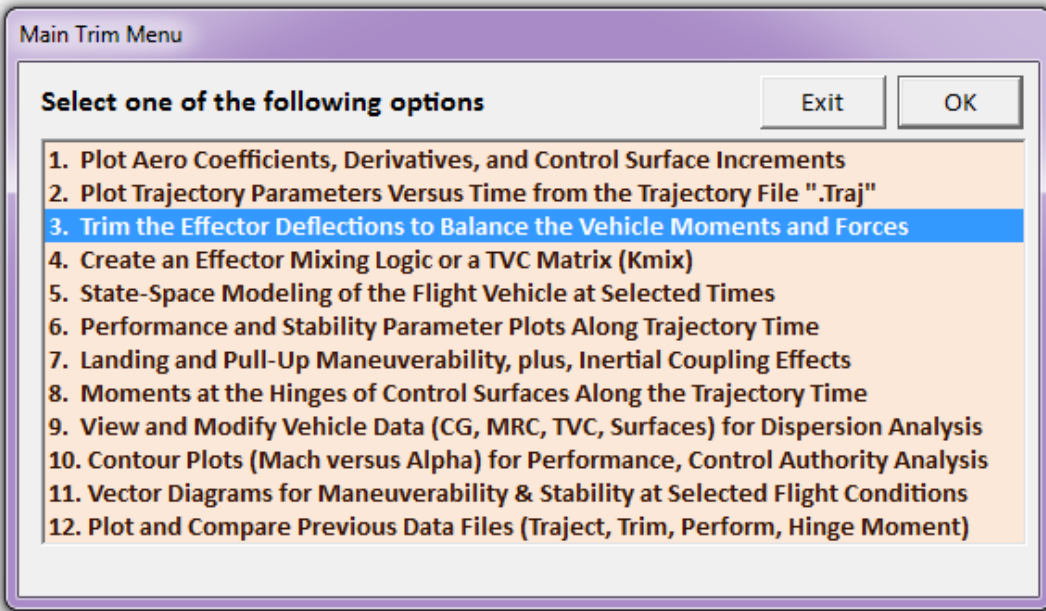
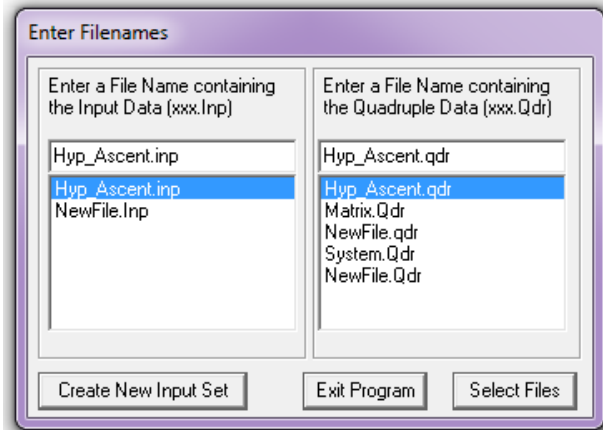
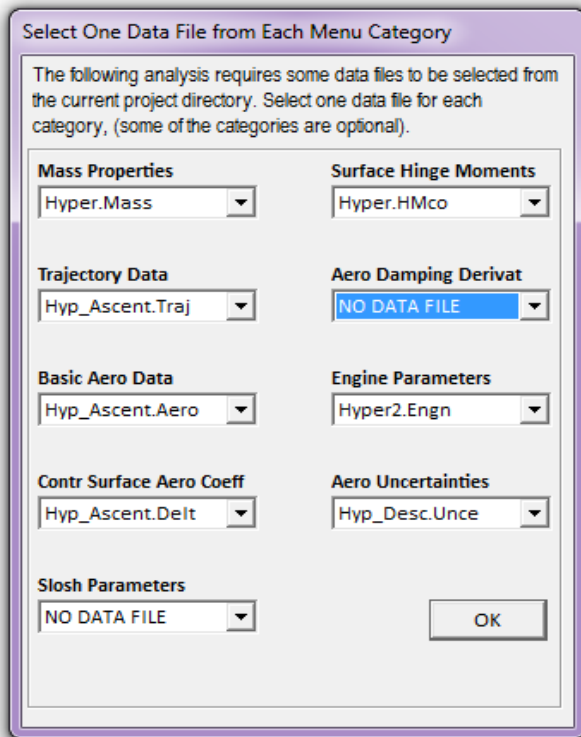
## 1.9 Manually Adjusting the Trimming Conditions

The Trim program calculates the effector trim positions by taking into consideration the control capability of each effector in the trim directions, at each point along the trajectory. When multiple effectors are present it is more likely that multiple directions will be affected and trimming along those directions becomes easier. The pseudo-inversion algorithm in the Trim program favors the effectors that have greater control authority in a certain direction and it allows them to contribute more in that direction than those that have less authority. There are situations, however, where the analyst may wish to bypass the automatic allocation of control authority and to manually trade the activity of one effector against another. In the longitudinal direction, for example, a vehicle may have an Elevon, Body-Flap, Speed-Brake, and Thrusters. It may be possible to eliminate, or to reduce activity in some of the controls, like for example the body-flap, by keeping it at a fixed position and trimming with the other effectors. In the example that follows we will show how to modify the trimming conditions in the program. This of course is only possible when the vehicle is equipped with other effectors that can provide sufficient controllability in the trim directions. If the effectors system is incapable or barely sufficient to span all directions the program will not allow the user to make any adjustments in the effector trim positions or permit very small adjustments. So in order to demonstrate this trimming adjustment feature and the ability to trade control authority among effectors we must first include some more effectors in our vehicle.

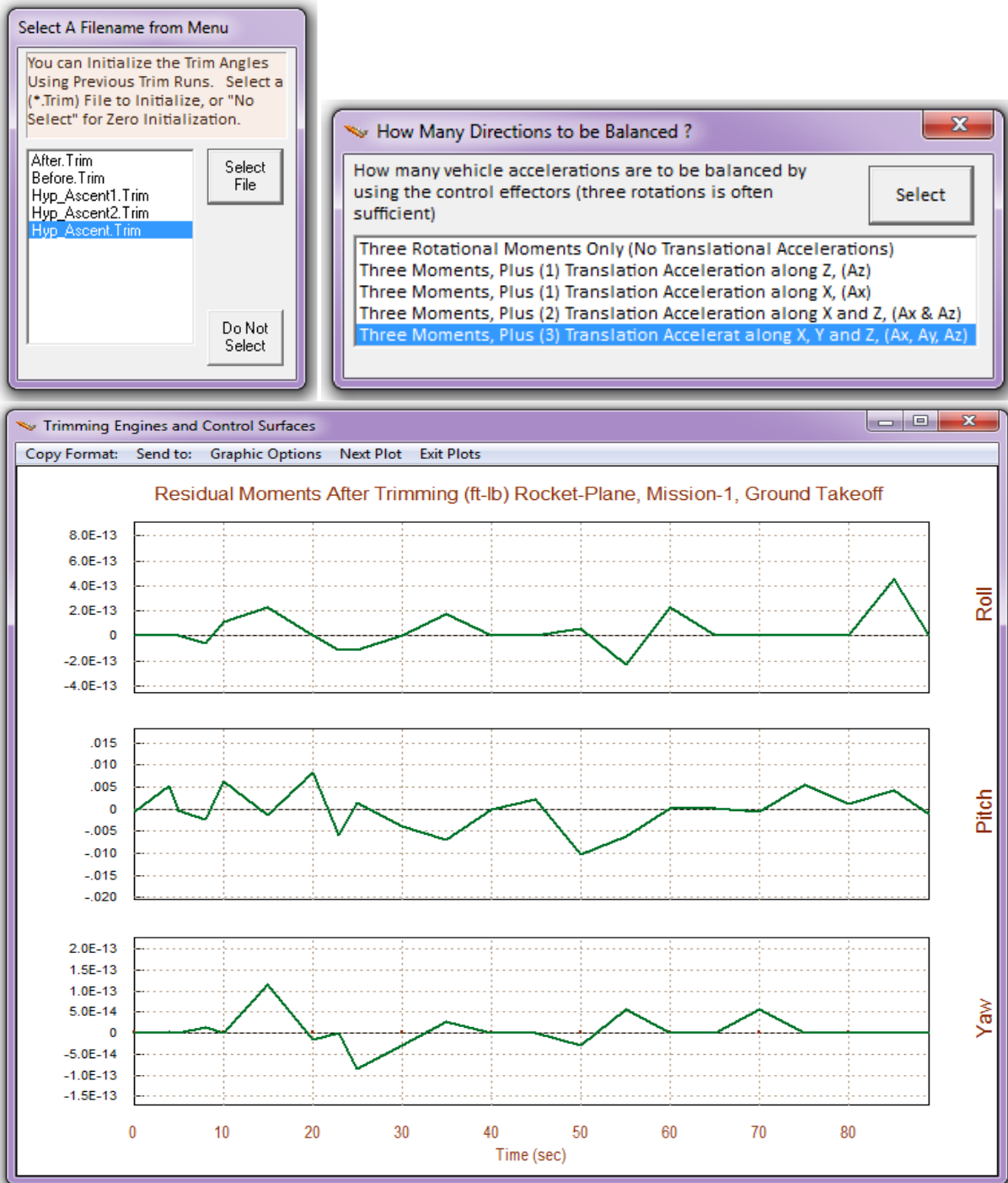
The additional effectors are specified in a new propulsion file "*Hyper2.Engn*". We will replace the previous file "*Hyper1.Engn*" that used only one fixed but throttling main engine with this new file that includes the same main engine but it is also allowed to gimbal its nozzle  $\pm 5^\circ$  in the pitch and yaw directions relative to its mounting position, which is along the x-axis. This is in addition to the  $\pm 40\%$  throttling capability that we have already included. This offers us some additional pitch and yaw control capability. Furthermore, in addition to the main engine gimbaling we have included two reaction control thrusters located near the front end of the vehicle. They are firing in the  $\pm Z$  and in the  $\pm Y$  directions and are capable to produce up to  $\pm 4,000$  (lb) of thrust. These thrusters will give us some acceleration control in the normal and lateral directions and since we already have x-acceleration control provided by the main engine, with the additional effectors we should be able to trim in all 6 directions. This means that we should be able to balance in all 6 directions and set our trimming conditions to include all 3 translational accelerations in addition to the 3 rotational accelerations, assuming of course that the trajectory is achievable.

Notice that there is a difference in the throttle control definitions between the main engine and the lateral jets and how the throttle parameter is interpreted by the program. In file "*Hyper2.Engn*" the max throttle control parameter for the main engine is 0.4 (which is less than 1). It means that the nominal engine thrust is 62,000 (lb) and it varies from 37,000 (lb) to 89,000 (lb) depending on the throttle control variable that is calculated when trimming along the trajectory, and the engine thrust is always positive. When the thruster is defined to be a reaction control jet the throttle parameter is defined to be exactly 1. A thruster represents a pair of back-to-back firing

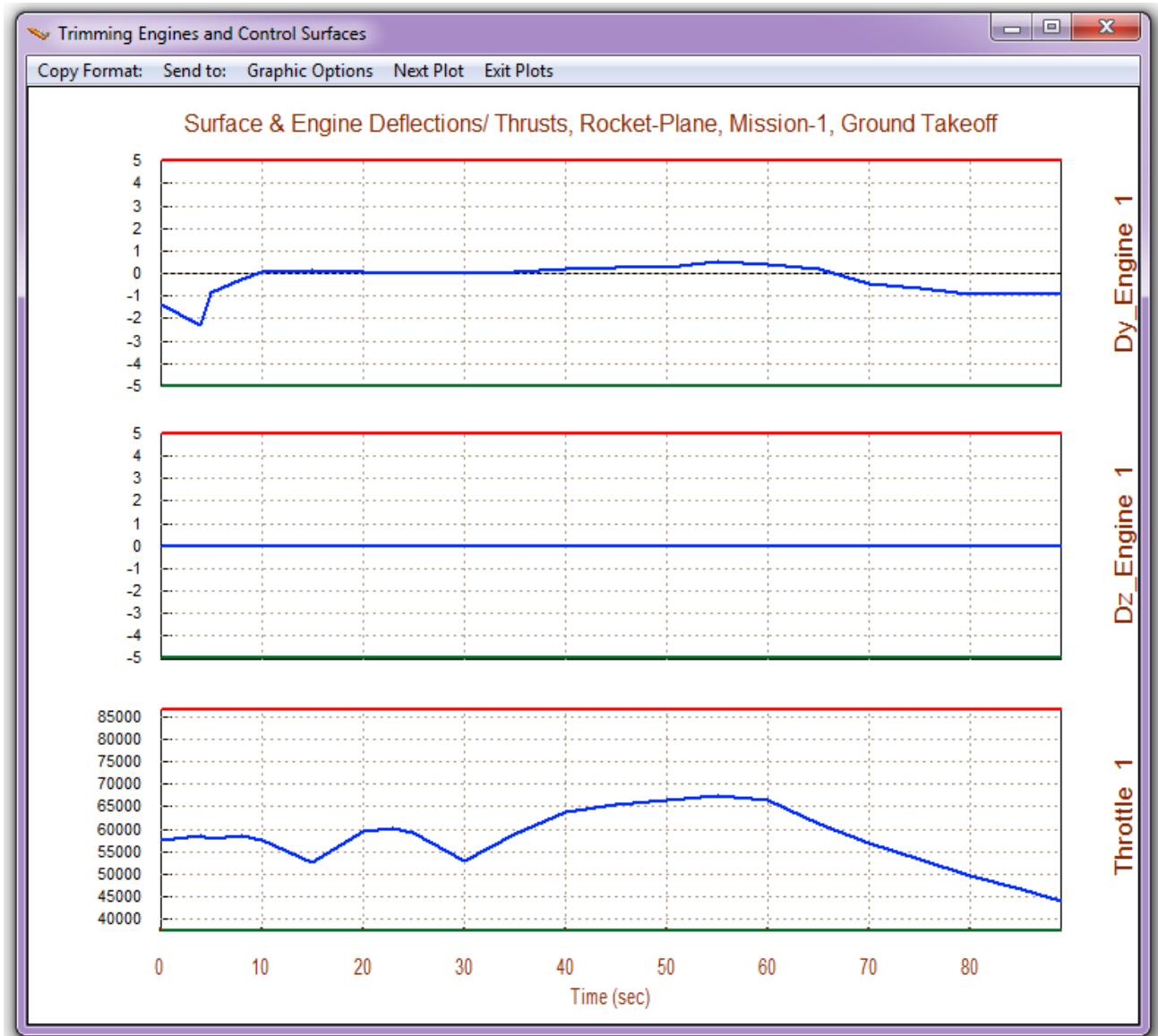
jets that can generate zero, positive or negative force such as jets firing in the  $\pm Z$  and  $\pm Y$  directions. It means that the nominal jet thrusts are zero when the throttle command is zero, and when the throttle control varies between zero and  $\pm 1$  during trimming the jet thrusts will vary between zero and  $\pm 4,000$  (lb) as a function of the throttle input. So let us begin the Trim program again by selecting files, the same trajectory and most other files are the same, except for the engines file. We also select an input and an output systems file, although we are not going to need them in this example. The first thing we must to do is to trim.

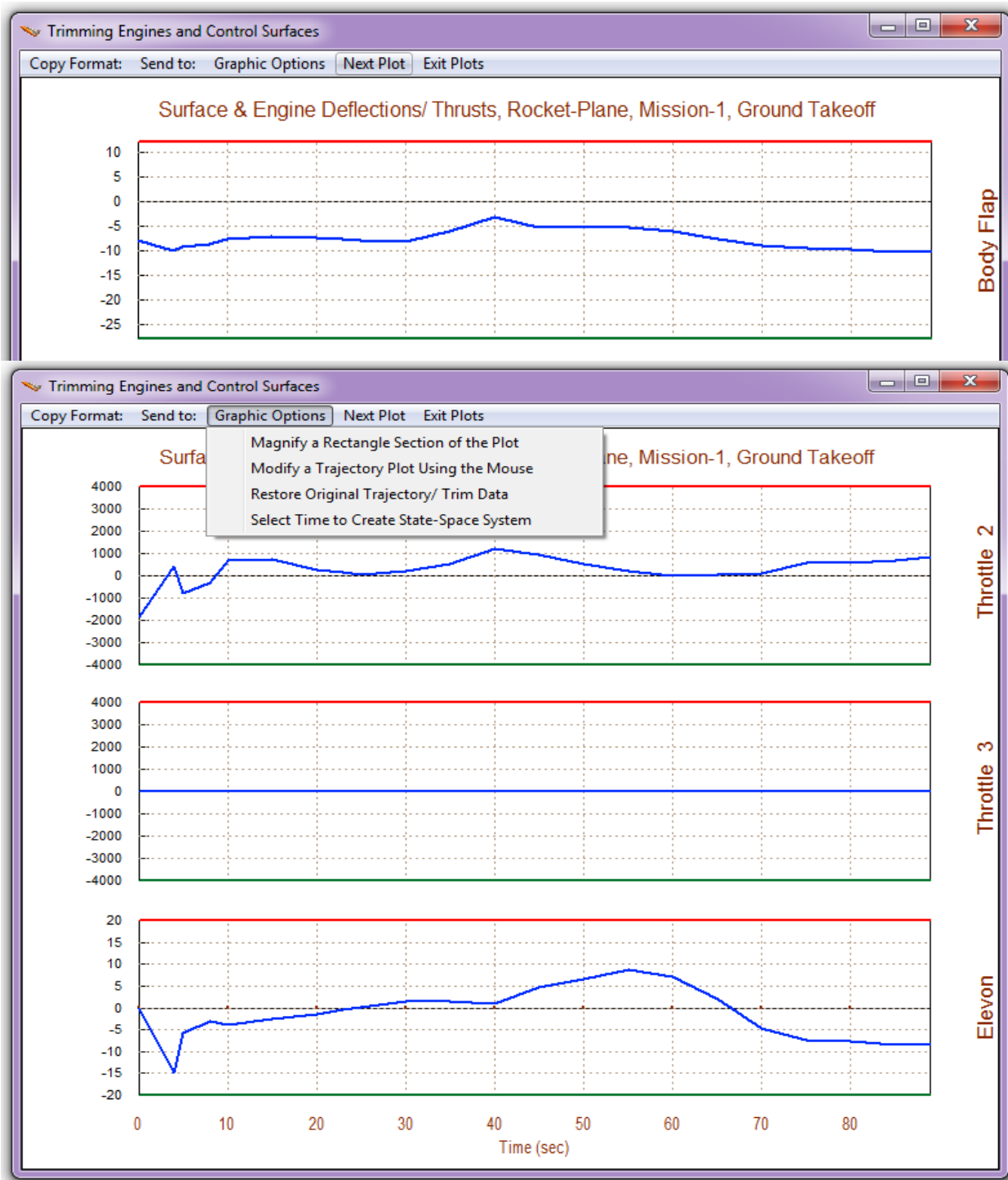


We initialize "Trim" from zero since our trimming conditions have changed and we do not have an initialization file yet. This time we have sufficient effectors to be able to trim along more degrees-of-freedom. So in the next dialog click on "Do Not Select" an initialization filename and in the directions menu select the last option to trim in all 6 directions. The first two plots are telling us that the trimming calculation was good because all residual forces and moments are practically zero.



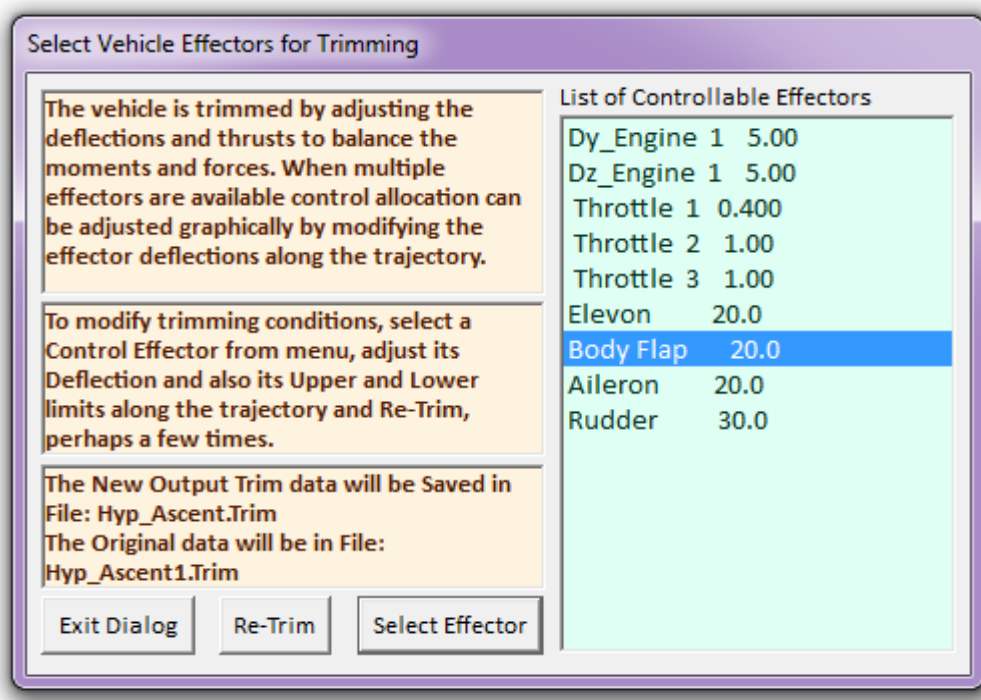
The first two plots in the figure below show the main engine deflections in pitch and yaw ( $\delta_{ye}$ ,  $\delta_{ze}$ ). The main engine is now contributing in pitch trim along with the Elevon and the Body-Flap. Only 1° or 2° of pitch gimbaling is used out of the  $\pm 5^\circ$  max availability. The main engine is not gimbaling in yaw because there are no lateral disturbances present yet. The third curve below shows the thrust variation of the main engine. The thrust is varied during trim in order to match the axial acceleration defined in the trajectory. The next plot shows the Body-Flap that has a  $-8^\circ$  bias that was defined in the aerosurface data file. The Trim program calculates the aerosurface deflections relative to this bias value and not relative to zero. The next two plots "Throttle 2 and 3" show the two RCS jet thrusts. "Throttle-2" is the thrust in the  $\pm Z$  direction, and "Throttle-3" is in the  $\pm Y$  direction, which is zero in this case. During take-off the normal  $\pm Z$  thruster is assisting by applying a negative thrust to provide a positive pitching moment. The last plot shows the Elevon activity. The aileron and rudder are not shown because they are zero.





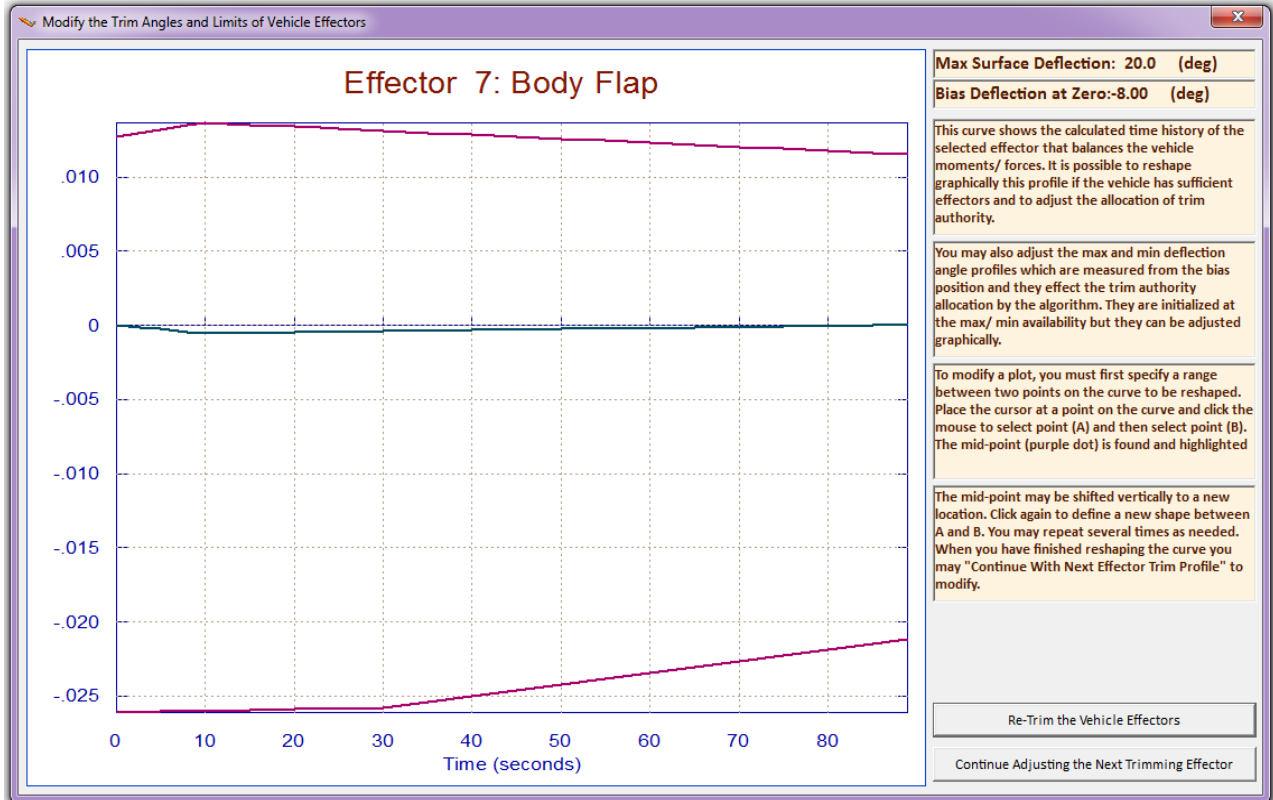
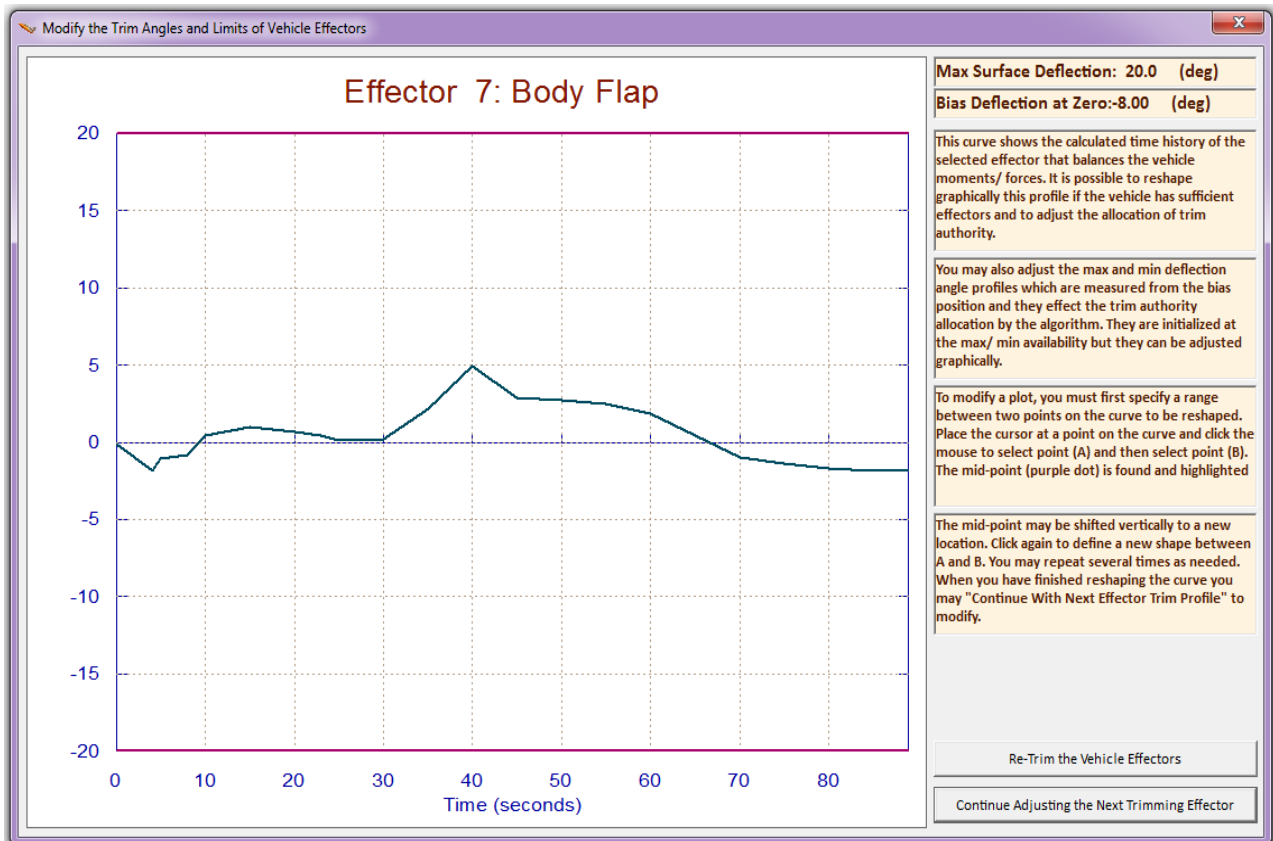
The trim angles and engine throttles shown above were calculated by the Trim program based on the control authority of each individual effector without any assistance from the user. The results show that we have plenty of pitch effectors. Let us see if we can eliminate at least one of them, for example, the Body-Flap and set it at a fixed position. We will try to keep it constant at its  $-8^\circ$  bias position by constraining its activity during trimming and consequently allowing other effectors to contribute more. Technically, it means that perhaps we may be able to replace the Body-Flap with a fixed aero-surface and to eliminate the actuator and pivot mechanisms.

From one of the trim plots, go to the horizontal menu bar on the top and click on "*Graphic Options*", and from the vertical pop-up menu select "*Modify a Trajectory Plot*", as shown. The following dialog/ menu shows a list of the 9 effectors. We may select one of them to modify graphically, in this case the Body-Flap, and click on "*Select Effector*" button. A modification dialog comes up showing the trim history of the Body-Flap that was calculated by Trim during the 89 seconds of ascent trajectory (green line). Notice that the  $-8^\circ$  bias is taken out because the trim angle is calculated relative to the bias, and the modifications made to this effector will be defined relative to the bias position. The two magenta lines define the upper and lower bounds of the trim history which are initially two horizontal lines at  $+20^\circ$  and at  $-20^\circ$ . They were originally defined in the aero-surface aero data. The upper and lower limits define the amount of effector usage and they also bound the deflections or the engine throttles during trimming. They can be modified graphically using the mouse.



### Setting a Surface at a Fixed Position

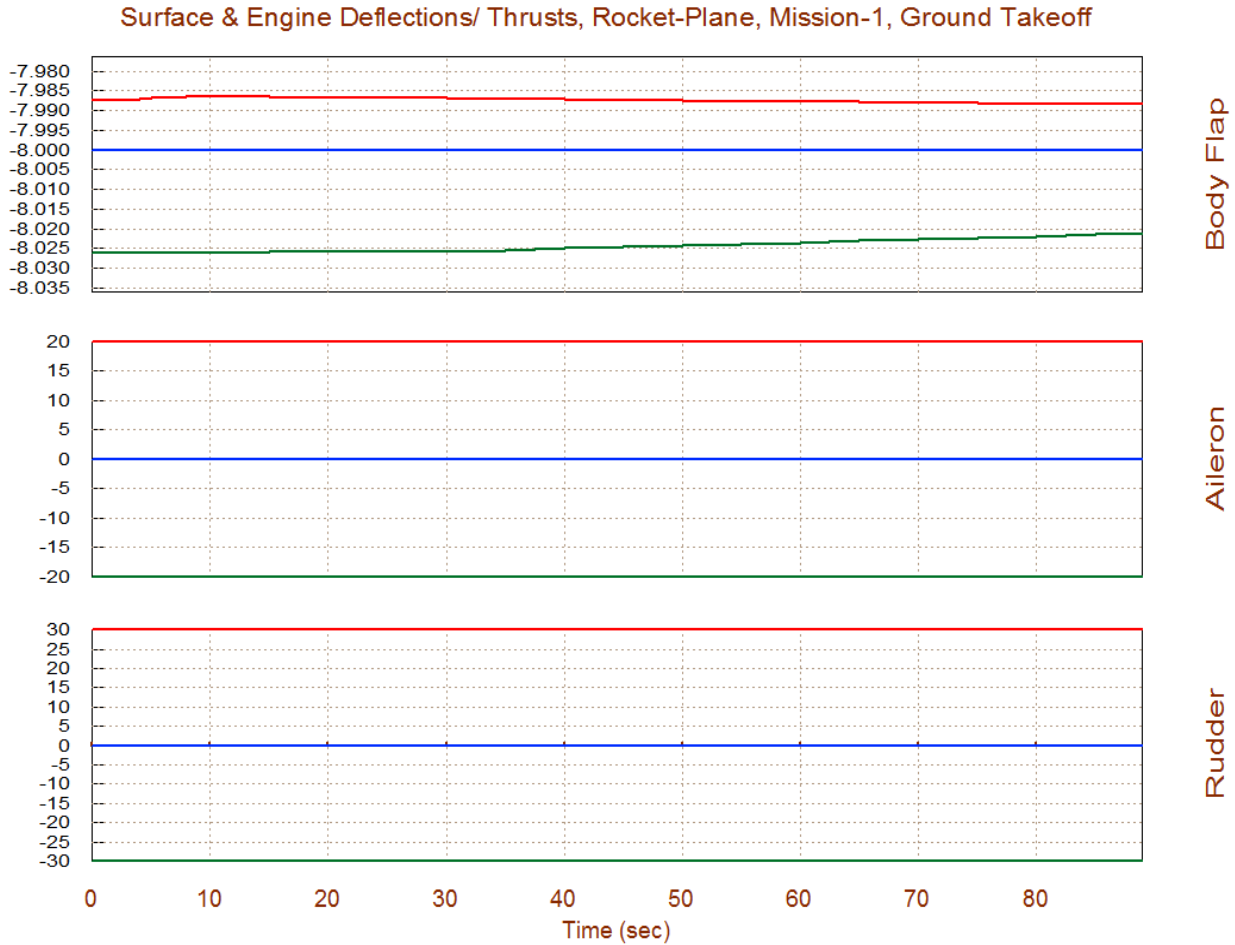
If we reduce the amplitude of the two limit lines towards zero (bias) it is telling the algorithm that the Body-Flap effector in this case is less capable and that it should not be used as much during the next trim. The program will then seek assistance from other effectors to provide control in the directions where the Body-Flap was contributing. We not only reduce the upper and lower deflection limits (magenta lines) but we must also reduce the amplitude of the surface deflection curve (green line). This is done using the mouse, by modifying one section at a time. When we are satisfied with the shape and size of the trim curve and its limit lines, we click on "*Re-Trim*", either on the menu or the plot dialogs, and obtain the new trim results. This may take a couple of re-trims until the Body-Flap activity is sufficiently reduced. This will happen, of course, at the expense of other effectors usage.



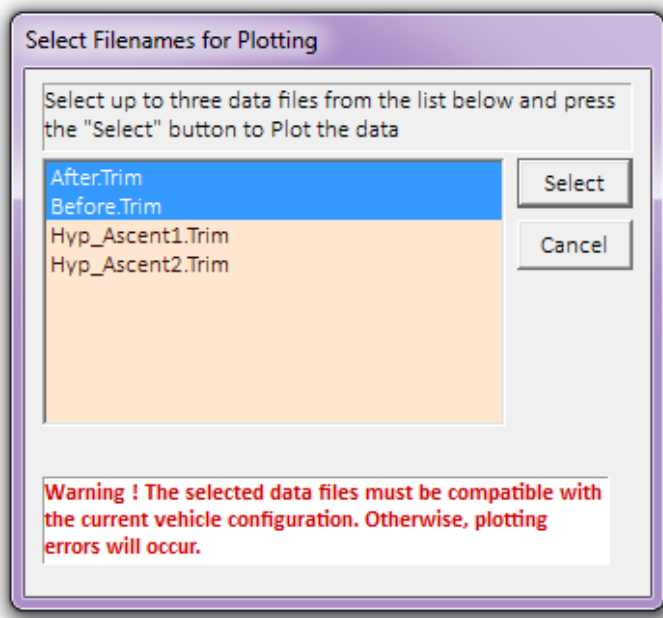
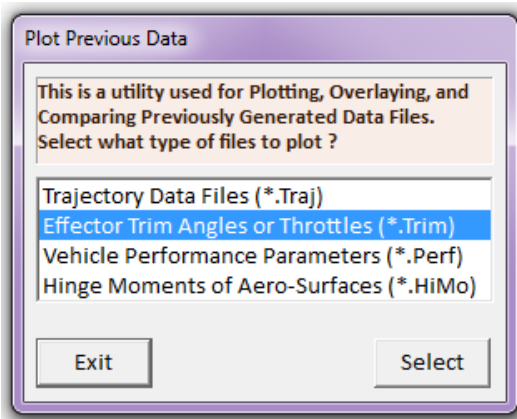
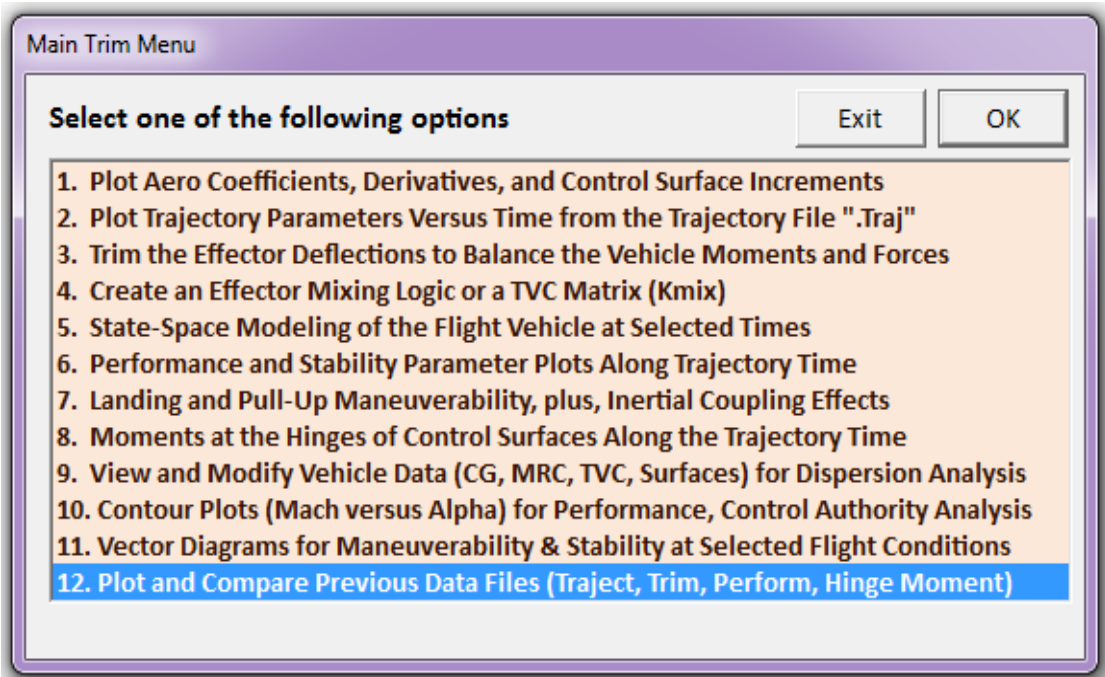
Modify the trim history using the mouse and Re-Trim the effectors a few times until the size of the curve is sufficiently reduced



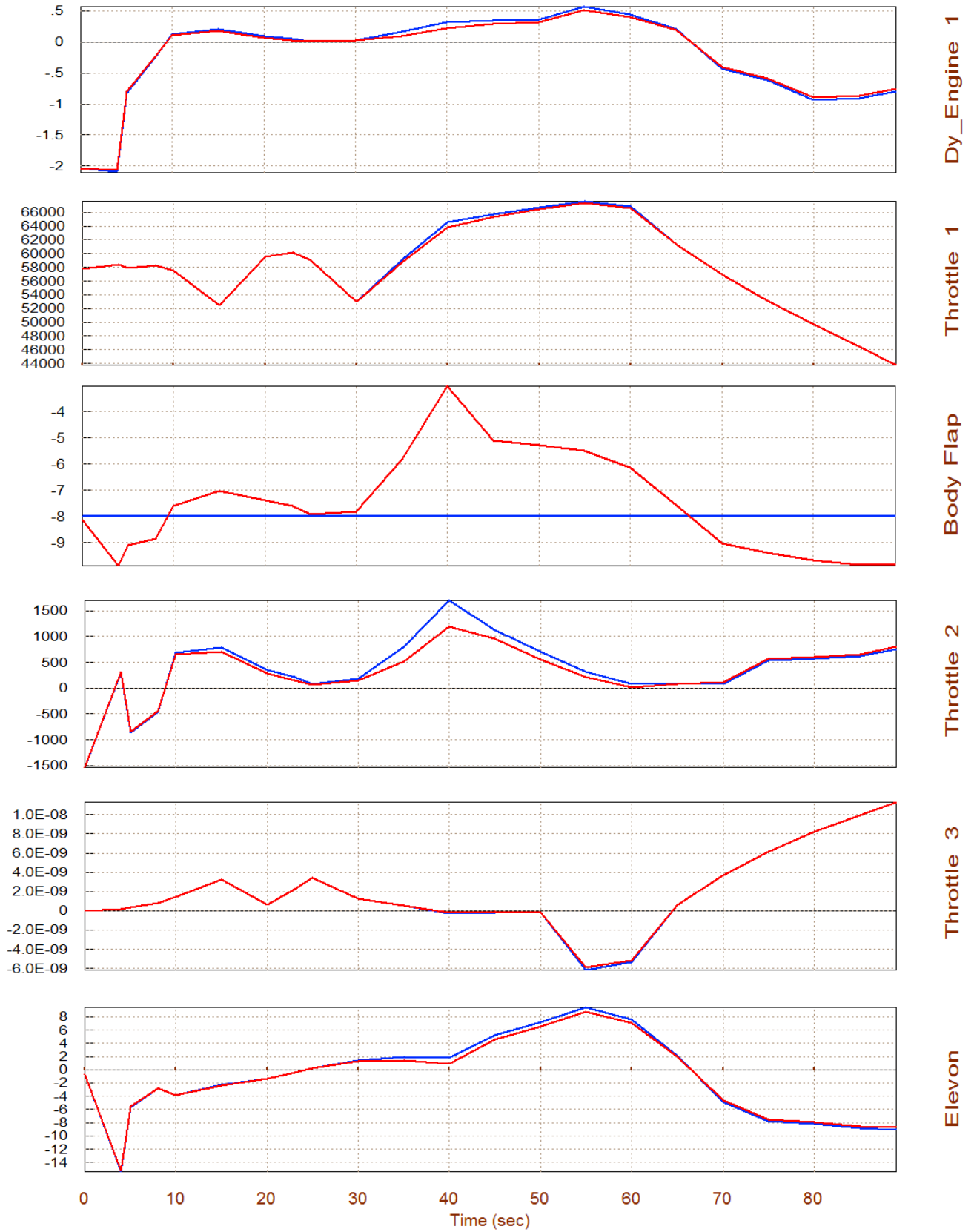
After re-trimming the Body-Flap activity is significantly reduced to its bias position as shown by the blue line below. Notice also the modified upper and lower limits shown by the red and green lines. The Body-Flap bias is included in this plot unlike the previous modification dialog/plots where the bias is not shown for convenience.



Now let us compare the before and after Body-Flap trim adjustment results and see which effectors took over the contributions after freezing the Body-Flap. The trim files are continuously updated with new data, so in order to save the results we renamed the trim files to "before.trim" and "after.trim", meaning, before and after the trim adjustment. From the main menu go to option (12) to compare the trim results and select the two files. The original trim is shown in red and the modified trim results are shown by the blue lines. The reduction in the Body-Flap activity is very obvious in this plot which is now fixed at its bias position  $-8^\circ$ . It appears that the normal force  $\pm Z$  thruster "Throttle 2" activity has increased in order to compensate for the elimination of the Body-Flap motion. The main engine pitch deflection and also the Elevon deflections were also affected at a lesser extent.

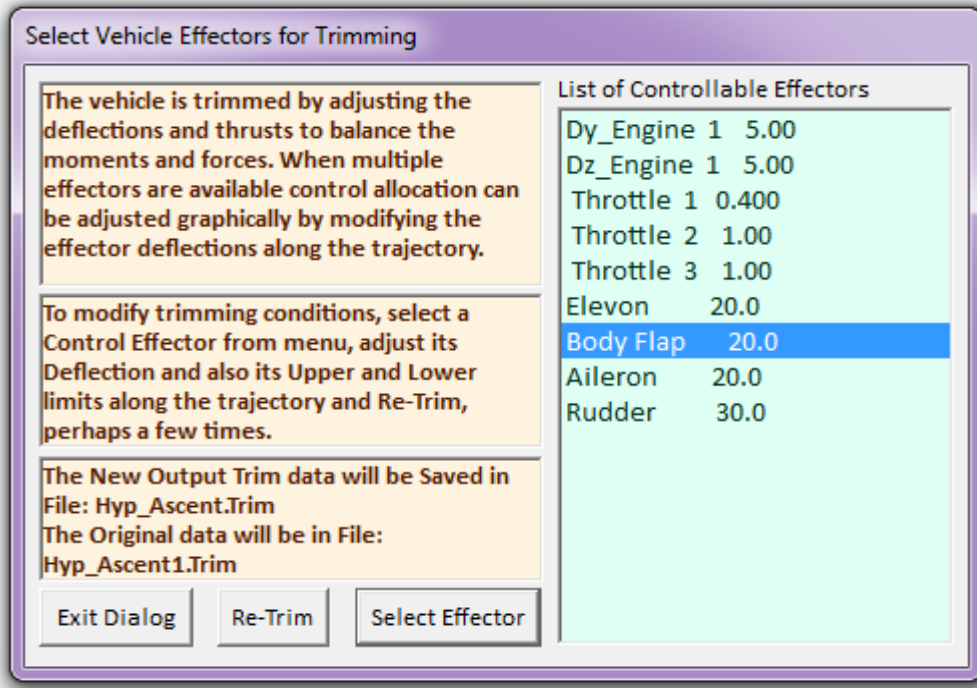


Surface & Engine Deflections/ Thrusts, Rocket-Plane, Mission-1, Ground Takeoff

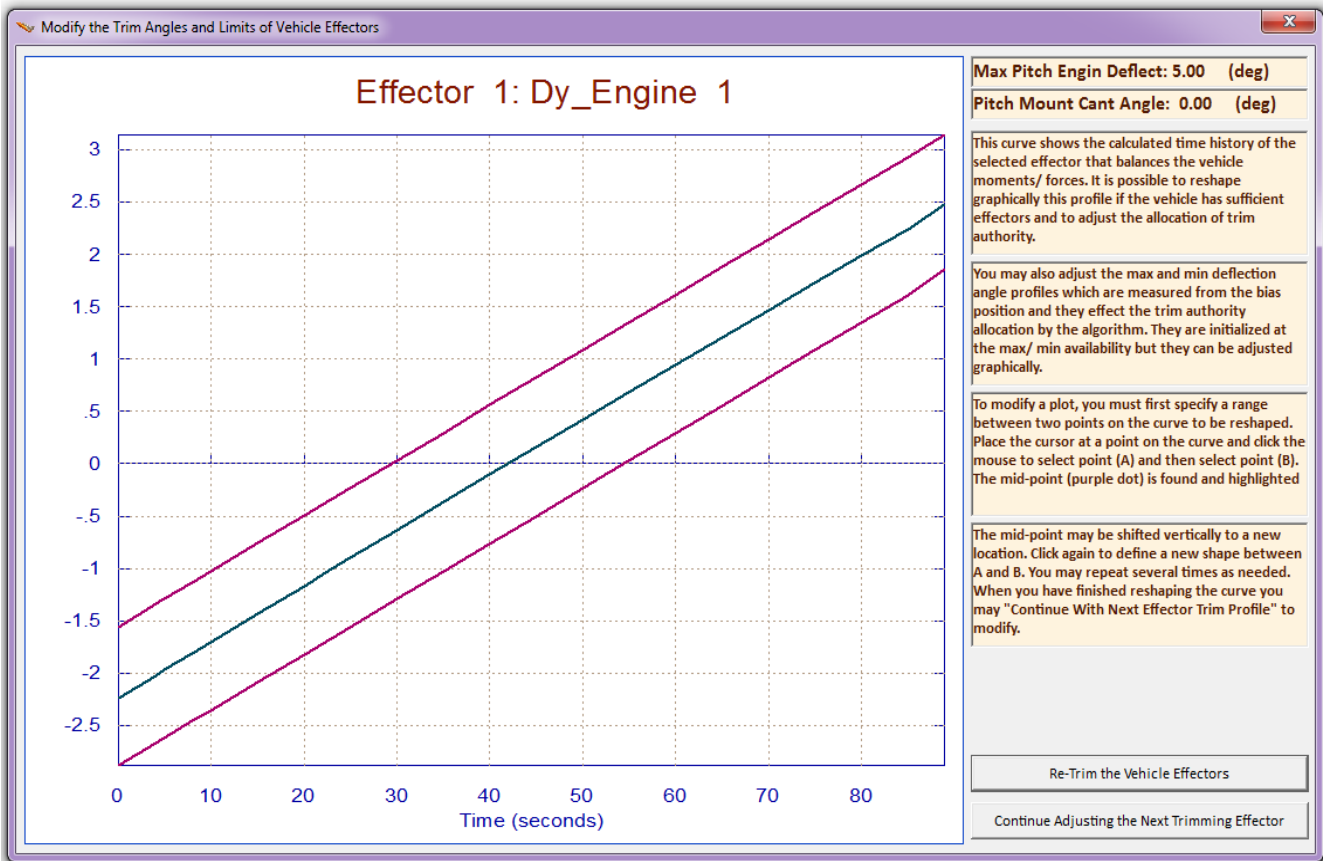
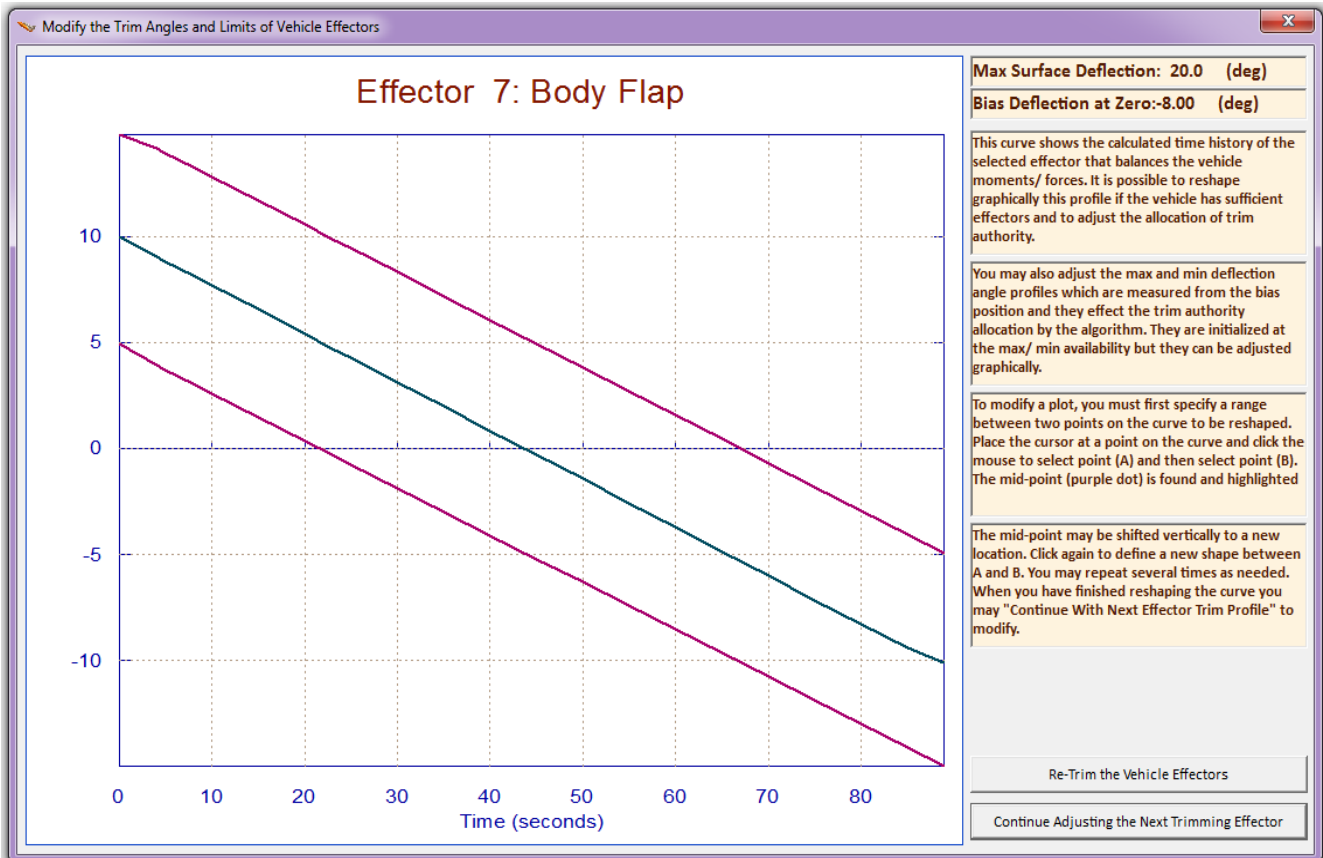


## Scheduling the Effectors

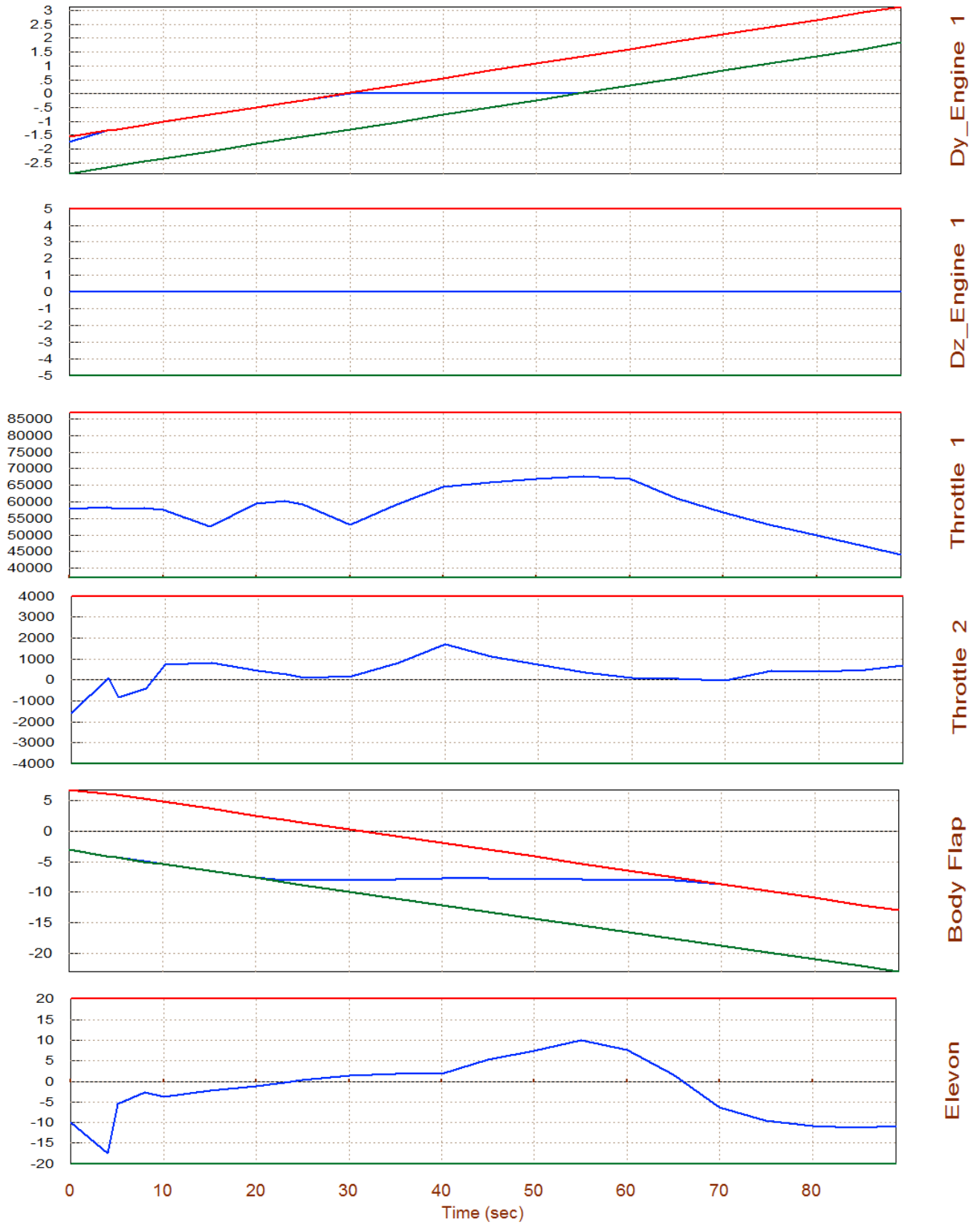
It is also possible to use the same process in order to schedule (vary as a function of time) the positions of more than one effectors simultaneously by constraining their deflections graphically, as shown below, assuming of course that there are other effectors that can be manipulated by the Trim algorithm in order to balance the moments and forces. In this example we are still using the same effectors and after the first trim we go to the top menu, select "*Modify a Trajectory Plot*", but instead of fixing the position of the Body-Flap we are now scheduling it as shown in the next figure (green line). We also modify the upper and lower limits (magenta lines) parallel to the schedule line allowing  $\pm 5^\circ$  space for adjustment. When the Body-Flap modification is complete we do not re-trim yet but click on "*Continue Adjusting the Next Effector*". From the effectors menu this time choose the pitch TVC deflection of the main engine "*Dy\_Engine 1*". The dialog-plot shows the pitch engine deflection from the previous (unconstrained) trim. Using the mouse and the dialog graphic capabilities this plot can be modified and scheduled as shown in the next figure. The upper and lower limits are also set parallel to the scheduled pitch engine deflection line with a small space in between. When the adjustments are complete, click on "*Re-Trim*".

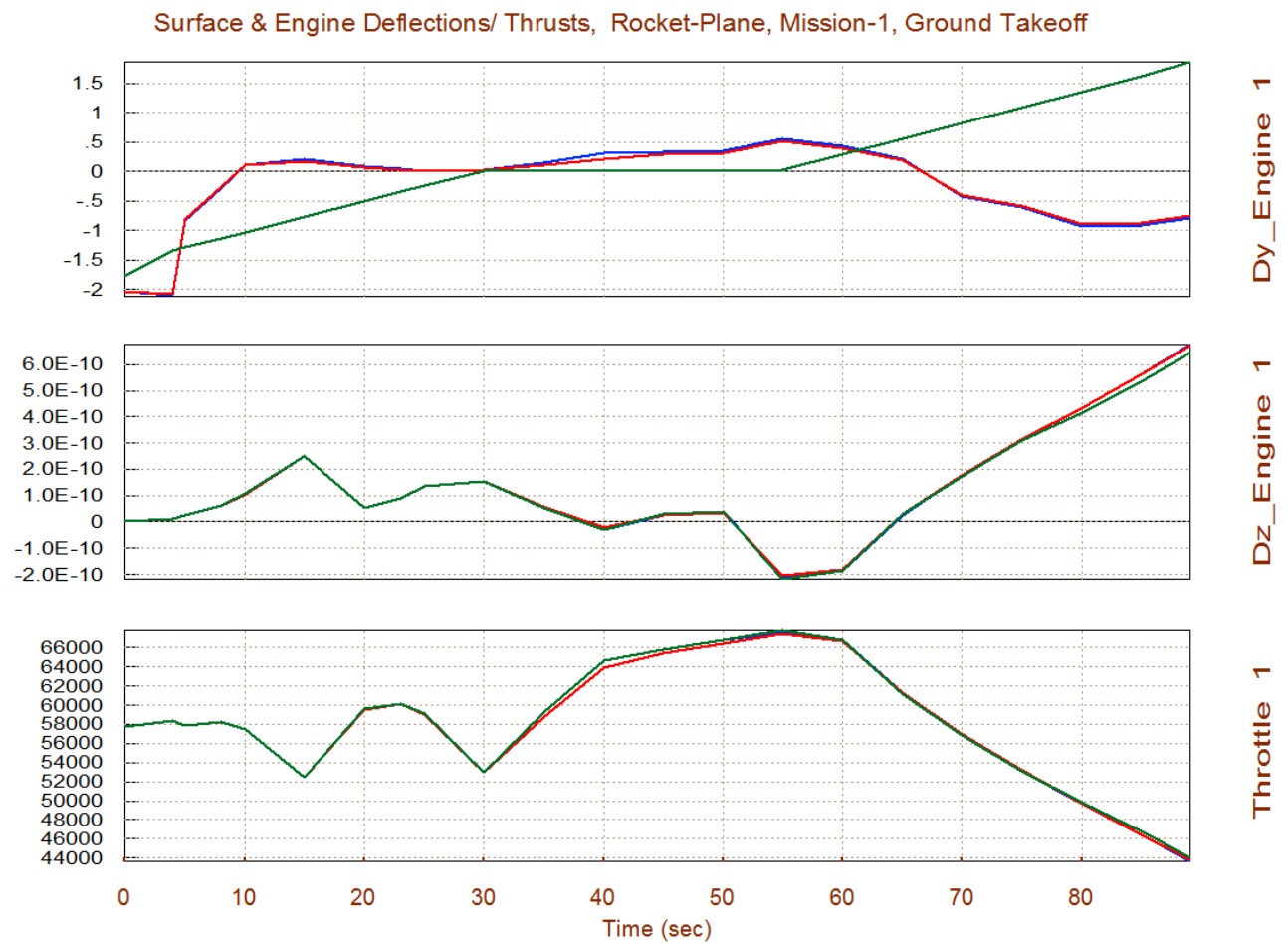
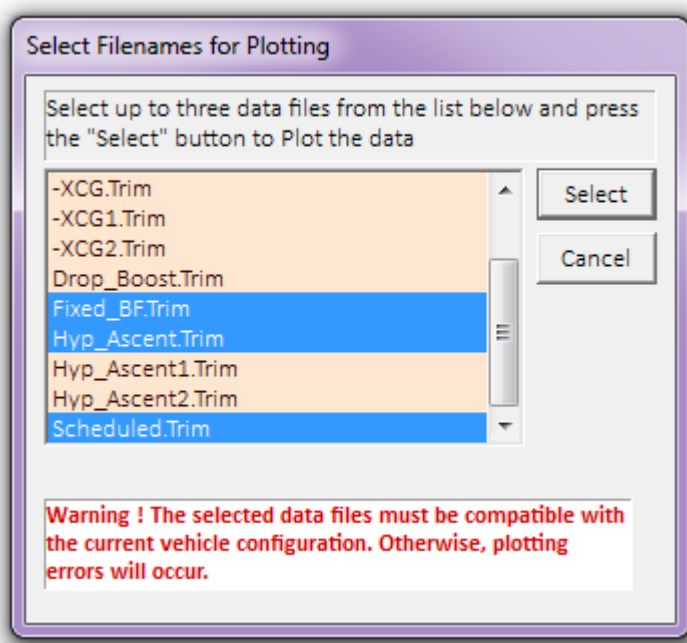


After re-trimming the following plots show the new trim results with the bounded Body-Flap and pitch engine deflections. The new trim positions (blue lines) are now constrained to lie in between the upper and lower limits (red and green lines). To summarize, let us plot all 3 cases together and compare them using option (12), as shown in the following plots. The red curves are from the original unconstrained file "Hyp\_Ascent.Trim". The blue curves correspond to the previous case where we fixed the Body-Flap at its bias and saved it in file "*Fixed\_BF.Trim*". The green curves are from our latest trim where we scheduled the pitch TVC and the Body-Flap and saved it in file "*Scheduled.Trim*".

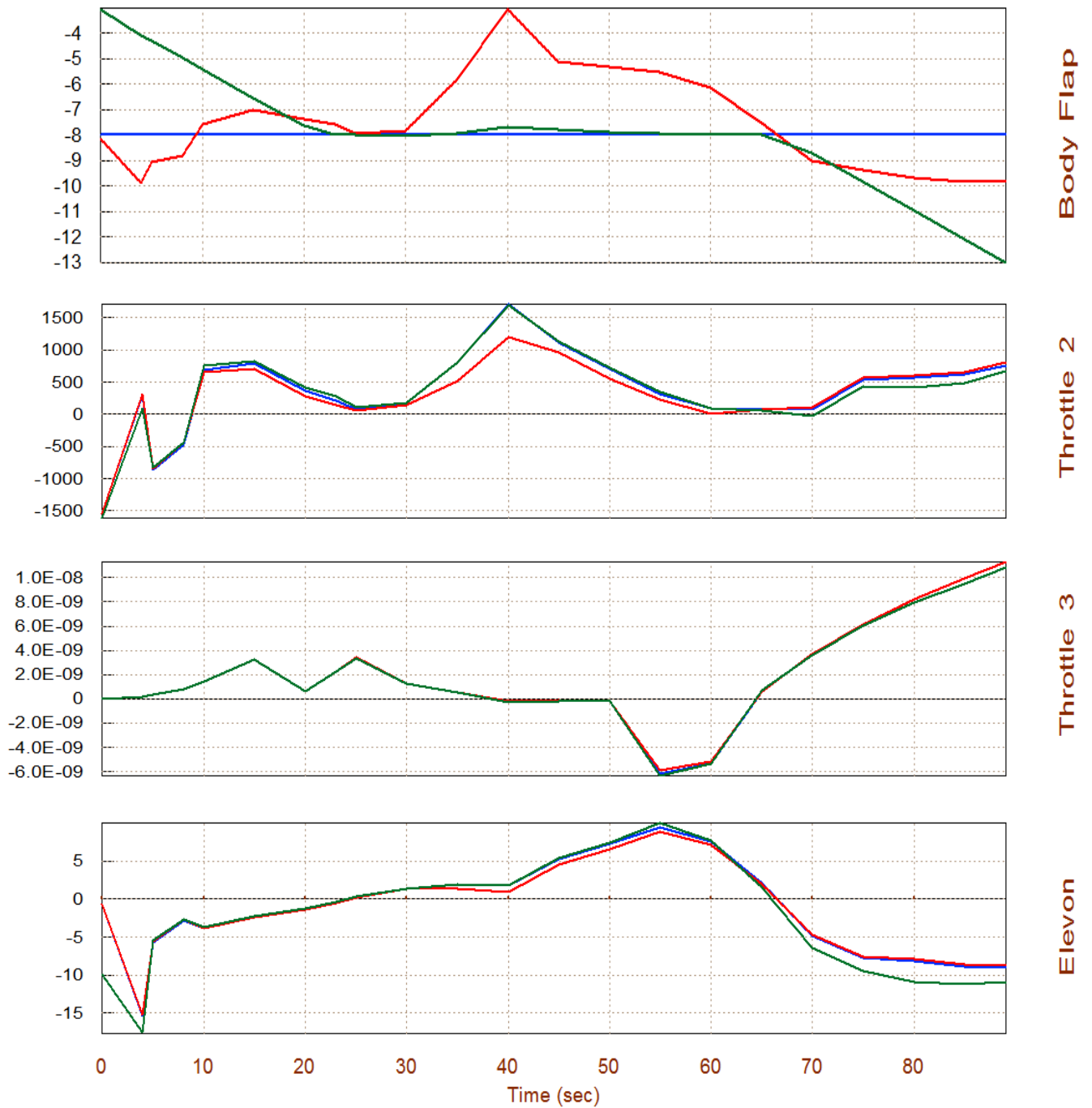


Surface & Engine Deflections/ Thrusts, Rocket-Plane, Mission-1, Ground Takeoff





Surface & Engine Deflections/ Thrusts, Rocket-Plane, Mission-1, Ground Takeoff



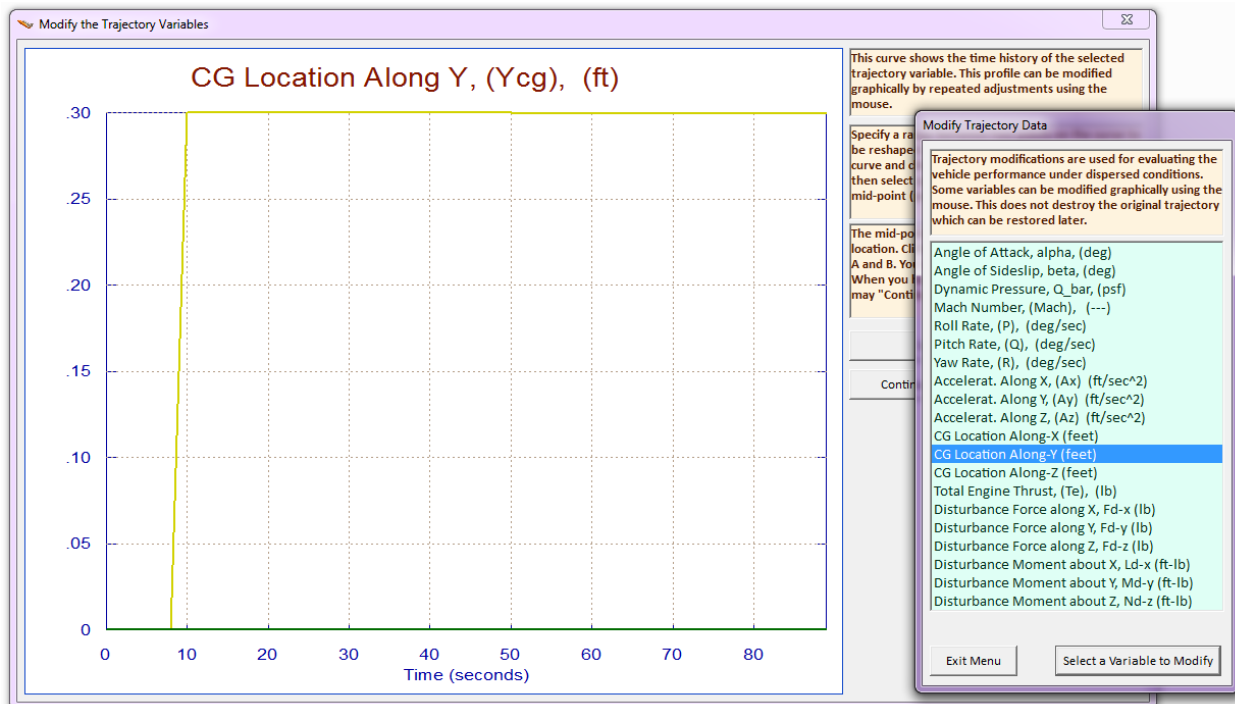
The Body-Flap and the pitch engine deflection (green lines) are now within the schedule limits. It appears that the Elevon and the  $\pm Z$  RCS jet "Throttle 2" are now compensating in pitch for the scheduling of the two effectors.



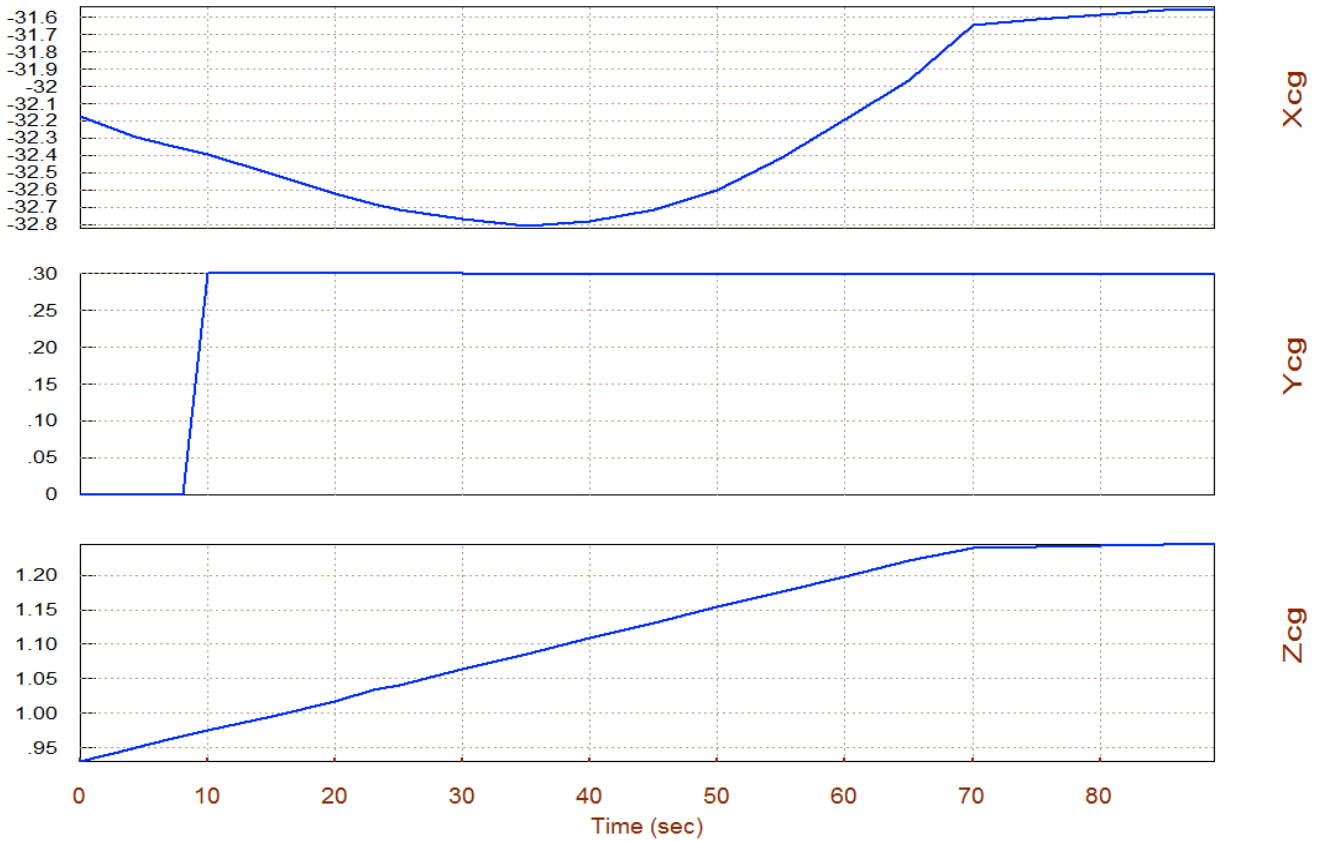
## 1.10 Lateral Trimming with a $Y_{CG}$ Offset

In our previous example, using file "Hyper2.Engn", the vehicle was trimmed in all 6 directions, but we did not really examine the effects of an excitation in the lateral directions. Being able to trim in all 6 directions means that the vehicle has the control authority to counteract not only disturbance moments but also forces along the x, z and y directions. It means that it can maintain the accelerations required by the trajectory even in the event of a disturbance or a CG shift because it has the effectors authority to compensate against the disturbances directly without having to modify its trajectory and its  $\alpha$  and  $\beta$  angles. One way to upset the balance of moments and forces in the lateral axes is to shift the CG in the y direction towards the right wing. The main engine thrust through the vehicle center axis will create a positive yawing moment and the other effectors must react against it.

The first thing to do is modify the YCG. Continuing from section 1.9 and after returning to the Trim main menu, go to option (2) which plots the trajectory data and then modify the CG as it was described in Section (1.3), using the "Graphic Options", and then from the vertical pop-up menu select "Modify a Trajectory Plot Using the Mouse". From the menu select "CG Location Along Y". The YCG initially is at zero (green line at the bottom) but its time history can be modified using the mouse, by shifting it 0.3 (ft) to the right, starting at 10 sec. Then click on "Save the Modified Trajectory" and the modification will remain in memory for the trim analysis. Click on "Exit Menu" and return to the trajectory plots to check that the YCG has been changed and also the trajectory title is now "User Modified Trajectory". The original trajectory may later be restored by going back to the "Graphic Options" menu and clicking on "Restore the Original Trajectory".



Vehicle CG in (feet), User Modified Trajectory ...



Main Trim Menu

Select one of the following options Exit OK

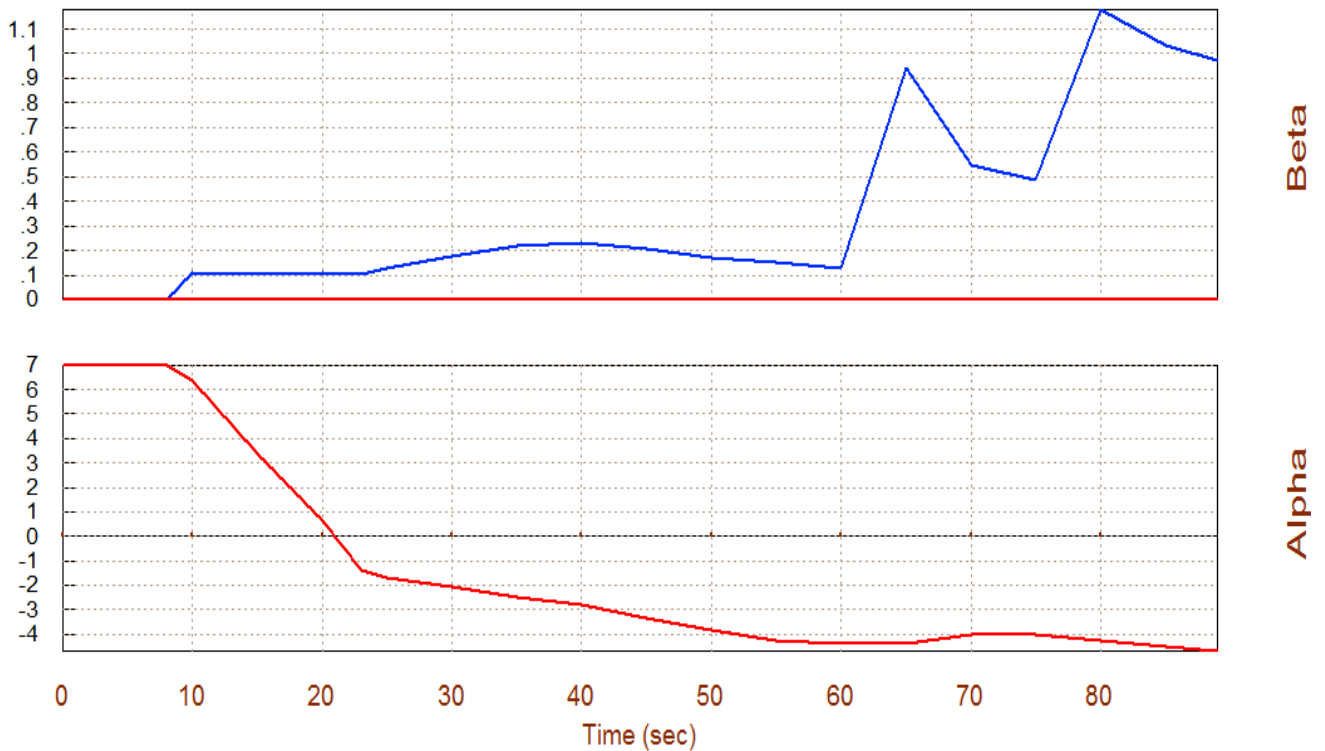
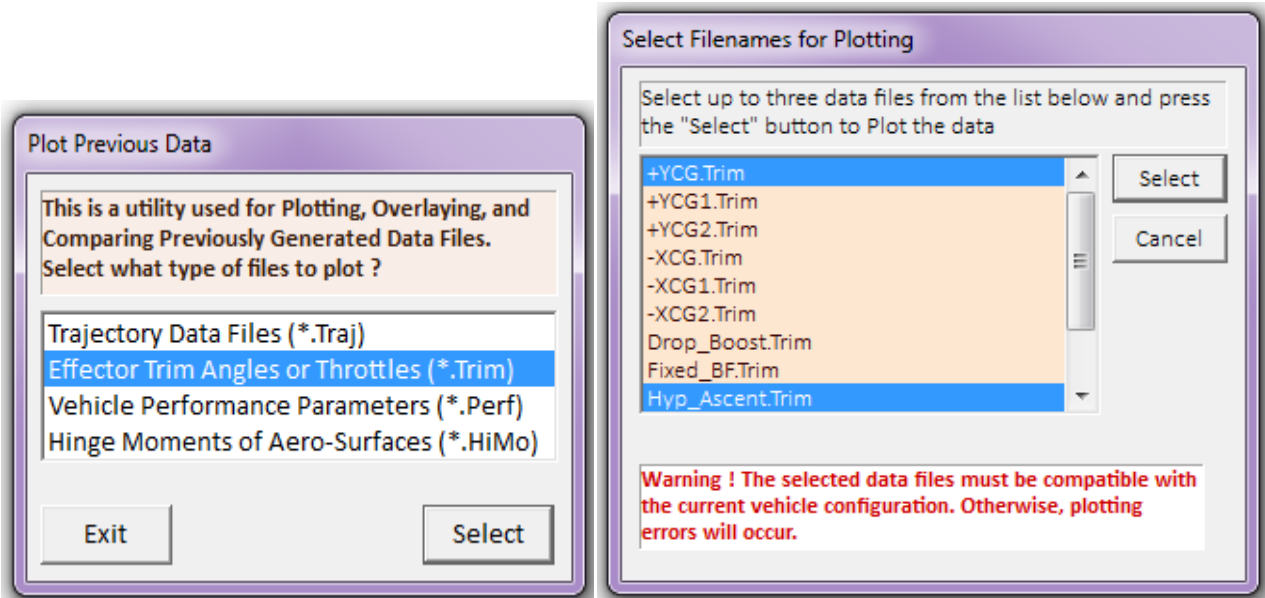
1. Plot Aero Coefficients, Derivatives, and Control Surface Increments
2. Plot Trajectory Parameters Versus Time from the Trajectory File ".Traj"
3. Trim the Effector Deflections to Balance the Vehicle Moments and Forces
4. Create an Effector Mixing Logic or a TVC Matrix (Kmix)
5. State-Space Modeling of the Flight Vehicle at Selected Times
6. Performance and Stability Parameter Plots Along Trajectory Time
7. Landing and Pull-Up Maneuverability, plus, Inertial Coupling Effects
8. Moments at the Hinges of Control Surfaces Along the Trajectory Time
9. View and Modify Vehicle Data (CG, MRC, TVC, Surfaces) for Dispersion Analysis
10. Contour Plots (Mach versus Alpha) for Performance, Control Authority Analysis
11. Vector Diagrams for Maneuverability & Stability at Selected Flight Conditions
12. Plot and Compare Previous Data Files (Traject, Trim, Perform, Hinge Moment)

How Many Directions to be Balanced ? X

How many vehicle accelerations are to be balanced by using the control effectors (three rotations is often sufficient) Select

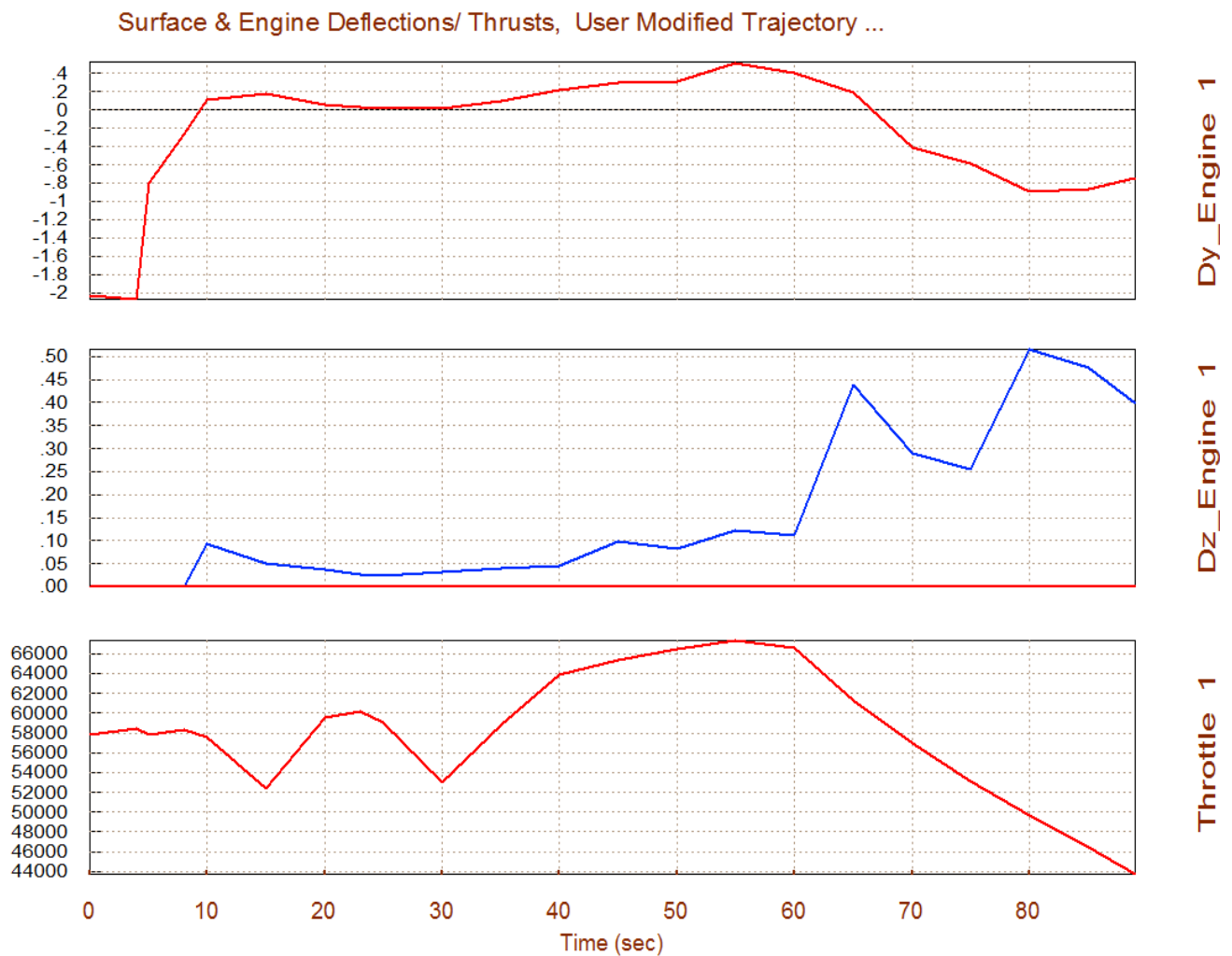
- Three Rotational Moments Only (No Translational Accelerations)
- Three Moments, Plus (1) Translation Acceleration along Z, (Az)
- Three Moments, Plus (1) Translation Acceleration along X, (Ax)
- Three Moments, Plus (2) Translation Acceleration along X and Z, (Ax & Az)
- Three Moments, Plus (3) Translation Accelerat along X, Y and Z, (Ax, Ay, Az)

Exit the trajectory plots and from the Trim menu select option (3) to re-trim the effectors with the +YCG modified trajectory. Do not select an initialization file, and from the "directions to balance" menu choose the last option for trimming in all 6 directions, as in section 1.9. The program now calculates the new effector trim positions required to balance the lateral CG shift. After trimming we go back to option (12) and compare the trim results from the two 6-dof trim cases, (a) the +Y<sub>CG</sub> offset case and (b) the original unconstrained trim with the Y<sub>CG</sub> centered, obtained in section 1.9.

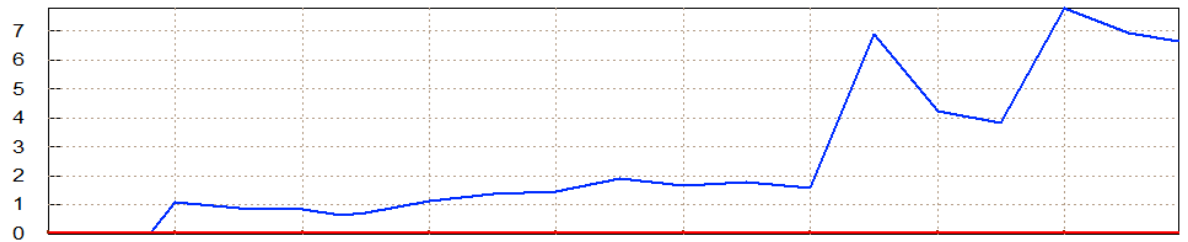
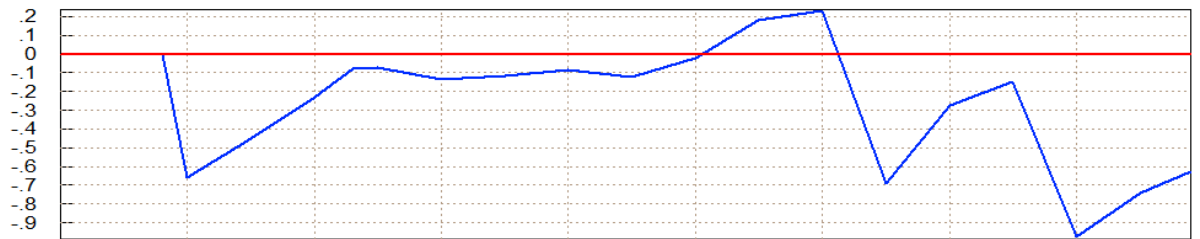
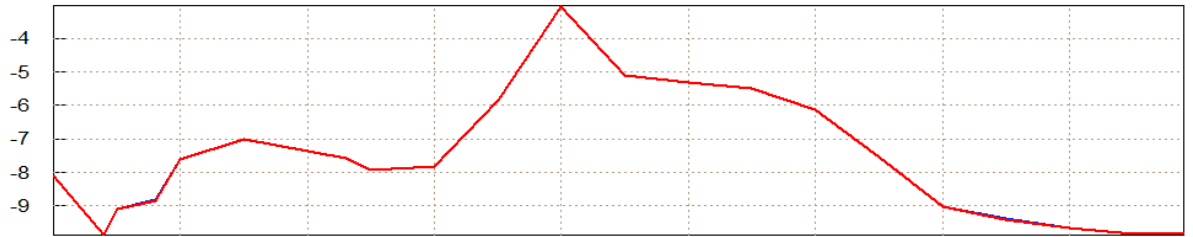
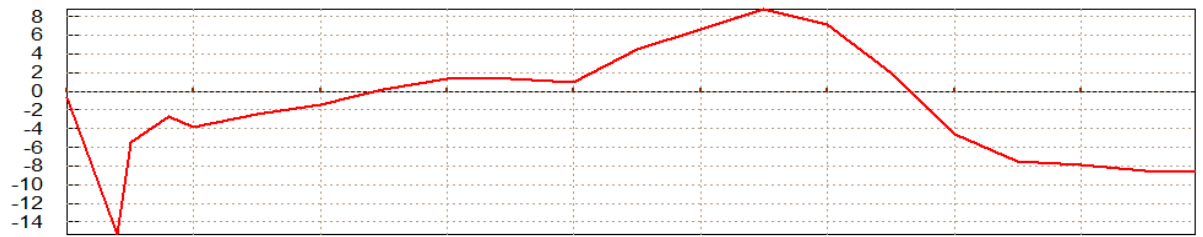
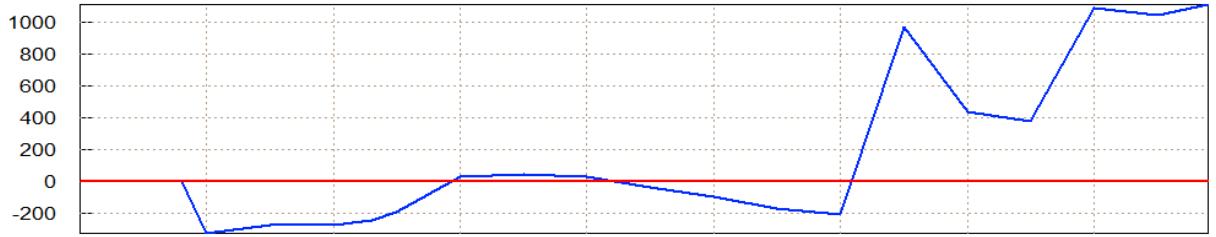
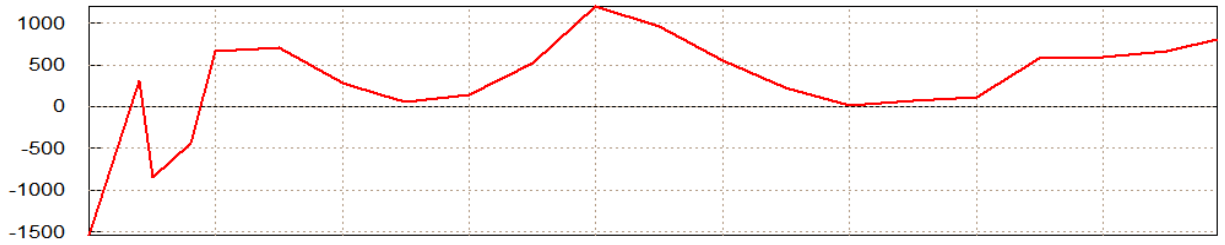


The main engine thrust, the pitch engine and the elevon deflections are the same as when the  $Y_{CG}$  is centered, but now we have positive yaw TVC and rudder rotations that produce a negative yawing moment. This is to counteract against the positive yawing moment generated by the main engine coupling with the  $+Y_{CG}$  offset. The positive yaw deflections of the rudder and of the main engine also produce a positive side-force along  $+Y$  creating a positive sideslip  $\beta$ . The side-force produced by the jet in the  $\pm Y$  direction "Throttle 3" is also contributing towards balancing the sideforce due to  $\beta$  and the yawing moment. The jet in the Z direction "Throttle 2" is also active in trimming the pitch and Z directions. Both the rudder and the TVC produce a positive rolling moment, and therefore, the aileron deflection is mostly negative to balance this moment.

In conclusion, we proved that this augmented system of effectors is capable of producing the moments and forces required to trim not only 3 moments, but also all 3 accelerations along the trajectory including CG dispersions. We also demonstrated how to analyze dispersions by modifying some of the trajectory variables graphically and re-trimming.



Surface & Engine Deflections/ Thrusts, User Modified Trajectory ...



Time (sec)

## 2.0 Descent Phase



When the vehicle reaches an altitude of 76,000 (feet), the descent phase begins. During descent the engine and RCS jets are turned off, the vehicle banks to the left and it glides and lands on a runway like a space shuttle controlled by the four aero-surfaces. We are going to use the "Trim" program to analyze the descent part of the trajectory, calculate the trim angles, and evaluate the vehicle performance along the trajectory and also by using contour plots and vector diagrams. We will also show how to use the Trim program to schedule the position of the Body-Flap as a function of time. Dynamic modeling and control design will also be performed at a selected flight condition. The control gains will be used in a 6-dof non-linear simulation model.

### 2.1 Descent Data Files

The data for this hypersonic vehicle example during descent is in folder "*C:\Flixan\Trim\ Examples\ Hypersonic Vehicle \Descent*". This folder includes data files which are inputs to the Trim program and also the files which are generated by Trim. Let us first describe the input data files.

- The descent trajectory file is "*Hyp\_Desc.Traj*". It starts from an altitude of 76,000 (ft), at Mach: 4.5, and with a negative  $\alpha = -1.7^\circ$ . Then it banks to the left to change direction and align itself with the landing site. It maintains a positive  $\alpha = 5^\circ$  during most of the descent flight. The max dynamic pressure is 1000 (psf) and the flight duration is approximately 1100 sec. There is also a modified version of this trajectory that has the  $Y_{CG}$  off-centered towards the right "*Hyp+Ycg.Traj*".
- The basic aerodynamic data is in file "*Hyp\_Desc.Aero*".

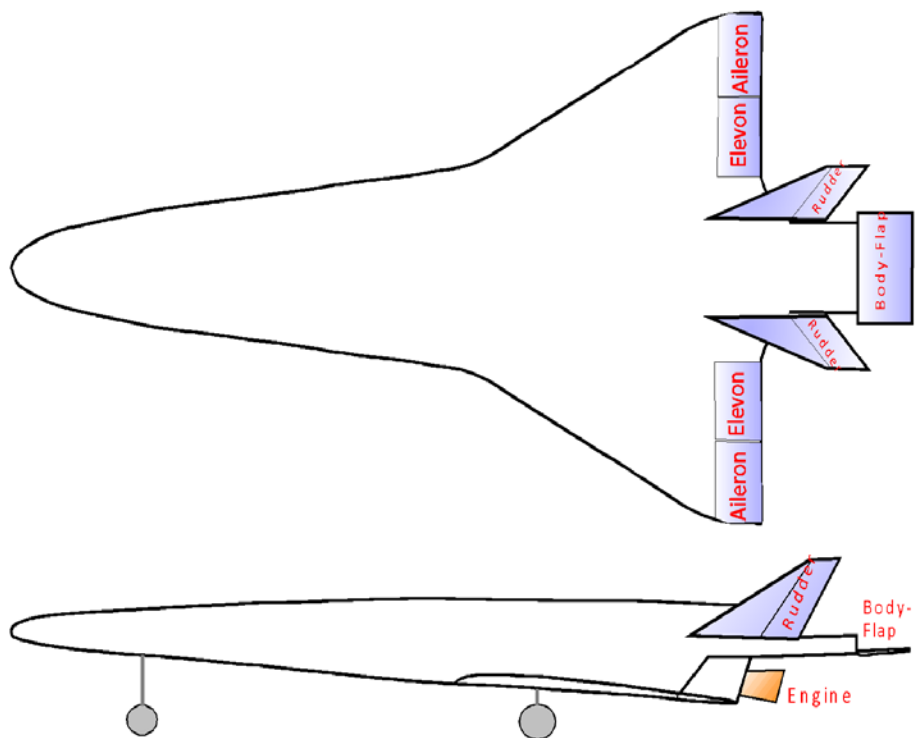
- The increment coefficients for the 4 aero-surfaces are in file: "*Hyp\_Desc.Delt*".
- The file "*Hyper.HMco*" includes the hinge moment coefficients data for the four aero-surfaces. It contains a 4-dimensional array of coefficients for each aerosurface (as a function of Mach, alpha, beta, and delta) and it is very similar to the aero-surface coefficients file.
- The file "*Hyp\_Desc.Unce*" includes the aerodynamic uncertainties data for the basic coefficients, the derivatives, and the aerosurface derivatives.
- The mass properties are in file "*Hyper.Mass*". The first column contains the vehicle mass in (slugs), and the remaining columns contain the corresponding inertias and CG location. Only the last mass point is used from this file (450 slugs) because the weight is not changing during descent.

There is no engine and jet data file in this example because the vehicle does not use any gimbaling engines or RCS jets during descent. The files generated by "Trim" are:

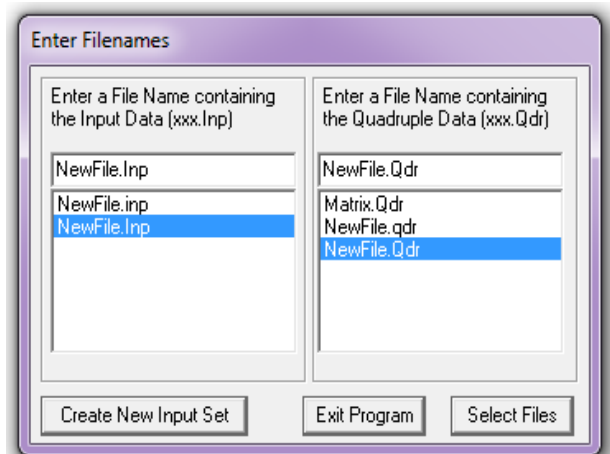
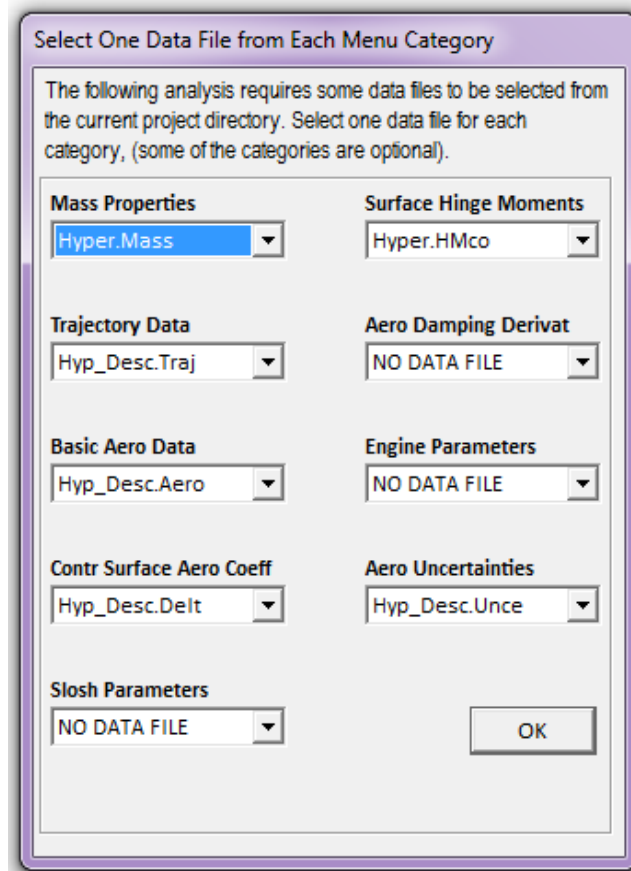
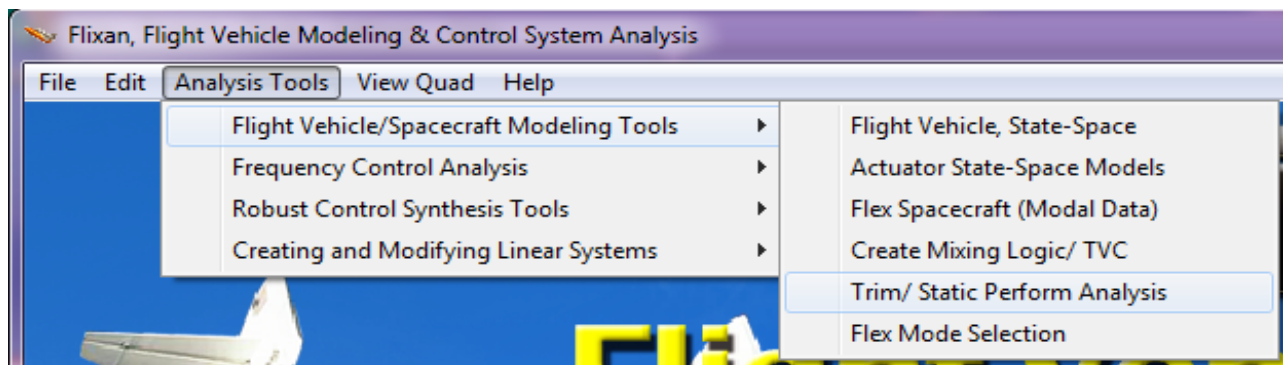
- File "*Hyp\_Desc.Trim*" contains the control surface trim positions along the trajectory. When additional trims are generated from the same trajectory file, the two previous trim trajectories are saved in files "*Hyp\_Desc1.Trim*" and "*Hyp\_Desc2.Trim*".
- File "*Hyp\_Desc.Perf*" contains the performance parameters calculated along the trajectory. The two previous performance parameter data are saved in files: "*Hyp\_Desc1.Perf*" and "*Hyp\_Desc2.Perf*".
- File "*Hyp\_Desc.HiMo*" contains the moments in (ft-lb) at the aerosurface hinges.

## 2.1 Checking the Trajectory Data

Let us begin the analysis by checking out the trajectory. Start the Flixan program and select the descent project folder "*C:\Flixan\Trim\Examples\Hypersonic Vehicle\ Descent*" that contains the analysis files. Then, from the Flixan main menu select "*Analysis Tools*", from the drop-down menu select "*Flight Vehicle/Spacecraft Modeling Tools*", and then "*Trim/Static Performance Analysis*".



From the following dialog select the input files that will be used by the Trim program, as shown below. From the input/ output filenames menu select the default "*NewFile*" names because we are not using any specific files for the time being.



The following is the Trim program main menu. Select the 2nd option for plotting the trajectory variables versus time in multiple window-plots. The first plot shows the CG location which is constant in this case because the vehicle does not deplete any fuel during descent. The next two figures show the angles of attack, sideslip, flight-path angle, relative velocity, Mach number, and dynamic pressure.

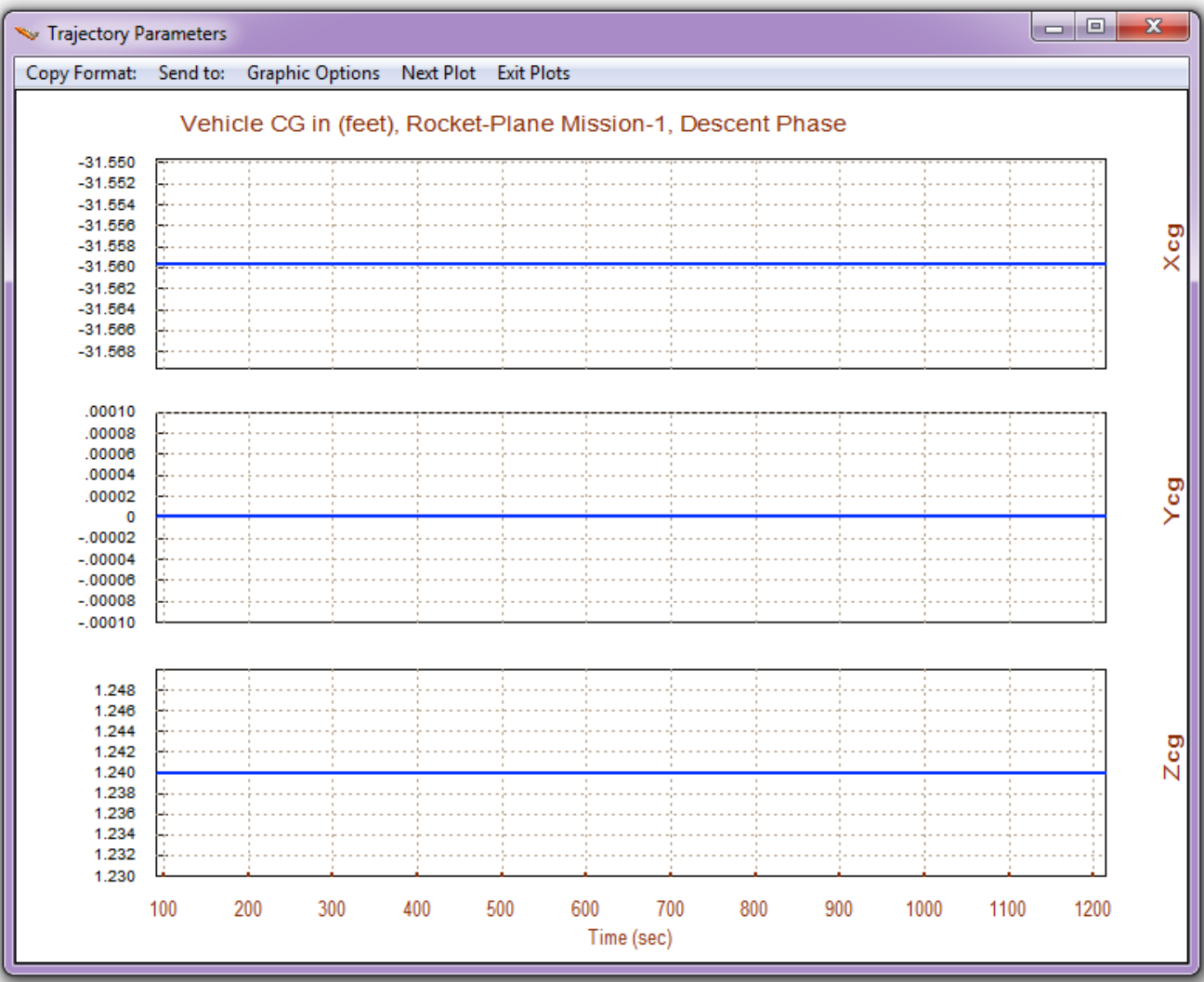


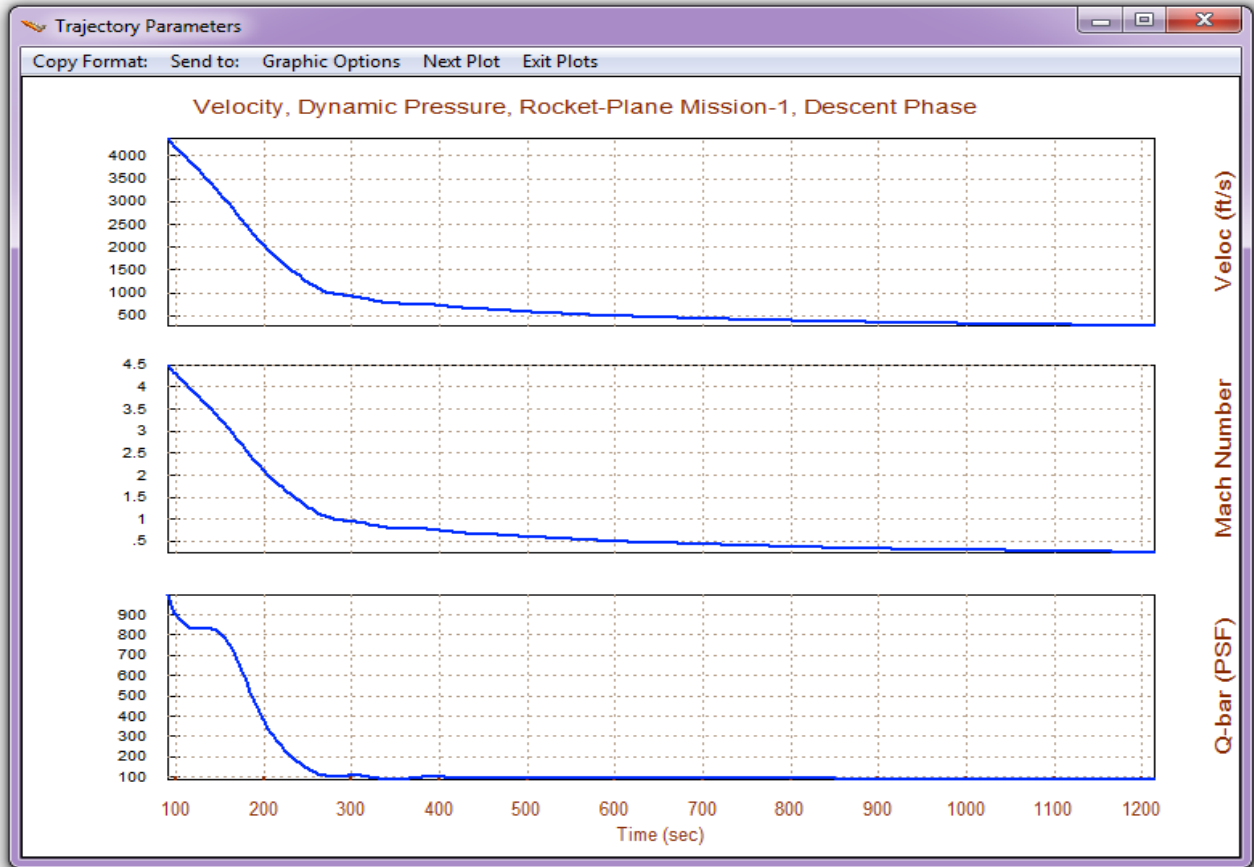
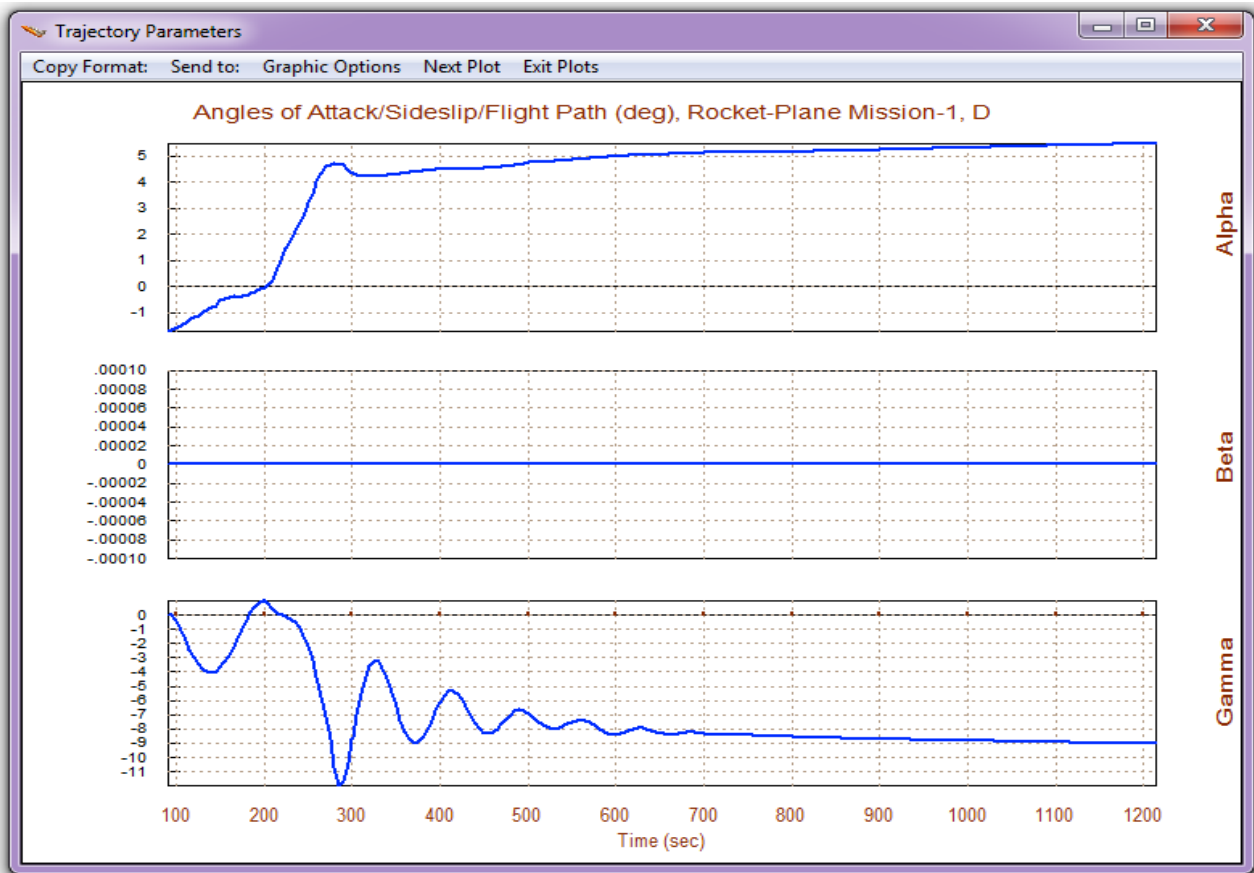
Main Trim Menu

Select one of the following options

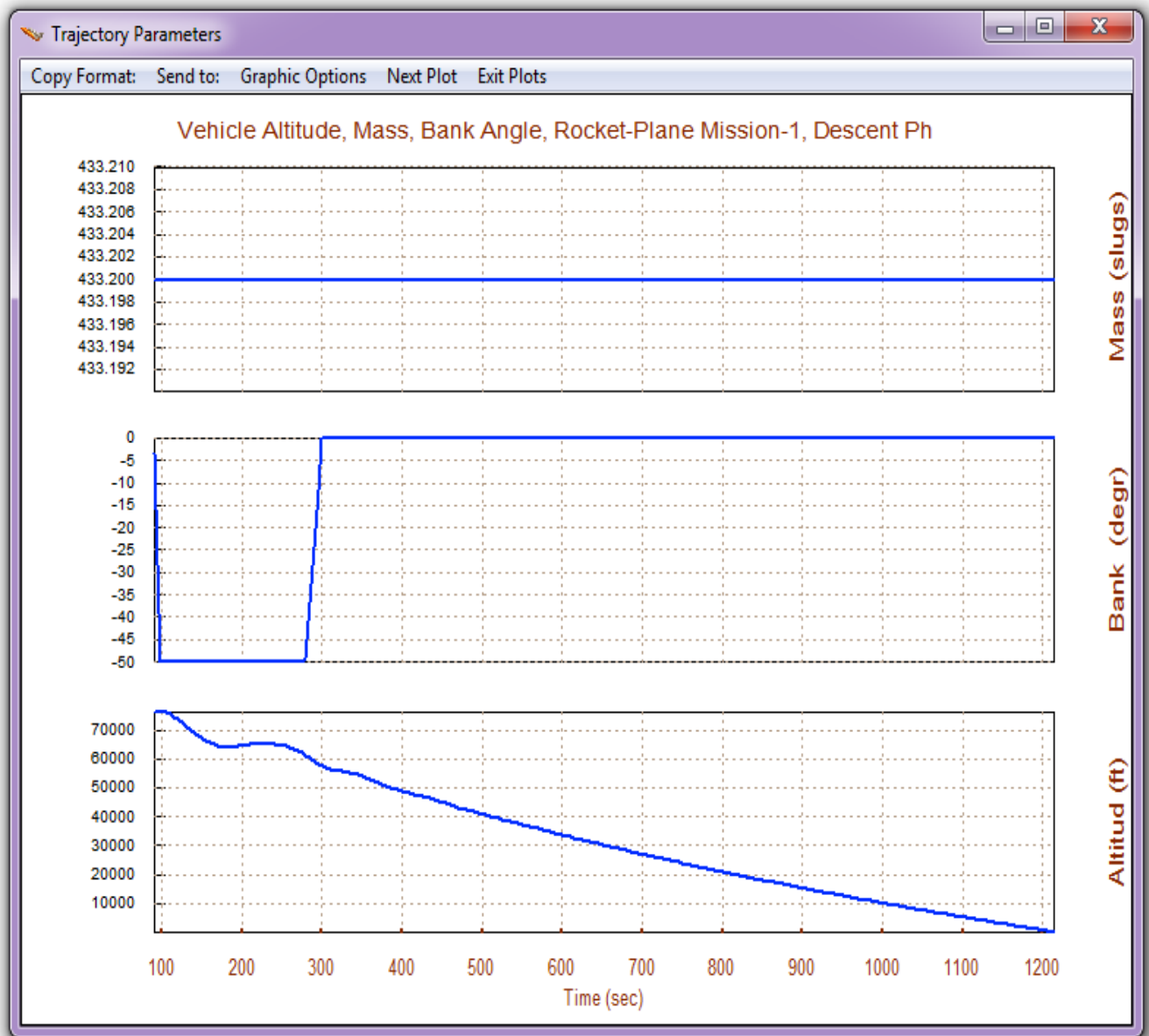
Exit OK

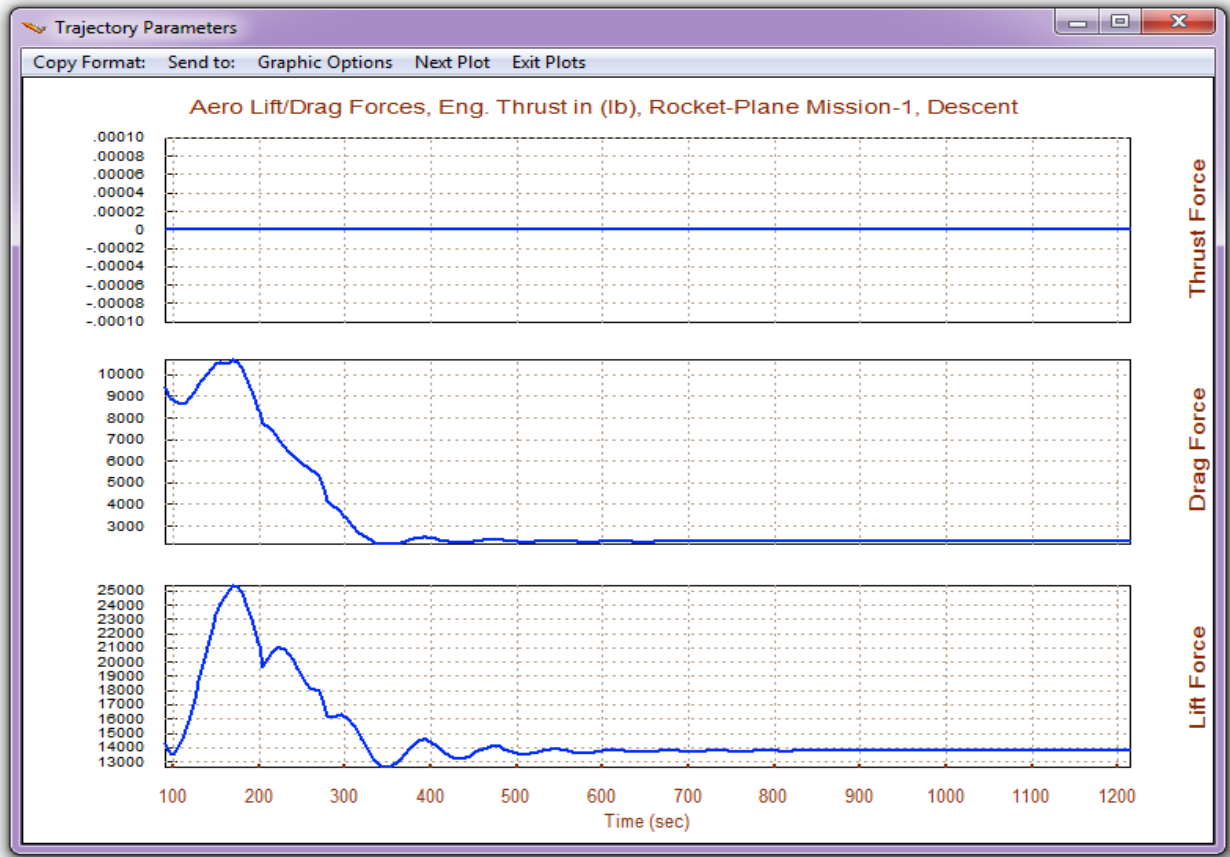
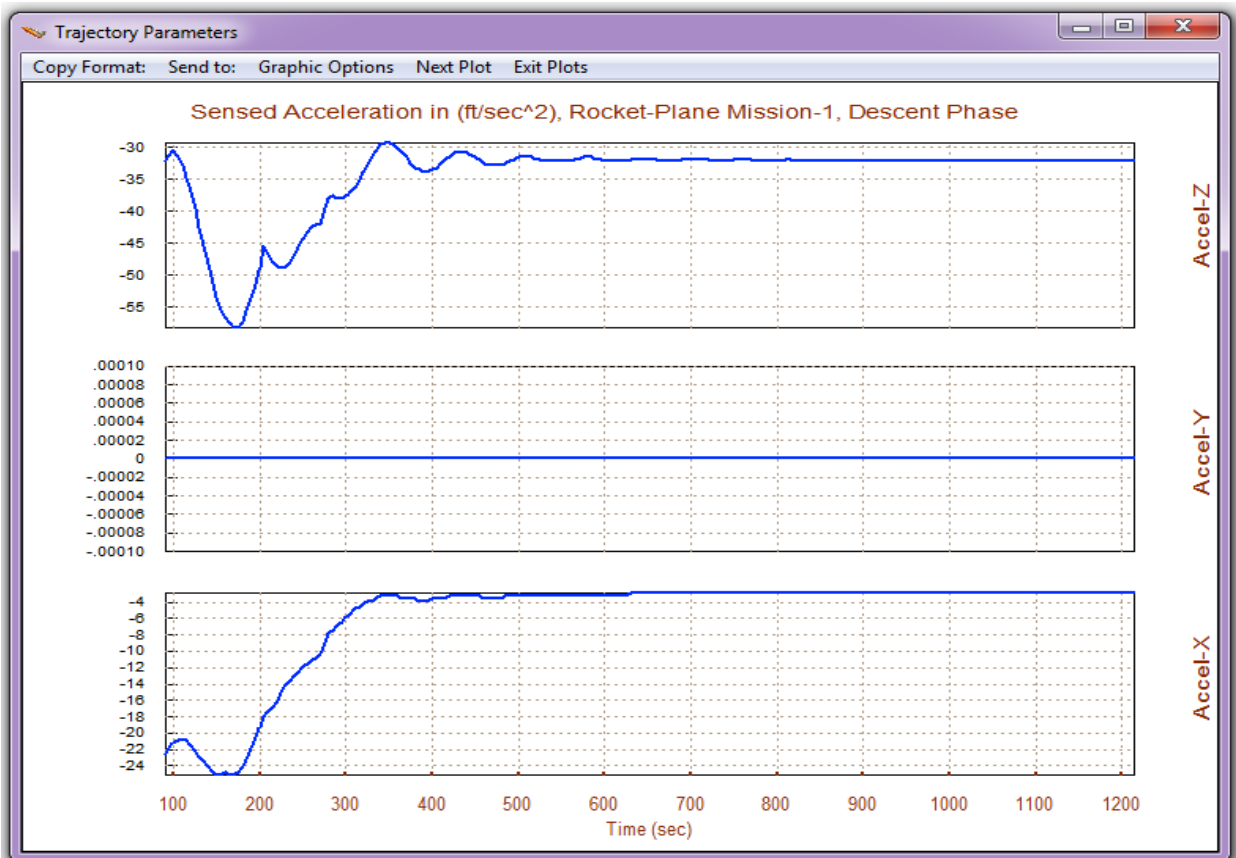
1. Plot Aero Coefficients, Derivatives, and Control Surface Increments
2. Plot Trajectory Parameters Versus Time from the Trajectory File ".Traj"
3. Trim the Effector Deflections to Balance the Vehicle Moments and Forces
4. Create an Effector Mixing Logic or a TVC Matrix (Kmix)
5. State-Space Modeling of the Flight Vehicle at Selected Times
6. Performance and Stability Parameter Plots Along Trajectory Time
7. Landing and Pull-Up Maneuverability, plus, Inertial Coupling Effects
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10. Contour Plots (Mach versus Alpha) for Performance, Control Authority Analysis
11. Vector Diagrams for Maneuverability & Stability at Selected Flight Conditions
12. Plot and Compare Previous Data Files (Traject, Trim, Perform, Hinge Moment)





The next figures show the vehicle mass in (slugs) which is constant, the bank angle that initially rotates  $-50^\circ$  for a period of 200 (sec) for the vehicle to maneuver and align itself with the runway. The altitude begins at 76,000 (ft) and decreases to approximately 100 (ft) above sea level near the landing site. The next 3 plots show the acceleration in  $(\text{ft}/\text{sec}^2)$  measured at the CG which stabilizes to approximately  $-1$  (g) in the  $-Z$  direction (lift) prior to landing. The thrust is zero because the engine is not used during descent. We finally have the lift and drag forces which are shown only for reference and not used by the trimming program. The  $(x, y, z)$  accelerations are used instead for trimming.



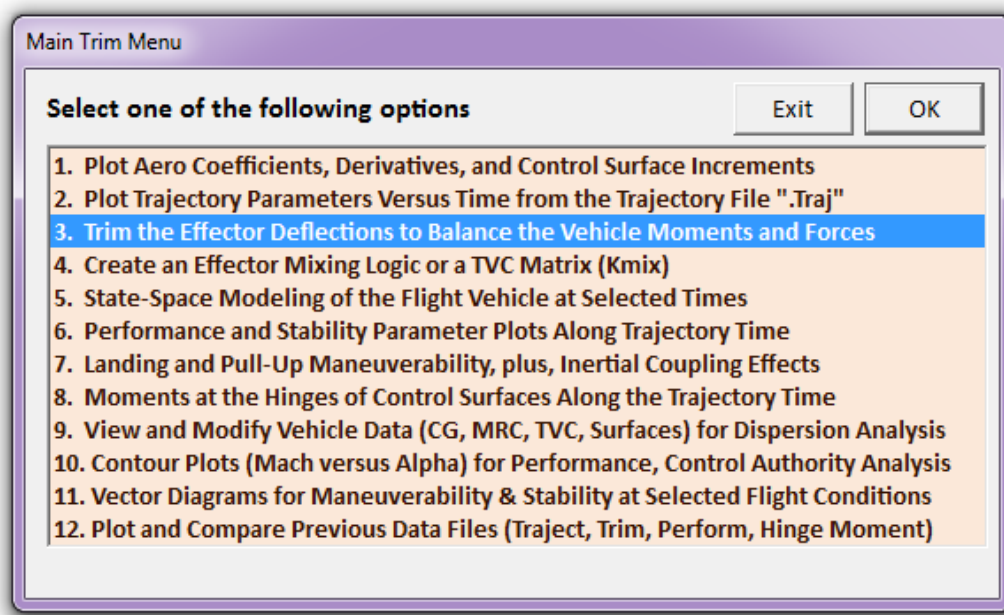


## 2.2 Trimming and Performance Analysis during Descent

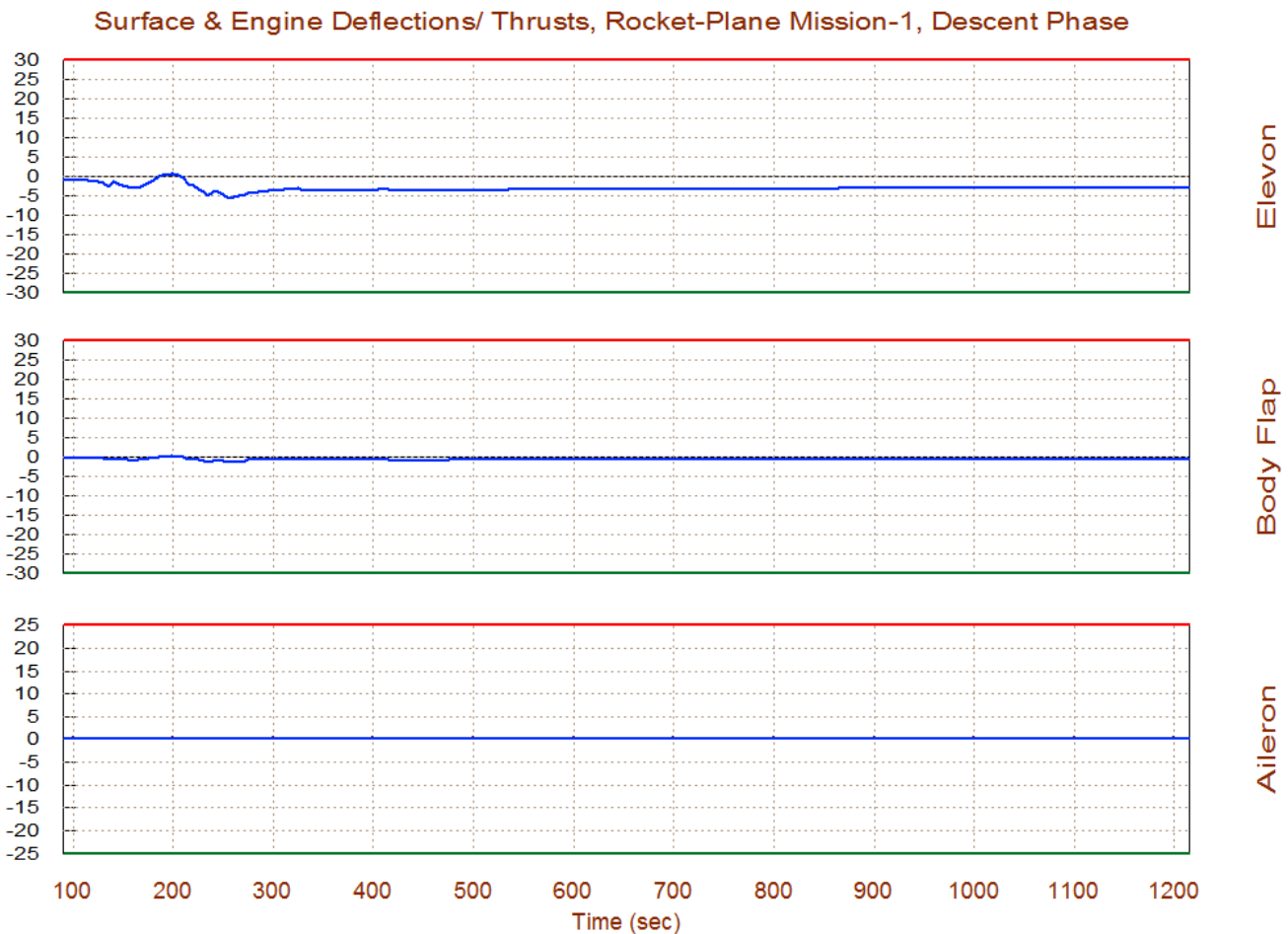
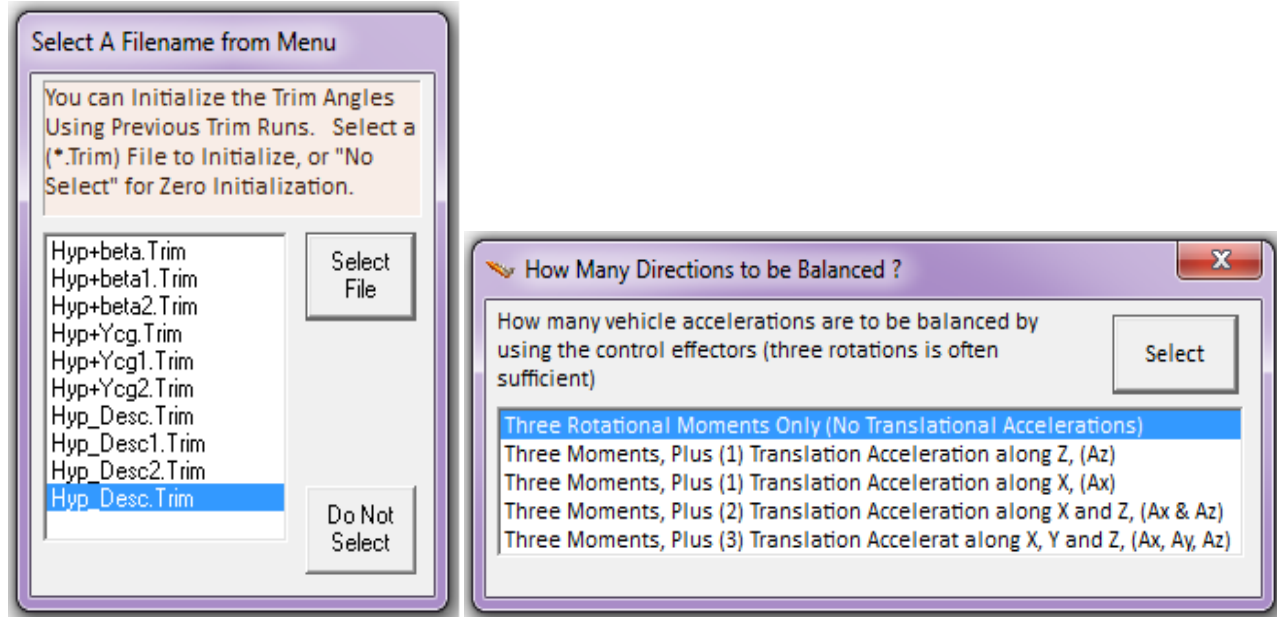
The vehicle during descent uses the Elevon and the Body-Flap for trimming and control in the longitudinal directions. Having two aerosurfaces it may be possible to independently control and trim not only in pitch but also in the z direction. However, before we decide on how to allocate longitudinal controllability we must first evaluate if the Body-Flap (BF) can provide sufficient controllability to trim the vehicle in the z-acceleration direction, independently of pitch which is trimmed mainly by the Elevon. If the BF is sufficiently effective in the z-direction we may decouple the pitch and z-acceleration directions and to control them both independently. Otherwise, the only use for the BF would be to assist the Elevon in pitch trimming and possibly also for pitch control. It may be used, for example, to bias the trim position of the Elevon so that it is more centered and improve its effectiveness. Plus controlling and trimming the normal acceleration independently of pitch may not be a very valuable feature because normal acceleration can also be achieved by means of alpha pitching. In this a case the BF can be used alongside with the Elevon by scheduling it to increase the Elevon effectiveness and also assisting parallel with the Elevon in pitch control by deflecting it relative to its scheduled position. We must, therefore, trim and analyze the vehicle controllability against aero-disturbances in both cases: (a) when the BF is operating in parallel with the Elevon to trim and control in pitch, and (b) when the BF is used independently of the Elevon to trim and control normal acceleration. We must examine both cases to determine which approach is better fitting.

### Trimming and Controlling Only 3 Moments

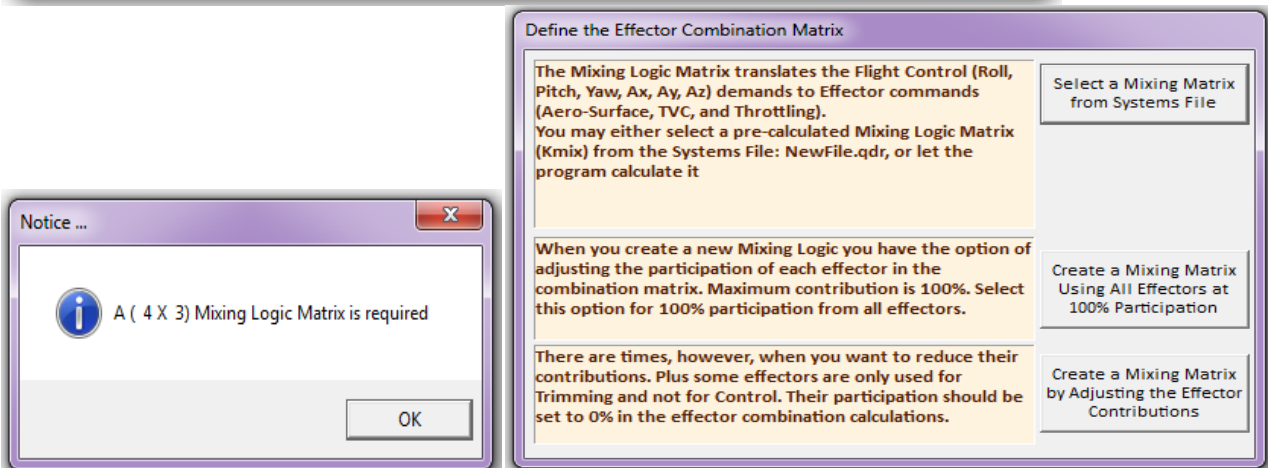
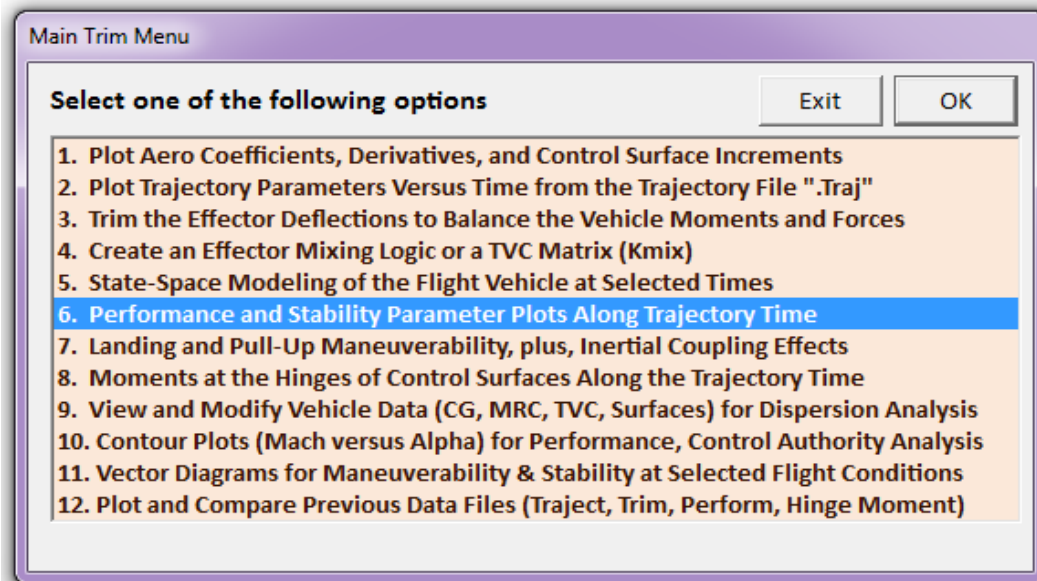
Let us first trim along the 3 rotations. During descent the bias in the Body-Flap aero-surface data file "*Hyp\_Desc.Delt*", is set to zero. Return to the Trim main menu and select the third option for trimming the control surfaces along the selected trajectory.



In the next menu "Do Not" initialize Trim from a previous trim file. From the menu that selects the degrees-of-freedom to be trimmed choose the first option that trims only the 3 moments, as shown below, and the Trim algorithm will attempt to trim along the 3 rotations using the 4 aero-surfaces.

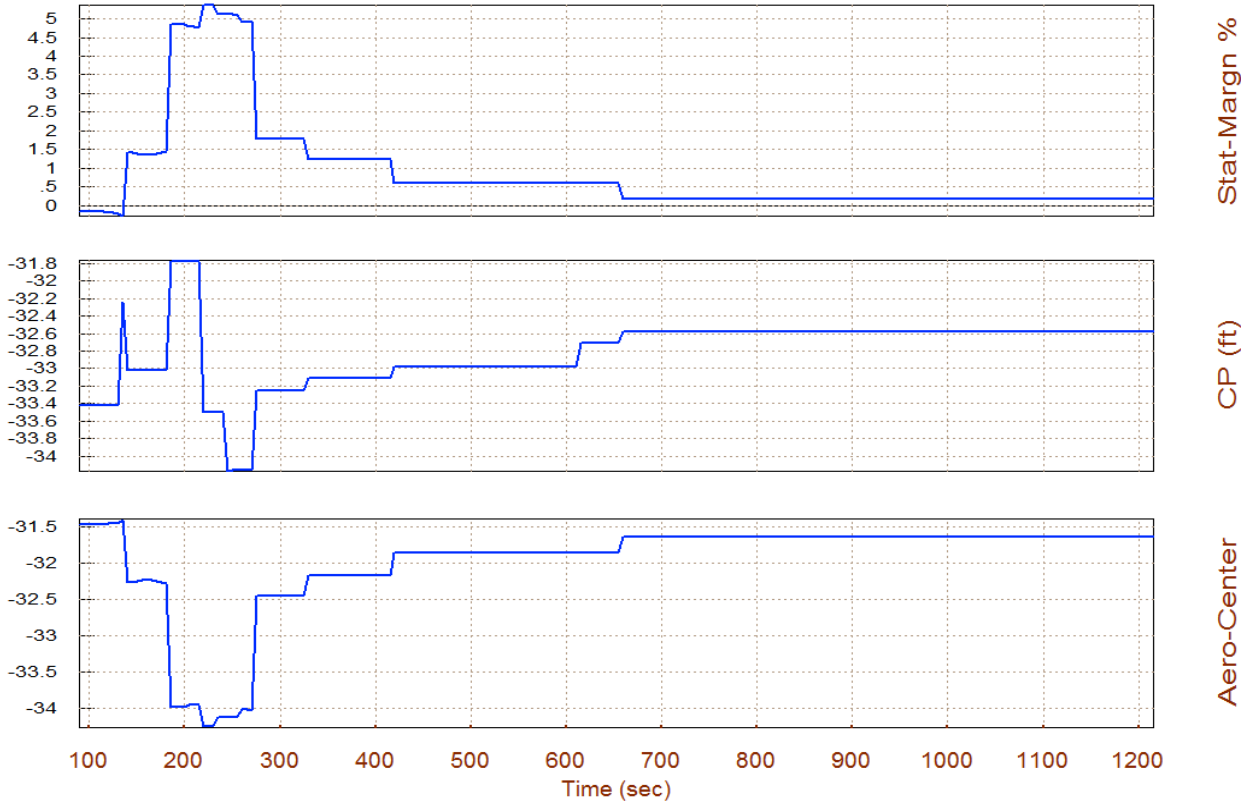


The above plot shows the trim deflections of the Elevon and the Body-Flap. The aileron and rudder deflections are zero because there is no lateral excitation due to roll/yaw symmetry. Now let us analyze the vehicle performance parameters using the above trimming condition. Return to the Trim main menu and select option (6) for analyzing the performance parameters along the descent trajectory. The Trim program requires a (4x3) control surface combination matrix that allocates the 3 control demands efficiently among the surfaces. The 3 matrix inputs are the (roll, pitch, and yaw) directions to be trimmed and also controlled. The 4 matrix outputs drive the control surfaces (elevon, body-flap, aileron and rudder). Since we do not have a mixing matrix defined yet we should allow the program to calculate it along the trajectory by choosing the second option in the effector combination dialog below.

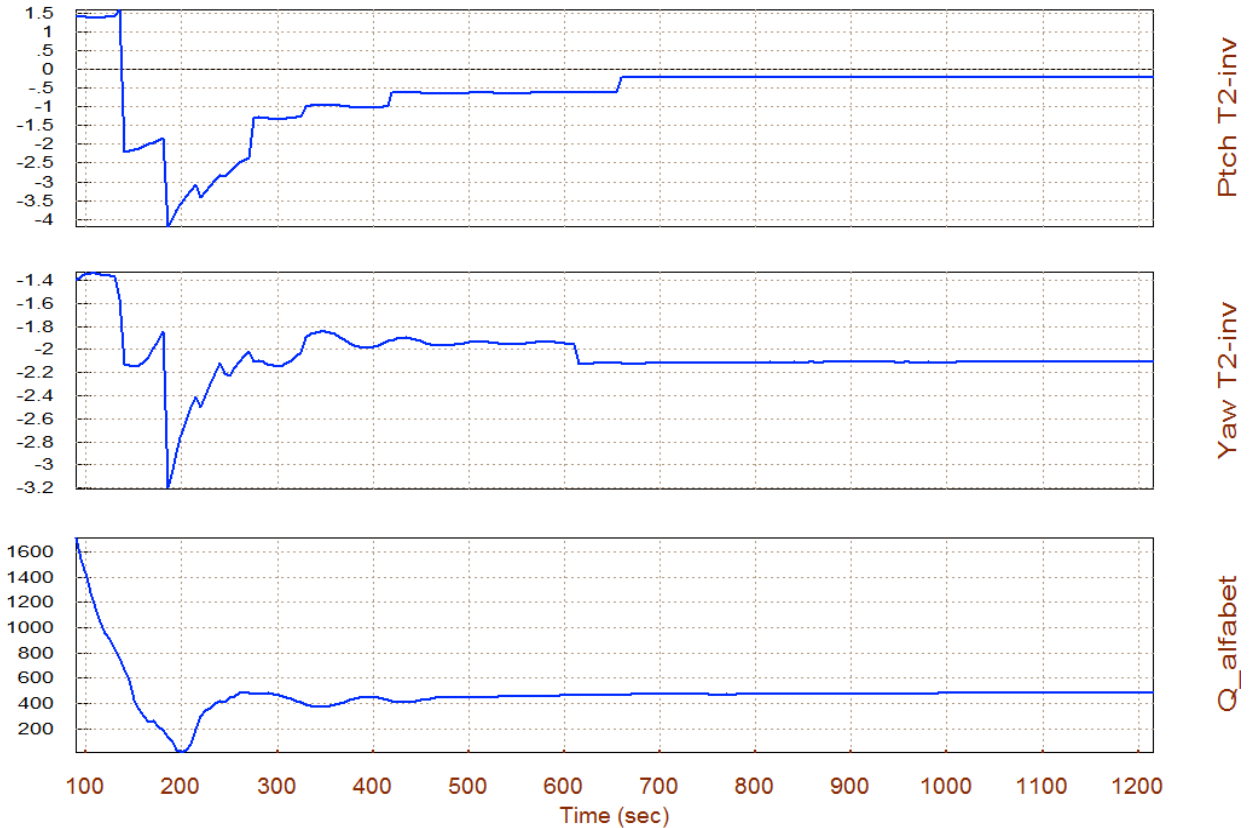


In the next dialog we must also define the wind-shear disturbance in terms of the maximum  $\alpha$  and  $\beta$  dispersions from trim, which is  $3^\circ$  in both  $\alpha$  and  $\beta$ .

Static Margin, Center of Pressure, Aero-Center (ft), Rocket-Plane Mission-1, Des

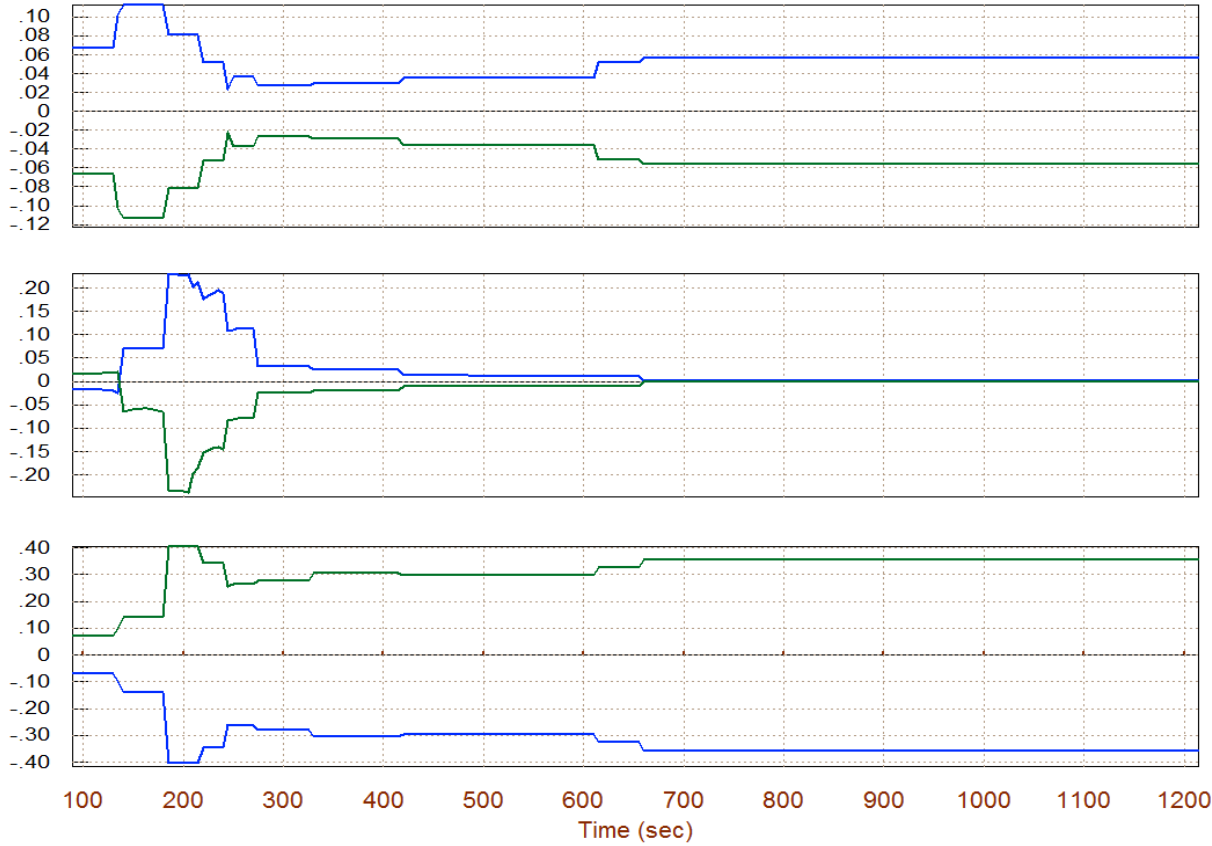


Short-Period ( $w$ )/ Time-to-Double-Ampl-Inverse (/sec),  $Q_{\alpha\beta}$  (deg-lb/ft<sup>2</sup>)

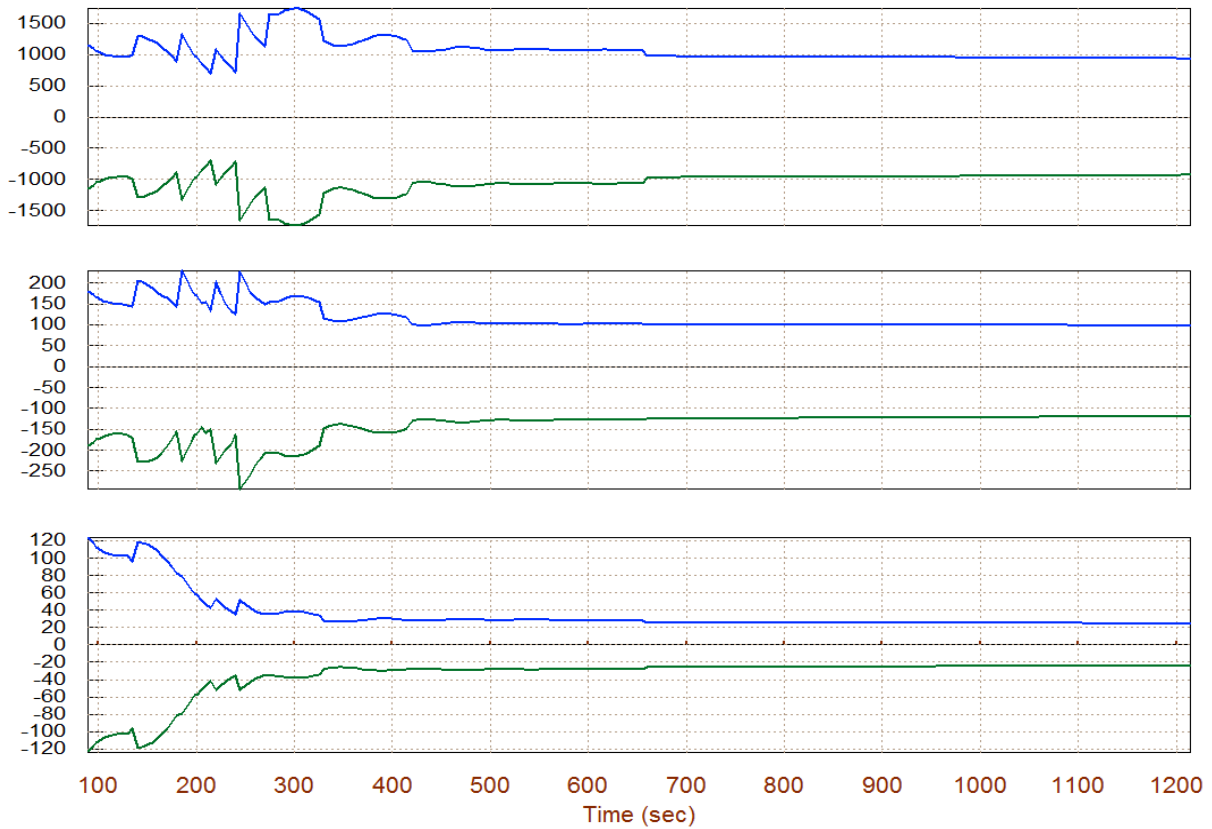




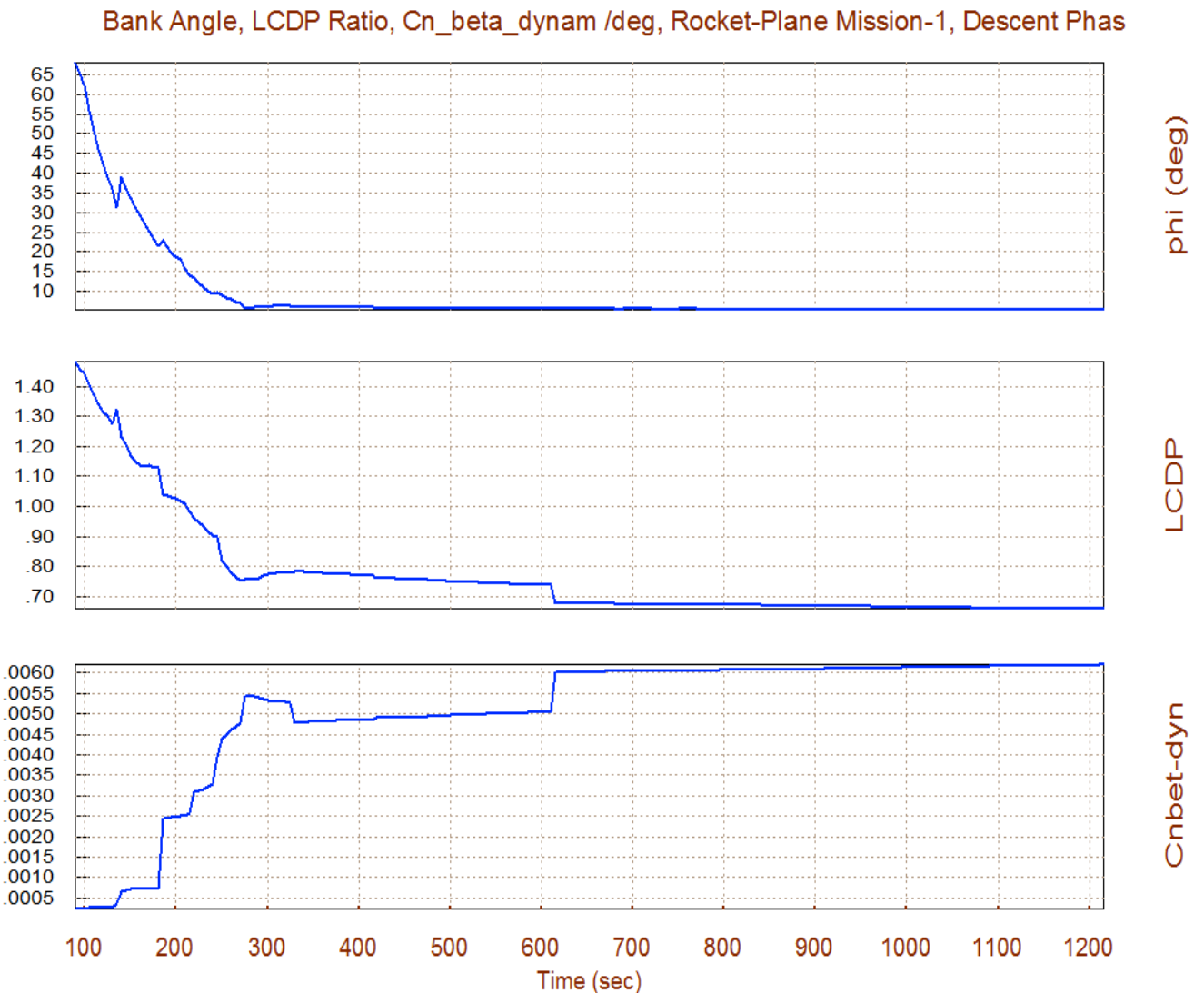
Rotation Control Authority  $|dQ/dQ_{max}| < 1$  for 3 (deg) of Alpha & Beta Variation



Max Angular Accelerations (rad/sec<sup>2</sup>), at Maximum +ve and -ve Control Demands

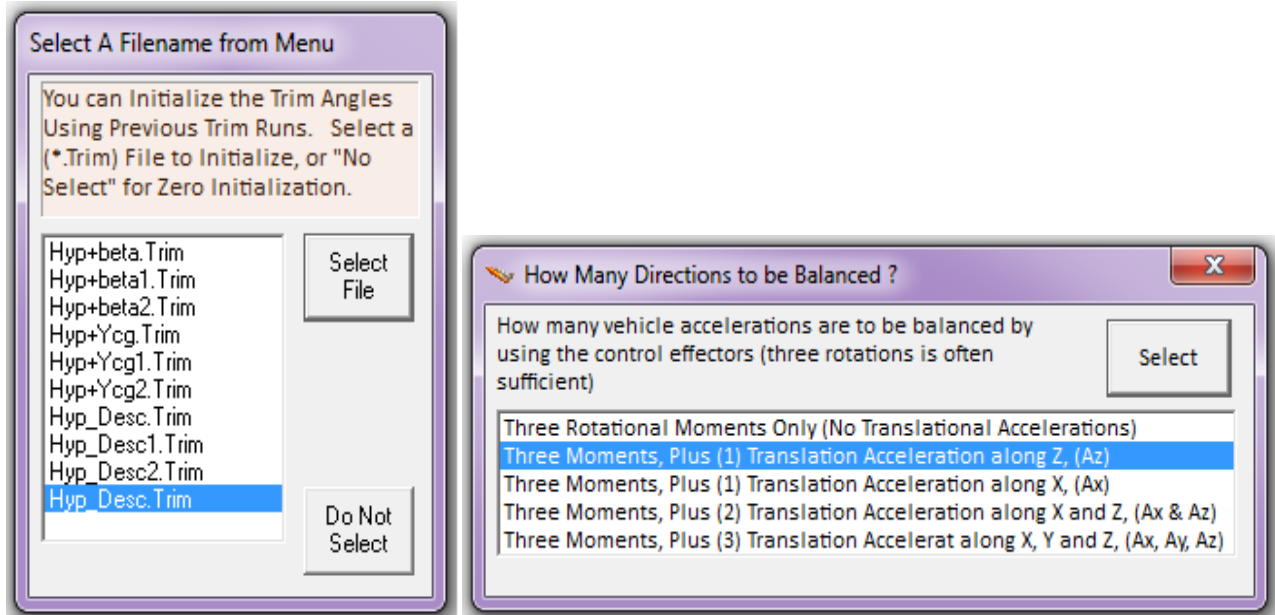


The static margin shows that for the most part the descent vehicle is statically stable in pitch. It is slightly unstable during the first 40 sec. A previous plot shows that the short-period and the Dutch-roll resonances peak at 4 and 3.2 (rad/sec) respectively. The short period (pitch resonance) is very small (0.24 rad/sec) during the second half of the flight which is a sign that the vehicle is close to neutral stability in pitch. The  $(Q_\alpha, Q_\beta)$  plot measures the normal and lateral load. Its peak value at  $3^\circ$  of  $(\alpha_{\max} \ \& \ \beta_{\max})$  is 1,600 (psf-deg), which is acceptable. The  $C_n\beta$ -dynamic is positive throughout showing lateral stability. The LCDP ratio is also very good. The bank angle  $\phi < 2.5^\circ$  near landing shows that there is no problem with cross-wind. It means that  $3^\circ$  of sideslip due to a steady cross-wind-shear will cause less than  $2.5^\circ$  of roll. The control authority against  $\pm\alpha_{\max}$  and  $\pm\beta_{\max}$  dispersions is also very good because the magnitude of the control effort parameter is much less than one in all 3 axes for both positive and negative dispersions. It means that the vehicle has the control authority to trim in roll, pitch, and yaw against  $\pm 3^\circ$  of  $\alpha$  and  $\beta$  dispersions from trim. The figures also show the maximum accelerations achieved when the controls are maximized in the positive (blue) and in the negative (green) directions.

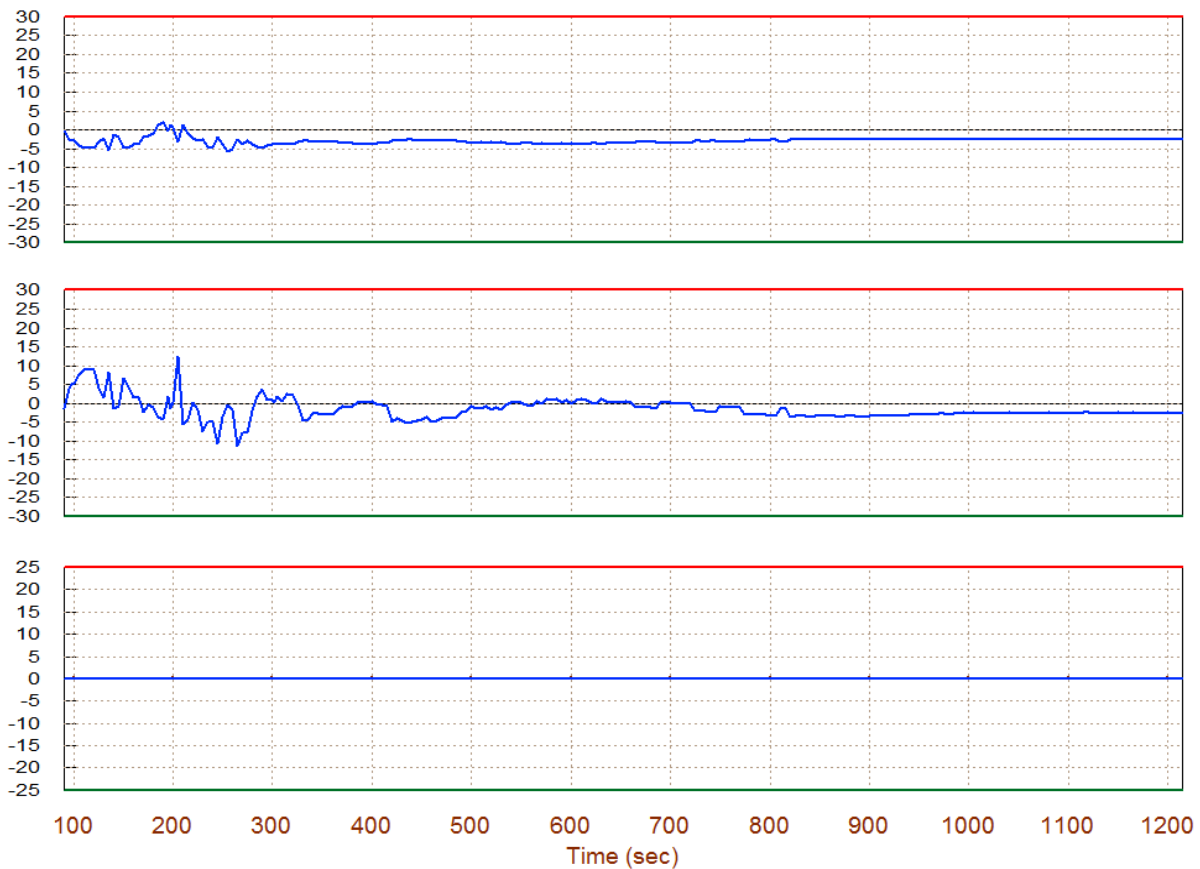


### Trimming and Controlling 3 Moments plus the Z-Acceleration

Now let us return to the Trim main menu and select option (3) again to re-trim. This time we will include the Z-acceleration, in addition to the three rotations, to see if it is possible to trim along the 4 directions. It means that the vehicle will use the Elevon and mainly the BF to also trim along the normal acceleration which is defined in the trajectory.

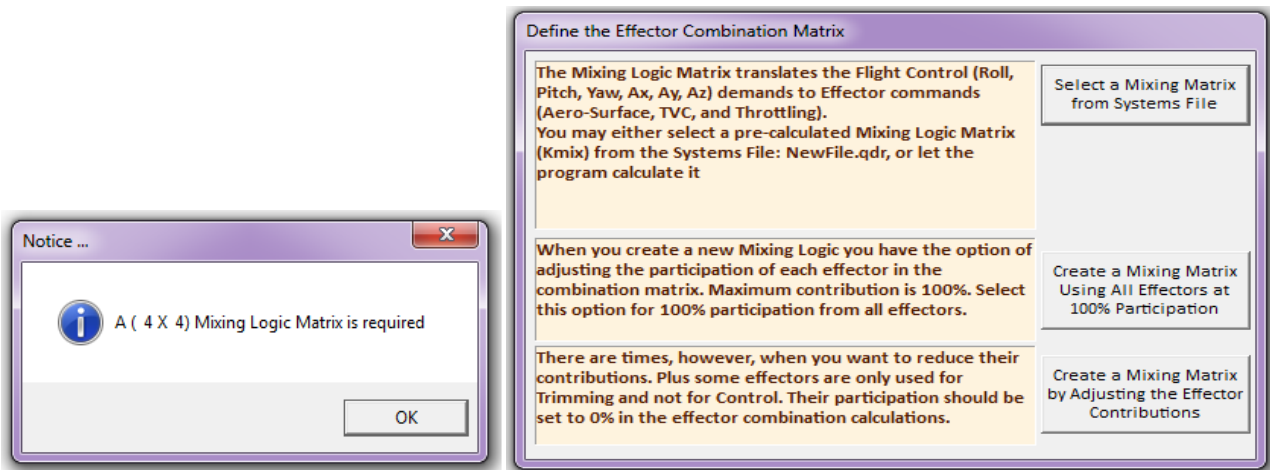


Surface & Engine Deflections/ Thrusts, Rocket-Plane Mission-1, Descent Phase

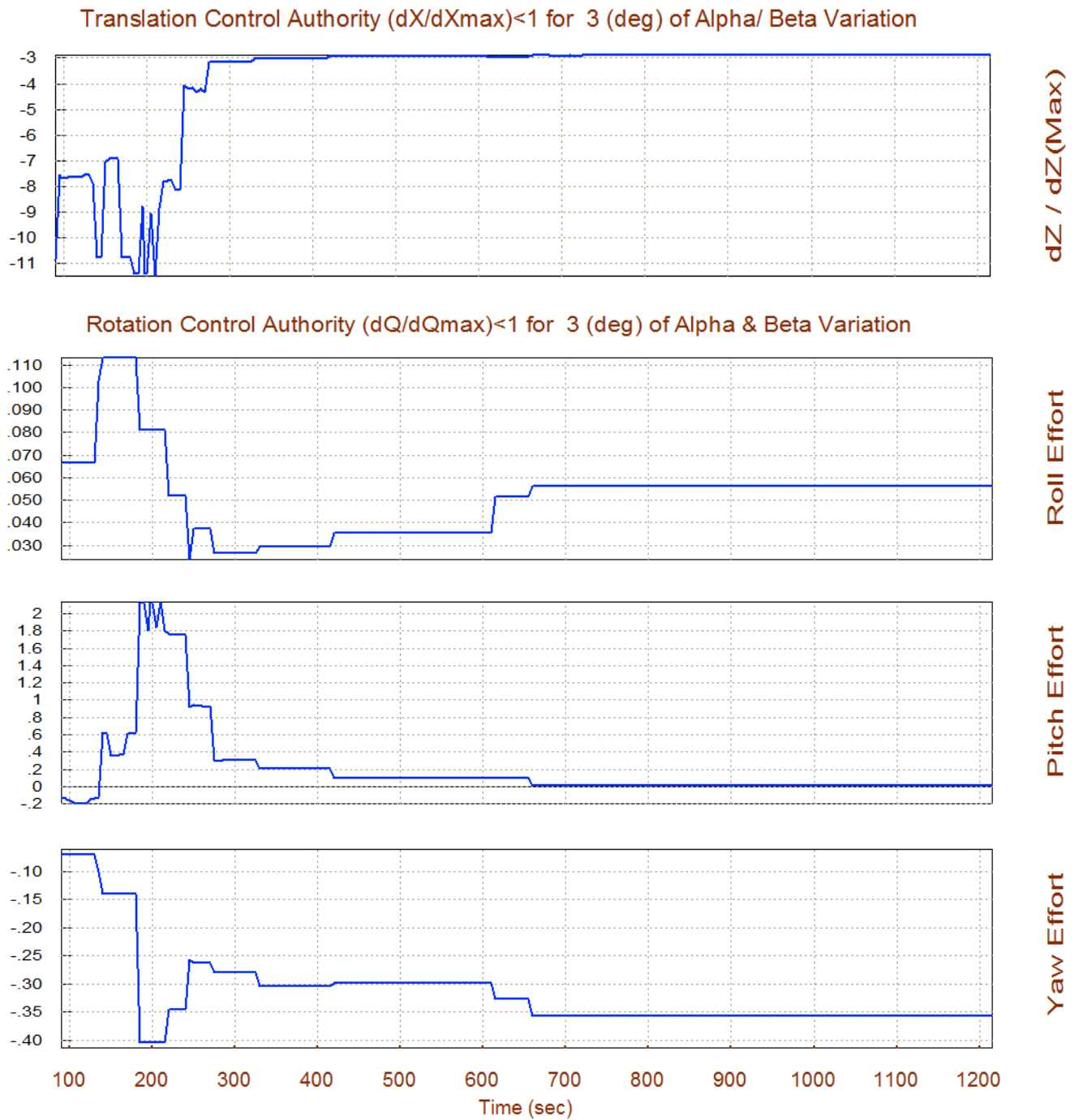


The aerosurface deflections plot shows that although it is possible to trim in 4 axes, the activity of the Body-Flap in trimming the z-acceleration is much bigger than in the previous case where we trimmed only moments and the BF was only assisting the Elevon in pitch. The noise in the Body-Flap signal in the high Qbar region is attributed to the fact that small angle of attack variations require big Body-Flap deflections in order to balance the z-acceleration. This may be a sign perhaps that the Body-Flap effectiveness is not sufficient to independently control and trim in the z-direction. It requires too much deflection to balance small variations in alpha. However, this must be further investigated using controllability analysis and we must repeat the performance analysis using the latest trim conditions.

Return to the Trim main menu and select option-6 again to analyze performance parameters along the descent trajectory using the latest trimming conditions. Now the Trim program requires a (4x4) control surface combination matrix. It is assumed that the flight control system is expected to independently control roll, pitch, yaw, plus normal acceleration. The 4 matrix inputs are the (roll, pitch, yaw, and Nz-acceleration) directions to be trimmed and also controlled. The 4 matrix outputs go to the 4 aero-surfaces. Since we do not have a mixing-logic matrix defined yet we should again let the program to calculate it along the trajectory by choosing the second option in the effector combination dialog.



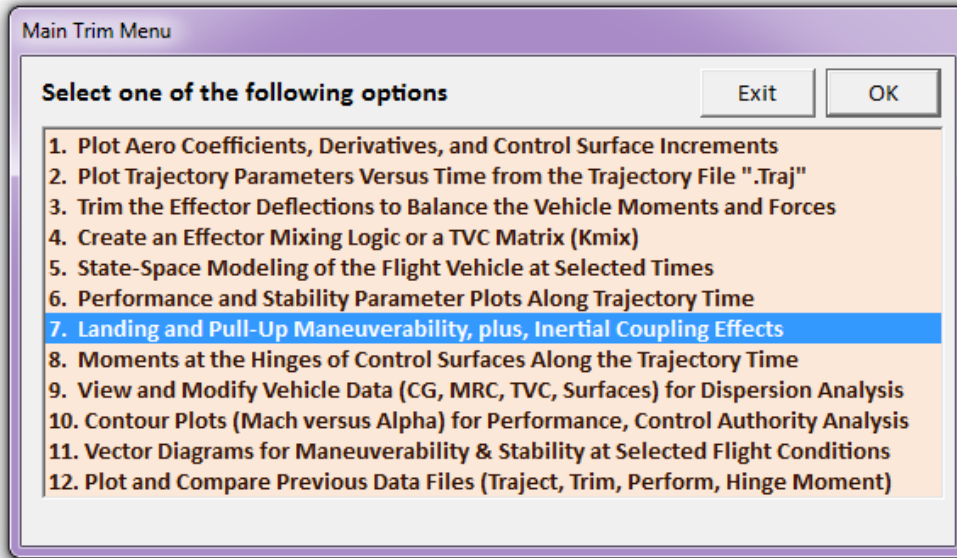
The following plot shows the control effort required to trim along the 4 selected directions. There is no problem in lateral but in the longitudinal directions it clearly proves that the control effort required exceeds the control capability of the effectors system. It is not possible to control both directions ( $\dot{q}$  &  $\ddot{z}$ ) against alpha dispersions, independently from each other using the two aerosurfaces. Theoretically, it may be possible to trim and to control in all four directions independently if the disturbances are very small, but this would not be robust. The vehicle will not be able to handle any significant amount of alpha disturbance or to maneuver. So let us now forget about using the Elevon and the Body-Flap independently to control pitch and Nz and let us reduce our expectations to trimming only in 3 directions (roll, pitch, and yaw). We should go back to option (2) and re-trim selecting the first option in the trim directions menu, "*Three rotational moments only (no translations)*".



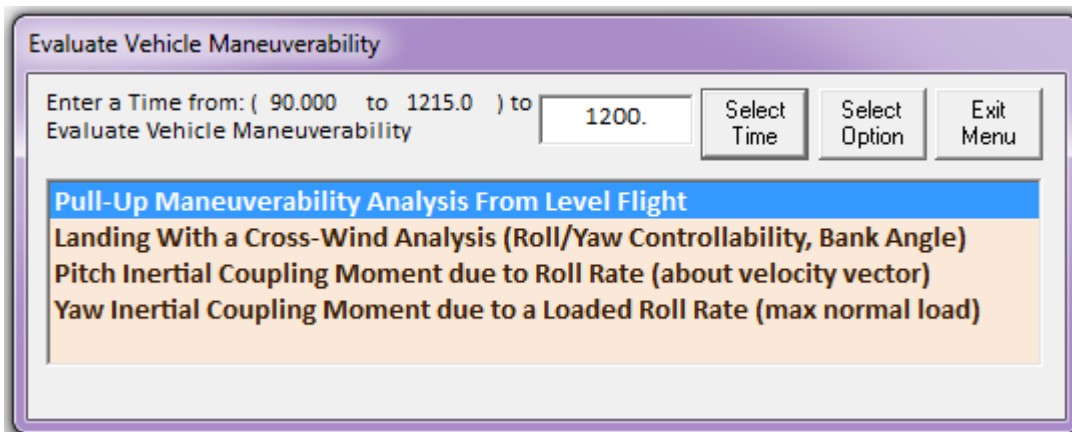
The longitudinal controllability is greatly reduced when attempting to independently control Nz and pitch directions. The control effort in the pitch axis is violated between times 180 to 250 sec. It means that 3° of alpha dispersion from trim is too much for pitch control system to tolerate. The authority in the Nz control loop, shown at the top plot, is even worse. The magnitude of the z-axis control effort ( $\delta\ddot{z}/\delta\ddot{z}_{max}$ ) greatly exceeds the max allowable effort that should be less than 1. When operating independently, the two controllers are incapable of producing the control authority required to overcome wind disturbances in neither direction. They can, however, join forces together to control the pitch direction, which also controls Nz indirectly via  $\alpha$ .

## 2.4 Additional Performance Parameters

Let us now return to the Trim main menu and select option-7 to analyze some situations which are mostly applicable to a maneuvering aircraft. This analysis is performed at selected fixed flight conditions and not along the entire trajectory.

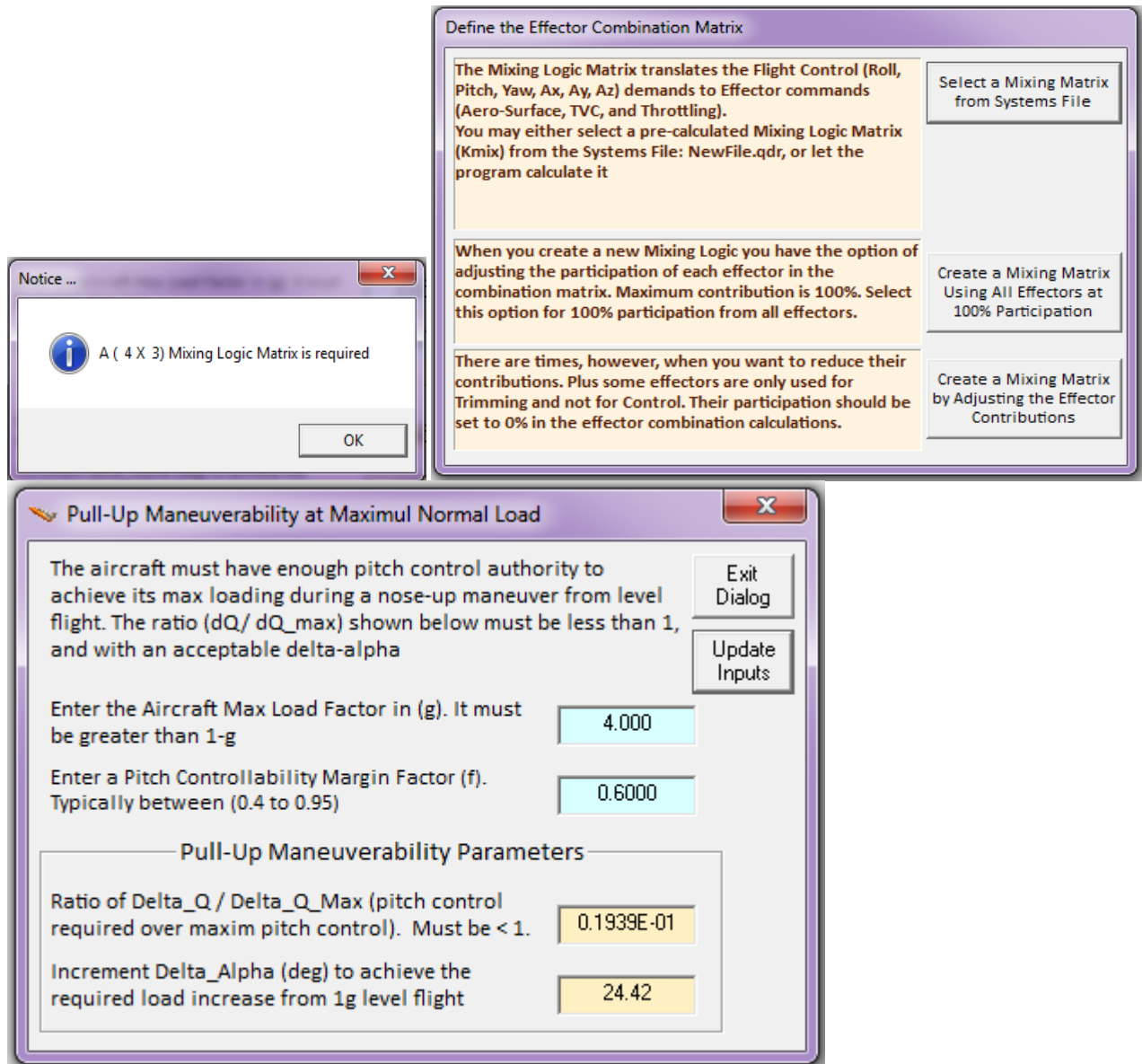


The following menu consists of 4 maneuverability analysis options, as it is described in Section 3. It includes a flight time input field where the user must enter the analysis flight time within the trajectory time range, and click on "Select Time". You may then select one of the options in the menu, beginning with the first option, which analyzes the control authority of the vehicle's pitch effectors to perform a pull-up maneuver from level flight, as described in equations (3.41 and 3.42), and then click on "Select Option".



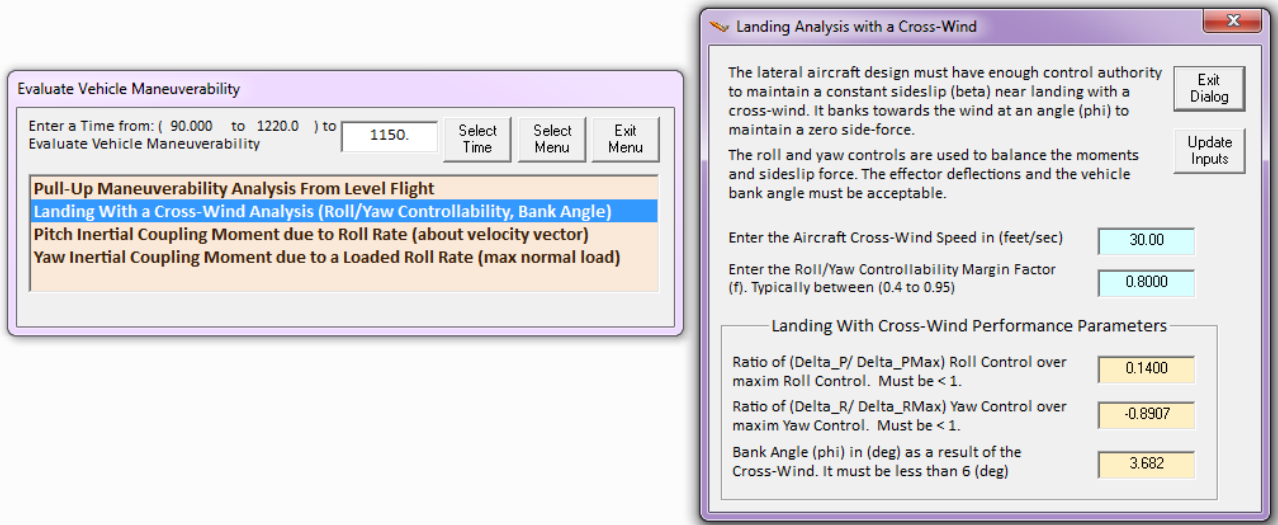
The program requires a (4x3) effector combination matrix, like before, so we let the program calculate a matrix by clicking on the second option in the next dialog "Create a Mixing Matrix Using All Effectors at 100% Participation". Another dialog comes up that is specific to the pull-up maneuverability option and where the user enters and receives data. In the light cyan fields you

must enter the vehicle maximum load factor, let's say 4-g's, and also a margin factor for pitch controllability (say 0.6). It means that you don't want to allocate more than 60% of pitch control for this maneuver because you want to leave some control capability for other functions, like gusts, etc, and click on the "Update Inputs" button on the right.

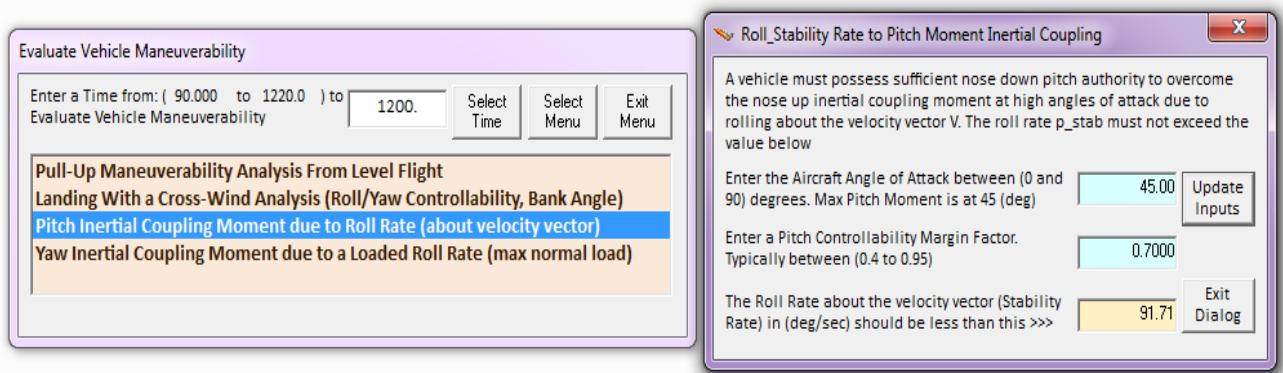


The results in the yellow fields at the bottom show that in order to pull 4 g's from level flight, the angle of attack must be raised to an additional 24.4° from trim. This, however, requires a trivial amount of pitch control effort (only 0.0194). This must be less than one because one corresponds to the total allocated pitch control, which in this case is 60% of the total available. This small amount of effort is due to the fact that the vehicle is almost neutrally stable in this flight condition.

Now let us try the second option that has to do with analyzing the vehicle's ability to land with cross-winds. This test is useful at this time prior to landing because we must analyze the cross-wind effects in roll. Let us select a time near landing and click on "Select Options". On the RHS dialog we enter the cross-wind 30 (ft/sec), a controllability margin factor less than one to allow some control space for other functions, and click on "Update Inputs". The results in the yellow fields show that the aileron control effort is small. The rudder effort is not so small but marginal because it is close to 0.9. The bank angle due to the cross-wind is 3.7°, which is acceptable.

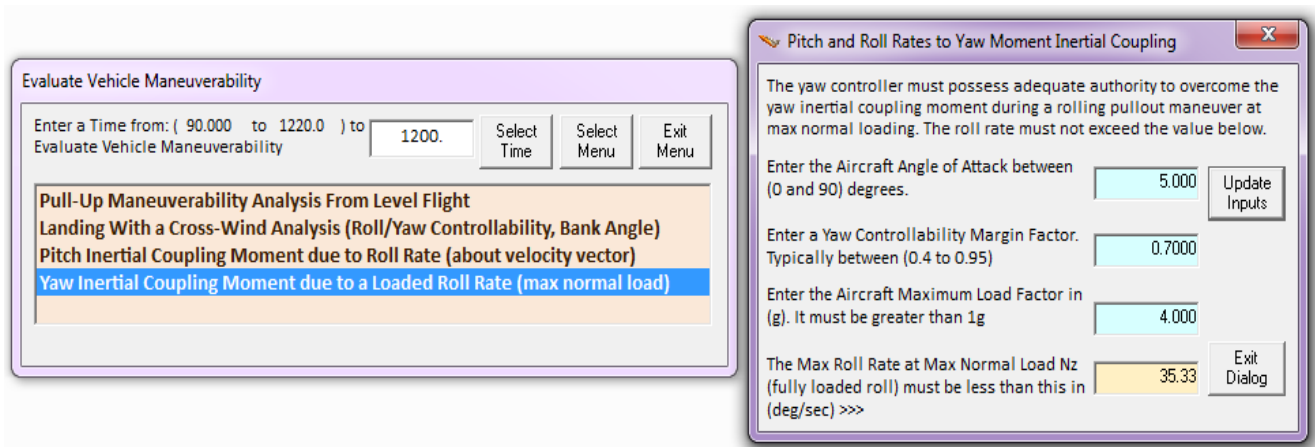


The third option analyzes the roll to pitch inertial coupling. When the vehicle is rolling about its velocity vector it produces a pitch torque that has to be taken out by the elevon. This test of course does not have any meaningful application in this vehicle because it is not maneuvering much. On the RHS dialog enter the angle of attack about which the vehicle is rotating (45° produces the worst pitching moment), enter the pitch controllability margin factor (0.7), and click on "Update Inputs". The result in the yellow field at the bottom is telling us that the vehicle may roll at rates as high as 92 (deg/sec) about the velocity vector (which is at 45° of alpha) without saturating the pitch (elevon) controllability.





In the fourth test we assume that the vehicle is capable of maneuvering like a fighter aircraft and it is rolling under max normal load. During such a loaded pullout maneuver there is a significant amount of yawing moment generated due to the inertial pitch/roll coupling that has to be taken out by the rudder (or yaw control). We are interested to calculate what is the max roll rate under peak normal load that the vehicle is able to perform without saturating the rudder. Select the fourth option from the menu and the corresponding dialog comes up on the right. In the cyan fields we must enter the angle of attack ( $5^\circ$ ), the yaw controllability margin factor (0.7), and the max normal load factor (4-g's). Click on the "Update Inputs" button and the yellow field at the bottom is showing us that a maximum roll rate of 36 (deg/sec), is a safe pullout roll rate under max loading that will keep the rudder within range. This completes the additional performance tests.

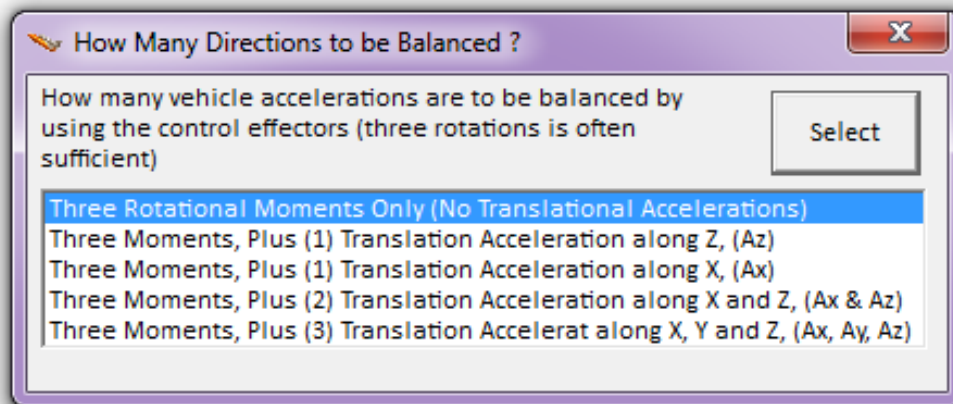
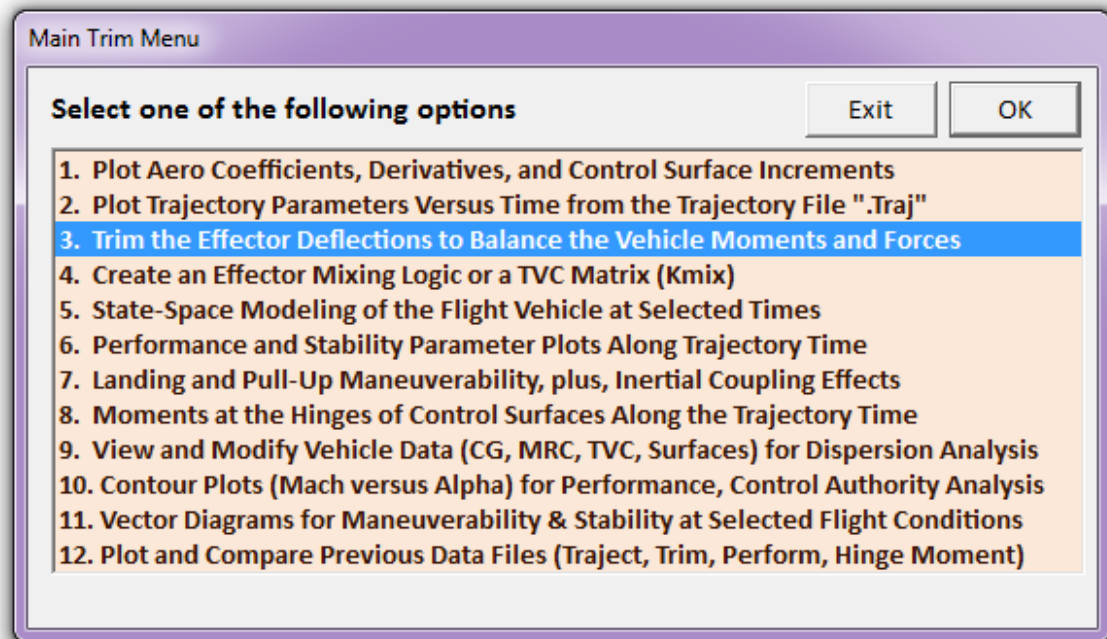


## 2.5 Scheduling the Body-Flap

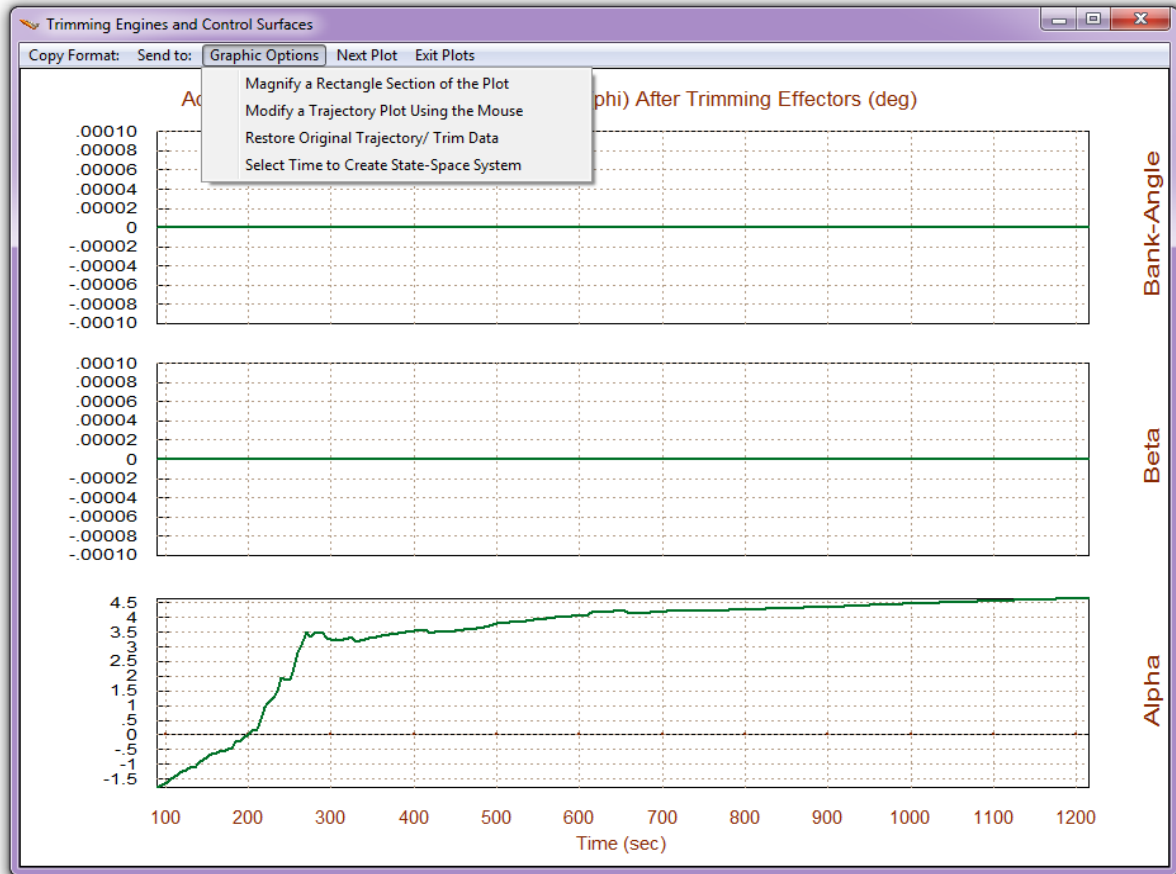
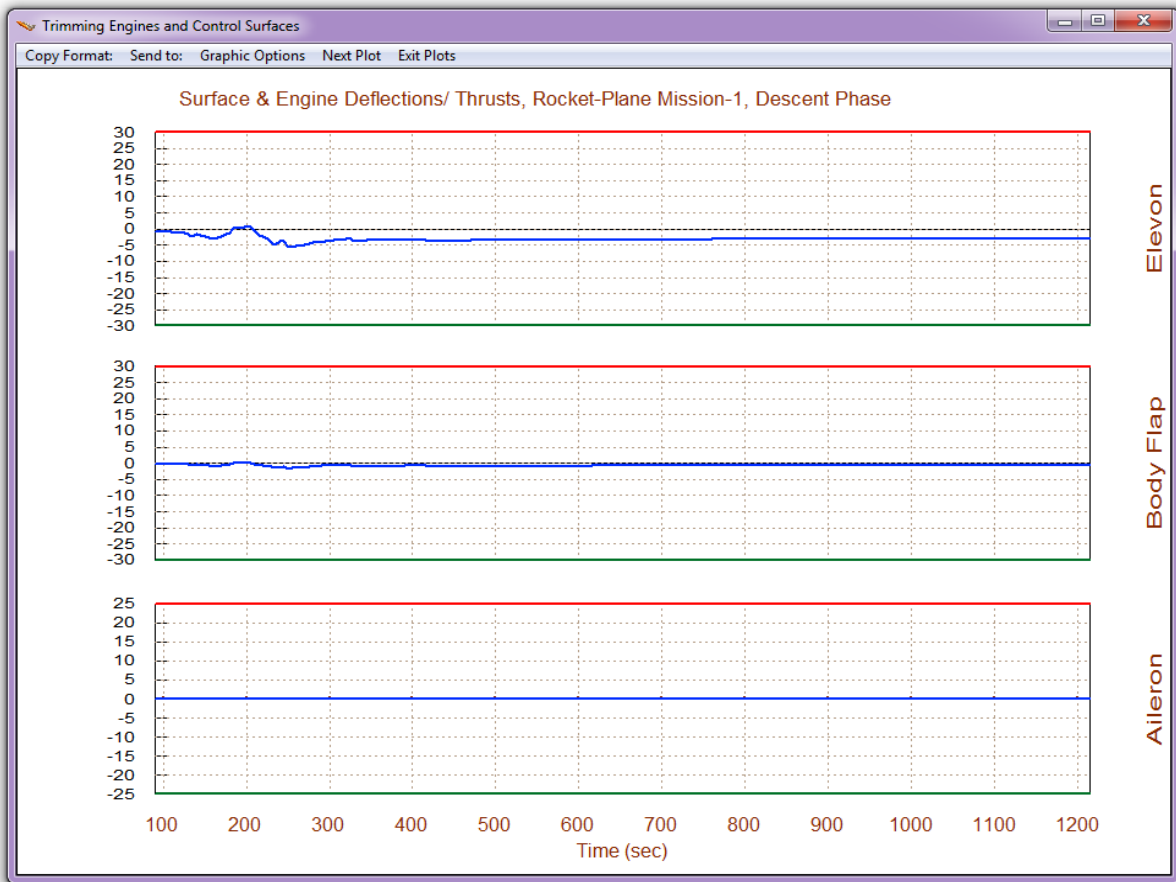
In section (2.2) we attempted to use the Body-Flap as a means to control and trim the vertical acceleration independently of  $\alpha$ . We concluded that although this is theoretically possible, it is not feasible, however, because it does not produce the controllability required to counteract the normal forces generated when the vehicle deviates a couple of degrees from nominal  $\alpha_0$ . The pitch axis controllability was also degraded because the effectors control authority was divided in two directions. We also concluded that normal acceleration can be controlled indirectly by controlling  $\alpha$  via pitch control and, therefore, we compromised by trimming 3-rotational dof and a flight control system using 3 control loops. So is there any use for the Body-Flap? Yes, it can be used for pitch control in combination with the Elevon. We can design a mixing logic matrix that combines them in parallel for pitch control. But in addition and most importantly it can be biased or scheduled to offer better trimming conditions

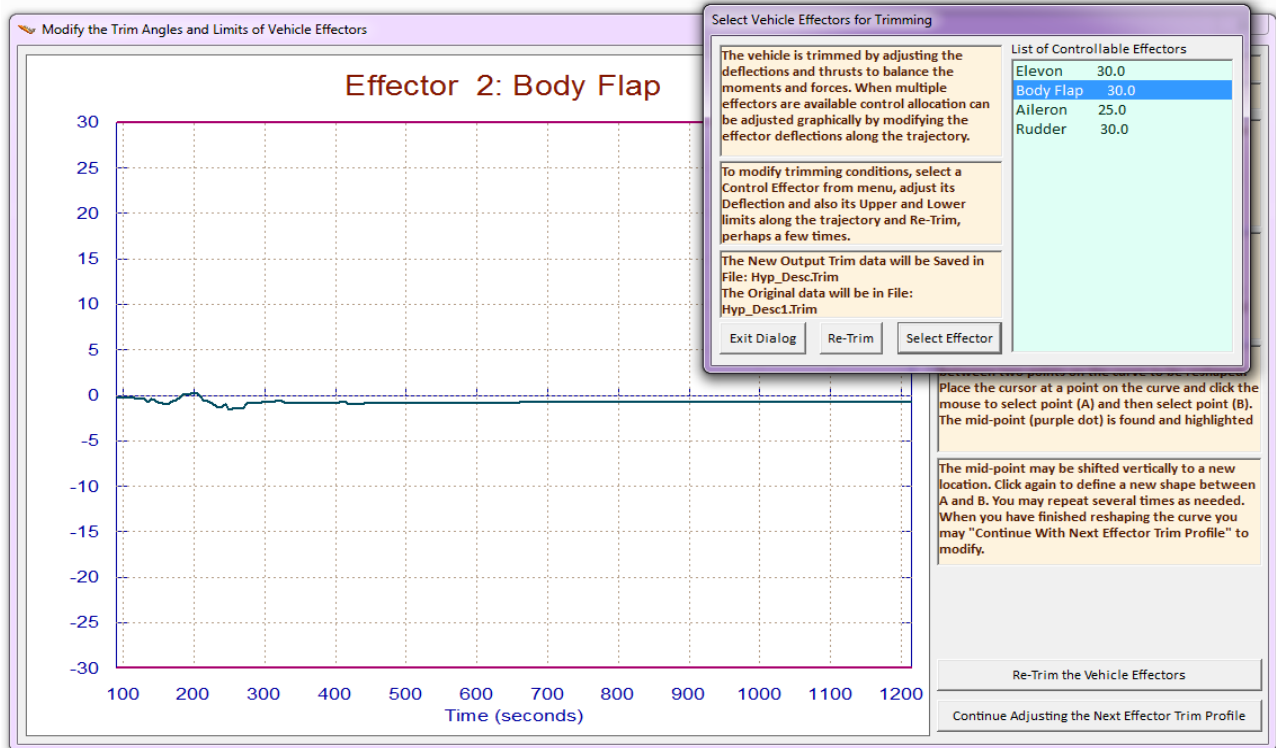


for the Elevon. Let us return to the main menu, re-trim the vehicle in roll, pitch and yaw, and we shall attempt to re-schedule the Body-Flap from its default trim trajectory.



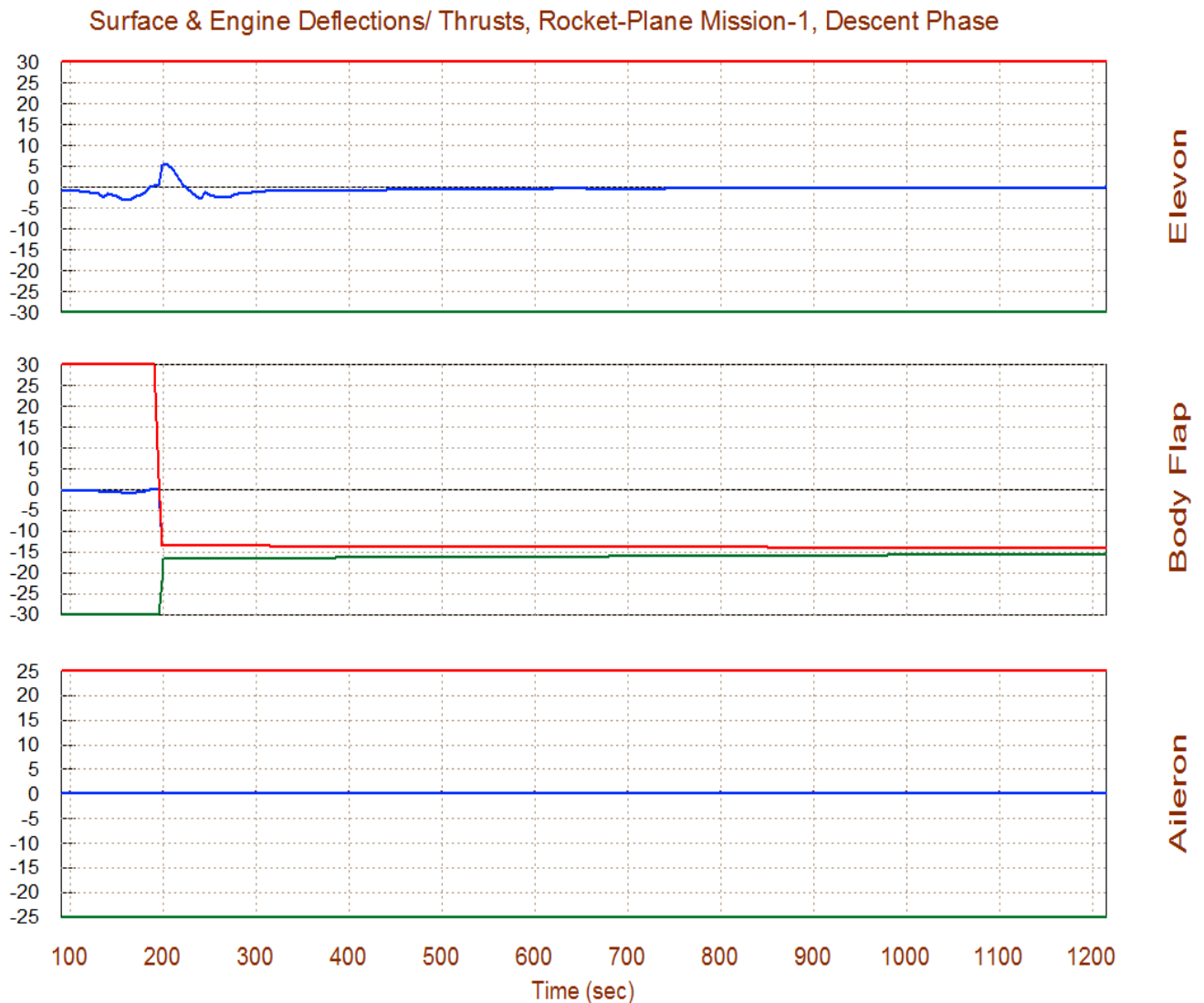
Let us re-trim along only the 3 rotations. The unbiased Trim results show that during most of the flight, pitch trimming is accomplished mainly by the Elevon which is deflecting at  $-4^\circ$ , and the Body-Flap is not contributing very much. We may be able to bias or schedule the Body-Flap in order to shift the elevon deflection closer to zero. By doing so the Elevon will have a wider deflection capability for control, maneuvering, and reacting to gusts. We can modify the position of the Body-Flap and constrain its limits graphically in Trim and then re-trim the control surfaces using the modified Body-Flap trajectory. The Elevon will re-adjust itself to a new position to balance the pitch moment. From the horizontal menu located near the top of a Trim window, select "Graphic Options" and then "Modify a Trajectory Plot Using the Mouse", as shown.

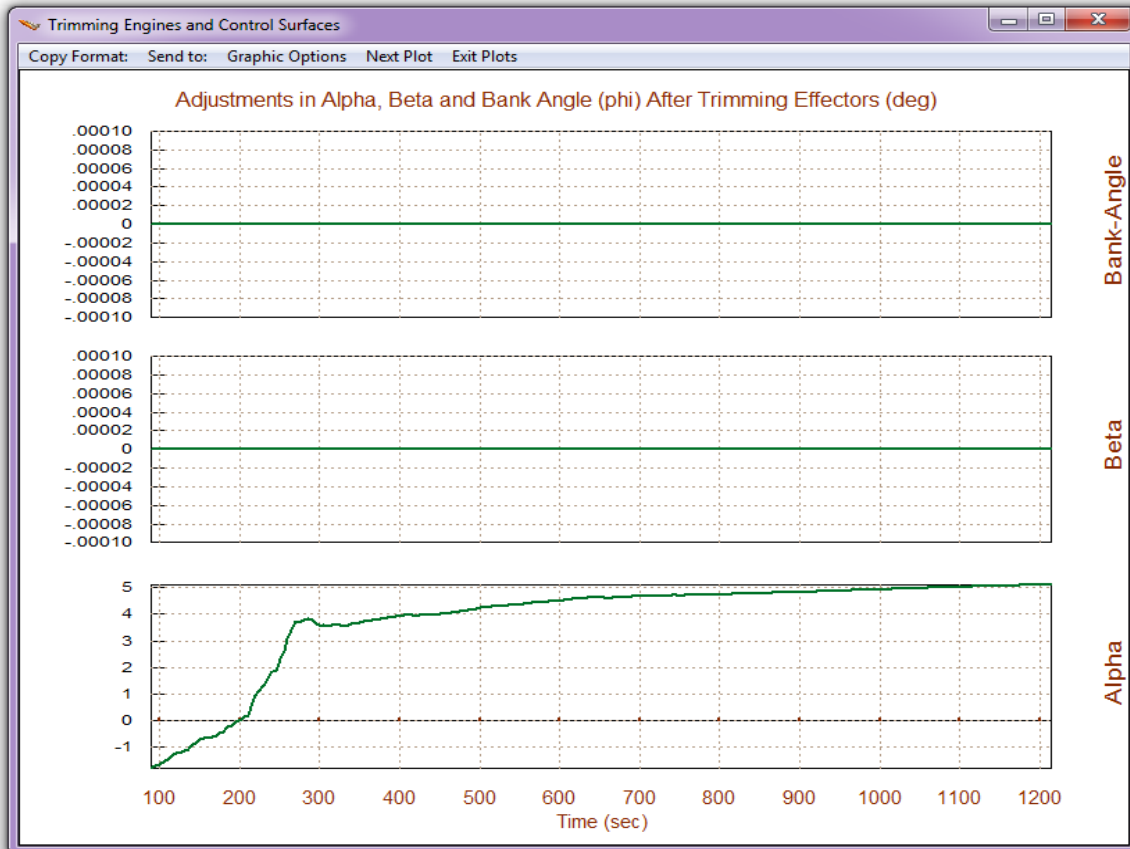
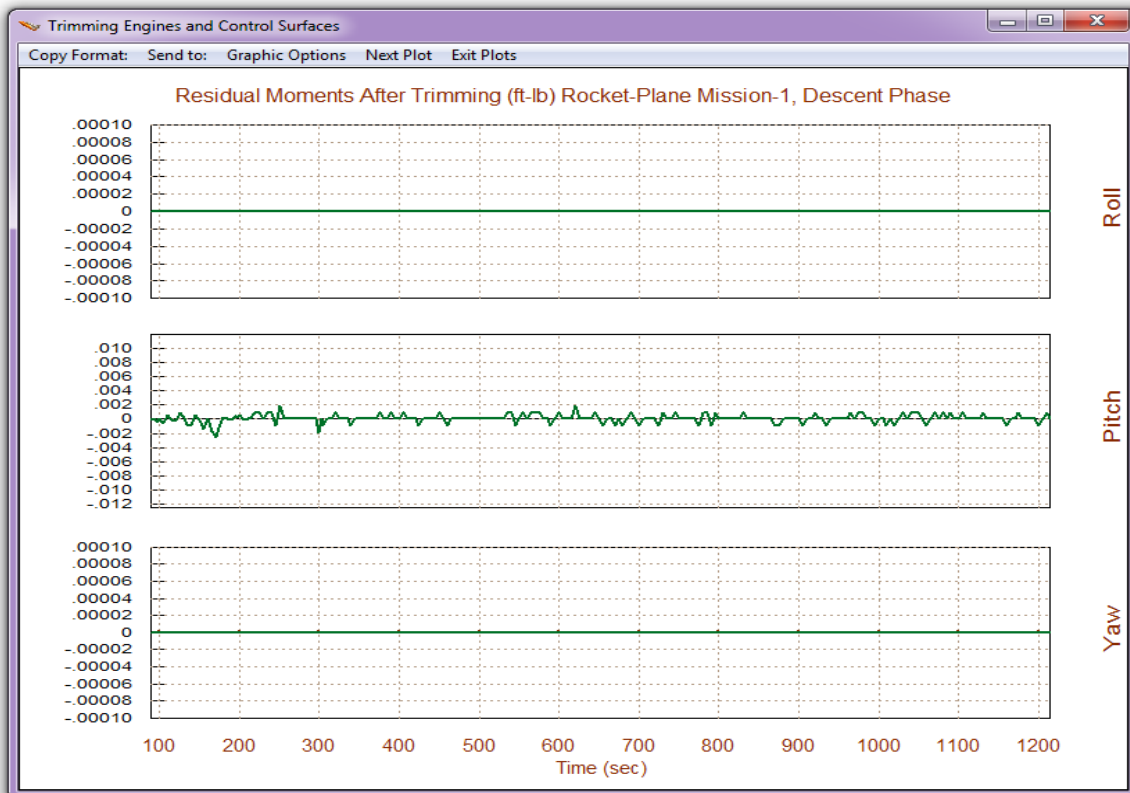




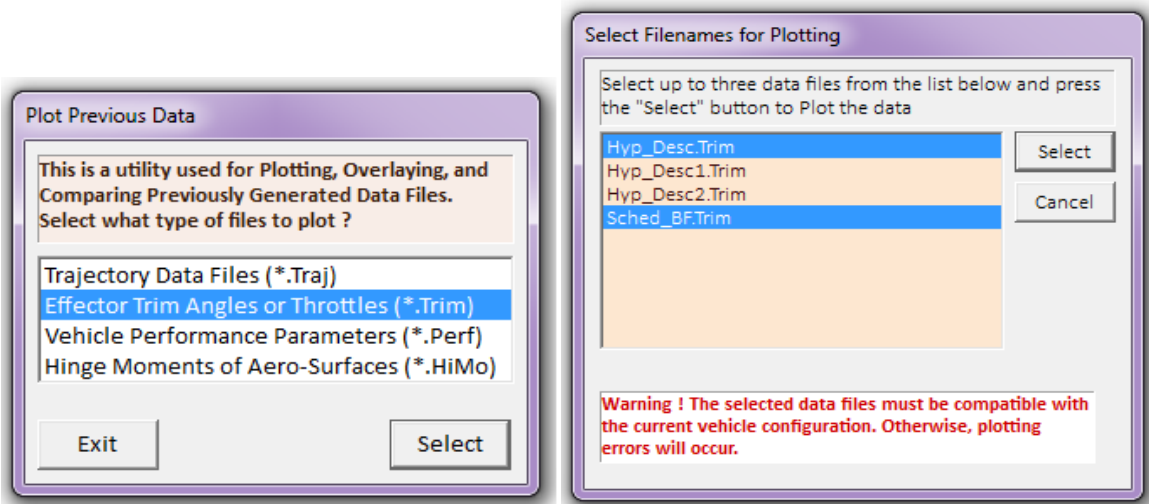
A menu comes up showing a list of the four aero-surfaces. We select the Body-Flap to modify and click on the "Select Effector" button. The dialog/plot on the top shows the original trim history of the Body-Flap (green) as it was calculated by the Trim program the first time. It also shows its upper and lower limits (magenta lines) which initially they are  $\pm 30^\circ$  according to the aero-surface data file. We would like to re-schedule this trim trajectory and make the Body-Flap more negative which, hopefully, will force the Elevon

to trim closer to zero. The second plot shows the modified Body-Flap trajectory after it was reshaped by the analyst using the mouse. The modified BF deflection was set at a constant value  $-14^\circ$ , after  $t=200$  seconds, as an attempt to reduce the Elevon deflection in that range. The upper and lower BF limits are also reduced from  $\pm 30^\circ$  to smaller values, because reducing the limits it bounds and de-emphasizes the effector priority during trim and the deflection will not change as much from its set value after re-trimming. When the user modifications are complete, click on "Re-Trim" on either dialog, and the program will generate a new trim history, as shown below. As you can see, the user modified Body-Flap trim history was not altered by re-trimming, but the Elevon deflection was reduced to zero, as planned. The next plot is for evaluating the trim success, showing that a perfect moments balance was achieved without any residuals. We should now compare the latest trim results against the original trim, so we should save the latest trim file before it gets over-written by another re-trim and we rename it from "Hyp\_Desc.Trim" to "Sched\_BF.Trim".

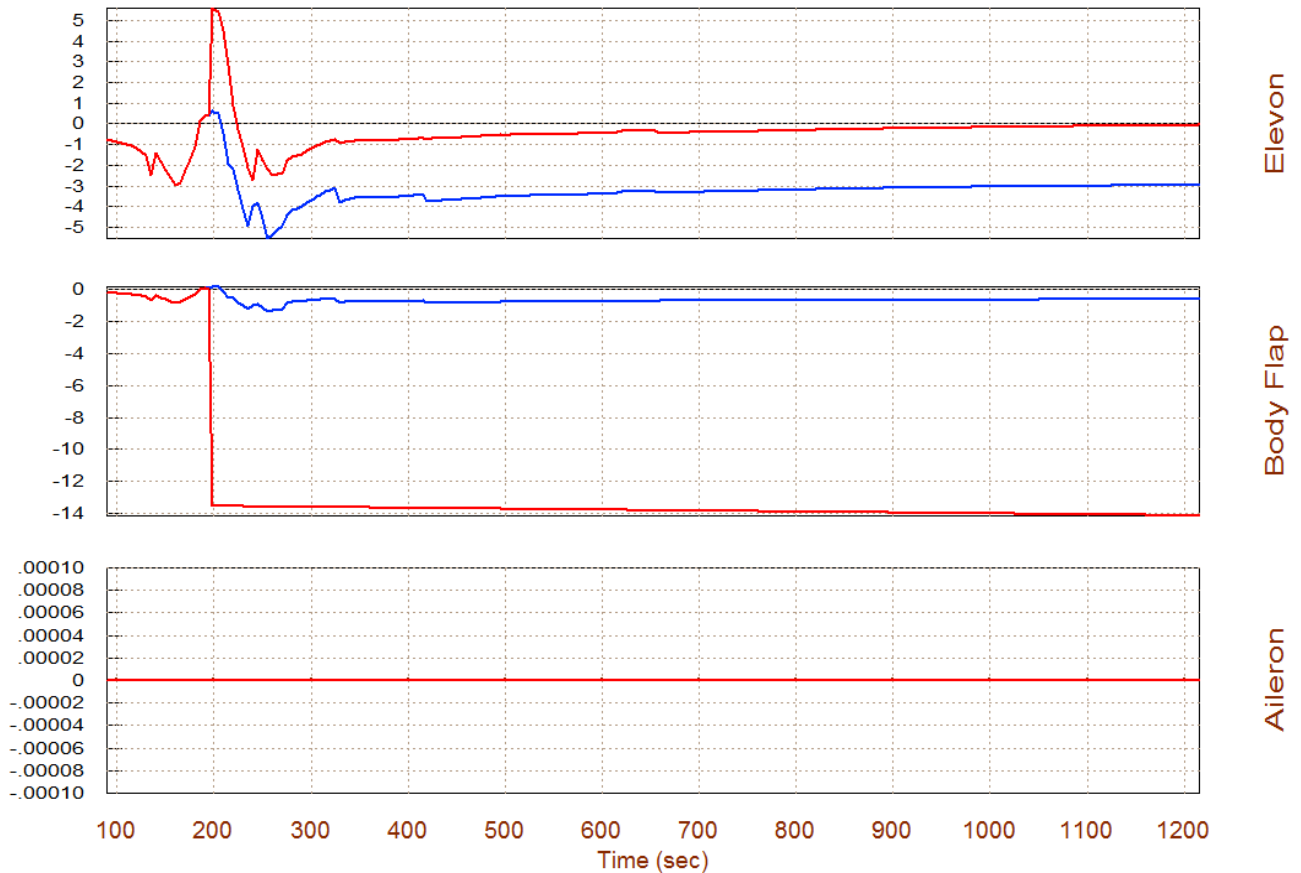




Let us now co-plot the two trim histories together for comparison, that is, the original 3-dof trim against the re-scheduled BF. Return to the Trim main menu, select option (12), and from the next menu on the left select the second option for plotting ".Trim" files, and from the right menu select the two trim files: "*Hyp\_Desc.Trim*" and "*Sched\_BF.Trim*", which contain the original and the modified trim trajectories. The blue curves are from the original trim and the red are from the modified BF trajectory. It is obvious that by re-scheduling the Body-Flap deflection further negative it causes the Elevon to re-trim closer to zero, which is better for control authority and, therefore, we conclude that the Body-Flap can be useful in improving the trimming conditions.

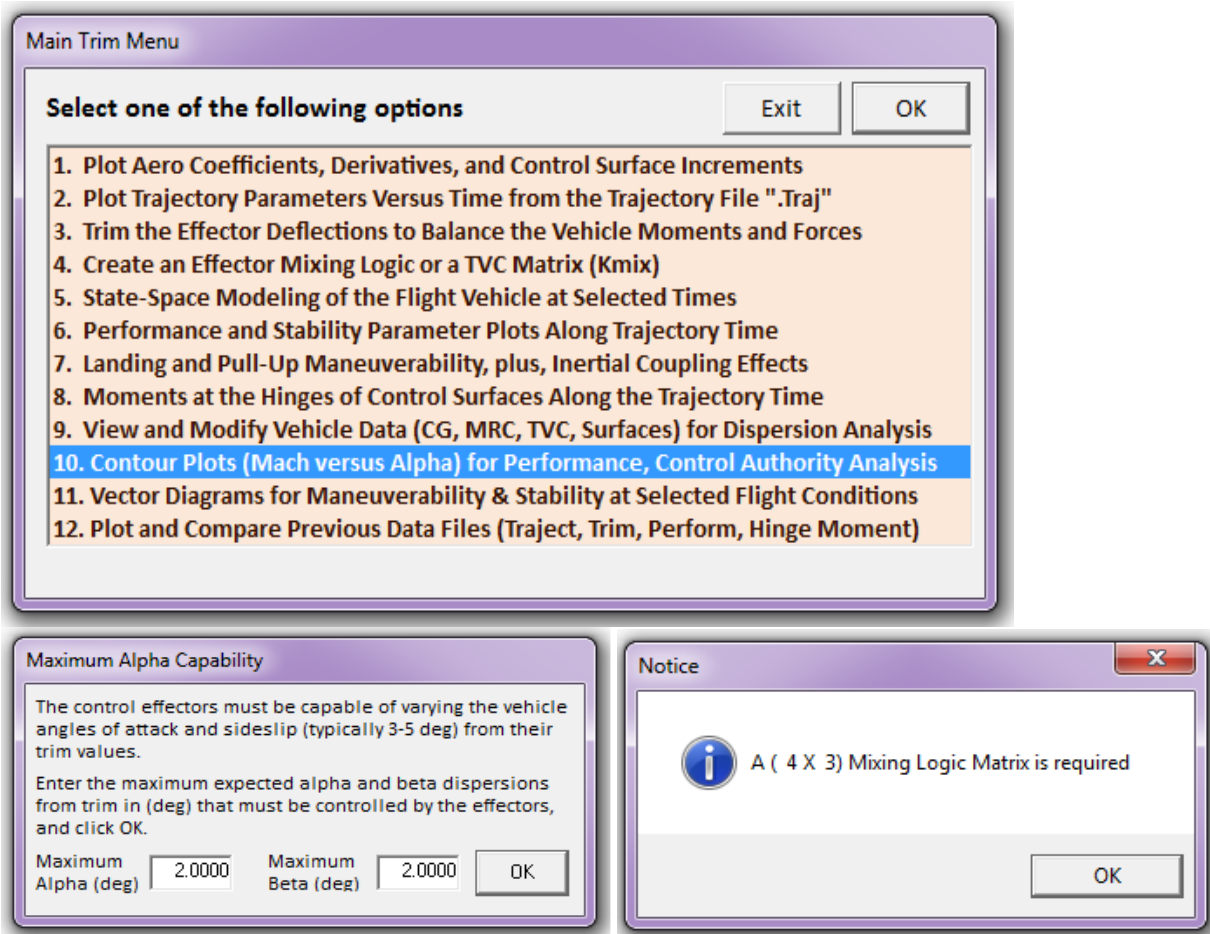


Surface & Engine Deflections/ Thrusts, Rocket-Plane Mission-1, Descent Phase



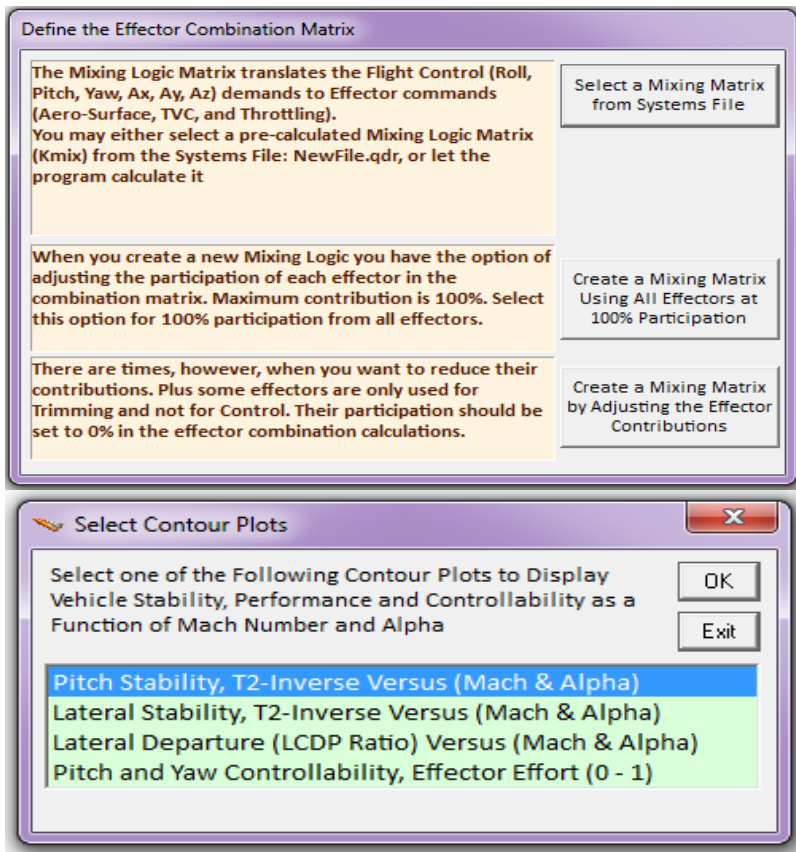
## 2.6 Stability & Performance Analysis Using Contour Plots

As we already described, contour plots are 3-dimensional plots that present us a wider perspective of vehicle performance in the entire range of Mach and alpha rather than in the vicinity of the trajectory. To create contour plots of some performance parameters against Mach and Alpha start the Trim program and select option-10 from the main menu. In the next dialog we must specify the aero disturbances which in steady-state are defined by the maximum dispersion angles ( $\alpha_{\max}$  and  $\beta_{\max}$ ) relative to the velocity vector. Enter  $2^\circ$  for both  $\alpha_{\max}$  and  $\beta_{\max}$ , as shown below. The next step is to create a control surface combination matrix. We will again allow the program to calculate a mixing matrix along the trajectory by selecting the second option in the Mixing Logic selection dialog (all effectors at 100% participation). The (4x3) effector combination matrix converts the 3 flight control demands (roll, pitch, and yaw) to 4 aerosurface commands.



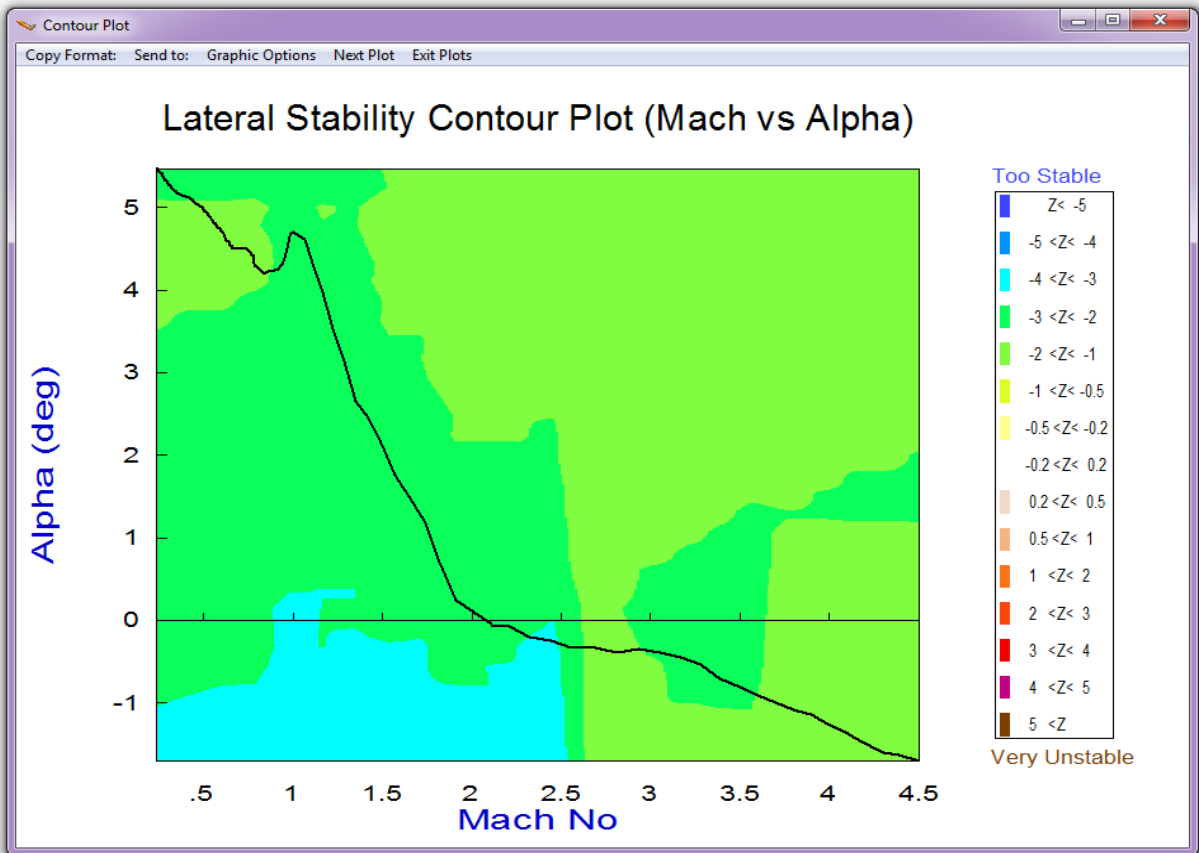
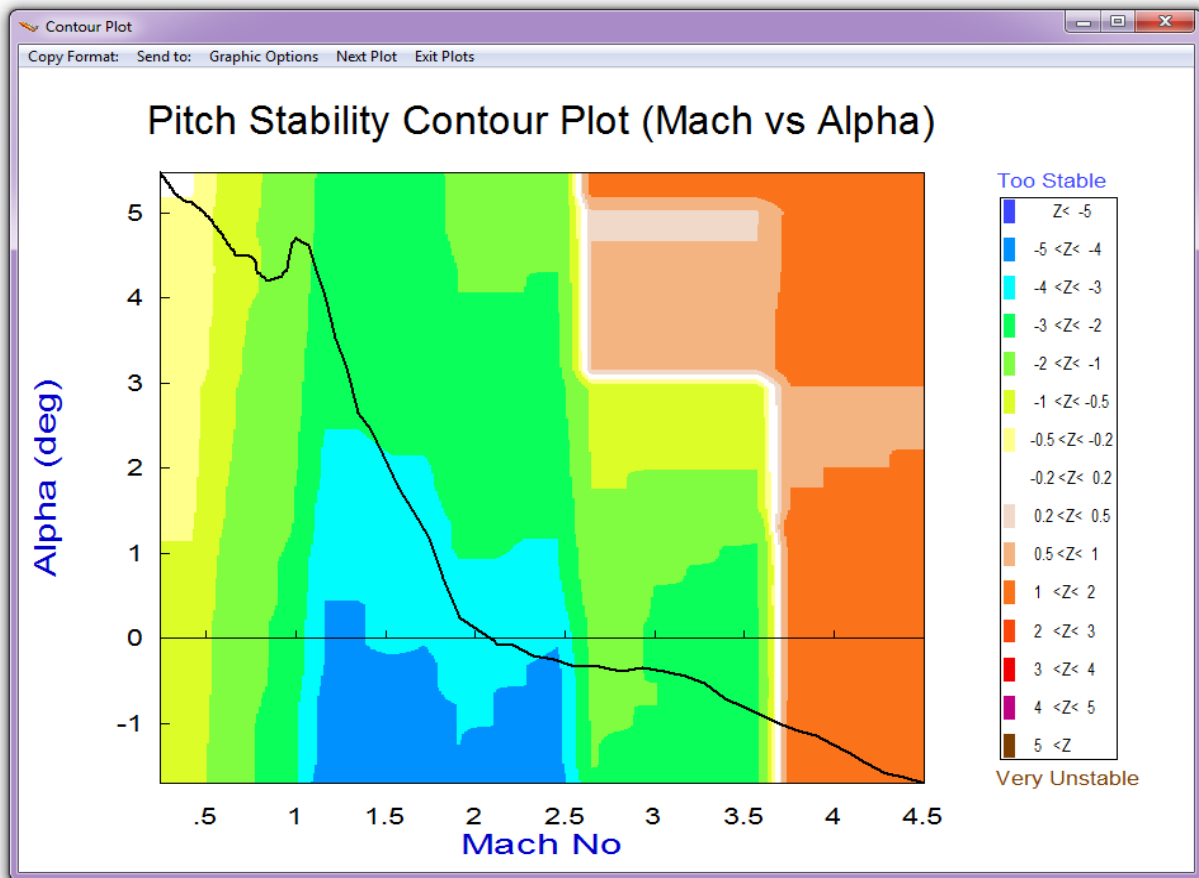
The next menu is used for selecting the type of contour plot that plots one of the performance parameters as a function of Mach versus Alpha. The first parameter in the menu is the pitch stability parameter that was described in equations (3.14 & 3.15).



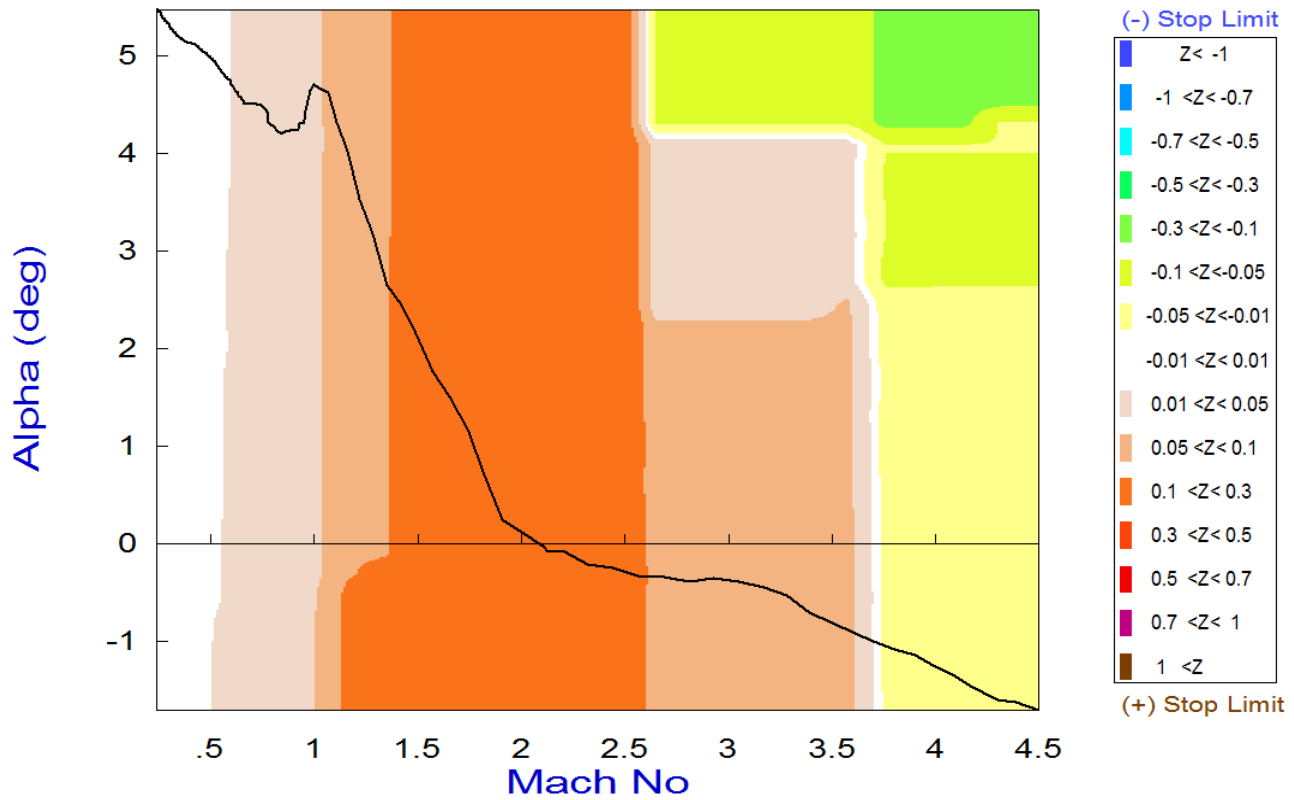


The trajectory is shown as a black line crossing through the Mach vs Alpha range. It starts at  $T=90$  (sec) in the lower right-hand corner where ( $\alpha=-1.7^\circ$ , Mach=4.5) and it ends up at  $T=1220$  (sec) in the upper left-hand corner where ( $\alpha=5.3^\circ$ , Mach=0.2). The color coding represents the value of the pitch stability parameter. Initially, at high Mach the trajectory passes through a statically unstable (divergent) region where the time-to-double amplitude  $T_2=0.67$  (sec). Then it passes through a stable region with maximum short period oscillations at 4 rad/sec, and it ends up close to neutrally stable or rather slightly stable at ( $\alpha=5.3^\circ$ , Mach=0.2). In the lateral axes the vehicle is statically stable with a maximum Dutch-roll resonance at 3.2 (rad/sec). The Roll-Departure (LCDP-ratio) is also very good because the trajectory is mostly in the white region indicating an excellent turn-coordination capability.

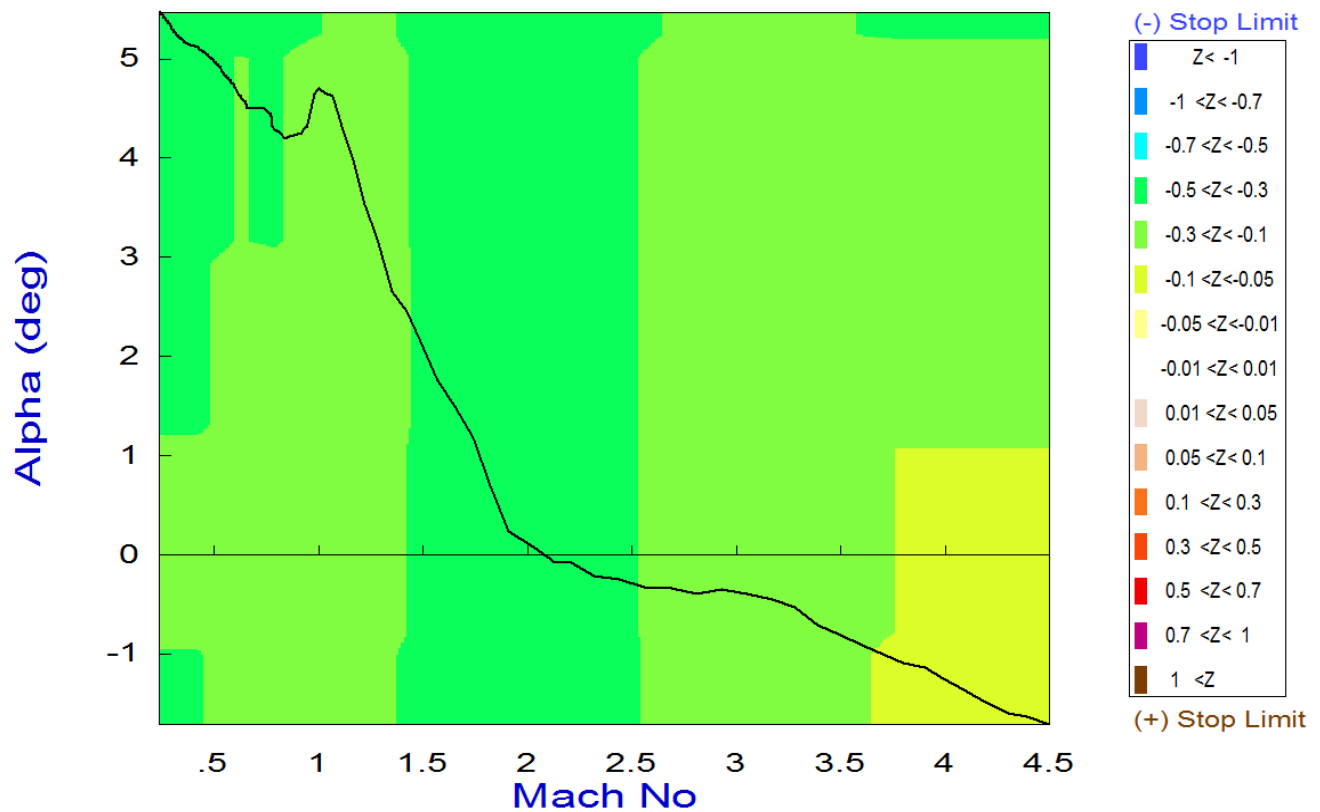
The next 3 plots show the control authority of the effectors against an aero disturbance created by a wind-shear that is defined by the dispersion angles ( $\alpha_{max}= 2^\circ$  and  $\beta_{max}= 2^\circ$ ). The control surfaces have the control authority to counteract the disturbances when the control effort parameter is less than one across the trajectory, or even better it should be less than 0.5, to allow some control for steering, gusts and other functions. In this case, the effector system satisfies the requirement in all 3 axes. Notice that the control effort parameters in pitch and roll are small. When the trajectory crosses the white region it means that the vehicle is close to neutral stability and the effort to maneuver becomes negligible. The magnitude of the yaw effort is not as small, but it is still good.



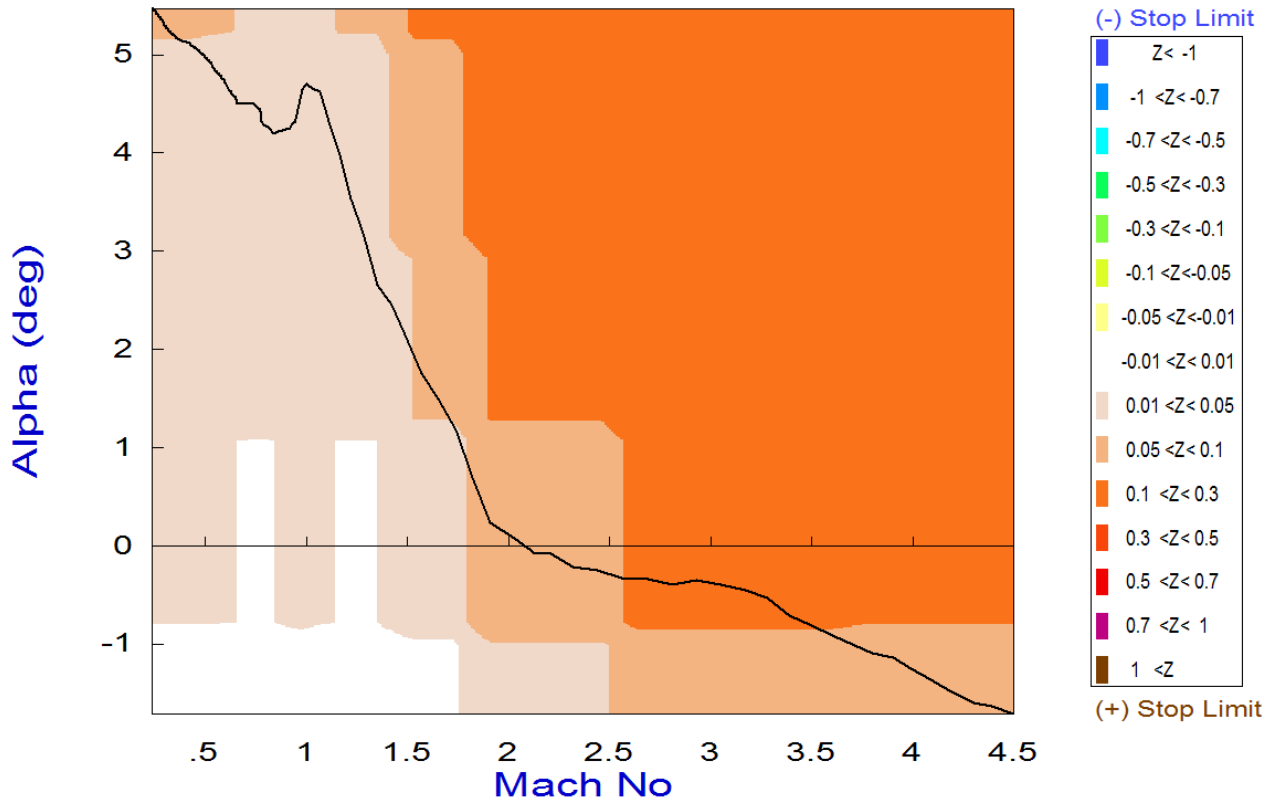
Pitch Control Effort Contour Plot (Mach vs Alpha)



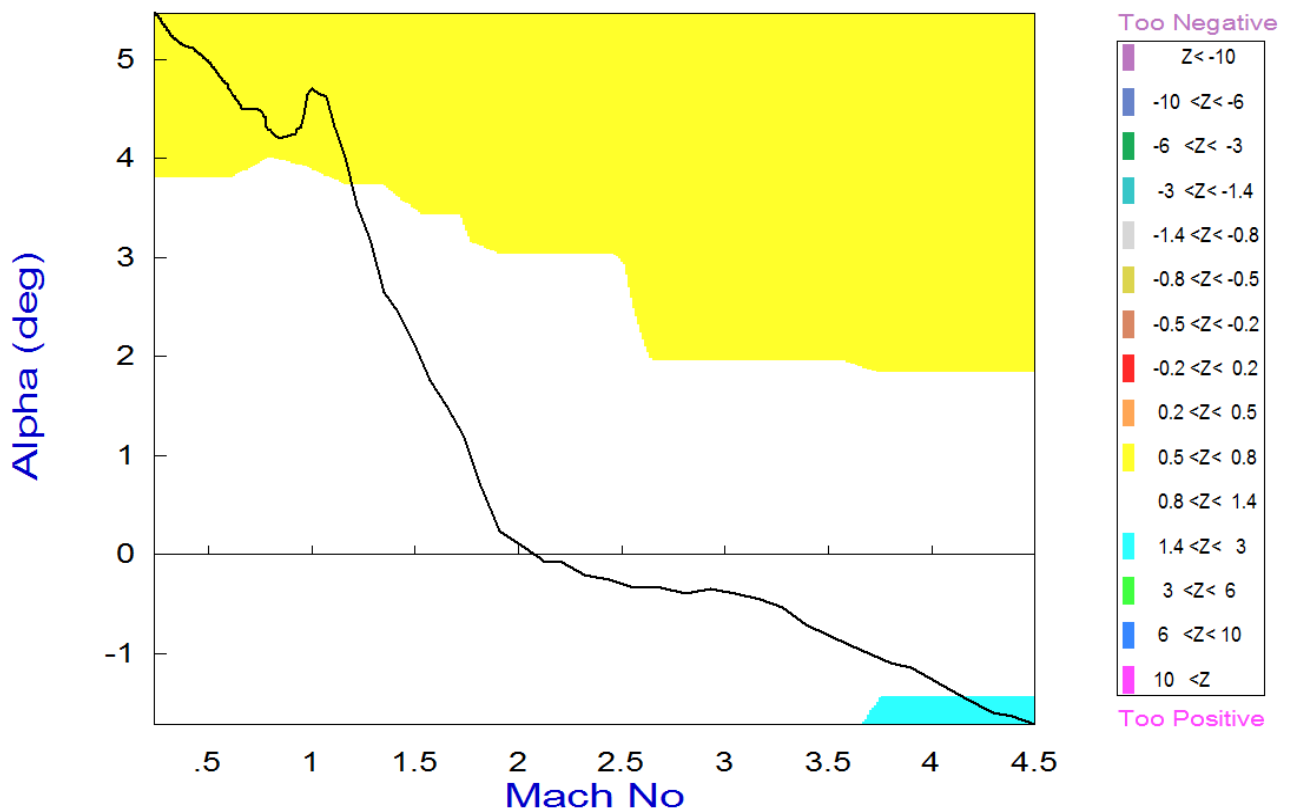
Yaw Control Effort Contour Plot (Mach vs Alpha)



Roll Control Effort Contour Plot (Mach vs Alpha)

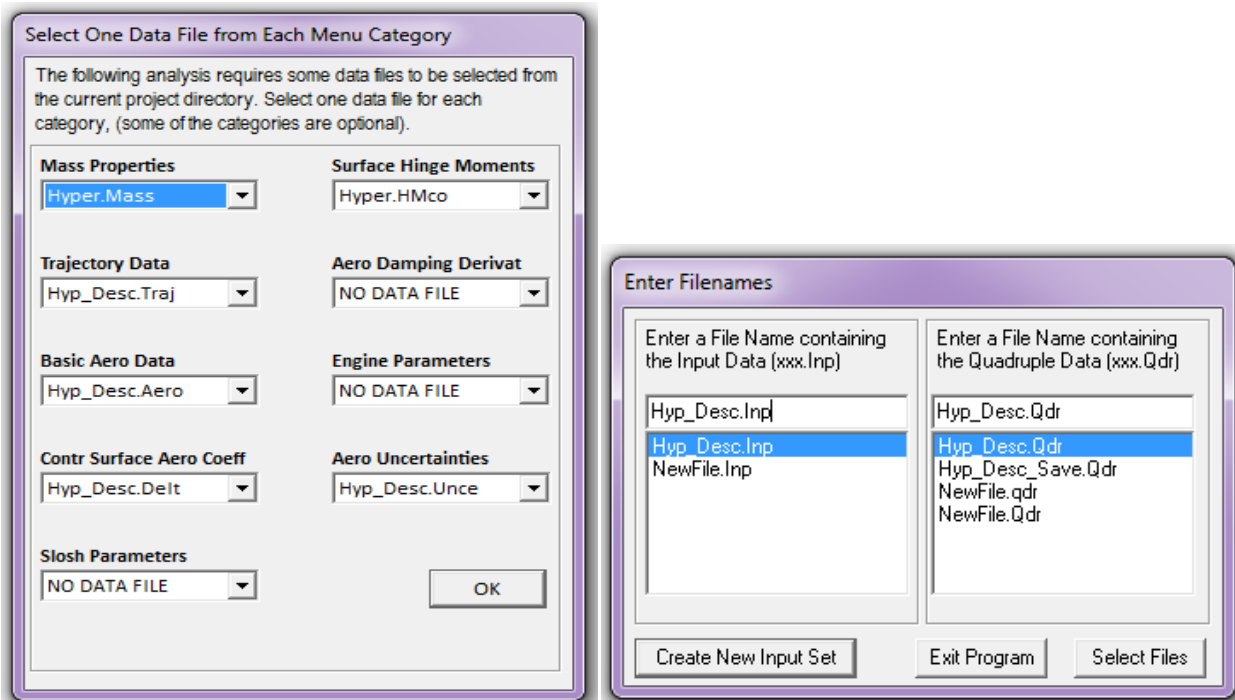


Roll Departure (LCDP) Contour Plot (Mach vs Alpha)

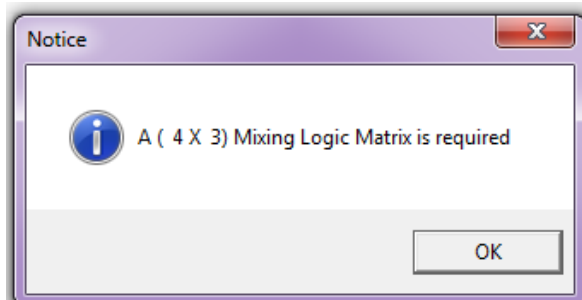
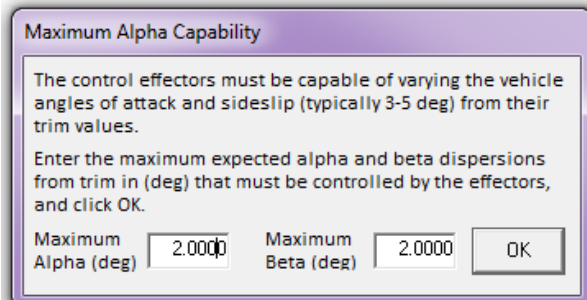
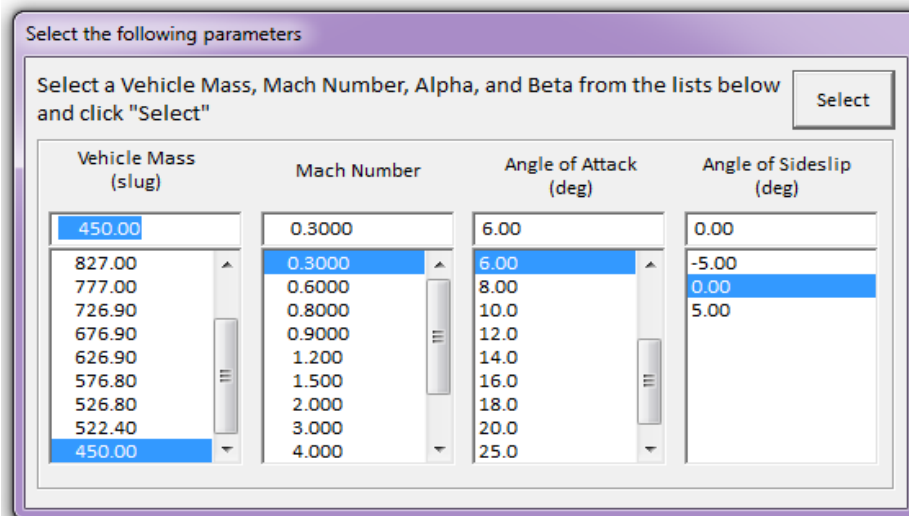
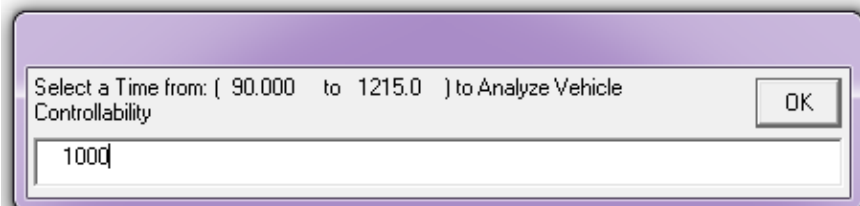
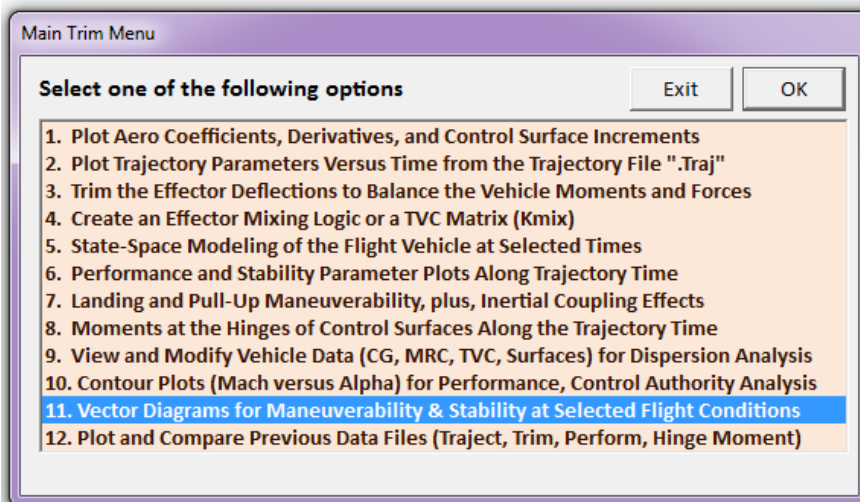


## 2.7 Vector Diagrams Analysis

Vector diagrams are 2-dimensional plots used for analyzing vehicle controllability at fixed flight conditions. We compare the control authority of the aerosurfaces in two directions against the effects on the vehicle of a wind-shear disturbance that is defined by dispersions in the angles of attack and sideslip and we determine if the effectors provide sufficient control authority to counteract the disturbance moments and forces. This is not just a magnitude comparison but it also allows us to examine the directions of the controls against the disturbance. It helps us evaluate the orthogonality of the control system, compare the acceleration magnitudes generated by the controls and compare them against the accelerations generated by the dispersions and determine if the controls are powerful enough and their directions are capable of counteracting the disturbance moments along the roll, pitch, and yaw, in this case. Start the Trim program and from the menus select the data files as shown below.



From the Trim menu select option-11 for plotting vector diagrams, and then enter an arbitrary flight condition at time=1000 sec. The next dialog consists of four menus used for selecting the vehicle mass, Mach number, alpha, and beta. The default values correspond to the selected flight time, but they may be changed if the user desires to analyze a different flight condition, such as, a combination of parameters that does not correspond to a trajectory time. In this case we select the default values and click on "Select". The wind disturbances are defined by the maximum alpha and beta dispersions from trim. In the next dialog enter the maximum dispersion angles ( $\alpha_{max}$  and  $\beta_{max}$ )=2°. The program also requires an aero-surface combination matrix. This time we will select a pre-calculated mixing logic matrix "Kmix\_1000" from file "Hyp\_Desc.Qdr" which is created in Section 2.8.



From the mixing-logic matrix selection dialog click on the first option to select the already existing matrix from file "Hyp\_Desc.Qdr". The matrix selection menu below shows the gain matrices which are saved in the systems file "Hyp\_Desc.Qdr". Select "Kmix\_1000" and click on "View Matrix" to take a look at the matrix in color coded form. Its inputs are: roll, pitch and yaw FCS demands, and

its outputs are: Elevon, Body-Flap, Aileron, and Rudder deflections. Click on "Exit" to return to the menu and then click on "Select Matrix" to continue. The program uses the mixing matrix plus other inputs to calculate the vector diagrams, some of which are shown in the following pages.

**Define the Effector Combination Matrix**

The Mixing Logic Matrix translates the Flight Control (Roll, Pitch, Yaw, Ax, Ay, Az) demands to Effector commands (Aero-Surface, TVC, and Throttling). You may either select a pre-calculated Mixing Logic Matrix (Kmix) from the Systems File: Hyp\_Desc.qdr, or let the program calculate it

When you create a new Mixing Logic you have the option of adjusting the participation of each effector in the combination matrix. Maximum contribution is 100%. Select this option for 100% participation from all effectors.

There are times, however, when you want to reduce their contributions. Plus some effectors are only used for Trimming and not for Control. Their participation should be set to 0% in the effector combination calculations.

Select a Mixing Matrix from Systems File

Create a Mixing Matrix Using All Effectors at 100% Participation

Create a Mixing Matrix by Adjusting the Effector Contributions

**Select a Gain Matrix**

Select one of the following Matrices from the Systems File

KMIX\_1000 : Mixing Logic Matrix for the Rocket-Plane during Descent , S

**Display or Modify a Matrix**

**K** Matrix Name: KMIX\_1000   **Mixing Logic Matrix for the Rocket-Plane during Descent** Matrix Title

**Inputs** | **Outputs** | **Matrix Description**

Mixing Logic Matrix for the Rocket-Plane during Descent at T=1000

Select an Input or Output variable from the menu above and press Edit. Then you may type in a new description for that variable in the field above.

Repeat to change other Inputs, Outputs, or Matrix Elements. Then click on "Save Changes" to save the new values and titles or Exit the dialog.

Matrix Element: ( 1, 1)= 0.0000000

To Create a New Matrix, Click on the Elements, Enter New Values and Push on "Save Changes"

Color Code for Magnitudes between Zero (black) to One (white)

Color Code for Magnitudes between One (white) to Infinity (blue)

The vector diagrams include a menu bar above the title. From this menu, click on: "Select Vector Diagrams". A vertical menu comes up and from this menu select: "Moments/ Forces at Max Controls versus Moments/ Forces at Max Alpha/ Beta". The vector diagrams in Figure 2.7.1 show the roll/ yaw moments and side-force, which are non-dimensional ( $C_l$ ,  $C_n$ ,  $C_Y$ ), produced when the roll and yaw FCS demands are maximized by saturating the effectors system. The solid blue vector corresponds to max positive yaw FCS demand ( $\delta R_{+FCS\_Max}$ ) from trim position, and the dashed blue vector to max negative yaw demand ( $\delta R_{-FCS\_Max}$ ) from trim. The moment is exactly in the demanded yaw direction. Similarly, the green vectors are created by maximizing the roll FCS demands in the positive and negative directions from trim ( $\delta P_{\pm FCS\_Max}$ ). Their directions are mostly in the demanded roll direction with some coupling in yaw. The plot below shows the effect that the yaw FCS demand ( $\delta R_{\pm FCS\_Max}$ ) has in yaw and also in side-force  $C_Y$ . Positive yaw produces negative side-force, as expected.

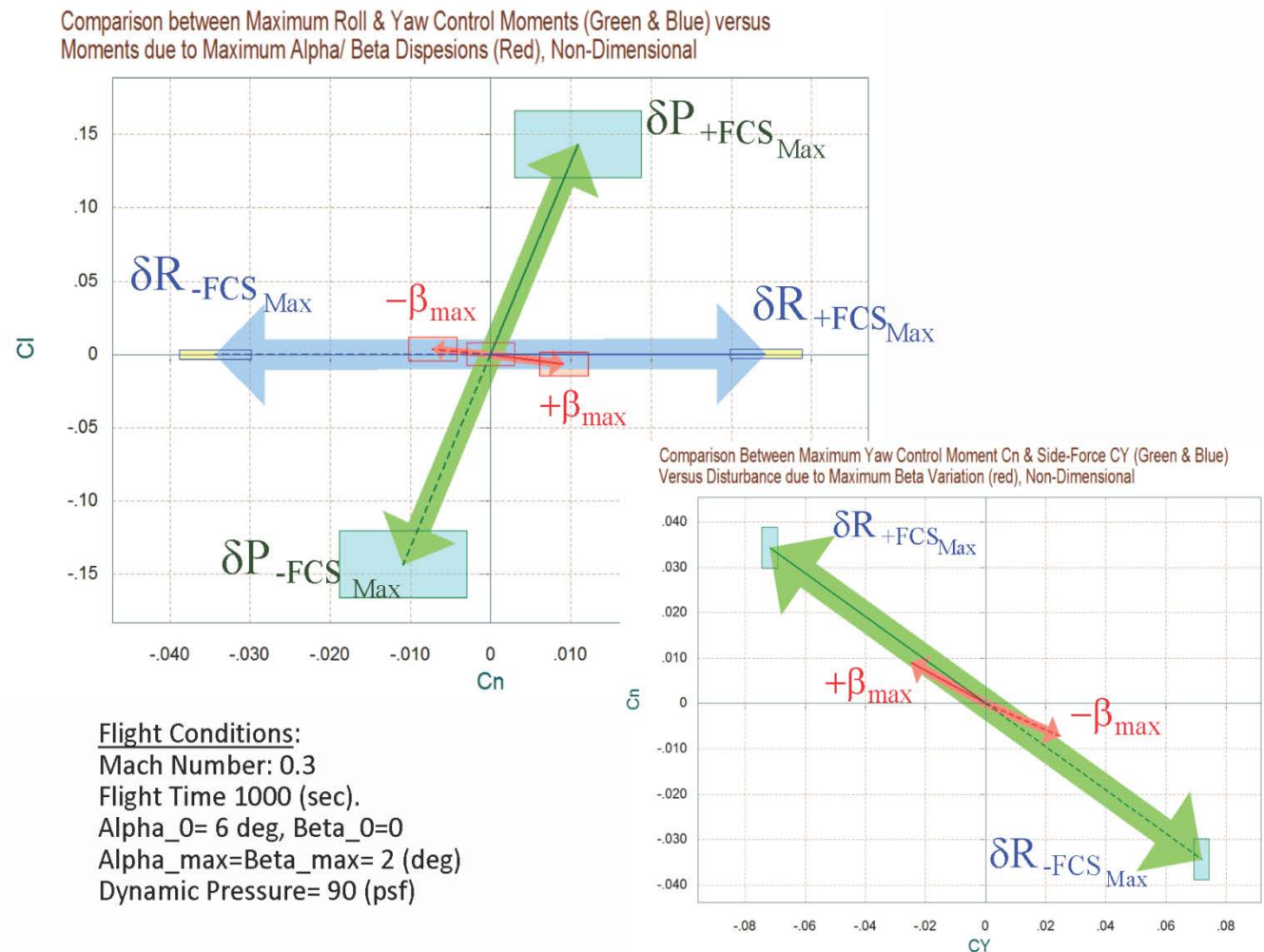


Figure 2.7.1 Maximum Roll and Yaw Moments and Side-Force produced due to  $\pm\beta_{max}$  and Controls

The two red vectors in the top diagram show the roll and yaw moments produced by the variations in the angles of attack and sideslip ( $\pm\alpha_{max}$  and  $\pm\beta_{max}$ ) from their trim positions ( $\alpha_0$  and  $\beta_0$ ). The moment is mainly in yaw due to  $\beta$  variations. A positive  $\beta_{max}$  generates a positive yawing

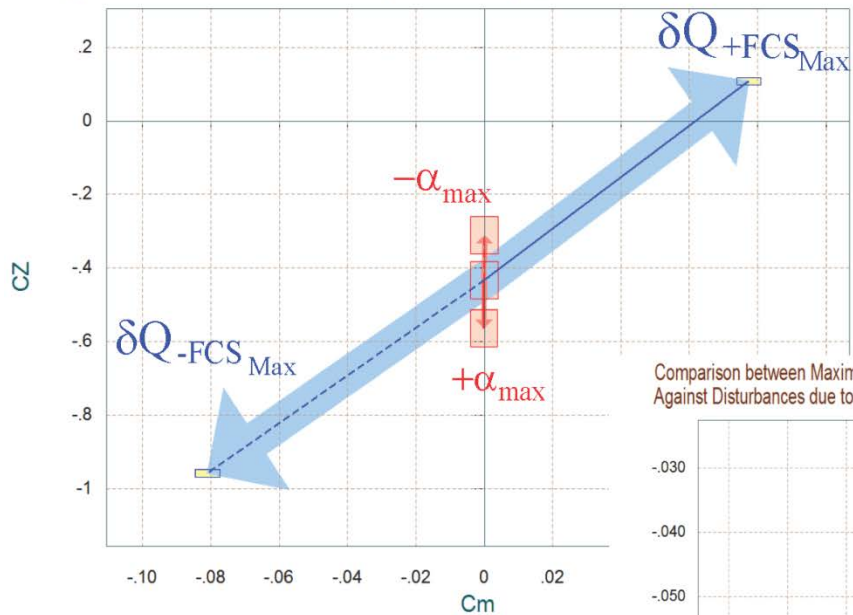


moment because the vehicle is statically stable in yaw. It also produces a negative side-force. The red rectangles at the tips of the red arrows show the moment uncertainties in this flight condition due to variations in the aero coefficients. The yellow and cyan rectangles at the tips of the control vectors represent the control uncertainties. The uncertainties are obtained from file "*Hyp\_Desc.Unce*". Controllability is good in both directions because the magnitudes of the control vectors are larger than the red disturbance vectors in both directions. The control vectors are also almost orthogonal to each other which provide controllability in both directions. The accelerations plot shows the roll and yaw accelerations in (rad/sec<sup>2</sup>) due to max  $\pm$ roll and  $\pm$ yaw control. The accelerations are more decoupled and orthogonal than the moments.

Figures 2.7.2 shows the controllability in the longitudinal directions when the pitch control demand ( $\delta Q_{FCS}$ ) is maximized. There is only one control in the longitudinal axes. The blue vectors in the diagram show the maximum pitch moment  $C_m$  plotted against the  $C_z$  and the  $C_x$  force coefficients when the pitch control demand is maximized to saturation. The solid blue vector shows the moments and forces produced when the pitch control is maximum positive ( $\delta Q_{+FCS\_Max}$ ), and the dashed blue vector is when the pitch control is maximum negative ( $\delta Q_{-FCS\_Max}$ ). The pitch control, in addition to pitching moment, it also produces force variations in the x and z directions due to aerosurface deflections. But unlike in the lateral directions the vectors here are not symmetrical. At trim condition when there is no control being applied the axial force coefficient  $C_x$  has a nominal value of -0.037 and the  $C_z$  coefficient is -0.45. It implies negative acceleration due to drag since this vehicle has no thrust.

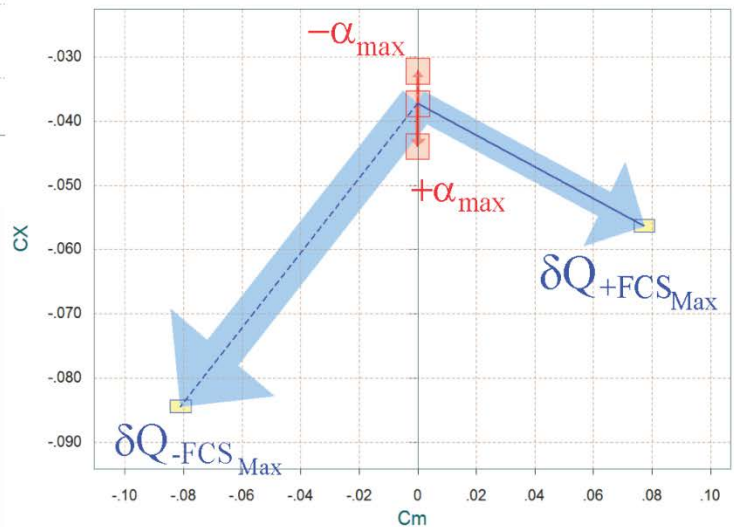
The vehicle is trimmed in pitch because  $C_m=0$  when the control  $\delta Q_{FCS}=0$ , and it is accelerating in the -x and -z directions because  $C_x<0$  and  $C_z<0$  when  $\delta Q_{FCS}=0$ . Either max positive or max negative pitch control demand ( $\delta Q_{\pm FCS\_Max}$ ) produces an increase in the aft force ( $-C_x$ ). A +pitch control demand increases both the aft force ( $-C_x$ ) and the  $C_z$  force because the Elevons deflect negative (up). A negative pitch control demand deflects the Elevons positive (down), that also increases the aft force ( $-C_x$ ) but it decreases the  $C_z$  force (lift). The red vectors represent the pitch moment, axial and z forces generated by the variations in the angles of attack and sideslip ( $\pm\alpha_{max}$  and  $\pm\beta_{max}$ ) which were defined to be  $\pm 2^\circ$ , relative to the trim angles. The disturbance in this case is mainly due to the  $\pm\alpha_{max}$  variations. Figure 2.7.2 shows that a reduction in the angle of attack (negative  $\alpha_{max}$ ) generates a less negative z-force and a reduction in the aft force coefficient (increase in  $C_x$ ) because the vehicle is trimmed with  $\alpha_0= +6^\circ$ . It does not affect the pitching moment because the vehicle is neutrally stable in this flight condition. The rectangles show the possible vector variations due to the uncertainties in the aero-coefficients.

Comparison between Maximum Pitch Control Moment and Normal-Z Force (Blue & Green)  
Against Disturbance due to Maximum Alpha Variation (red), Non-Dimensional



**Flight Conditions:**  
 Mach Number: 0.3  
 Flight Time 1000 (sec).  
 Alpha\_0= 6 deg, Beta\_0=0  
 Alpha\_max=Beta\_max= 2 (deg)  
 Dynamic Pressure= 90 (psf)

Comparison between Maximum Pitch Control Moment and Axial X-Force (Blue & Green)  
Against Disturbances due to Maximum Alpha Variation (red), Non-Dimensional

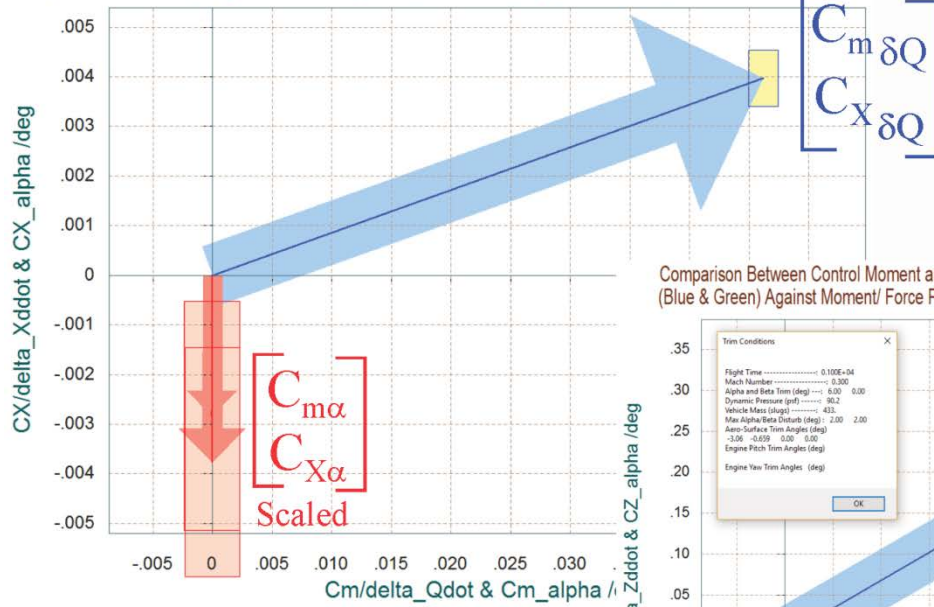


**Figure 2.7.2 Maximum Pitch Moment, Normal and Axial Forces produced due to  $\pm\alpha_{max}$  and Controls**

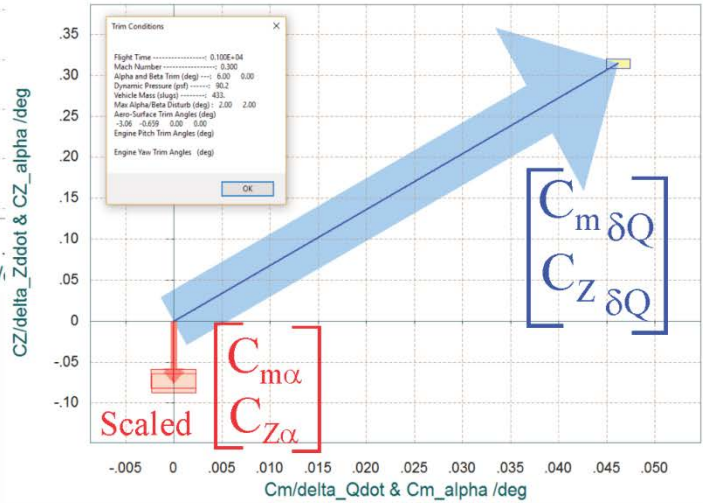
The vector diagram in Figures 2.7.3 and 2.7.4 show the partials in the longitudinal directions. The system has only pitch control and the blue vectors show the pitch moment, X and Z forces partials  $\{Cm\delta Q_{FCS}, CX\delta Q_{FCS}, CZ\delta Q_{FCS}\}$  per pitch acceleration demand in  $(rad/sec^2)$ . It shows that the pitch control couples in the normal and axial forces due to the aerosurface deflections. The red vectors in Figure 2.7.3 are the scaled partials  $\{Cm\alpha, CX\alpha, CZ\alpha\}$ . They are two because they are calculated at the two extreme  $\pm\beta_{max}$  positions. Notice that  $Cm\alpha$  is very small because the vehicle is close to being neutrally stable in this flight condition. An increase in alpha causes a reduction in both  $C_X$  and  $C_Z$ . The red rectangles centered at the tips of the  $\{Cm\alpha, CX\alpha, CZ\alpha\}$  vectors represent the spreading of the vector due to the aero uncertainties. Similarly the yellow rectangle at the tip of the pitch control partial is due to the uncertainties in  $\{Cm\delta Q_{FCS}, CX\delta Q_{FCS}, CZ\delta Q_{FCS}\}$ . The uncertainties are obtained from file "Hyp\_Desc.Unce".

Figure 2.7.4 compares the moment and forces partials per pitch control demand against the partials per airspeed variation. Alpha and beta do not change but only the airspeed changes along  $V_0$  as a result of wind variation. It shows that an increase in airspeed ( $\delta V$ ) increases drag (-CX) and lift (-CZ). It also causes a small negative pitching moment. The airspeed variation partials are scaled in order to be comparable to the control partials.

Comparison Between Control Moment & Force Partial:  $\{C_m/\delta_Q \text{ \& } C_X/\delta_X\}$  (Blue and Green), Against Partial:  $\{C_m/\alpha \text{ \& } C_X/\alpha\}$  (Red Vectors)



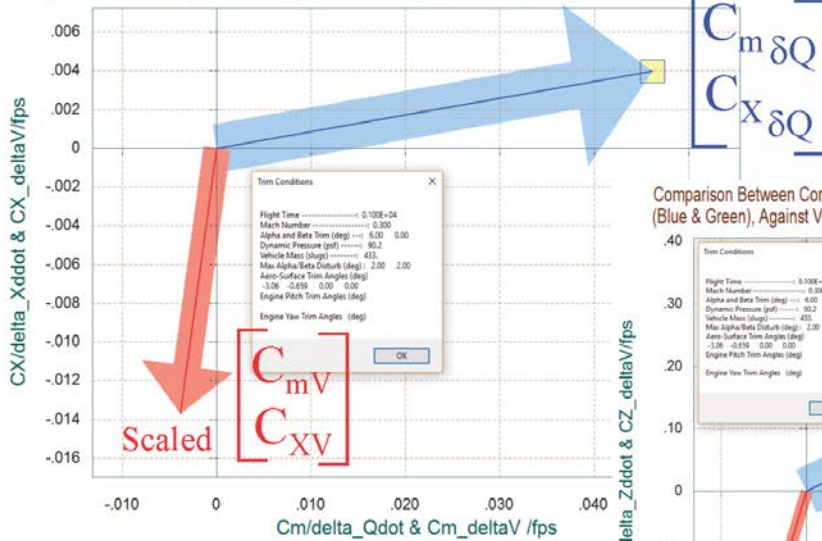
Comparison Between Control Moment and Normal Force Partial:  $\{C_m/\delta_Q \text{ \& } C_Z/\delta_Z\}$  (Blue & Green) Against Moment/ Force Partial:  $\{C_m/\alpha \text{ \& } C_Z/\alpha\}$  (Red Vectors)



**Flight Conditions:**  
 Mach Number: 0.3  
 Flight Time 1000 (sec).  
 Alpha<sub>0</sub>= 6 deg, Beta<sub>0</sub>=0  
 Alpha<sub>max</sub>=Beta<sub>max</sub>= 2 (deg)  
 Dynamic Pressure= 90 (psf)

Figure 2.7.3 Pitch Moment and Forces Partial, Controls against Alpha Variations

Comparison Between Control Moment & X-Force Partial:  $\{C_m/\delta_Q \text{ \& } C_X/\delta_X\}$  (Blue and Green), Against Velocity Variat. Partial:  $\{C_m/\delta_V \text{ \& } C_X/\delta_V\}$  (Red)



Comparison Between Control Moment and Normal Force Partial:  $\{C_m/\delta_Q \text{ \& } C_Z/\delta_Z\}$  (Blue & Green), Against Velocity Variation Partial:  $\{C_m/\delta_V \text{ \& } C_Z/\delta_V\}$  (Red)

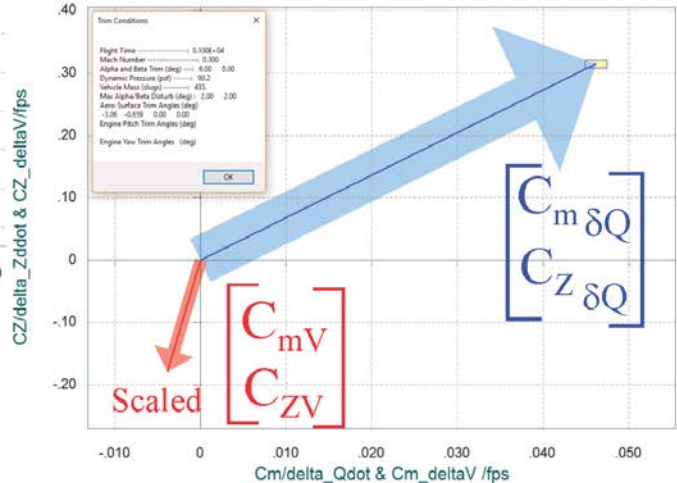
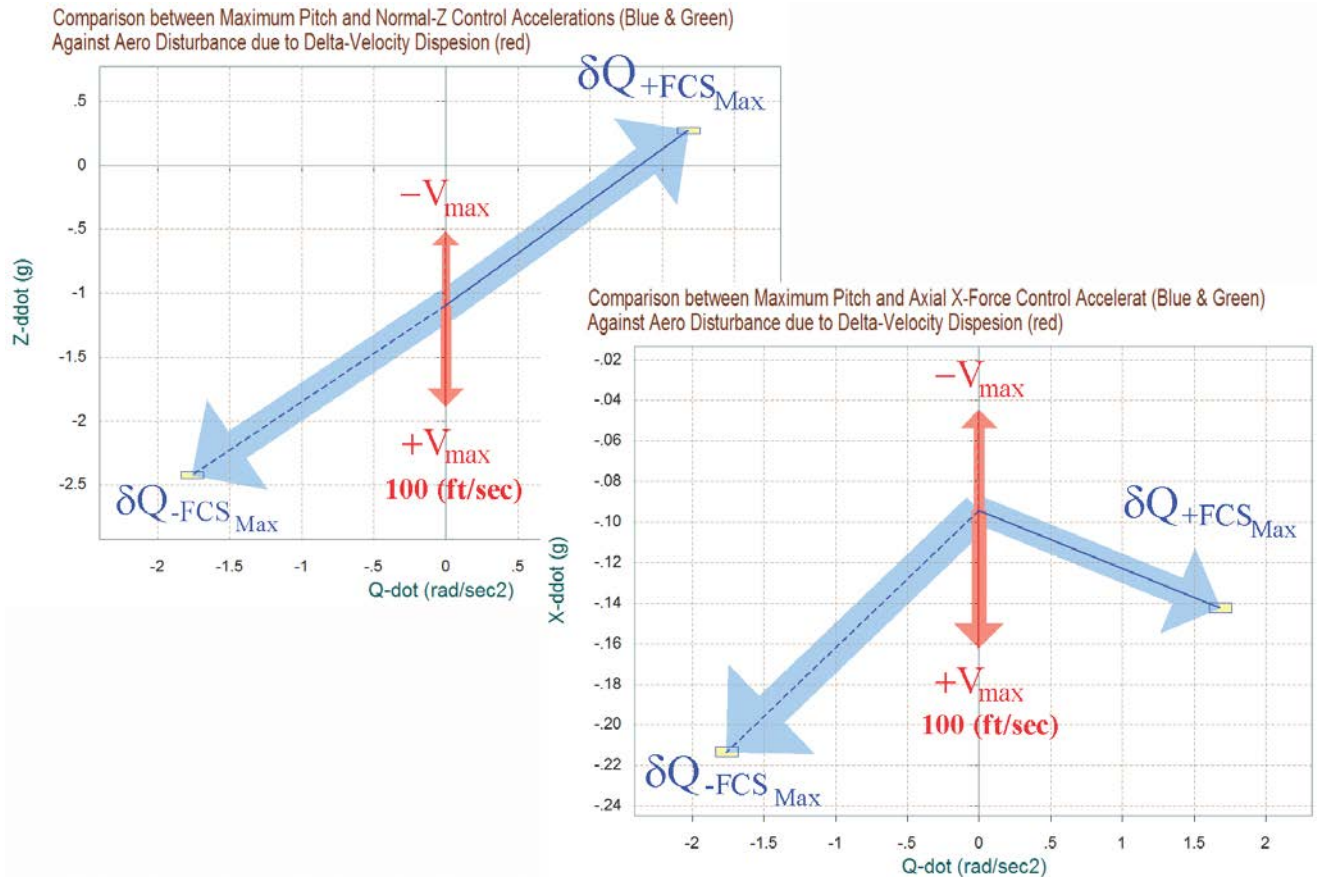


Figure 2.7.4 Pitch Moment and Forces Partial, Controls versus Airspeed Variations



**Figure 2.7.5 Maximum Pitch, X, and Z Accelerations Obtained by Maximizing the Pitch Control ( $\delta Q$ ) and also from  $\pm 100$  (feet/sec) Air-Speed Variations**

Figure 2.7.5 shows the effects on the pitch, X, and Z accelerations when the pitch control ( $\delta Q_{FCS}$ ) is maximized in the positive and negative directions from trim. At trim the vehicle has negative acceleration that can be varied with the pitch control. A variation in the airspeed  $\pm V_{max}$  also causes a variation in the X and Z accelerations but has a very small impact in the pitch acceleration.

The vector diagram in Figure 2.7.6 shows the roll and yaw moment partials. The blue vector is the moment partials per yaw control demand  $\{C_n\delta R_{FCS}, C_l\delta R_{FCS}\}$  and it is pointing mainly in the yaw direction. The green vector is the moment partials per roll FCS demand  $\{C_n\delta P_{FCS}, C_l\delta P_{FCS}\}$  and it is pointing entirely in the roll direction. The red vectors are the  $\{C_n\beta, C_l\beta\}$  partials. In this flight condition  $C_n\beta$  is positive and stronger than  $C_l\beta$  because the vehicle is directionally very stable. The second figure below shows also the side-force variation per yaw demand  $\{C_y\delta R_{FCS}\}$ . The two red vectors are calculated at the two extreme  $\pm\alpha_{max}$  positions. The solid red vector represents  $\{C_n\beta \& C_l\beta\}$  at  $+\alpha_{max}$ , and the dashed red vector is  $\{C_n\beta \& C_l\beta\}$  at  $-\alpha_{max}$ . The second figure also shows the variation of the side-force per beta  $C_{y\beta}$ . The red vector per beta partials are scaled in order to be comparable with the control partials as already described in equations (8.1 through 8.4). The red rectangles centered at the tip of the  $\{C_n\beta, C_l\beta\}$  vectors are due to the uncertainties in the two partials. Similarly the yellow rectangle at the tip of the yaw control partial is due to the uncertainties in  $\{C_n\delta R, C_l\delta R\}$ , and the cyan rectangle at the tip of the roll control partial is due to the uncertainties in  $\{C_n\delta P, C_l\delta P\}$ .

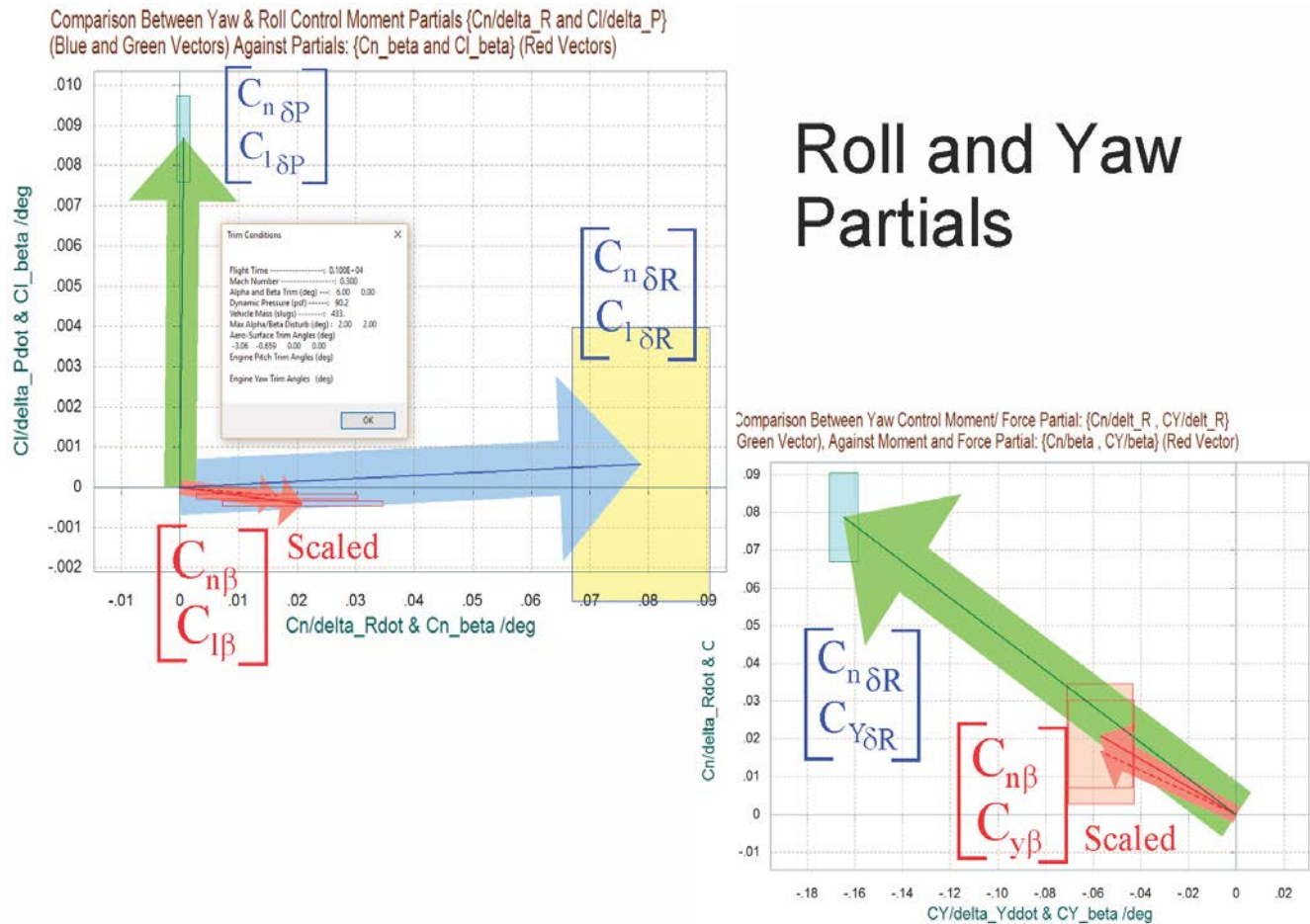


Figure 2.7.6 Roll and Yaw Moment Partial, Controls against Beta Dispersions

The vector diagram in Figure 2.7.7 shows the partials of accelerations per acceleration demands in roll and yaw. The green vector is  $\{\dot{P}/\delta P_{FCS}, \dot{R}/\delta P_{FCS}\}$ , and the blue vector is  $\{\dot{P}/\delta R_{FCS}, \dot{R}/\delta R_{FCS}\}$ . The axis units are in  $(\text{rad}/\text{sec}^2)$  per  $(\text{rad}/\text{sec}^2)$ . They are unit vectors perfectly aligned with the commanded directions, orthogonal to each other and are completely decoupled from each other. This diagonalization is achieved by the effector mixing logic ( $K_{mix}$ ) which couples the controls in order to counteract the  $I_{xz}$  product of inertia.

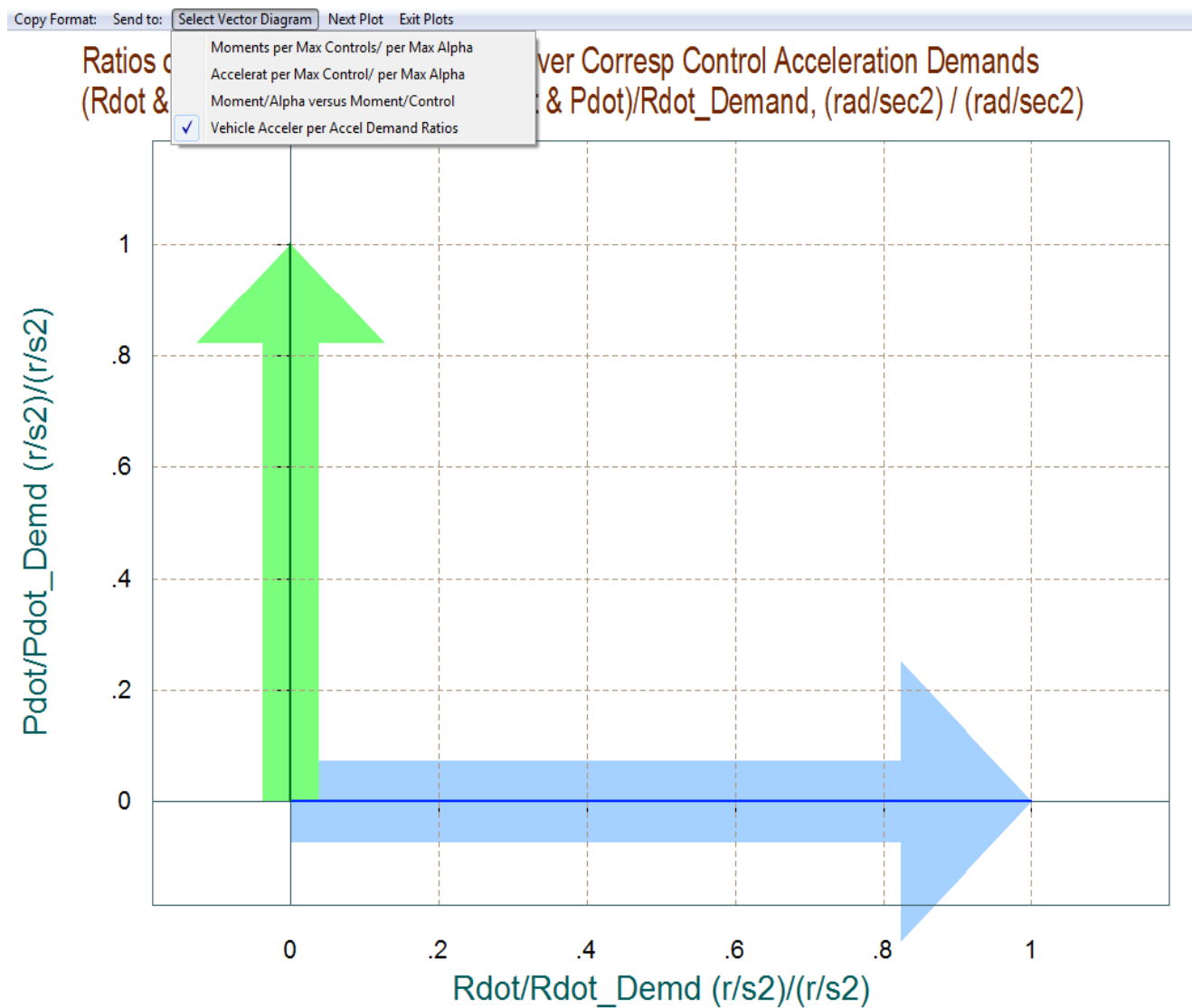
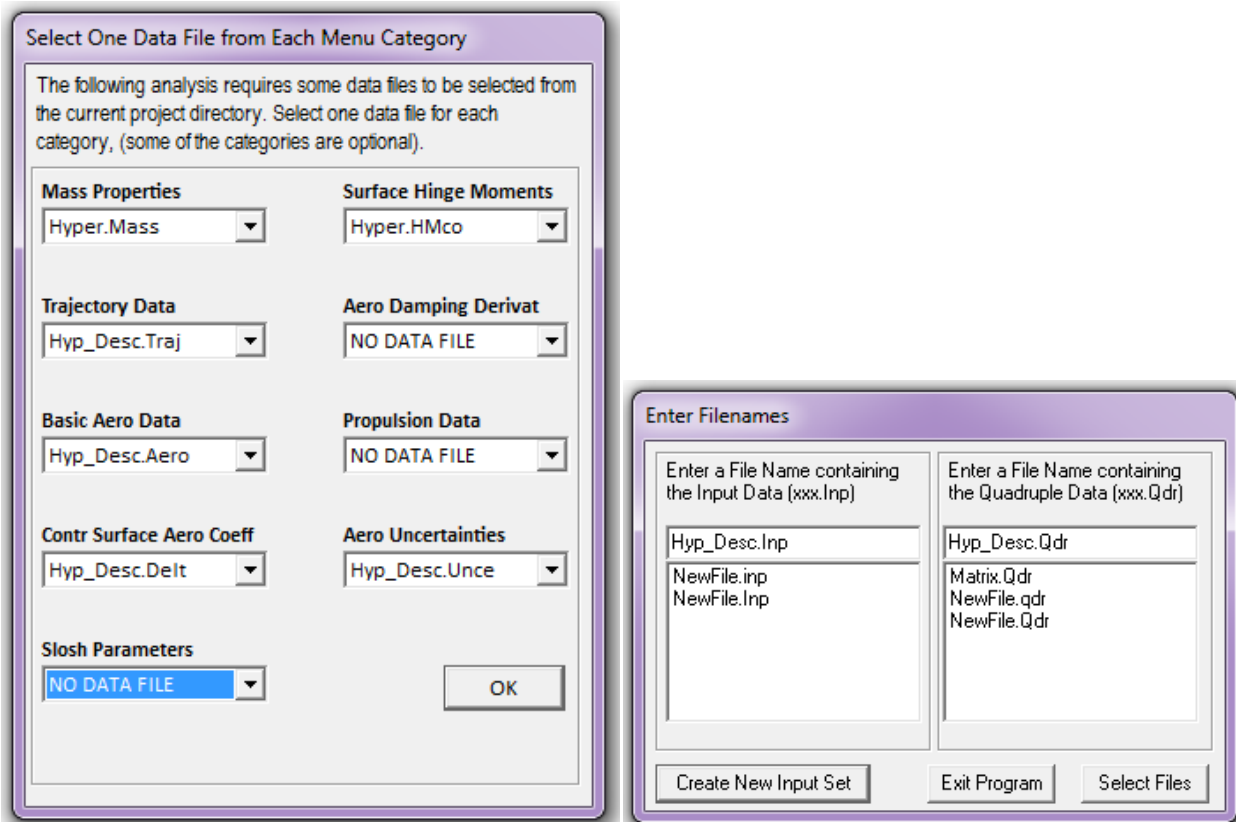


Figure 2.7.7 Roll and Yaw Moment Partial and Acceleration Partial

## 2.8 Preliminary Dynamic Analysis

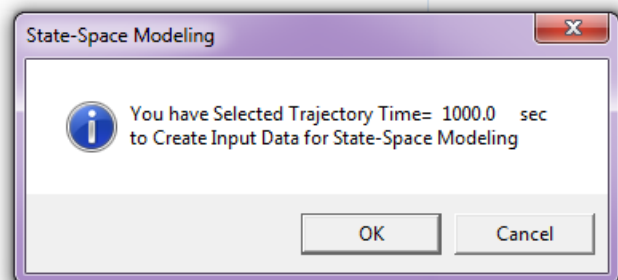
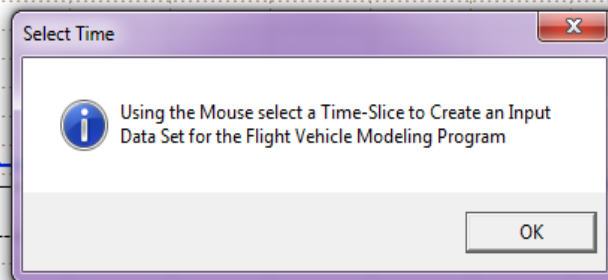
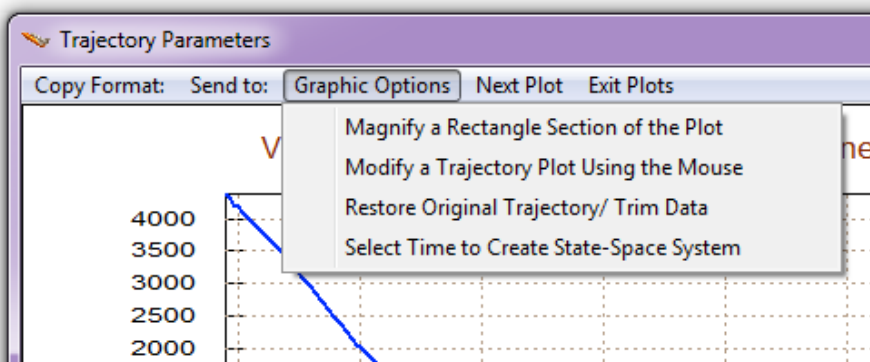
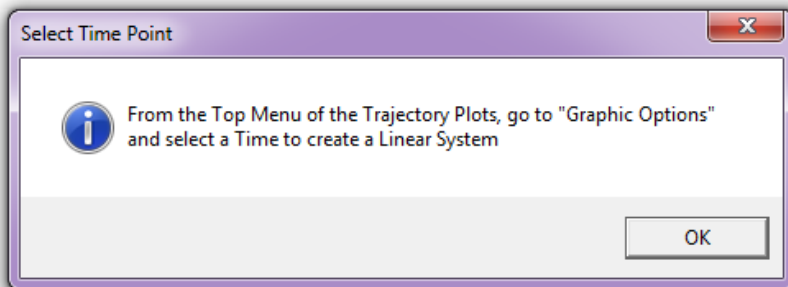
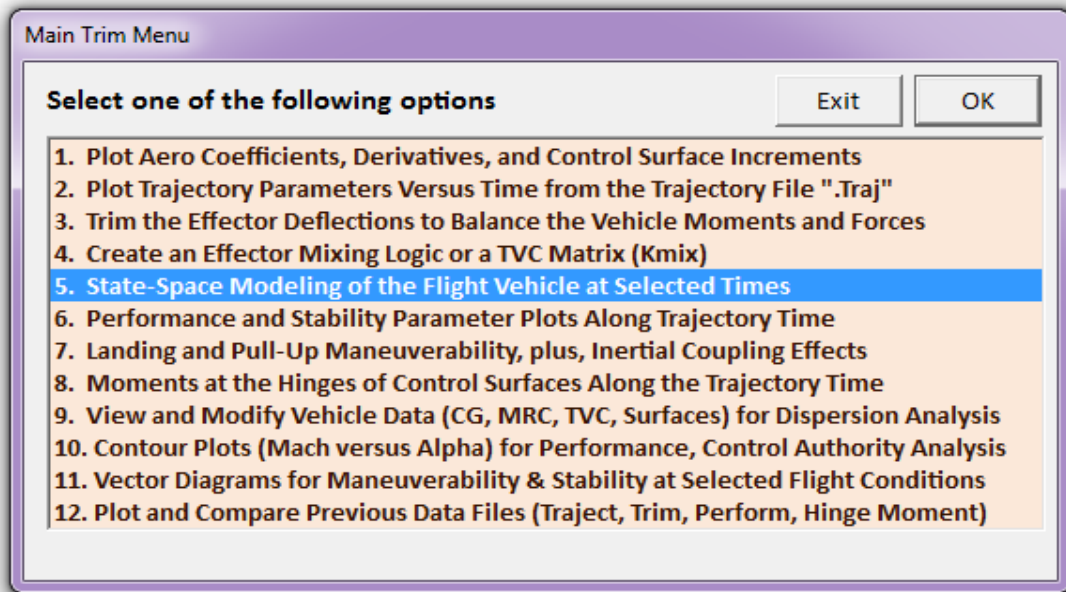
We will now demonstrate how to use the Trim program to create linear models for control design and analysis. We will select an arbitrary flight time 1000 (sec) from take-off and generate a linear state-space system using the Flixan flight vehicle modeling program (FVMP). We will also generate a control surface mixing matrix that corresponds to the same flight condition. We will separate the pitch and lateral subsystems, check their open-loop eigenvalues, and see how they compare with our static analysis. So let us begin the Trim program, as before, select the Trim data files, select also the Flixan input data file "*Hyp\_Desc.Inp*" and the systems file "*Hyp\_Desc.Qdr*".



The Trim program will generate the flight vehicle input data in file "*Hyp\_Desc.Inp*", and the vehicle dynamic model will be created by the FVMP and it will save it in "*Hyp\_Desc.Qdr*". This input data file has already been prepared, and in addition to the vehicle data it contains also data sets that will be used by Flixan to further refine the vehicle modeling and analysis. The Trim program first generates the vehicle input data in file "*Hyp\_Desc.Inp*". The additional data-sets in that file are introduced by the analyst and processed by other Flixan utilities, for the purpose of separating the pitch and lateral systems, generating a surface combination matrix (Kmix), and re-formatting the systems for Matlab.

From the Trim main menu we select Option (5) to generate the vehicle data for the dynamic model. The program activates the trajectory plotting option from where the user is prompted to select a flight time across the plot using the mouse. From the top menu bar, click on "*Graphic*

Options" and from the vertical drop-down menu click on "Select Time to Create a State-Space System".





Slide the cursor along the horizontal time axis and click at  $t = 1000$  sec. Another dialog confirms the trajectory time selected to generate the linear model. Click "OK", or if you made an error click on "Cancel" and select another time point from the trajectory plot. At this point the flight vehicle modeling program (FVMP) dialog comes up showing the vehicle data (coming from Trim) categorized in tabs. This dialog allows you edit some of the data or titles in the various fields and tabs. You must click on "Update Data" every time you modify a group of data before changing tabs. When you are done editing click on on "Save in File" button to save the vehicle data in "Hyp\_Desc.Inp". You may also edit that file directly by clicking on "Edit Input File". Finally you may click on "Run" to generate the vehicle system in "Hyp\_Desc.Qdr". But you are not ready to run it yet before including the remaining of the Flixan data sets for the additional processes described, so click on "Exit" for now, and take a look instead in the already prepared file.

**Flight Vehicle Parameters**

**Vehicle System Title**  
**Rocket-Plane Mission-1, Descent Phase/ T= 1000 sec**

**Number of Vehicle Effectors**  
 Gimbal Engines or Jets. Include Tail-Wags-Dog?  WITH TWD / WITHOUT TWD  
 Rotating Control Surfaces. Include Tail-Wags-Dog?  4 WITH TWD / WITHOUT TWD  
 Reaction Wheels?  0  
 Single Gimbal CMGs?  0  
**Momentum Control Devices**  
 Include a 3-axes Stabilized Double Gimbal CMG System?  Yes / No

**Number of Sensors**  
 Gyros  0  
 Acceleromet  3  
 Aero Vanes  0  
 External Torques  0

**Modeling Options (Flags)**  
 Output Rates in: Body Axes / Stability Axes  
 Turn Coordination?  Include Turn Coordin / Without Turn Coordin  
 Aero-Elasticity Options: Include GAFD, H-param Flex Coupl. data only / Neither Gafd nor Hpar  
 Attitude Angles: Euler Angles / Integrals of Rates / LVLH Attitude

**Number of Modes**  
 Structure Bending  0  
 Fuel Sloshing:  0

Buttons: Edit Input File, Exit, Update Data, Run, Save in File

---

Reaction Wheels | Single Gimbal CMGs | Double Gimbal CMG System | Slewing Appendages | Gyros | Accelerometer | Aero Sensors | Fuel Slosh | Flex Modes | User Notes  
 Mass Properties | Trajectory Data | Gust/ Aero Paramet. | Aero Force Coeffs | Aero Moment Coeffs | Control Surfaces | Gimbal Engines/ RCS | External Torques

**This Vehicle has 4 Control Surfaces** | **Control Surface No: 1 Elevon** | Surface Definition | Next Surface

**Surface Rotation Angles**  
 Surface Trim Position (deg)  -3.040600  
 Largest Positive Deflection from Trim (deg)  30.00000  
 Largest Negative Deflection from Trim (deg)  -30.00000

**Surface Location (ft) and Hinge Orientation Angles (deg)**  
 Xcs  -41.40000  
 Ycs  0.000000  
 Zcs  2.000000  
 Phi\_cs  0.000000  
 Lambda\_cs  0.000000

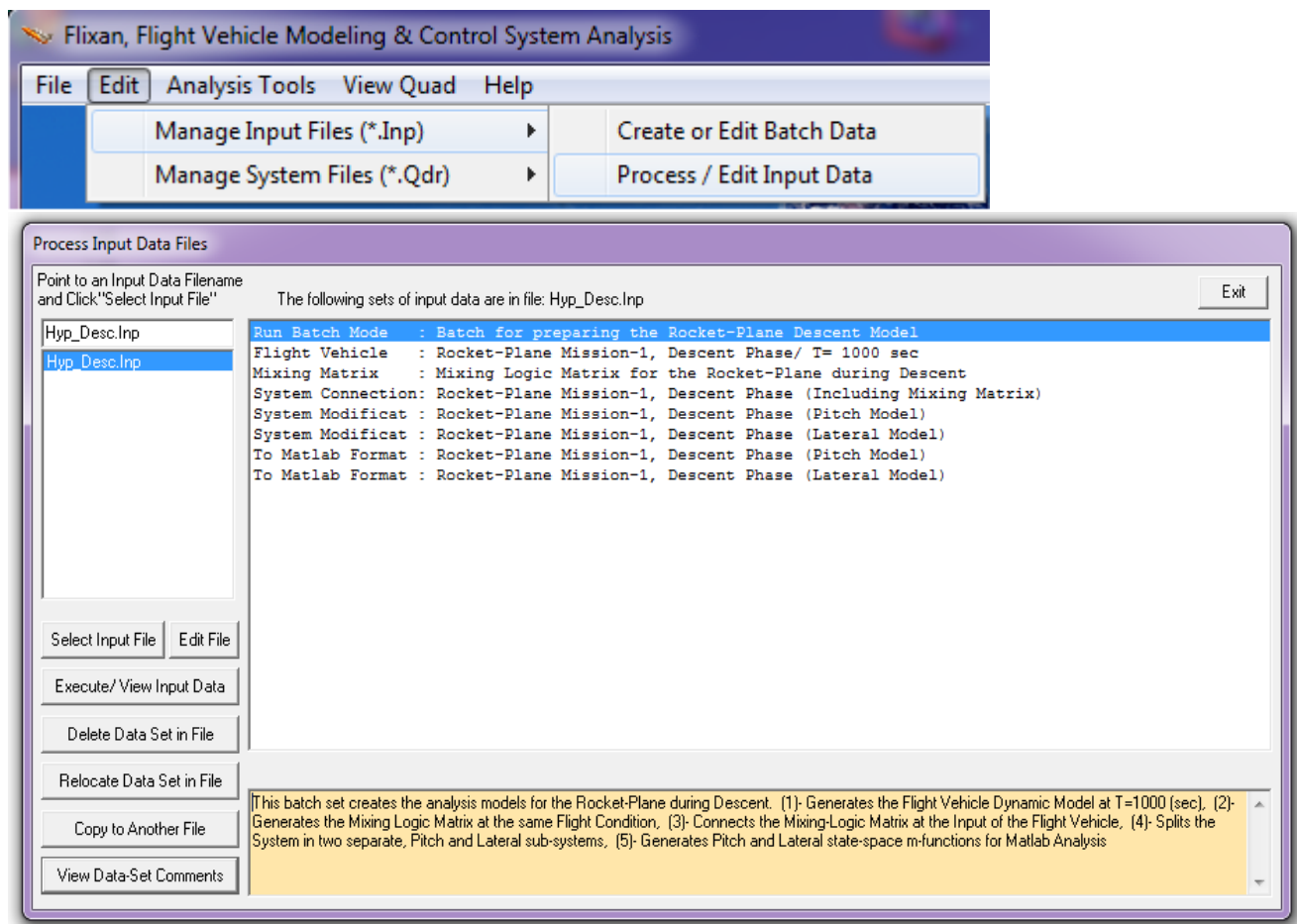
**Control Surface Mass Properties**  
 Control Surface Mass in (Slugs)  10.00000  
 Moment of Inertia about Hinge (slug-ft<sup>2</sup>)  100.0000  
 Moment Arm (feet), Surface CG to Hinge  2.000000  
 Control Surface Chord (feet)  2.360000  
 Control Surface Reference Area (ft<sup>2</sup>)  11.81000

**Aero Force Derivatives**  
 Ca\_delta  0.6722500E-03  
 Ca\_delta\_dot  0.000000  
 Cy\_delta  0.000000  
 Cy\_delta\_dot  0.000000  
 Cz\_delta  -0.1732500E-01  
 Cz\_delta\_dot  0.000000  
 Control Force Derivatives due to Surface Deflection (1/deg) and Control Force Derivatives due to Surface Rate (1/deg/sec)

**Aero Moment Derivatives**  
 Cl\_delta  0.000000  
 Cl\_delta\_dot  0.000000  
 Cm\_delta  -0.2565000E-02  
 Cm\_delta\_dot  0.000000  
 Cn\_delta  0.000000  
 Cn\_delta\_dot  0.000000  
 Control Moment Derivatives due to Surface Deflection (1/deg) and Control Moment Derivatives due to Surface Rate (1/deg/sec)

**Hinge Moment Derivatives**  
 Chm\_Alpha  -0.6450001E-02  
 Chm\_Beta  0.000000  
 Chm\_Delta  -0.1644900E-01  
 Chm\_Mach  0.000000  
 Hinge Moment Derivatives with respect to Changes in: Alpha, Beta, Surf Deflection (1/deg) and changes in Mach Number

The already created input file "*Hyp\_Desc.Inp*" includes several sets of data and each set is processed by a separate Flixan utility. On the top of the file there is a batch set included that processes the remaining data-sets together. It creates a vehicle dynamic model and an aero-surface mixing matrix at  $t=1000$  sec. The vehicle model is then combined with the mixing logic matrix and separated into a pitch and a lateral subsystems. The input to the pitch system is pitch FCS demand rather than elevon, and the inputs to the lateral system are roll and yaw FCS demands rather than aileron and rudder. The two subsystems are also converted to Matlab m-functions so they can be loaded into Matlab. To run this batch from Flixan go to "*Manage Input Files (\*.Inp)*" and then click on "*Process/ Edit Input Data*". From the following dialog, first select the input file from the left menu, and then from the right menu select the batch set, which is the top title in the menu, and click on "*Execute*".



The Flixan program creates the pitch and lateral subsystems in Matlab format in files "*vehi\_pitch.m*" and "*vehi\_lateral.m*" respectively. They are in folder "*C:\Trim\Examples\Hypersonic Vehicle\Descent\Mat-Eigen*" and are loaded into Matlab using the Matlab script "*init.m*" which calculates their eigenvalues, as shown in the next page. Both systems are dynamically unstable with unstable poles because they are open-loop. They are statically stable, however, according to our previous static performance analysis in section (2.3), see figure, with a pitch short-period resonance 0.25 (rad/sec) and a Dutch-Roll resonance 2.1 (rad/sec) in the lateral axes. These resonances are the same as those obtained by calculating the eigenvalues of the two systems, as shown below.

```

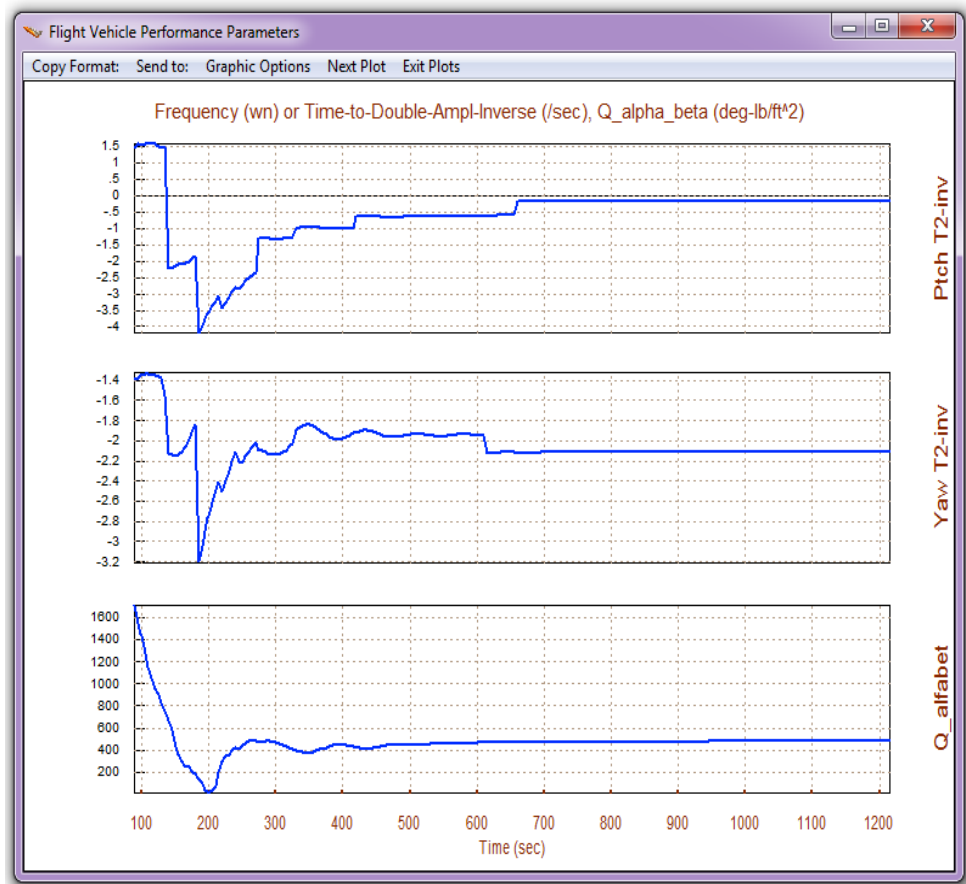
Pitch_Eigen =

-0.8358
 0.3610
-0.2375 + 0.2476i
-0.2375 - 0.2476i
 0.0000

Lateral_Eigen =

 0
 0.1109 + 2.0965i
 0.1109 - 2.0965i
-0.3985
-0.0000

```



## 2.9 Flight Control Design

The directory: "C:\Flixan\Trim\Examples\Hypersonic Vehicle\Descent\Mat\_Design" contains the data used for a preliminary flight control design using vehicle models generated at some critical flight conditions along the trajectory. The folder contains input data files used by Flixan at 7 flight conditions and the corresponding system files. The LQR method was used for the creation of state-feedback gains. Each input data file, for example "T95\_Design.Inp", contains the following:

- A batch data-set which calls the various Flixan utilities and creates the various matrices and systems.
- A set of flight vehicle data, title: "Rocket-Plane Mission-1, Descent Phase/ T= 95.0 sec", which is processed by the flight vehicle modeling program.
- A mixing logic set, title: "Mixing Logic Matrix for the Rocket-Plane during Descent" which generates an effector combination matrix "Kmix\_95", corresponding to the same flight condition.
- A systems interconnection set "Rocket-Plane Mission-1, Descent Phase (Including Mixing Matrix)" which combines the vehicle model "Rocket-Plane Mission-1, Descent Phase/ T= 95.0 sec" with the mixing matrix "Kmix\_95", and creates a new system.
- A data set "Rocket-Plane Mission-1, Descent Phase (Pitch Model)" which extracts only the pitch dynamics from the combined system.

- A data set "Rocket-Plane Mission-1, Descent Phase (Lateral Model)" which extracts only the roll/yaw dynamics from the combined system.
- Two data sets which convert the pitch and lateral subsystems to Matlab m-function format, "pitch\_95.m" and "later\_95.m".

## Pitch Design

The pitch design is performed in sub-directory "Mat\_Design\Pitch LQR" by running the file "pitch\_design.m". The Matlab script loads the pitch state-state system "Pitch\_95.m" and augments the state-vector by introducing one additional state, alpha-integral. The state-vector consists of: theta ( $\theta$ ), pitch rate ( $q$ ),  $\alpha$ , change in altitude ( $h$ ), change in velocity ( $v$ ), and  $\alpha$ -integral. The design script uses LQR to calculate the state-feedback gain Kq\_95.mat. Then it performs a frequency response analysis to show stability margins.

```
% Pitch LQR Design for Rocket-Plane
[Ap,Bp,Cp,Dp]= pitch_95; % Load the Pitch Design Model
[Api,Bpi,Cpi,Dpi]= linmod('Pitch_Design'); % Augment Pitch Simulink model
sys=SS(Api,Bpi,Cpi,Dpi);
% Weights[thet, q, alpha, h, V, alf_int]
Q= diag( [0.0001, 0.2, 0.001, 0.03,0.01,0.0001]); % Weights(thet,q,alpha,h,v,alf-int)
R=12; % Control Weights R=2
[Kq,S,E] = LQR(sys,Q,R)
save Kq_95.mat Kq -ascii % Save the LQR gains in Kpqr.mat

w=logspace(-3, 3, 4000); % Define Frequ Range
[Al,B1,C1,D1]= linmod('Freq_Anal'); % Linearize Open-Loop model
sys= SS(Al,B1,C1,D1); % Create SS System
%figure(1); Nyquist(sys,w) % Plot Nichol's Chart
figure(2); Nichols(sys,w) % Plot Nichol's Chart
figure(3); Bode(sys,w); grid on % Plot Bode Plot
```

## Lateral Design

Similarly, the lateral design is performed in sub-directory "Mat\_Design\Lateral LQR" by executing file "Later\_design.m". The Matlab script loads the lateral state-state system "Later\_95\_lqr.m" and augments the state-vector by introducing one additional state, phi-integral. The lateral state-vector consists of: roll attitude ( $\phi$ ), roll rate ( $p$ ), yaw rate ( $r$ ), sideslip ( $\beta$ ), and  $\phi$ -integral. The design script uses LQR to calculate the lateral state-feedback gain Kpr\_95.mat. Then it performs a frequency response analysis to show stability margins.

```
% LQR Lateral Design for Hypersonic Rocket-Plane
[Al,B1,C1,D1]= later_95_lqr; % Load the Lateral Design Model
[Ali,Bli,Cli,Dli]= linmod('Lat_des'); % Linearize Lateral Simulink model
sys=SS(Ali,Bli,Cli,Dli);
% states: phi, p, r, beta, phi_int
Q=diag([ 8, 2, 2, 0.01, 0.05]);
R=diag([1 1]*3); % Control Weights
[Kpr,S,E] = LQR(sys,Q,R)
save Kpr_95.mat Kpr -ascii % Save the LQR gains in Kpqr.mat
```

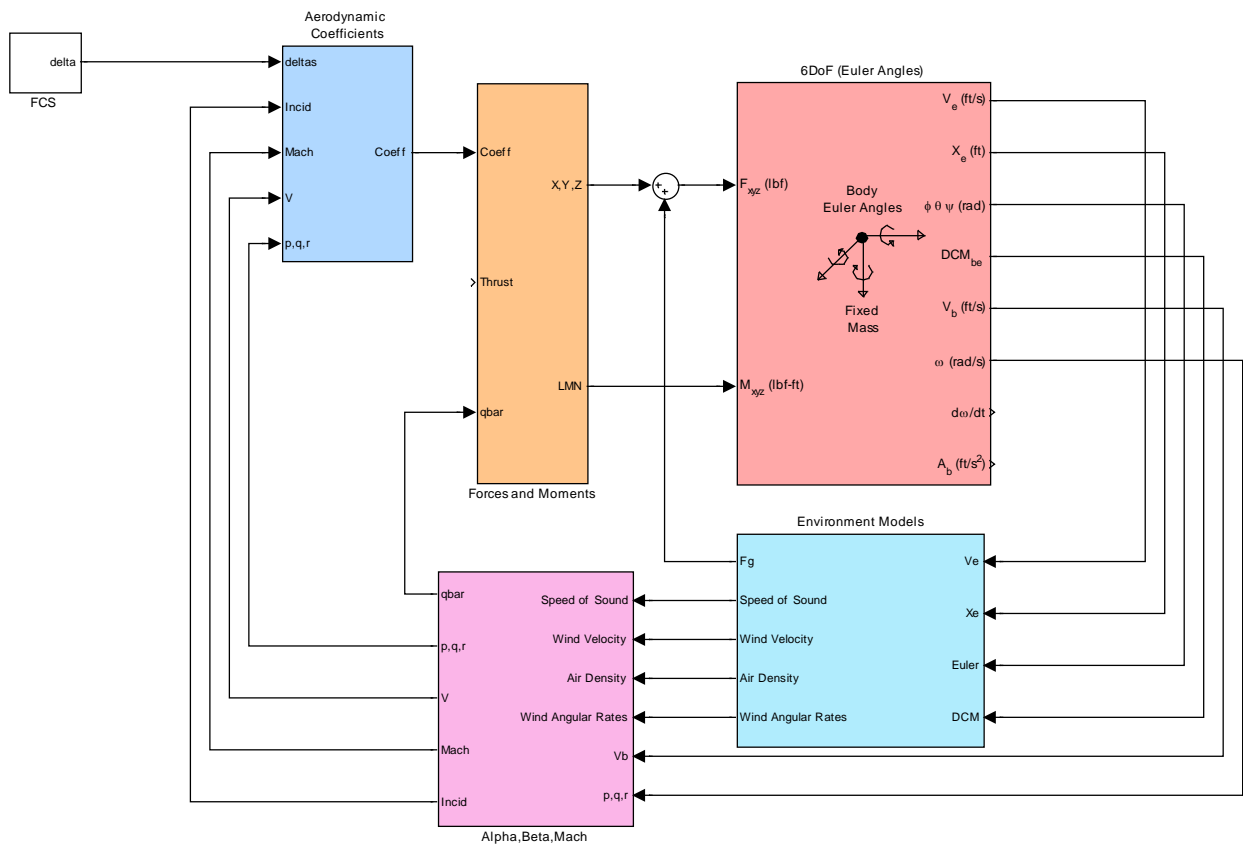
```

% Frequency Domain Analysis for Hypersonic Vehicle
w=logspace(-3, 3, 4000); % Define Freq Range
[Al,B1,C1,D1]= linmod('Freq_Anal'); % Linearize Open-Loop model uses combined plant
sys= SS(Al,B1,C1,D1); % Create SS System
% %figure(1); Nyquist(sys,w) % Plot Nichol's Chart
figure(2); Nichols(sys,w) % Plot Nichol's Chart
figure(3); Bode(sys,w); grid on % Plot Bode Plot

```

## 2.10 Non-Linear 6-dof Descent Simulation

The LQR gains and the aero-surface mixing matrices from section 2.9 are used in a 6-dof Matlab simulation. The simulation files are in subdirectory "C:\Flixan\Trim\Examples\Hypersonic Vehicle\Descent\Descent 6-dof Matlab Simulation", and the Simulink model is "Descent\_Sim.mdl", shown below.



The simulation is initialized from file "init.m", which loads the aero data from file "Base\_Aero\_Coeff.m" and the aero-surface aero coefficients from files "Elevon.m", "BodyFlap.m", "Aileron.m", and "Rudder.m". It also loads the control gains and mixing matrices that were computed in section 2.9, from file FCS.mat. The simulation tracks a pre-calculated altitude versus time trajectory which is slightly different from the point-mass trajectory.