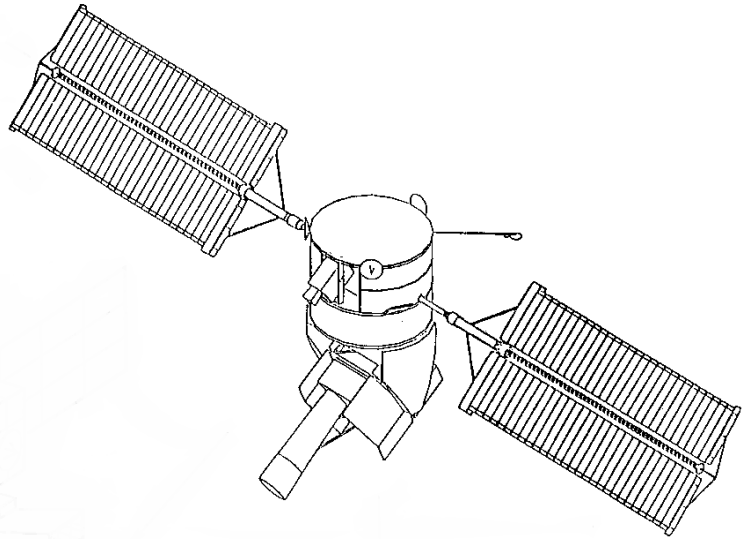


3 Rigid-Body Spacecraft Equations

This section describes the equations of motion for a rigid body spacecraft that is controlled by reaction control jets (RCS), and also an array of momentum exchange devices, such as, reaction wheels (RWA), single-gimbal control moment gyros (SG-CMG), and double-gimbal control moment gyros (DG-CMG). Equations of a spacecraft with rotating appendages are also included. The Flixan program provides the capability to implement any spacecraft configuration consisting of: RCS, RWs, CMGs, and gimbaling appendages, or any combination of the above devices. Structural flexibility can also be included and implemented as a separate flex system coupled



with the rigid-body model. It requires using the flexible spacecraft modeling program and modal data with nodes at the actuator and sensor locations, as demonstrated in the examples.

3.1 Rigid Body Spacecraft with Reaction Wheels

Reaction wheels consist of a spinning rotor whose spin axis is fixed relative to the spacecraft and its spin rate is maintained close to zero. Its speed is increased or decreased to generate a reaction torque about the spin axis. They are inexpensive but their control torque capabilities are small and are typically used to control small 3-axes stabilized satellites. Figure 1a shows a typical reaction wheel (RW) with the control electronics. Its angular rate can be varied by applying a torque to the motor that rotates it about its spin axis. As the wheel accelerates it applies an equal and opposite reaction torque to the spacecraft that is used to control attitude. In general, a three axes stabilized spacecraft requires a RW array consisting of at least three reaction wheels.

Figure 1b represents a rigid-body spacecraft with a RW and an externally applied force \mathbf{F} . The spacecraft origin is at the CG. The unit vector $\underline{\mathbf{a}}_i$ defines the spin direction of wheel (i). A force vector $\underline{\mathbf{F}}_j$ defines an external force (j), for example, due to an RCS jet firing. The displacement $\underline{\mathbf{d}}_j$ represents the distance between the force \mathbf{F}_j application point, and the vehicle CG (assuming the mass properties include the wheels). By combining the reaction wheel assembly (RWA) spin column vectors $\underline{\mathbf{a}}_i$ together we can create a transformation matrix C_w^b that transforms the wheel momentum from individual wheel axis to spacecraft body axes (x, y, z). For example, if we are using four reaction wheels the transformation from wheel to body is shown in equation 1.1. The transformation from body to wheel is the pseudo-inverse of C_w^b .

$$C_w^b = \begin{bmatrix} \underline{\mathbf{a}}_1 & \underline{\mathbf{a}}_2 & \underline{\mathbf{a}}_3 & \underline{\mathbf{a}}_4 \end{bmatrix} \quad (1.1)$$

The following equations describe the non-linear dynamics of a spacecraft with reaction wheels and reaction control jets that can be used for large angle simulations. The rate of change of system momentum (total momentum of spacecraft plus wheels) is not affected by the internal wheel control torques but it is only affected by the external torques due to disturbances or the RCS jets torques.

$$\dot{\underline{H}}_{sys} = -\underline{\omega}_b \times \underline{H}_{sys} + \underline{T}_{ext} \quad \textit{System Angular Momentum Rate} \quad (1.2)$$

\underline{T}_{ext} is the sum of all external moments applied to the spacecraft. It consists of three terms: aerodynamic disturbance torques T_D , gravity gradient torques T_{GG} , and also the control torques generated by the reaction control jets $\underline{F}_{RCS(j)}$.

$$\underline{T}_{ext} = \underline{T}_D + T_{GG} + \sum_j^{nf} \underline{d}_j \times \underline{F}_{RCS(j)} \quad (1.3)$$

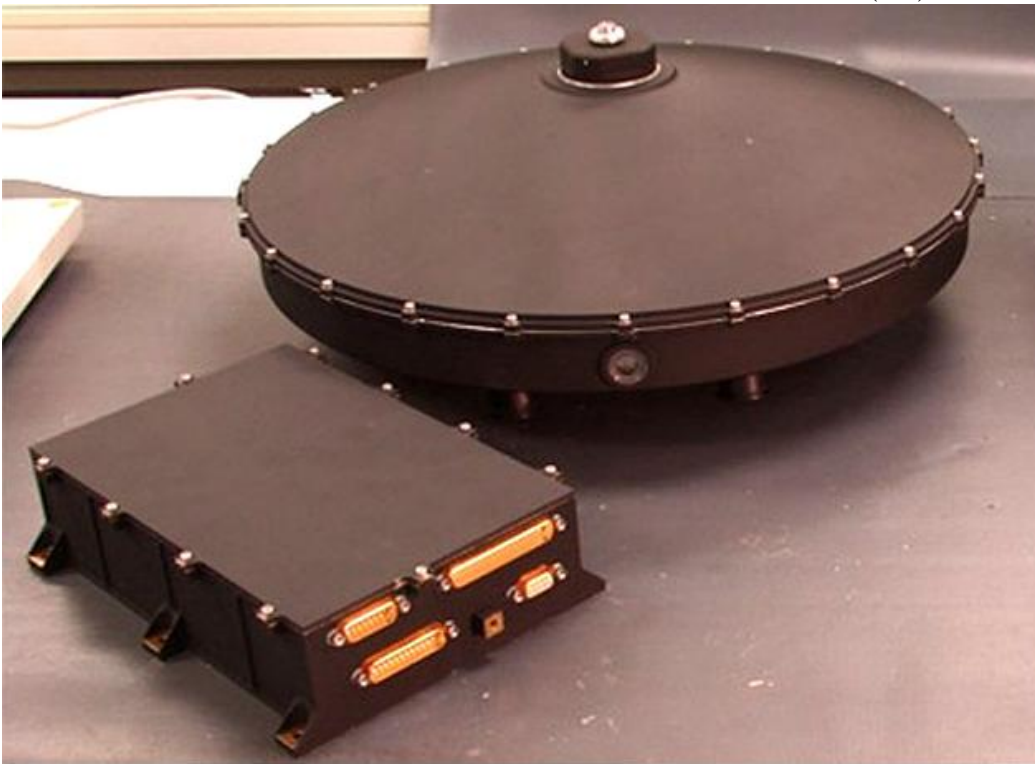


Figure 1a Typical Reaction Wheel with Control Electronics

When the spacecraft has an orbital rate ω_0 (rad/sec) the gravity gradient torque is a function of the LVLH Euler angles (ϕ , θ , ψ), as described in equation 1.4.

$$T_{GG} = 3\omega_o^2 (c \times J_v c) \quad , \quad c = \begin{bmatrix} -\sin \theta \cos \psi \\ \cos \phi \sin \theta \sin \psi + \sin \phi \cos \theta \\ -\sin \phi \sin \theta \sin \psi + \cos \phi \cos \theta \end{bmatrix} \quad (1.4)$$

The spacecraft with reaction wheels model can be implemented in two forms. In the simplest form the momentum of the reaction wheel array is calculated in the body frame. This model does not keep track of the individual wheel speeds and it does not calculate the torque produced by each individual wheel on the vehicle. Only the combined RWA torque is calculated, which is fine if the wheels are collocated. The following equation calculates the spacecraft rotational acceleration $\dot{\omega}$ as a function of the RWA internal torque vector T_w and also the external torques T_{ext} . The second equation calculates the rate of change in RW momentum as a function of the torque generated by the wheels, which is opposite to spacecraft torque.

$$\begin{aligned} J_v \dot{\omega} &= -\omega \times J_v \omega + \underline{T}_{RW} + \underline{T}_{ext} \\ \dot{h}_b + \omega \times h_b &= -\underline{T}_{RW} \end{aligned} \tag{1.5}$$

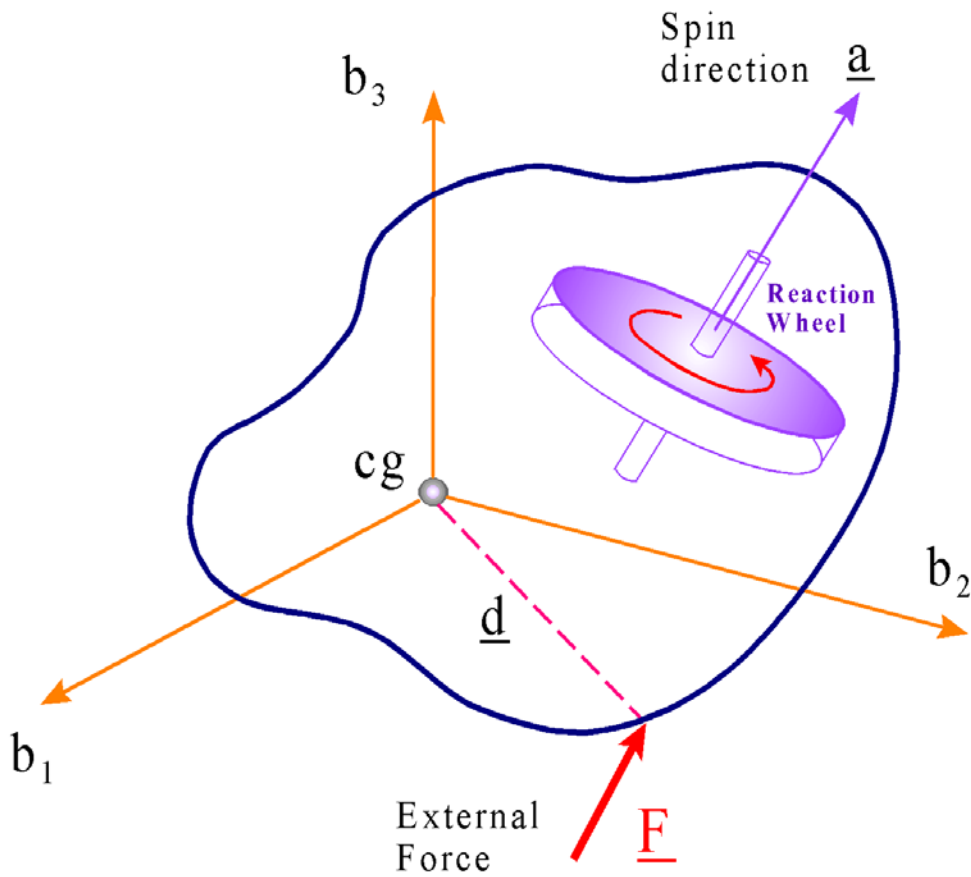


Figure 1b Spacecraft with a Reaction Wheel and an External Force

Where:

- h_b is the RW array momentum in the spacecraft body frame.
- T_{RW} is a vector of RW torques in spacecraft body axes
- T_{ext} is the external torques applied to the spacecraft in body axes
- J_v is the spacecraft moment of inertial dyadic

The total system momentum \underline{H}_{sys} consists of the spacecraft momentum plus the combined wheel momentum. During maneuvering the RW momentum varies significantly but the system momentum is maintained small by frequent momentum dumps. In the absence of external torques the system momentum is constant and close zero.

$$\underline{H}_{sys} = \underline{J}_{sc} \underline{\omega} + \underline{h}_b \quad ; \quad \underline{h}_b = \underline{C}_w^b \underline{h}_w \quad (1.6)$$

The torque applied to the spacecraft body axes by the reaction wheels array is

$$\underline{T}_{RW} = \underline{C}_w^b \underline{T}_{wi} - \underline{\omega} \times \underline{C}_w^b \underline{h}_{wi} \quad (1.7)$$

Where the first term is due to the torque motors that accelerate the RWs and the second term is a gyroscopic torque due to coupling between the i^{th} wheel momentum with the spacecraft rate. Resolving this equation for each individual wheel, the torque applied to the spacecraft body axis \underline{T}_{rwi} by the i^{th} RW is a combination of the motor torque T_{wi} applied about the wheel spin vector \underline{a}_{wi} plus the gyroscopic torque due to the coupling between the wheel momentum and the spacecraft rate ω . The torque at each individual wheel, equation 1.8, can be used to excite structural flexibility.

$$\underline{T}_{rwi} = \underline{a}_{wi} T_{wi} - \underline{\omega} \times \underline{a}_{wi} \underline{h}_{wi} \quad (1.8)$$

The spacecraft rate of change of momentum under the influence of an array of RW torques and the external torques is obtained from equation 1.9

$$\underline{J}_v \dot{\underline{\omega}} = -\underline{\omega} \times \underline{J}_v \underline{\omega} + \underline{C}_w^b \underline{T}_{wi} - \underline{\omega} \times \underline{C}_w^b \underline{h}_{wi} + \underline{T}_{ext} \quad (1.9)$$

Assuming that the system momentum is zero

$$\underline{H}_{sys} = \underline{J}_v \underline{\omega} + \underline{C}_w^b \underline{h}_{wi} = 0 \quad (1.10)$$

$$\underline{J}_v \dot{\underline{\omega}} = \underline{C}_w^b \underline{T}_{wi} + \underline{T}_{ext}$$

The inertial rate of change of the i^{th} RW momentum about its spin axis is

$$\dot{\underline{h}}_{wi} = -\underline{T}_{wi} \quad (1.11)$$

The measured angular rate of the i^{th} reaction wheel relative to the spacecraft is equal to its inertial rate minus the spacecraft rate resolved in the wheel spin axis, that is

$$\underline{w}_i = \frac{\underline{h}_{wi}}{\underline{J}_{wi}} - \underline{a}_{wi}^T \underline{\omega} \quad (1.12)$$

The Euler angles are measured relative to the rotating LVLH frame,

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \frac{1}{\cos \psi} \begin{bmatrix} \cos \psi & -\cos \phi \sin \psi & \sin \phi \sin \psi \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi \cos \psi & \cos \phi \cos \psi \end{bmatrix} \omega + \begin{pmatrix} 0 \\ \omega_o \\ 0 \end{pmatrix} \quad (1.13)$$

Where:

ω_o is the orbital rate

J_w is the individual wheel moment of inertia about their spin axis

J_v is the spacecraft inertia and it includes the weights of the reaction wheels.

The matrix J_w is diagonal consisting of the individual wheel inertias about their spin axis. It is assumed that the vehicle mass properties and inertias matrix (J_v) include the weights of the reaction wheels. The non-linear reaction wheel equations described above are not used in the Flixan program. The Flixan equations are all linearized at fixed steady-state conditions. The linearized dynamics of a spacecraft with reaction wheels are presented in the following section. In the examples section, however, we have included some large angle spacecraft modeling examples with reaction wheels and reaction control jets, using the above non-linear equations implemented in Matlab/ Simulink programs.

1.1 Linearized Equations for an Orbiting Spacecraft with Reaction Wheels and Momentum Bias

The following equations are obtained by linearizing the previous equations relative to the LVLH frame. We assume that the spacecraft has a steady pitch rate that is equal to the negative orbital rate $-\omega_o$. Equation 1.14 calculates the rate of change of angular momentum as a function of RW and external torques. The roll, pitch and yaw attitude is measured relative to the LVLH frame.

$$\begin{bmatrix} I_{XX} & I_{XY} & I_{XZ} \\ I_{XY} & I_{YY} & I_{YZ} \\ I_{XZ} & I_{YZ} & I_{ZZ} \end{bmatrix} \begin{pmatrix} \dot{\omega}_X \\ \dot{\omega}_Y \\ \dot{\omega}_Z \end{pmatrix} = \omega_o \begin{bmatrix} I_{XZ} & 2I_{YZ} & I_{ZZ} - I_{YY} \\ -I_{YZ} & 0 & I_{XY} \\ I_{YY} - I_{XX} & -2I_{XY} & -I_{XZ} \end{bmatrix} \begin{pmatrix} \omega_X \\ \omega_Y \\ \omega_Z \end{pmatrix} \\ + 3\omega_o^2 \begin{bmatrix} I_{ZZ} - I_{YY} & I_{XY} & 0 \\ I_{XY} & I_{ZZ} - I_{XX} & 0 \\ -I_{XZ} & -I_{YZ} & 0 \end{bmatrix} \begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} + \omega_o^2 \begin{bmatrix} -2I_{YZ} \\ 3I_{XZ} \\ -I_{XY} \end{bmatrix} + T_{RW} + \sum T_{ext} \quad (1.14)$$

1. The first term on the right side is the linearized $\omega \times J\omega$ term.
2. The second term on the right side is the gravity gradient torque which is a function of attitude.
3. The third term is a bias torque due to the linearization caused by the products of inertia.
4. T_{RW} is the RW array control torques in body axes, and
5. T_{ext} is other external torques.

Equation 1.15 calculates the rate of change of RW momentum resolved in the body axes. Where: $(\omega_{x0}, \omega_{y0}, \omega_{z0})$ is the nominal (steady) spacecraft rate.

$$\begin{pmatrix} \dot{h}_x \\ \dot{h}_y \\ \dot{h}_z \end{pmatrix} = - \begin{pmatrix} \omega_{x0} \\ \omega_{y0} \\ \omega_{z0} \end{pmatrix} \times \begin{pmatrix} h_x \\ h_y \\ h_z \end{pmatrix} - \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} H_{x0} \\ H_{y0} \\ H_{z0} \end{pmatrix} - \underline{T}_{RW} \quad (1.15)$$

The torque applied to the spacecraft by the reaction wheels array in body axes is

$$\underline{T}_{RW} = \underline{C}_w^b \underline{T}_{wi} - \underline{\omega}_0 \times \underline{C}_w^b \underline{h}_{wi} - \underline{\omega} \times \underline{C}_w^b \underline{h}_{w0} \quad (1.16)$$

The torque applied to the spacecraft by each individual wheel \underline{T}_{rwi} is a combination of the motor torque T_{wi} applied about the wheel spin vector \underline{a}_i plus the gyroscopic torque due to the coupling between the wheel momentum and the spacecraft rate. The torque at each individual wheel can be used to excite structural flexibility.

$$\underline{T}_{rwi} = \underline{a}_{wi} T_{wi} - \underline{\omega}_0 \times \underline{a}_{wi} \underline{h}_{wi} - \underline{\omega} \times \underline{a}_{wi} \underline{h}_{wi0} \quad (1.17)$$

The rate of change of momentum of the i^{th} reaction wheel about its spin axis is

$$\dot{h}_{wi} = -T_{wi} \quad (1.18)$$

The measured i^{th} wheel angular rate relative to the spacecraft is equal to its inertial rate minus the spacecraft rate resolved in the i^{th} wheel spin axis, that is

$$\omega_i = \frac{h_{wi}}{J_{wi}} - \underline{a}_{wi}^T \underline{\omega} \quad (1.19)$$

Where:

| | |
|--------------------------------|---|
| $\underline{\omega}_0$ | is the nominal spacecraft body rate about x, y, z axes: $\omega_{x0}, \omega_{y0}, \omega_{z0}$ |
| $\omega_x, \omega_y, \omega_z$ | is the variation in vehicle rate. |
| H_{x0}, H_{y0}, H_{z0} | is the nominal reaction wheel momentum of the RW assembly. |
| h_x, h_y, h_z | is the variation in RW momentum in body axes. |
| \underline{T}_{wi} | is the motor torque vector in reaction wheel axes. |
| \underline{h}_{w0} | is the nominal wheels momentum vector: $h_{w01}, h_{w02}, h_{w03}, \dots, h_{w0n}$ |
| \underline{a}_{wi} | is the unit vector of the i^{th} RW spinning direction |
| \underline{C}_w^b | is the wheel axes to body transformation matrix. |

3.2 Spacecraft with Single Gimbal CMGs

A single gimbal control moment gyroscope (SGCMG) is shown in Figure 2.1. It consists of a spinning rotor that is mounted on a structure that can be gimballed perpendicular to the rotor axis. The rotor spin rate is maintained at a constant speed by a small motor that produces a constant angular momentum h . It is the precession of this vector that produces a useful output torque that is substantially greater than a reaction wheel torque, and for this reason it is very attractive in applications that require high torque and in fast maneuvering or agile spacecraft. The direction of the spinning rotor and hence the flywheel momentum can be rotated relative to the spacecraft by a stronger motor and a gimbal. The gimbal motor controls the gimbaling rate, and hence the output torque. By commanding the gimbal to rotate, by means of a servo system that receives rate command from the steering logic, high precession torques are generated by changing the orientation of the angular momentum vector. The reaction torque on the spacecraft T is equal and opposite to the rate of change in momentum vector \dot{h} , which is orthogonal to the momentum vector h and also to the gimbaling vector according to the right hand rule. However, the torque direction at any instant is a function of the gimbal position. The SGCMG essentially acts as a torque amplification device because the output torque magnitude is equal to the CMG momentum multiplied by the gimbal rate. In addition, this does not require very much power, because, a small input torque from the gimbal actuator produces a much greater torque in a plane formed by the rotating momentum vector.

2.1 CMG Array Geometry

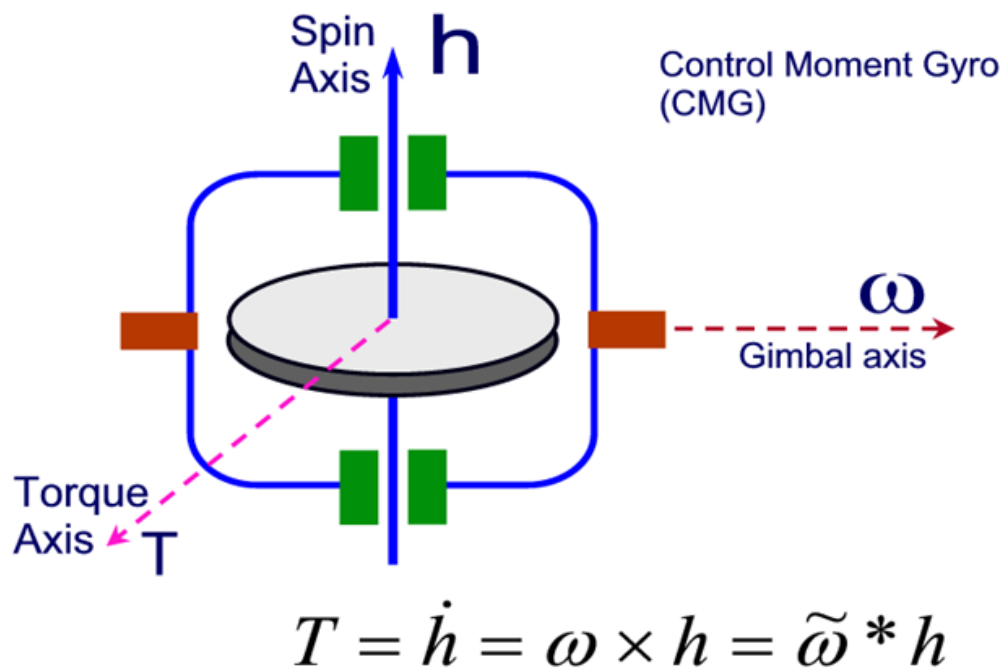


Figure 2.1a Single-Gimbal Control Moment Gyro

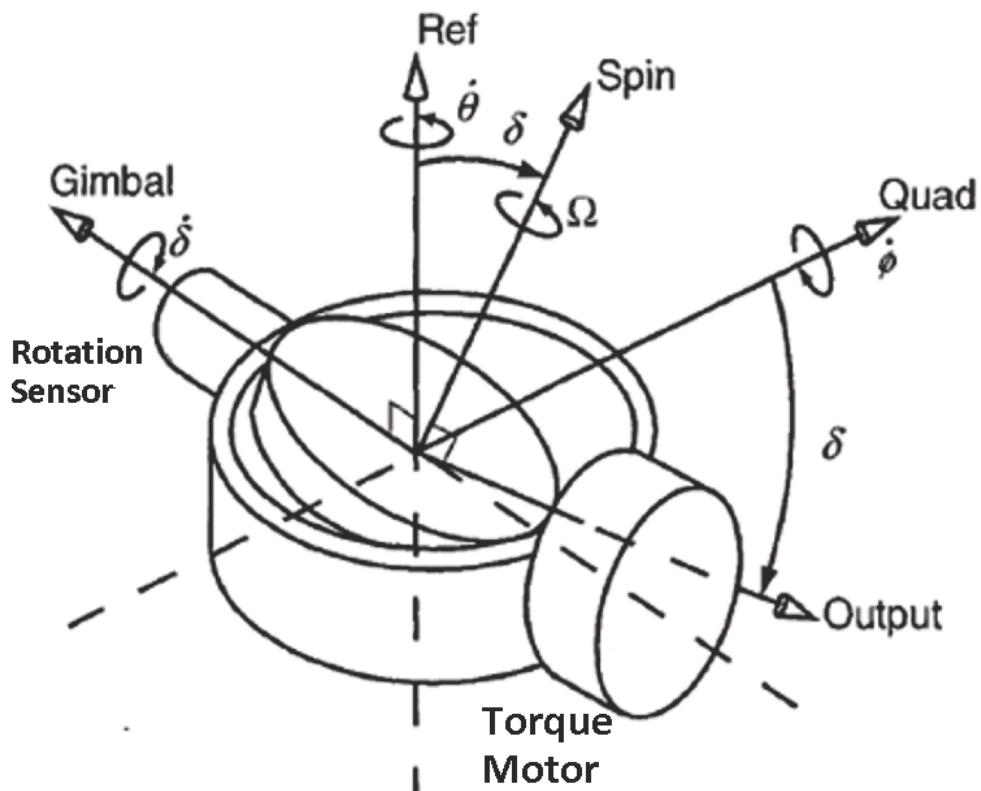
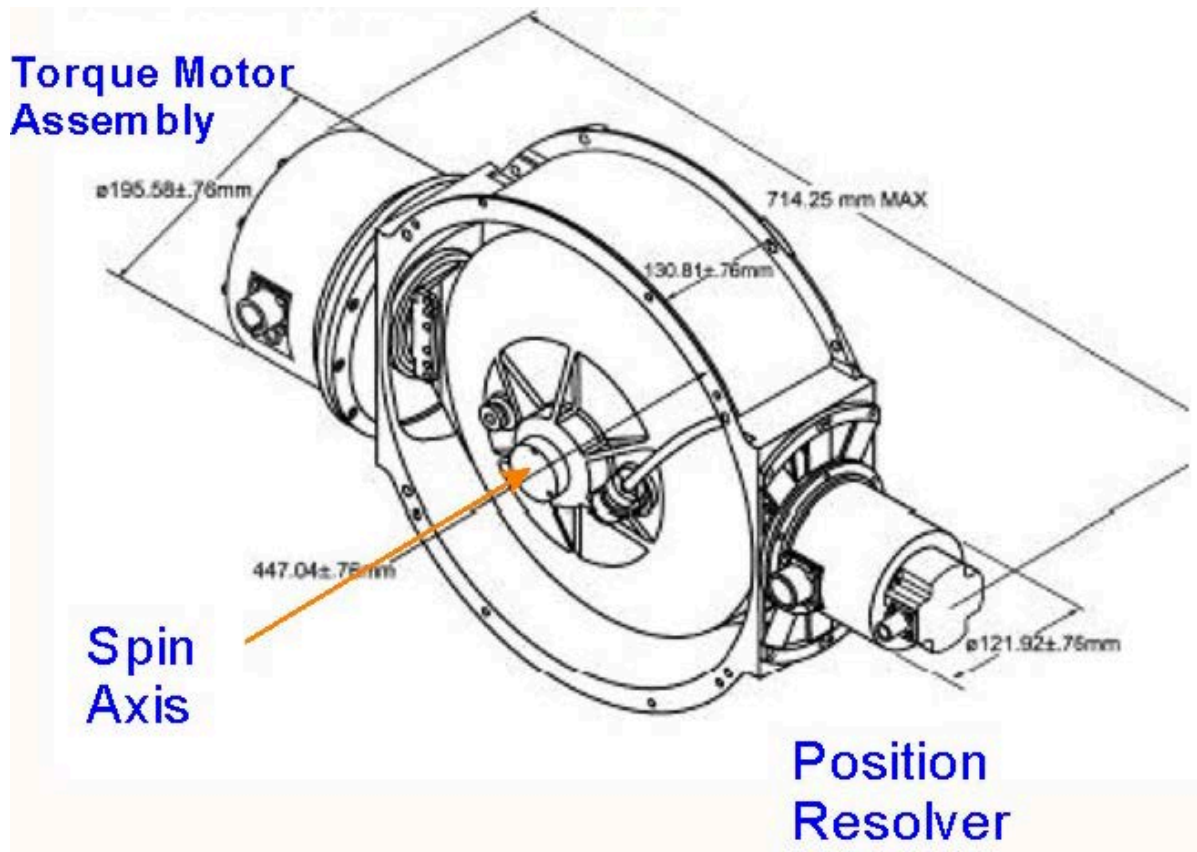


Figure 2.1b Single Gimbal Control Moment Gyro

This large torque amplification plus their capability of storing large amounts of angular momentum over long periods of time makes them especially advantageous as attitude control actuators for agile space spacecraft that require fast maneuvering, and high precision. They are also used in large space structures, such as a space station. Single-gimbal and double-gimbal CMGs have been used for attitude control of the Skylab, the MIR and the International Space Station (ISS). Another attractive feature of CMGs compared with reaction wheels is that the rotor in a CMG spins at a constant rate which places the vibrations at known frequencies while in a RW, the rotor speed changes, thus, exciting the spacecraft structure in multiple frequencies which may not be desirable in precision applications. CMGs, however, are complex systems, expensive, and require complex controls with singularity avoidance algorithms.

Figure 2.2 shows the local coordinates of a Single Gimbal CMG vectors which are defined relative to the spacecraft axes. They are the Gimbal, Reference, and Quad axes (\underline{m} , \underline{r} , \underline{q}) which are fixed relative to the spacecraft. The Gimbal (\underline{m}) is the axis perpendicular to the plane about which the rotor is gimballed by the servo. Ref (\underline{r}) is the direction of the spin vector when the gimbal angle δ_i is zero. The spin or momentum vector remains always in the plane at any gimbal angle δ_i . Quad (\underline{q}) is the quadrature formed by the cross-product of the Gimbal and Ref axes.

The momentum of one CMG about the Gimbal, Output, and Spin axes can be calculated in the CMG axes as a function of the spinning rotor momentum and the spacecraft rotation rate in CMG axes.

$$\begin{pmatrix} h_g \\ h_o \\ h_s \end{pmatrix} = \begin{pmatrix} J_g \dot{\delta}_i \\ J_o (\dot{\phi} \cos \delta - \dot{\theta} \sin \delta) \\ h_0 + J_s (\dot{\theta} \cos \delta + \dot{\phi} \sin \delta) \end{pmatrix} \quad (2.1)$$

Equation 2.2 calculates the rate of change of momentum by one CMG which is the moment generated by the CMG in the Gimbal, Output, and Spin axes respectively as a result of gimbaling and base motion. The Gimbal axis is fixed, but the Spin and the Output torque axes vary as a function of the gimbal angle.

$$\begin{bmatrix} M_G \\ M_O \\ M_S \end{bmatrix} = \begin{bmatrix} T_{gi} \\ J_o (\ddot{\phi} \cos \delta - \dot{\phi} \dot{\delta}_i \sin \delta - \dot{\theta} \dot{\delta}_i \cos \delta - \ddot{\theta} \sin \delta) + h_0 \dot{\delta}_i + \dot{\delta}_i (J_s - J_g) (\dot{\theta} \cos \delta + \dot{\phi} \sin \delta) \\ J_s (\ddot{\Omega} + \ddot{\theta} \cos \delta + \dot{\phi} \sin \delta + \dot{\phi} \dot{\delta}_i \cos \delta - \dot{\theta} \dot{\delta}_i \sin \delta) + \dot{\delta}_i (J_g - J_o) (\dot{\phi} \cos \delta - \dot{\theta} \sin \delta) \end{bmatrix}$$

Equation 2.2 SGCMG Momentum along the: Gimbal, Output, and Spin axes

The torques in the Spin and the Output axes vary as a function of the gimbal angle δ . The reaction torque that is applied on the spacecraft is in the opposite direction $[-M_G, -M_O, -M_S]$. Note that the angular rotation of the spacecraft couples with the stored CMG momentum vector and produces additional torque at the interface. The rotational body rates of the spacecraft can be converted to rates about the reference coordinates axes of each CMG. If $\underline{\omega}$ is the spacecraft roll, pitch and yaw body rate vector ($\omega_x, \omega_y, \omega_z$), equations 2.3 resolve the spacecraft rates about the CMG axes: (\underline{r} , \underline{q} , \underline{m}). $\dot{\theta}$ and $\dot{\phi}$ are the spacecraft rates resolved about the CMG reference and quad axes. The gimbal inertial rate $\dot{\delta}_m$ consists of: the gimbal rate relative to spacecraft $\dot{\delta}$ plus $\dot{\xi}$ which is the spacecraft rotational rate resolved about the CMG gimbal vector.

$$\begin{aligned}
\dot{\theta} &= \omega_x \cos \gamma + \omega_y \sin \gamma \\
\dot{\phi} &= -\omega_x \cos \beta \sin \gamma + \omega_y \cos \beta \cos \gamma + \omega_z \sin \beta \\
\dot{\xi} &= \omega_x \sin \beta \sin \gamma - \omega_y \sin \beta \cos \gamma + \omega_z \cos \beta \\
\dot{\delta}_{in} &= \dot{\delta} + \dot{\xi}
\end{aligned} \tag{2.3}$$

Where:

- J_g is the CMG moment of inertia about its gimbal axis
- J_o is the CMG moment of inertia about its output axis
- J_s is the CMG moment of inertia about its spin axis
- T_{gi} is the torque applied by the torque motor at the gimbal
- $\ddot{\delta}_i$ is the inertial angular acceleration of the rotor about the gimbal including spacecraft
- δ is the CMG gimbal rotation about the \underline{m} axis measured from the Ref axis
- h_0 is the constant CMG momentum about its spin axis ($I_s \Omega$)
- $\dot{\Omega}$ is the rotor spin acceleration
- $\dot{\theta}$ is the vehicle rate about the CMG reference \underline{r} axis
- $\dot{\phi}$ is the vehicle rate about the CMG quad \underline{q} axis
- $\dot{\xi}$ is the vehicle rate about the CMG gimbal \underline{m} axis

The following projection matrix P in equation 2.4 transforms the CMG torques from CMG reference frame to the spacecraft (x, y, z) axes.

$$\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{bmatrix} \sin \beta \sin \gamma & -\sin \delta \cos \gamma - \cos \delta \cos \beta \sin \gamma & \cos \delta \cos \gamma - \sin \delta \cos \beta \sin \gamma \\ -\sin \beta \cos \gamma & -\sin \delta \sin \gamma + \cos \delta \cos \beta \cos \gamma & \cos \delta \sin \gamma + \sin \delta \cos \beta \cos \gamma \\ \cos \beta & \cos \delta \sin \beta & \sin \delta \sin \beta \end{bmatrix} \begin{pmatrix} M_G \\ M_O \\ M_S \end{pmatrix}$$

Equation 2.4 CMG Reference to Spacecraft Transformation Matrix

When a spacecraft is controlled by CMGs at least three CMGs are needed to provide 3 axes control. If we consider an array of SGCMG mounted on the surfaces of a pyramid with their gimbal axes directions (\underline{m}_i) perpendicular to one of the surfaces of the pyramid and the momentum direction (\underline{h}_i) always aligned with the surface of the pyramid as the gimbal δ_i rotates. The output torque from each CMG is equal to the rate of change of angular momentum which is in the ($\underline{m}_i \times \underline{h}_i$) direction and proportional to the gimbal rate $\dot{\delta}_i$. From the pyramid surfaces orientations we can calculate three important matrices that will be used in the equations of motion.

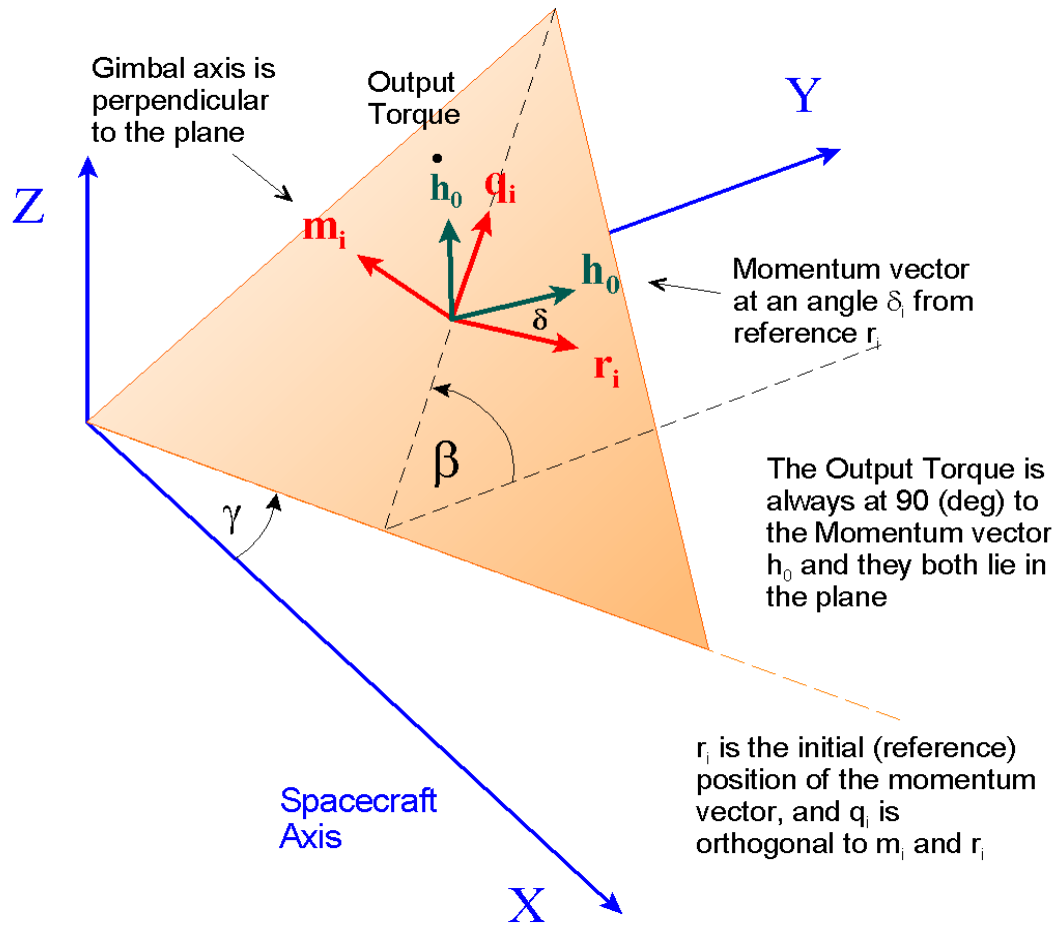
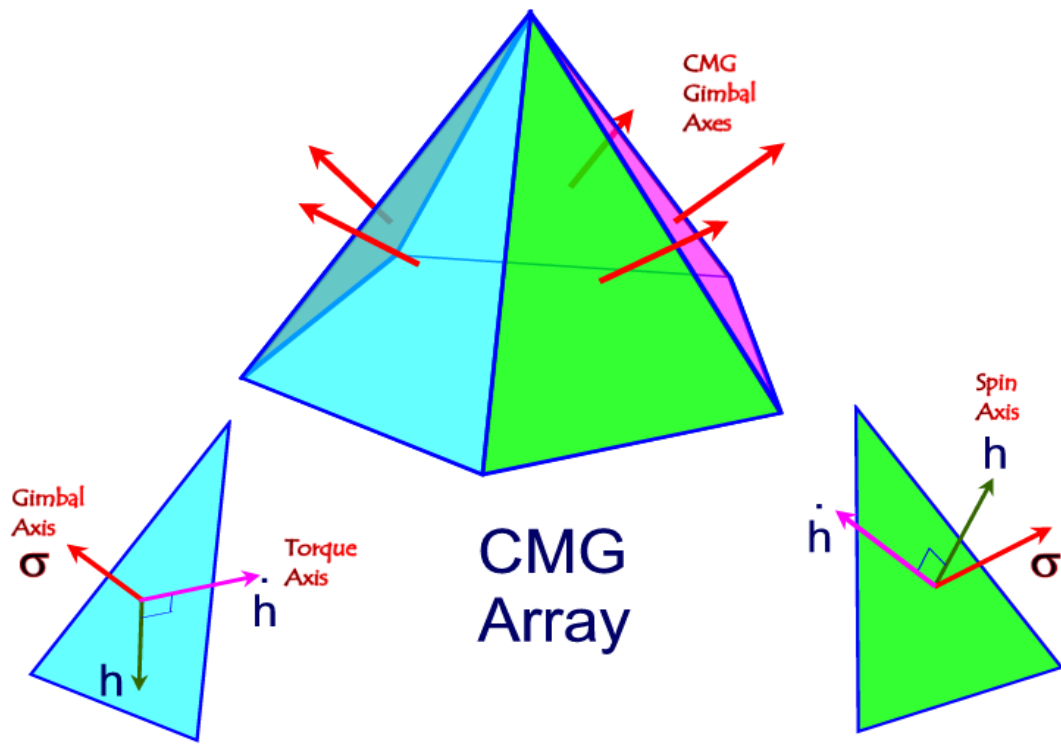


Figure 2.2 Orientation of a CMG in Spacecraft Coordinates

Let us consider a SGCMG pyramid arrangement shown in Figure 2.4b, where the spacecraft has four SGCMGs mounted onto the four faces of the four sided pyramid. All CMGs have the same constant angular momentum $h_0=1200$ (ft-lb-sec). Their CMG momentum vectors \underline{h}_i are initially (at zero gimbal, $\delta_{0i}=0$) parallel to the spacecraft X-Y plane producing zero total momentum. Their momentum vectors are constrained to lay parallel to the surface of the pyramid and they can be continuously rotated about the gimbal vectors $\underline{\delta}_i$, which are perpendicular to each surface. The orientation of each CMG relative to the spacecraft is shown in Figure 2.4b. The pyramid angle β is 68° , and the γ_i angles of the four surfaces according to Figure 2.2 are: 90° , 180° , 270° , and 0° . We can create the 3×4 gimbal to body transformation matrix M_g^b by stacking together the four gimbal direction column unit vectors \underline{m}_i as shown in equation 2.5.

$$M_g^b = [\underline{m}_1 \quad \underline{m}_2 \quad \underline{m}_3 \quad \underline{m}_4] \text{ where: } \underline{m}_i = \begin{bmatrix} \sin \beta_i \sin \gamma_i \\ -\sin \beta_i \cos \gamma_i \\ \cos \beta_i \end{bmatrix} \quad (2.5)$$

$$M_g^b = \begin{bmatrix} \sin \beta & 0 & -\sin \beta & 0 \\ 0 & \sin \beta & 0 & -\sin \beta \\ \cos \beta & \cos \beta & \cos \beta & \cos \beta \end{bmatrix} = \begin{bmatrix} 0.927 & 0 & -0.927 & 0 \\ 0 & 0.927 & 0 & -0.927 \\ 0.375 & 0.375 & 0.375 & 0.375 \end{bmatrix}$$



σ is the CMG gimbal axis direction
 h is the initial CMG momentum direction
 \dot{h} is the initial CMG torque direction

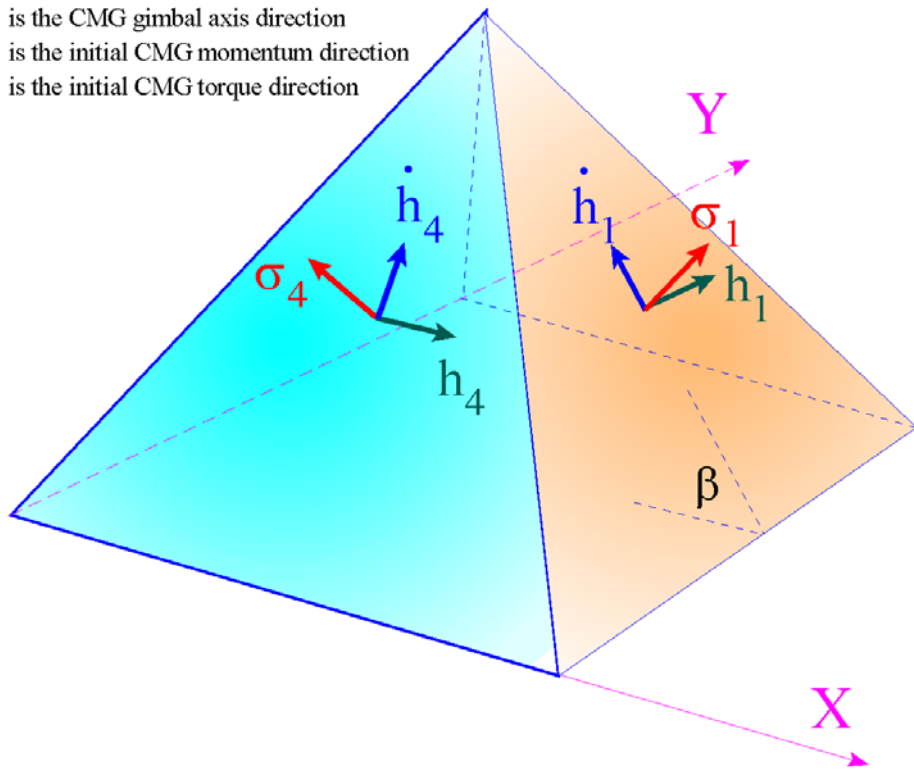


Figure 2.4 Arrays of five and four CMGs in Pyramid Configuration

Similarly, we create the 3x4 reference directions matrix R by stacking together the initial directions unit vectors \underline{r}_i of the momentum vectors \underline{h}_i when the gimbal angles are zero, $\delta_{0i}=0$. In this orientation the initial gimbal angles produce zero momentum bias.

$$R = [\underline{r}_1 \quad \underline{r}_2 \quad \underline{r}_3 \quad \underline{r}_4] \text{ where: } \underline{r}_i = \begin{bmatrix} \cos \gamma_i \\ \sin \gamma_i \\ 0 \end{bmatrix}; R = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.6)$$

We must also define a third 3x4 Quad-matrix Q that contains column vectors of the cross product direction unit vectors \underline{q}_i .

$$Q = [\underline{q}_1 \quad \underline{q}_2 \quad \underline{q}_3 \quad \underline{q}_4]; \underline{q}_i = (\underline{m}_i \times \underline{r}_i);$$

$$Q = \begin{bmatrix} -0.375 & 0 & 0.375 & 0 \\ 0 & -0.375 & 0 & 0.375 \\ 0.927 & 0.927 & 0.927 & 0.927 \end{bmatrix} \quad (2.7)$$

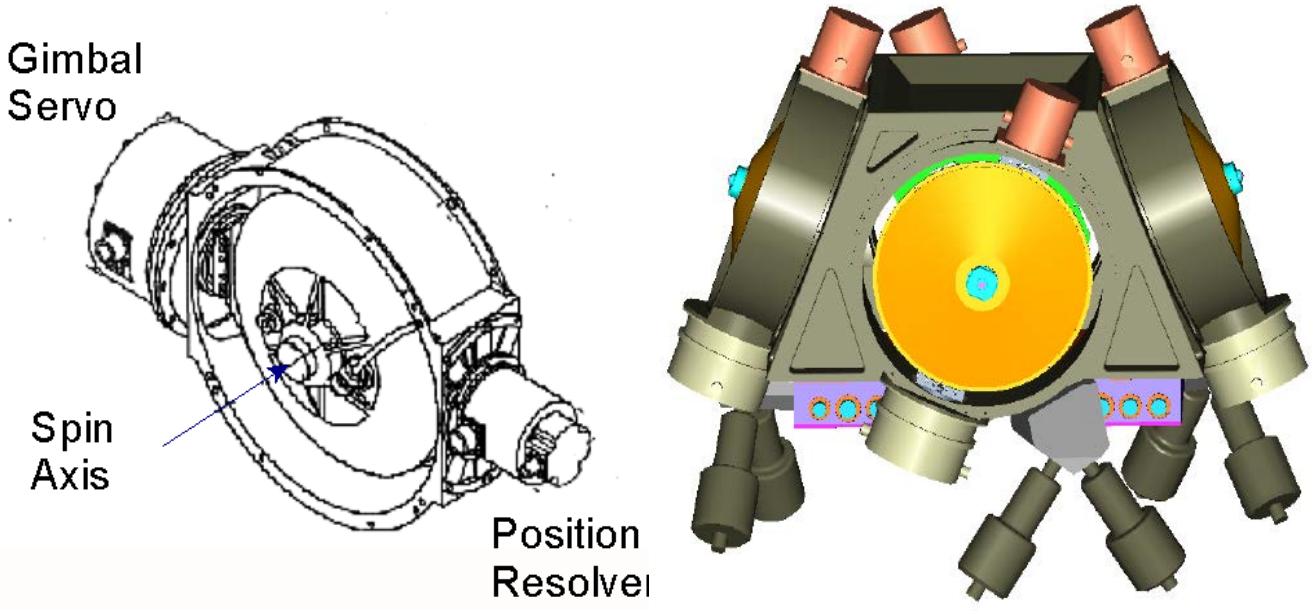


Figure 2.6 A Single-Gimbal Control Moment Gyro and a Cluster of four Single Gimbal CMGs mounted on a pyramid structure which is isolated from the spacecraft by means of disturbance isolation struts

Notice, that the pyramid structure is only used for visualization. The CMGs do not have to be physically mounted on the four surfaces of an actual pyramid, as in Figure 2.4, but they can be translated anywhere on the spacecraft as long as their gimbal axes (\underline{m}_i) and their reference momentum vectors (\underline{r}_i) are parallel to the directions shown in the pyramid. See, for example, the CMG cluster in Figure 2.6. The CMGs are typically mounted on a structure that it is mechanically isolated from the spacecraft by means of vibration isolation struts, as shown in Figure 2.6, that attenuate mechanical vibrations from the CMGs.

2.2 Controlling a Spacecraft with an Array of SGCMG

In order to generate the required torques that will control the spacecraft by using an array of Single Gimbal CMGs it is necessary to develop a steering logic. An optimal steering logic is one that maneuvers the CMG gimbal rates to generate spacecraft torques that are equal to the commanded control torques. However, one of the principal difficulties in using SGCMGs for spacecraft attitude control is the geometric singularity problem in which no control torque is generated for the commanded gimbal rates. The development of a SGCMG steering logic must also consider the avoidance of the singularities. The equations are similar to the reaction wheels, the combined spacecraft plus CMG rate of change of momentum is not affected by the CMG torque but it is affected only by the external torques.

$$\dot{\underline{H}}_{sys} + \underline{\omega} \times \underline{H}_{sys} = \underline{T}_{ext} \quad (2.8)$$

Where:

\underline{H}_{sys} is the combined CMG plus spacecraft system momentum,
 \underline{T}_{ext} is the external torque vector and
 $\underline{\omega}$ is the spacecraft angular rate.

By introducing the internal CMG torque we separate the spacecraft and CMG rate of change of momentum equations and solve for the spacecraft rate $\underline{\omega}$ as a function of internal plus external torques.

$$\begin{aligned} J_v \dot{\underline{\omega}} &= -\underline{\omega} \times J_v \underline{\omega} + \underline{T}_{cmg} + \underline{T}_{ext} \\ \dot{\underline{H}}_{cmg} + \underline{\omega} \times \underline{H}_{cmg} &= -\underline{T}_{cmg} \end{aligned} \quad (2.9)$$

Where: the total system momentum consists of spacecraft plus CMG momentum.

$$\underline{H}_{sys} = J_v \underline{\omega} + \underline{H}_{cmg} \quad (2.10)$$

The internal CMG torque \underline{T}_{cmg} applied to the spacecraft is equal and opposite to the torque applied to the CMG array. It consists of two terms: the control torque \underline{T}_{con} intended to control the spacecraft which is also the rate of change in the CMG momentum plus a component that cancels the gyroscopic torque $\underline{\omega} \times \underline{H}_{cmg}$.

$$\underline{T}_{cmg} = \underline{T}_{con} - \underline{\omega} \times \underline{H}_{cmg} \quad (2.11)$$

The control torque \underline{T}_{con} is a non-linear function of the gimbal angles and gimbal rates. It is the rate of change in GMG Momentum.

$$\underline{T}_{con} = -[A(\delta)] \dot{\underline{\delta}} = -\dot{\underline{H}}_{cmg} \quad (2.12)$$

The angular momentum vector of the entire CMG array in body axes \underline{H}_{cmg} is obtained by combining the individual CMG momentum vectors. Each CMG momentum vector was defined in CMG axes: Gimbal, Output, and Spin axes from equation 2.1, and each must be transformed to spacecraft axes using the following transformation matrix.

$$\underline{H}_{cmg} = \sum_{i=1}^{N_{cmg}} P_i \begin{pmatrix} h_G \\ h_O \\ h_S \end{pmatrix}_i \quad (2.13)$$

Where: matrix P_i transforms the i^{th} CMG momentum from (Gimbal, Output, Spin) axes to spacecraft axes.

$$P_i = \begin{bmatrix} \sin \beta \sin \gamma & -\sin \delta \cos \gamma - \cos \delta \cos \beta \sin \gamma & \cos \delta \cos \gamma - \sin \delta \cos \beta \sin \gamma \\ -\sin \beta \cos \gamma & -\sin \delta \sin \gamma + \cos \delta \cos \beta \cos \gamma & \cos \delta \sin \gamma + \sin \delta \cos \beta \cos \gamma \\ \cos \beta & \cos \delta \sin \beta & \sin \delta \sin \beta \end{bmatrix}$$

A simplified model can be obtained when we ignore the momentum about the CMG gimbal and output axes, because they are small, and consider only the CMG momentum about their spin axes. The combined CMG angular momentum vector is calculated from equation 2.14, as a function of the individual CMG momentums h_{0i} and the gimbal angles which define the spin axis orientations.

$$\underline{H}_{cmg} = \sum_{i=1}^{N_{cmg}} \left(\cos \delta_i \underline{r}_i + \sin \delta_i \underline{q}_i \right) h_{0i} \quad (2.14)$$

Also, by combining equations 2.9 and 2.12 we can rewrite equation 2.9 in terms of only the control torque instead of the total CMG torque \underline{T}_{cmg} as shown in 2.15. T_{con} is the steering torque designed to shape the spacecraft rate as commanded by the Attitude Control System. Equations 2.14 and 2.15 can be used instead of 2.9 in simple 6-dof simulations that do not require gimbal torque dynamics and coupling with structural flexibility.

$$J_{sc} \dot{\underline{\omega}} + \left(\underline{\omega} \times \underline{H}_{sys} \right) = \underline{T}_{con} + \underline{T}_{ext}$$

$$\dot{\underline{H}}_{cmg} = -\underline{T}_{con} \quad (2.15)$$

Otherwise, we calculate the control torque from each CMG in the Gimbal, Output and Spin axes and transform it from CMG axes to spacecraft axes. Then they are combined to form the total CMG torque in body axes.

$$\underline{T}_{cmg} = \sum_{i=1}^{N_{cmg}} P_i \begin{pmatrix} M_G \\ M_O \\ M_S \end{pmatrix}_i \quad (2.16)$$

Where:

$$\begin{bmatrix} M_G \\ M_O \\ M_S \end{bmatrix} = \begin{bmatrix} T_{gi} \\ J_o (\ddot{\phi} \cos \delta - \dot{\phi} \dot{\delta}_i \sin \delta - \dot{\theta} \dot{\delta}_i \cos \delta - \ddot{\theta} \sin \delta) + h_{0i} \dot{\delta}_i + \dot{\delta}_i (J_s - J_g) (\dot{\theta} \cos \delta + \dot{\phi} \sin \delta) \\ J_s (\dot{\Omega} + \ddot{\theta} \cos \delta + \ddot{\phi} \sin \delta + \dot{\phi} \dot{\delta}_i \cos \delta - \dot{\theta} \dot{\delta}_i \sin \delta) + \dot{\delta}_i (J_g - J_o) (\dot{\phi} \cos \delta - \dot{\theta} \sin \delta) \end{bmatrix}$$

For a more accurate calculation of the CMG momentum \underline{H}_{cmg} that includes also the effects due to the gimbal torques we may integrate equation 2.9 using \underline{T}_{cmg} from equation 2.16. The matrix A used in equation 2.12 relates the gimbal rates to rate of change in CMG momentum which is the control torque. It is a time varying $3 \times N_{cmg}$ matrix consisting of N_{cmg} column vectors \underline{a}_i . Its elements \underline{a}_i are related to the Ref and Quad vectors \underline{r}_i and \underline{q}_i of each CMG and they vary with the positions of the gimbal angles δ_i .

$$A(\delta) = (\underline{a}_1 \quad \underline{a}_2 \quad \underline{a}_3 \quad \underline{a}_4) \text{ where :} \quad (2.19)$$

$$\underline{a}_i = (\cos \delta_i \underline{q}_i - \sin \delta_i \underline{r}_i) h_{0i}$$

- δ_i is the gimbal angle for CMG (i)
- \underline{r}_i is a unit vector of the initial momentum direction for CMG (i)
- \underline{m}_i is a unit vector of the gimbal direction for CMG (i)
- \underline{q}_i is the orthogonal direction ($\underline{m}_i \times \underline{r}_i$) for CMG (i)
- T_{con} is the control torque applied to the gimbal by the motor
- T_{gi} is the gyroscopic torque $-\omega \times H_{cmg}$ resolved in gimbal (i) direction
- M_g^b is the (3 x 4) transformation matrix from gimbal axis to body axis
- h_{0i} is the constant momentum of the i^{th} CMG about its spin axis.
- N_{CMG} is the number of CMGs used.

The CMG gimbal rates are controlled by servo systems that generate gimbal torques T_{gi} . The servo torque at the gimbal of each CMG is also counteracting the gyroscopic disturbance torque created by the spacecraft rates $\dot{\theta}$ and $\dot{\phi}$ resolved about the CMG reference and quad axes, as defined in equation 2.3. Ignoring friction, the inertial acceleration of each CMG gimbal is obtained from the gimbal moment equation 2.20, where T_{gi} is the motor torque applied at each gimbal. Even though the CMG moment of inertia about the gimbal J_g is relatively small, the $\omega \times h_0$ gyroscopic moment produced by the CMG momentum coupling with spacecraft rate is a big torque that requires a powerful gimbal servo-motor in order to be able to achieve the required gimbal rate.

$$J_g \ddot{\delta}_{In} + h_{0i} (\dot{\theta}_i \sin \delta - \dot{\phi}_i \cos \delta) = T_{gi} \quad (2.20)$$

In the simple models we can assume that the gimbal rates are equal to the commanded rates $\dot{\delta}_i = \dot{\delta}_{cmd(i)}$. The relative gimbal angle δ for each CMG is obtained by integrating: $\delta = \delta_{In} - \xi$

Single Gimbal CMG Steering

The control system calculates the spacecraft acceleration commands $\dot{\omega}_{cmd}$ and it is the steering control law that generates the gimbal rate commands that drive the servo system which controls the gimbal rates. The spacecraft attitude error is used by the control law to calculate the spacecraft acceleration command. The attitude error is obtained from the quaternion. The quaternion is updated by integrating the quaternion rate which is a function of the body rate and the previous quaternion, as shown in equation 2.21.

$$\underline{\dot{Q}} = 0.5 \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} \underline{Q} \quad (2.21)$$

Equation 2.22 is the steering law which is updated at each iteration. It requires the pseudo-inverse of matrix [A] which is a function of the gimbal angles δ and used for calculating the SGCMG gimbal rate commands. The steering law makes the spacecraft acceleration to be equal to the commanded acceleration.

$$\dot{\delta}_{com} = -A^+(\delta) \left(J_v \dot{\omega}_{com} + (\omega \times H_{sys})_{estim} \right) \text{ where } : A^+ = A^T (A A^T)^{-1} \quad (2.22)$$

$$\dot{\omega} = \dot{\omega}_{com}$$

The development of a CMG steering logic, however, should also consider the avoidance of the singularities. In the examples section we demonstrate the design of a space station attitude control system using CMGs.

2.3 Linearized Equations of Spacecraft with SGCMGs

In Section 2.2 we developed a non-linear model of a spacecraft controlled with an array of SG-CMGs. The following linearized model of a spacecraft with single-gimbal CMGs is applicable for small gimbal variations relative to the nominal gimbal angle positions ($\delta_{01}, \delta_{02}, \delta_{03}, \dots, \delta_{0n}$), and is used mainly for control analysis. We assume that the spacecraft is in circular orbit of orbital rate ω_0 and its angular acceleration is obtained from equation 1.14 as a function of variations in the CMG, gravity-gradient, and external torques.

Equation 2.23 calculates the rate of change in CMG momentum, where: (ω_{x0}, ω_{y0} , and ω_{z0}) are the nominal (steady) body rates, and (ω_x, ω_y , and ω_z) are the variations in vehicle rates. Similarly, (H_{x0}, H_{y0} , and H_{z0}) is the nominal (steady) CMG array momentum, and (h_x, h_y, h_z) is the variation in CMG momentum. \underline{T}_{cmg} is the reaction torque applied to the the CMG array.

$$\begin{pmatrix} \dot{h}_x \\ \dot{h}_y \\ \dot{h}_z \end{pmatrix} = - \begin{pmatrix} \omega_{x0} \\ \omega_{y0} \\ \omega_{z0} \end{pmatrix} \times \begin{pmatrix} h_x \\ h_y \\ h_z \end{pmatrix} - \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} H_{x0} \\ H_{y0} \\ H_{z0} \end{pmatrix} - \underline{T}_{cmg} \quad (2.23)$$

The CMG torque is mainly due to the precession of momentum h_0 coupling with the gimbal rate $\dot{\delta}_i$. It also has components due to the spacecraft rates from equations 2.3 and the gimbal angle δ_0 . It is converted to spacecraft axes by the transformation in equation 2.4. Equation 2.24 combines the contributions from all CMGs to calculate the total torque.

$$\underline{T}_{cmg} = - \sum_{i=1}^{N_{cmg}} P_i \begin{bmatrix} T_{gi} \\ h_{0i} \dot{\delta}_i + (J_s - J_g)(\dot{\theta}_0 \cos \delta_0 + \dot{\phi}_0 \sin \delta_0) \dot{\delta}_i \\ (J_g - J_o)(\dot{\phi}_0 \cos \delta_0 - \dot{\theta}_0 \sin \delta_0) \dot{\delta}_i \end{bmatrix}_i \quad (2.24)$$

The inertial gimbal acceleration in equation 2.25 is a function of the gimbal torque applied by the servo plus gyroscopic terms due to the CMG momentum h_0 coupling with the spacecraft rates. It is obtained by linearizing equation 2.20.

$$J_g \ddot{\delta}_{In} = T_{gi} - h_{0i} (\dot{\theta}_0 \cos \delta_0 \delta + \dot{\phi}_0 \sin \delta_0 \delta + \sin \delta_0 \dot{\theta} - \cos \delta_0 \dot{\phi}) \quad (2.25)$$

The gimbal angle δ for each CMG relative to the spacecraft is obtained by subtracting the spacecraft rate from the inertial gimbal rate and integrating equation 2.26 which calculates the spacecraft rates resolved about the CMG coordinate axes. The gimbal servo system must be capable of providing the necessary torque T_{gi} in order to control the gimbal rate as commanded by the steering logic. The steering logic converts the torque commands from the attitude control system to gimbal rate commands. By accurately controlling the gimbal rates, the CMG array achieves the required control torques

$$\begin{aligned}
\dot{\delta} &= \dot{\delta}_m - \dot{\xi} \\
\dot{\xi} &= \omega_x \sin \beta \sin \gamma - \omega_y \sin \beta \cos \gamma + \omega_z \cos \beta \\
\dot{\theta} &= \omega_x \cos \gamma + \omega_y \sin \gamma \\
\dot{\phi} &= -\omega_x \cos \beta \sin \gamma + \omega_y \cos \beta \cos \gamma + \omega_z \sin \beta
\end{aligned} \tag{2.26}$$

Where:

| | |
|---|---|
| $\dot{\delta}_m$ | is the inertial acceleration of a gimbal |
| δ_0 | is the nominal gimbal angle of a CMG (assumed constant in linear model) |
| δ | is the small variation of a CMG gimbal angle from δ_0 |
| $\dot{\xi}$ | is the variation in spacecraft rate about the gimbal axis \underline{m} |
| $\dot{\theta}, \dot{\phi}$ | are the variations in spacecraft rates about the \underline{r} and \underline{q} axes |
| $\dot{\theta}_0, \dot{\phi}_0$ | are the nominal spacecraft rates about the \underline{r} and \underline{q} axes |
| $\omega_{x0}, \omega_{y0}, \omega_{z0}$ | are the nominal spacecraft body rates about x, y, z |
| $\omega_x, \omega_y, \omega_z$ | are the variation in vehicle rate about the body axis. |
| H_{x0}, H_{y0}, H_{z0} | are the nominal CMG array momentum about x, y, z |
| h_x, h_y, h_z | are the variations in CMG momentum about x, y, z. |
| h_0 | is the CMG constant momentum about its spin axis |

The servo system for each gimbal must provide the necessary torque T_g that will regulate the gimbal rate $\dot{\delta}$ according to the gimbal rate command $\dot{\delta}_{cmd}$ received from the steering logic. The steering logic converts the torque commands from the attitude control system to gimbal rate commands. By accurately controlling the gimbal rates the SGCMG array achieves the demanded control torques.

SGCMG References:

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3. Singularity Robust Steering Logic for Redundant Single-Gimbal Control Moment Gyros, Bong Wie, Chris Heiberg, Dave Bailey, AIAA GN&C Conference, August 2000, AIAA-2000-4453
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8. Bong Wie, *Space Vehicle Dynamics and Control*, AIAA, 2nd edition, 2010.

3.3 Spacecraft with Generic Momentum Control System

Let us consider a spacecraft that is controlled with a centralized Momentum Control System using a cluster of momentum exchange devices, such as double-gimbal CMGs. A simplified model is to bypass the steering logic that converts torque commands to gimbal rates and to assume that the control torque is equal to the commanded torque. Unlike our previous CMG and RW models, this time we will ignore the mounting geometry details of the individual CMG devices and analyze them as a combined 3-axis momentum exchange system. This is a simplified approach that can be applied to any 3-axis momentum exchange system, such as, in a double gimbal CMG array, where there is no direct transfer of gyroscopic torques between the rotor and the spacecraft through the bearings. In a double gimbal CMG array there is no direct transfer of gyroscopic torques between the rotor and the spacecraft through the bearings. The torque between the rotor and the spacecraft is transmitted through the servo actuators which are assumed to be ideal (control torque is equal to the commanded torque).

The simulation models bypass the steering logic that converts torque commands to gimbal rates. Equations 3.1 calculate the rate of change of angular momentum for the spacecraft and also for the CMG cluster. It does not include the CMG steering equations. The first equation calculates the rate of change of spacecraft momentum as a function of the applied CMG and external torques.

$$\begin{aligned} J_{sc} \dot{\omega} &= -(\omega \times J_{sc} \omega) + T_{ext} + T_c \\ \dot{H}_{cmg} &= -(\omega \times H_{cmg}) - T_c \end{aligned} \quad (3.1)$$

The second equation calculates the combined CMG momentum in body axes H_{cmg} . It states that the rate of change of momentum in the CMG cluster is equal to the reaction control torque generated by the CMG cluster that is applied by the gimbals $-T_c$. It consists of the combined torque of the individual gimbals resolved in body axes regardless of how many CMGs are in the cluster. If you add up these two 3.1 equations together the gimbal torque T_c disappears, and you end up with the conservation of system momentum equation.

For an orbiting satellite with orbital rate ω_0 , the Euler angles relative to the rotating LVLH frame are obtained from equation 3.2.

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \frac{1}{\cos \psi} \begin{bmatrix} \cos \psi & -\cos \phi \sin \psi & \sin \phi \sin \psi \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi \cos \psi & \cos \phi \cos \psi \end{bmatrix} \omega + \begin{pmatrix} 0 \\ \omega_o \\ 0 \end{pmatrix} \quad (3.2)$$

For more details in using this momentum exchange spacecraft model the user is referred to study our Space Station example.

3.1 Linearized Equations of a Spacecraft with DG CMGs in LVLH Orbit

In this simple linearized model we assume that the CMGs are a collocated cluster and not individual units. The internal CMG steering dynamics and steering logic are not included in the equations. The CMG cluster generates control torques in the body axes and the CMG momentum is calculated in the spacecraft axes. The spacecraft x-axis is along the velocity vector, the z-axis is pointing towards the earth, and its attitude is measured relative to the Local-Vertical-Local-Horizontal frame. For small attitude deviations from the average LVLH attitude the rate of change of spacecraft angular momentum is given in equation 3.3.

$$\begin{aligned}
 & \begin{bmatrix} I_{XX} & I_{XY} & I_{XZ} \\ I_{XY} & I_{YY} & I_{YZ} \\ I_{XZ} & I_{YZ} & I_{ZZ} \end{bmatrix} \begin{pmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{pmatrix} = \omega_o \begin{bmatrix} I_{XZ} & 2I_{YZ} & I_{ZZ} - I_{YY} \\ -I_{YZ} & 0 & I_{XY} \\ I_{YY} - I_{XX} & -2I_{XY} & -I_{XZ} \end{bmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \\
 & + 3\omega_o^2 \begin{bmatrix} I_{ZZ} - I_{YY} & I_{XY} & 0 \\ I_{XY} & I_{ZZ} - I_{XX} & 0 \\ -I_{XZ} & -I_{YZ} & 0 \end{bmatrix} \begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} + \omega_o^2 \begin{bmatrix} -2I_{YZ} \\ 3I_{XZ} \\ -I_{XY} \end{bmatrix} + T_C + T_D
 \end{aligned} \tag{3.3}$$

Where: the second term on the right side is the gravity gradient torque. The products of inertia cause additional bias torques. The attitude kinematics become

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{bmatrix} \omega_o \psi + \omega_x \\ \omega_o + \omega_y \\ -\omega_o \phi + \omega_z \end{bmatrix} \tag{3.4}$$

The change in CMG momentum in body axes is calculated by integrating the following equation

$$\dot{\underline{h}}_{cmg} = -(\underline{\Omega}_0 \times \underline{h}_{cmg}) - (\underline{\omega} \times \underline{H}_0) - \underline{T}_C \tag{3.5}$$

Where: $\underline{\Omega}_0$ is the spacecraft average, steady-state rate. The second term is due to coupling of the spacecraft rate with the nominal CMG momentum \underline{H}_0 . If we assume that the steady-state spacecraft rate is in pitch and equal to $(-\omega_0)$, which is the negative orbital rate, the variation in CMG momentum equations become.

$$\begin{aligned}
 \dot{h}_x &= \omega_o h_z + \omega_z H_{Y0} - \omega_y H_{Z0} - T_{CX} \\
 \dot{h}_y &= -\omega_z H_{X0} + \omega_x H_{Z0} - T_{CY} \\
 \dot{h}_z &= -\omega_o h_x + \omega_y H_{X0} - \omega_x H_{Y0} - T_{CZ}
 \end{aligned} \tag{3.6}$$

Where: T_{CX} , T_{CY} , T_{CZ} are the roll, pitch, and yaw control torques.

3.4 Spacecraft with Gimbaling Appendages

The following equations describe the motion of a spacecraft that has N_{gimb} gimbaling appendages attached to the main spacecraft body that may be rigid or flexible. The motion of the attached bodies is dynamically coupling with the spacecraft bus by the reaction torques generated when the gimbals are pivoting. There are two approaches to model this type of spacecraft configuration.

Using H-parameters

The first and easier approach is to use the H-parameters matrix which couples the gimbal motion with flexibility. In this case the vehicle mass-properties include the appendage bodies and the structural modes are calculated with the gimbals locked. They are released in the equations of motion by the introduction of the inertial coupling coefficients matrix H_p . Equation 4.1 calculates the gimbal acceleration relative to the spacecraft as a function of the gimbal torque T_{gi} and the interaction with the flex modes. H_p is the H-parameters matrix that is calculated from the mass-matrix of the finite elements model.

$$I_\alpha \ddot{\underline{\alpha}} + H_p^T \ddot{\underline{\eta}} = \underline{T}_{gi} \quad (4.1)$$

Equation 4.2 describes how the flex mode generalized displacements (η) are excited by the external forces and torques and also by the gimbal accelerations via the H-parameters.

$$M_G (\ddot{\underline{\eta}} + 2\zeta \Omega \dot{\underline{\eta}} + \Omega^2 \underline{\eta}) + H_p \ddot{\underline{\alpha}} = \Phi^T \begin{bmatrix} F_{ext} \\ T_{ext} \end{bmatrix} \quad (4.2)$$

Using Reaction Forces and Torques

The second approach of describing the motion of a spacecraft with gimbaling appendages is to assume that the appendages are rigid and they are attached with hinges to the main spacecraft body that may be flexible. The pivoting of the rigid appendages excites both rigid and flex spacecraft motion. In this case the structural modes of the spacecraft core body are generated without the appendages, and its mass properties should not include the appendage weights and inertias because their effects are introduced by the reaction forces and torques. The motion of the attached bodies is coupling with the spacecraft bus by the action/ reaction forces and torques generated when they are pivoting.

Equation 4.3 calculates the inertial rotational acceleration of an appendage about its hinge generated by the hinge torque T_{gi} . Where: I_α is the moment of inertia of the appendage body about its pivot.

$$I_\alpha \ddot{\underline{\alpha}} = \underline{T}_{gi} \quad (4.3)$$

The rotational rate of a gimbaling body relative to the spacecraft $\dot{\underline{\alpha}}_r$, is obtained from equation 4.4, by subtracting the spacecraft rate resolved about the hinge vector (\underline{h}_i), from the inertial gimbal rate $\dot{\underline{\alpha}}_i$

$$\dot{\underline{\alpha}}_r = \dot{\underline{\alpha}}_i - (\underline{\omega}_b \bullet \underline{h}_i) \quad (4.4)$$

Equation 4.5 calculates the reaction force on the spacecraft at the i^{th} gimbal generated due to the inertial rotational acceleration $\ddot{\alpha}_i$ of the appendage body about the gimbal and also the spacecraft acceleration resolved in the hinge direction. Where: \underline{l}_i is the distance vector from the i^{th} pivot to the appendage CG, and \underline{d}_i is the distance vector between the spacecraft CG and pivot (i). F_{sc} is the combined force from all gimbals motion that drives the rigid body dynamics.

$$F_{gi} = m_i [(\underline{l}_i \times \underline{h}_i) \ddot{\alpha}_i - (\underline{d}_i \times \underline{h}_i) (\dot{\omega} \bullet \underline{h}_i)]$$

$$F_{sc} = \sum_{i=1}^{N_{gimb}} F_{gi} \quad (4.5)$$

Equation 4.6 calculates the combined torque on the spacecraft due to the individual gimbal torques T_{gi} of N_{gimb} slewing appendages, where \underline{h}_i is the direction of the i^{th} hinge vector. It is used to drive the rigid body dynamics

$$\underline{T}_{sc} = - \sum_{i=1}^{N_{gimb}} (T_{gi} \underline{h}_i) \quad (4.6)$$

Equation 4.7 is the torque on the spacecraft generated by the gimbal reaction forces combined.

$$\underline{T}_{gr} = \sum_{i=1}^{N_{gimb}} (\underline{d}_i \times \underline{F}_{gi}) \quad (4.7)$$

Equation 4.8 is the structural flexibility equation. The generalized flex mode displacements ($\underline{\eta}$) are excited by external forces and torques (F and T) which include external forces and torques in addition to the forces and torques generated at the gimbals.

$$M_G (\ddot{\underline{\eta}} + 2\zeta \Omega \dot{\underline{\eta}} + \Omega^2 \underline{\eta}) = \Phi^T \begin{bmatrix} F \\ T \end{bmatrix} \quad (4.8)$$

Where:

- \underline{l}_i is the moment arm between the pivot (i) and the appendage center of mass
- \underline{d}_i is the moment arm between the spacecraft cg and the appendage pivot (i)
- $\underline{\eta}$ is the Generalized Displacement vector for N modes
- M_g is a diagonal matrix consisting of the generalized masses of N-modes
- Ω is a diagonal matrix of N-mode frequencies in (rad/sec)
- F, T are external forces and torques that excite the bending equations
- H_p is the H-parameters (Nmod x Ngimb) matrix for the gimbaling bodies
- Φ is the mode shapes matrix at the excitation points
- \underline{h}_i is the direction unit vector for hinge (i)

3.5 Generating Spacecraft Models with Flixan

In this section we will demonstrate how to generate linear spacecraft systems using the Flixan Flight Vehicle Modeling program. We begin with a Space Station model that is controlled by a centralized momentum control system. Then we will analyze a spacecraft that is controlled with an array of 4 Single Gimbal CMGs. Our third example is a spacecraft that is controlled by an array of 3 Reaction Wheels and it includes 4 gimbaling appendages.

5.1 Flexible Space Station Model with a Centralized Momentum Control System

The following “Flight Vehicle” dataset was created for a Space Station model that is controlled by a 3-axis momentum control system such as a cluster of double-gimbal CMGs. In this case we will not model the detailed CMG steering but use the simple generic model described in Section 3. The Space Station Example is in folder: “C:\Flixan\Examples\Large Space Station” and the input file that contains the spacecraft data is in file “SpaceStation.Inp”. This file contains two spacecraft datasets: a rigid and a flexible model, to be processed by the flight vehicle modeling program.

The title of the flexible vehicle set is “Space Station with RCS and a Double-Gimbal CMG Array” and it includes the following components: 8 force inputs representing the reaction control thrusters, 3 external torques for roll, pitch, and yaw, a centralized momentum control system, 3 gyros, 3 rate-gyros, and 4 accelerometers. The input file also includes a dataset of 34 selected flex modes to be combined with the spacecraft model and other datasets that convert systems to Matlab format. The title of the modal dataset is also included at the bottom of the spacecraft dataset. The Space Station dataset is shown below.

```
FLIGHT VEHICLE INPUT DATA .....
Space Station with RCS and a Double-Gimbal CMG Array
!
! The Space Station state-space model is now created using the vehicle modeling program.
! The model uses 8 RCS jets, an array of double-gimbal control moment gyros, 3 rate
! gyros, 3 attitude sensors, and 4 accelerometers. The Station is initialized at the
! Local Vertical Local Horizontal (LVLH) attitude and it has a negative pitch rate
! -0.063 radians/sec which is equal to the orbital rate. The vehicle rates are with
! respect to the LVLH frame. A constant bias torque 7.40153 (ft-lb) is applied
! in the direction (-0.1969, 0.5963, 0.7781) to represent the steady-state gyroscopic
! and gravity-gradient torques due to the constant pitch rate
Body Axes Output, LVLH Attitude

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft) : 6200.0      32.174      0.20896E+08
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft) : 0.119416e+9  0.40408e+8  0.110166e+9, 0.476e+7, 0.1216e+7,
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet) : 0.0      0.0      0.0
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet) : 0.0      25500.0    0.0001      700000.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2) : 0.0      0.0      0.0      0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec) : 0.0      0.0      0.0      0.0
Vehicle Attitude Euler Angles, Phi_o,Theta_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec) : 0.0000    0.000    0.0000    0.0000    -0.063
W-Gust Azim & Elev angles (deg), or Torque/Force direction (x,y,z), Force Locat (x,y,z) : Torque    0.196972   -0.596376   -0.778163
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet) : 0.0      1.0      1.0
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} : 0.0      0.0      0.0      -0.0
Aero Force Coeff/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/FV,PCa/Ph,Ca_alfdot,Ca_g,Ca_bet} : 0.0      0.0      0.0      0.0      0.0      0.0
Aero Force Coeff/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V} : 0.0      -0.0      0.0000    0.0000    0.0000    0.0000
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_g,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV} : 0.0      -0.0      0.0000    0.0000    0.0000    0.0000
Aero Moment Coeff/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa} : 0.0      -0.0      0.0000    0.0000    0.0000    0.0000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/FV,PCm/Ph} : 0.0      -0.0      0.0000    0.0000    0.0000    0.0000
Aero Moment Coeff/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa} : 0.0      0.0      0.0000    0.0000    0.0000    0.0000
```


Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ? : 8

| | | | | |
|--|-----------------------------|------------|------------|---------|
| RCS Jet No: 1 | Upper Right, +X (20 lb jet) | : UR+X Jet | Throttling | Node 25 |
| Engine Nominal Thrust, and Maximum Thrust in (lb) | (for throttling) | : 0.0 | 20.0 | |
| Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): | | 0.0 | 0.0 | 0.0 |
| Eng Mass (slug), Inertia about Gimbal (lb-sec ² -ft), Moment Arm, engine CG to gimbal (ft): | | 0.0 | 0.0 | 0.0 |
| Thruster location with respect to the Vehicle Reference Axes, Xjet, Yjet, Zjet, (ft) | | : 0.0, | 73.7, | -185.0 |
| RCS Jet No: 2 | Upper Right, -Y (20 lb jet) | : UR-Y Jet | Throttling | Node 25 |
| Engine Nominal Thrust, and Maximum Thrust in (lb) | (for throttling) | : 0.0 | 20.0 | |
| Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): | | 0.0 | -90.0 | 0.0 |
| Eng Mass (slug), Inertia about Gimbal (lb-sec ² -ft), Moment Arm, engine CG to gimbal (ft): | | 0.0 | 0.0 | 0.0 |
| Thruster location with respect to the Vehicle Reference Axes, Xjet, Yjet, Zjet, (ft) | | : 0.0, | 73.7, | -185.0 |
| RCS Jet No: 3 | Upper Left, +X (20 lb jet) | : UL+X Jet | Throttling | Node 26 |
| Engine Nominal Thrust, and Maximum Thrust in (lb) | (for throttling) | : 0.0 | 20.0 | |
| Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): | | 0.0 | 0.0 | 0.0 |
| Eng Mass (slug), Inertia about Gimbal (lb-sec ² -ft), Moment Arm, engine CG to gimbal (ft): | | 0.0 | 0.0 | 0.0 |
| Thruster location with respect to the Vehicle Reference Axes, Xjet, Yjet, Zjet, (ft) | | : 0.0, | -65.6, | -185.0 |
| RCS Jet No: 4 | Upper Left, +Y (20 lb jet) | : UL+Y Jet | Throttling | Node 26 |
| Engine Nominal Thrust, and Maximum Thrust in (lb) | (for throttling) | : 0.0 | 20.0 | |
| Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): | | 0.0 | +90.0 | 0.0 |
| Eng Mass (slug), Inertia about Gimbal (lb-sec ² -ft), Moment Arm, engine CG to gimbal (ft): | | 0.0 | 0.0 | 0.0 |
| Thruster location with respect to the Vehicle Reference Axes, Xjet, Yjet, Zjet, (ft) | | : 0.0, | -65.6, | -185.0 |

Number of External Torques on the Vehicle : 3

| | | | | |
|-------------|---------------------|-------|-----|-----|
| Torque No 1 | Direction (x, y, z) | : 1.0 | 0.0 | 0.0 |
| Torque No 2 | Direction (x, y, z) | : 0.0 | 1.0 | 0.0 |
| Torque No 3 | Direction (x, y, z) | : 0.0 | 0.0 | 1.0 |

Double Gimbal Control Moment Gyro System (3-axes), Initial Momentum (x,y,z) (ft-lb-sec) : Yes 0.0 0.0 0.0

Number of Gyros, (Attitude and Rate) : 6

| | | | | | | |
|-----------|--|---------|----------|-----|------|-----|
| Gyro No 1 | Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 2 | : Roll | Rate | 0.0 | 82.0 | 0.0 |
| Gyro No 2 | Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 2 | : Pitch | Rate | 0.0 | 82.0 | 0.0 |
| Gyro No 3 | Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 2 | : Yaw | Rate | 0.0 | 82.0 | 0.0 |
| Gyro No 1 | Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 2 | : Roll | Attitude | 0.0 | 82.0 | 0.0 |
| Gyro No 2 | Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 2 | : Pitch | Attitude | 0.0 | 82.0 | 0.0 |
| Gyro No 3 | Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 2 | : Yaw | Attitude | 0.0 | 82.0 | 0.0 |

Number of Accelerometers, Along Axes: (x,y,z) : 4

| | | | | | | |
|------------------|--|----------|------------|-----|---------|------|
| Acceleromet No 1 | Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2 | : X-axis | Accelerat. | 0.0 | 232.35 | 18.0 |
| Acceleromet No 2 | Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2 | : Z-axis | Accelerat. | 0.0 | 232.35 | 18.0 |
| Acceleromet No 3 | Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4 | : X-axis | Accelerat. | 0.0 | -224.63 | 16.0 |
| Acceleromet No 4 | Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4 | : Z-axis | Accelerat. | 0.0 | -224.63 | 16.0 |

Number of Bending Modes : 34

Space Station with RCS and a Double-Gimbal CMG Array, 34 Modes

The flight vehicle modeling program will process the above spacecraft dataset, including the following dataset which contains the selected space-station flex modes, and is referenced by its title. It is located in the same input file and includes frequencies (rad/sec), damping coefficients (ζ), modal mass in (slugs), and mode shapes and slopes at specific locations. Only the first of the 34 modes is shown. The locations correspond to the locations specified in the spacecraft dataset above.

To process this dataset using the flight vehicle modeling program start the Flixan program, select the project directory, then go to “*Program Functions*”, “*Flight Vehicle/Spacecraft Modeling Tools*”, and then “*Flight Vehicle State-Space*”. From the following menu select the input and system filenames and click on “*Process Files*”.

SELECTED MODAL DATA AND LOCATIONS FOR : 34 Modes
 Space Station with RCS and a Double-Gimbal CMG Array, 34 Modes
 ! The same 34 modes were selected as in the previous examples from all directions

| MODE# | 1/ 7, Frequency (rad/sec), Damping (zeta), Generalized Mass= | 1.0097 | 0.50000E-02 | 12.000 | | | |
|--|--|--------------|--------------|--------------|--------------|--------------|--------------|
| DEFINITION OF LOCATIONS (NODES) | phi along X | phi along Y | phi along Z | sigm about X | sigm about Y | sigm about Z | |
| Modal Data at the 8 Engines, (x,y,z)... | | | | | | | |
| Top Right RCS Thruster Assembl | 1050 | 0.59628D-02 | -0.16325D-04 | 0.96510D-04 | 0.42971D-06 | -0.33256D-04 | 0.10894D-04 |
| Top Right RCS Thruster Assembl | 1050 | 0.59628D-02 | -0.16325D-04 | 0.96510D-04 | 0.42971D-06 | -0.33256D-04 | 0.10894D-04 |
| Top Left RCS Thrusters Assembl | 2050 | 0.54261D-02 | -0.62395D-05 | 0.74506D-04 | -0.10905D-06 | -0.31918D-04 | -0.18344D-04 |
| Top Left RCS Thrusters Assembl | 2050 | 0.54261D-02 | -0.62395D-05 | 0.74506D-04 | -0.10905D-06 | -0.31918D-04 | -0.18344D-04 |
| Bottom Right RCS Thruster Asse | 1150 | -0.35170D-02 | -0.59190D-04 | 0.96566D-04 | 0.13160D-06 | -0.31650D-04 | 0.15942D-04 |
| Bottom Right RCS Thruster Asse | 1150 | -0.35170D-02 | -0.59190D-04 | 0.96566D-04 | 0.13160D-06 | -0.31650D-04 | 0.15942D-04 |
| Bottom Left RCS Thrusters Asse | 2150 | -0.32580D-02 | -0.58408D-04 | 0.74465D-04 | 0.21757D-06 | -0.30306D-04 | -0.12670D-04 |
| Bottom Left RCS Thrusters Asse | 2150 | -0.32580D-02 | -0.58408D-04 | 0.74465D-04 | 0.21757D-06 | -0.30306D-04 | -0.12670D-04 |
| Modal Data at the Double Gimbal GMGs Cluster... | | | | | | | |
| Right Keel/Boom Intrsect, CMG | 1000 | 0.11384D-02 | -0.58736D-04 | 0.96470D-04 | -0.23552D-06 | -0.30827D-04 | 0.65746D-04 |
| Modal Data at the 3 External Torque Points... | | | | | | | |
| Center Habitat Module | 9007 | 0.29012D-02 | -0.26348D-04 | 0.31307D-04 | 0.30160D-06 | -0.43846D-05 | -0.97127D-07 |
| Center Habitat Module | 9007 | 0.29012D-02 | -0.26348D-04 | 0.31307D-04 | 0.30160D-06 | -0.43846D-05 | -0.97127D-07 |
| Center Habitat Module | 9007 | 0.29012D-02 | -0.26348D-04 | 0.31307D-04 | 0.30160D-06 | -0.43846D-05 | -0.97127D-07 |
| Modal Data at the 6 Gyros ... | | | | | | | |
| Right Boom (Sensor Assembly) | 6001 | -0.14682D-03 | -0.65626D-04 | 0.90529D-04 | -0.37958D-06 | -0.33769D-04 | 0.86246D-04 |
| Right Boom (Sensor Assembly) | 6001 | -0.14682D-03 | -0.65626D-04 | 0.90529D-04 | -0.37958D-06 | -0.33769D-04 | 0.86246D-04 |
| Right Boom (Sensor Assembly) | 6001 | -0.14682D-03 | -0.65626D-04 | 0.90529D-04 | -0.37958D-06 | -0.33769D-04 | 0.86246D-04 |
| Right Boom (Sensor Assembly) | 6001 | -0.14682D-03 | -0.65626D-04 | 0.90529D-04 | -0.37958D-06 | -0.33769D-04 | 0.86246D-04 |
| Right Boom (Sensor Assembly) | 6001 | -0.14682D-03 | -0.65626D-04 | 0.90529D-04 | -0.37958D-06 | -0.33769D-04 | 0.86246D-04 |
| Right Boom (Sensor Assembly) | 6001 | -0.14682D-03 | -0.65626D-04 | 0.90529D-04 | -0.37958D-06 | -0.33769D-04 | 0.86246D-04 |
| Modal Data at the 4 Accelerometers, along (x,y,z)... | | | | | | | |
| Right Solar Array Boom | 6070 | -0.25921D-01 | -0.73995D-04 | -0.57584D-04 | | | |
| Right Solar Array Boom | 6070 | -0.25921D-01 | -0.73995D-04 | -0.57584D-04 | | | |
| Left SA Boom, Extreme end | 5070 | -0.26691D-01 | 0.68332D-05 | -0.15103D-03 | | | |
| Left SA Boom, Extreme end | 5070 | -0.26691D-01 | 0.68332D-05 | -0.15103D-03 | | | |
| Modal Data at the Disturbance Point | | | | | | | |
| Module, (Front Dock) | 9018 | 0.28820D-02 | -0.73445D-04 | 0.35876D-03 | 0.49346D-06 | -0.54146D-05 | -0.28216D-06 |

Flixan, Flight Vehicle Modeling & Control System Analysis

Utilities | File Management | **Program Functions** | View Quad | Help Files

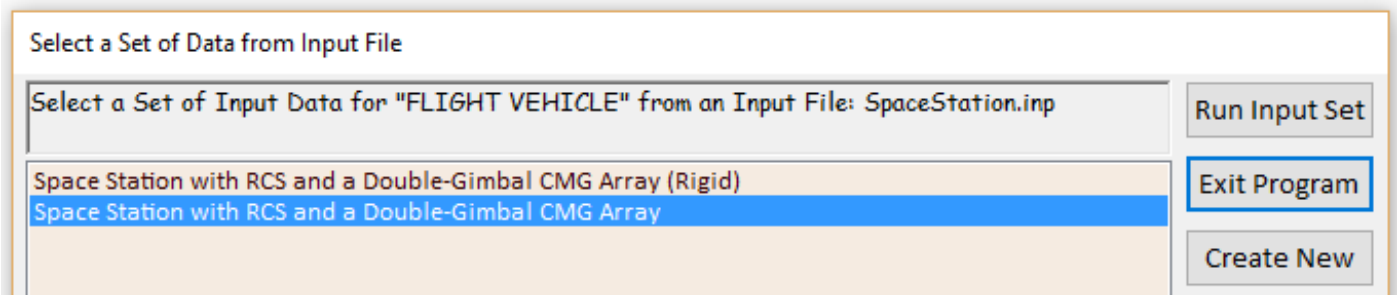
- Flight Vehicle/Spacecraft Modeling Tools > Flight Vehicle, State-Space
- Frequency Control Analysis > Actuator State-Space Models
- Robust Control Synthesis Tools > Flex Spacecraft (Modal Data)
- Creating and Modifying Linear Systems > Create Mixing Logic/ TVC
- Trim/ Static Perform Analysis
- Flex Mode Selection

Select Input and System Filenames

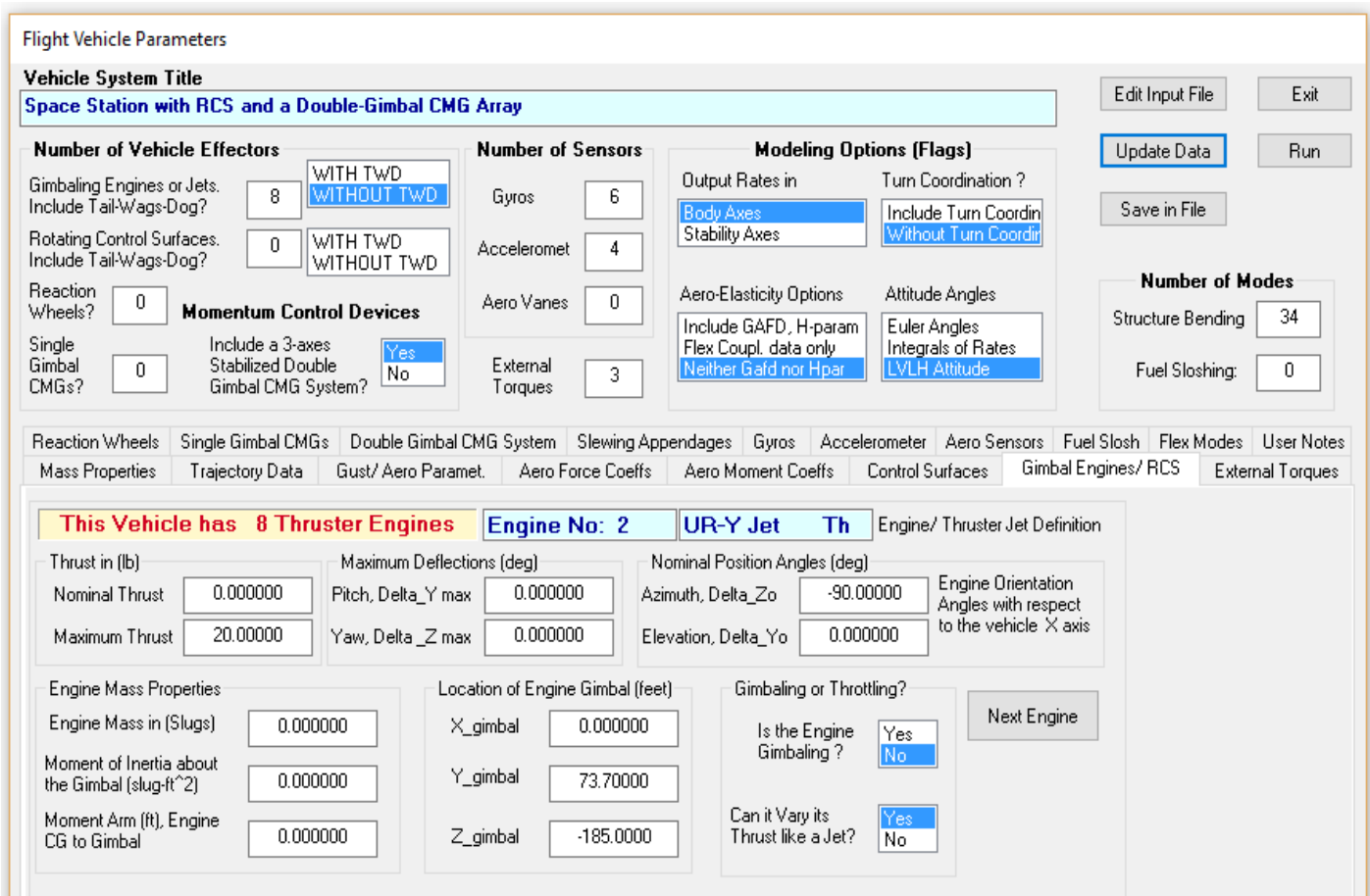
| | |
|--|---|
| Select a File Name containing the Input Data Set (x.Inp) | Select a File Name containing the State Systems (x.Qdr) |
| SpaceStation.Inp | SpaceStation.Qdr |
| SpaceStation.Inp | SpaceStation.Qdr |
| NewFile.Inp | NewFile.Qdr |

Create New Input Set | Exit Program | Process Files

The following menu shows the “Flight Vehicle” datasets which are in file “*SpaceStation.Inp*”. Select the second dataset that includes flexibility and click on “*Run Input Set*” to process it.



The flight vehicle modeling program reads the data and presents the following dialog that shows the spacecraft parameters in tabs, prior to processing it. Click on “*Run*” to process the dataset and the program will create the spacecraft state-space system in file “*SpaceStation.Qdr*”.



Flight Vehicle Parameters

Vehicle System Title
Space Station with RCS and a Double-Gimbal CMG Array

Number of Vehicle Effectors
 Gimballing Engines or Jets. Include Tail-Wags-Dog? 8 WITH TWD WITHOUT TWD
 Rotating Control Surfaces. Include Tail-Wags-Dog? 0 WITH TWD WITHOUT TWD
 Reaction Wheels? 0

Momentum Control Devices
 Single Gimbal CMGs? 0
 Include a 3-axes Stabilized Double Gimbal CMG System? Yes No

Number of Sensors
 Gyros 6
 Acceleromet 4
 Aero Vanes 0
 External Torques 3

Modeling Options (Flags)
 Output Rates in: Body Axes Stability Axes
 Turn Coordination?: Include Turn Coordin Without Turn Coordin
 Aero-Elasticity Options: Include GAFD, H-param Flex Coupl. data only Neither GaFd nor Hpar
 Attitude Angles: Euler Angles Integrals of Rates LVLH Attitude

Number of Modes
 Structure Bending 34
 Fuel Sloshing: 0

Mass Properties | Trajectory Data | Gust/ Aero Paramet. | Aero Force Coeffs | Aero Moment Coeffs | Control Surfaces | Gimbal Engines/ RCS | External Torques
 Reaction Wheels | Single Gimbal CMGs | Double Gimbal CMG System | Slewing Appendages | Gyros | Accelerometer | Aero Sensors | Fuel Slosh | Flex Modes | User Notes

This Vehicle Uses A Double Gimbal CMG Array

Initial CMG Momentum Vector
 Hx 0.000000
 Hy 0.000000
 Hz 0.000000

The table below is included in the output and it lists the Space Station system 15 inputs and 24 outputs. The system has 8 RCS jet force inputs providing forces in the directions defined in the input data. Actually, the inputs are defined as throttles rather than forces, which are the ratios of “force divided by max force”. There are 3 control torques in roll, pitch and yaw, provided by the centralized CMG cluster. We have also included 3 external torques for disturbance analysis. The last input is an additional disturbance torque in the direction defined in the input data.

The system outputs are the standard flight vehicle model outputs. Some of them are not useful for this application and they can be removed from the model. The set includes 3 gyros, 3 rate gyros, and 4 accelerometers at the locations defined in the input data. The last 3 outputs are the CMG momentum in spacecraft roll, pitch, and yaw axes. Actually they are variations in the CMG momentum from nominal momentum because this model is linear.

Definition of System Variables

Inputs = 15

| | | | | |
|----|--------------------------|---------------------------|----------------|-----|
| 1 | Throttle Input | dTh/Th for Engine No | 1 | (-) |
| 2 | Throttle Input | dTh/Th for Engine No | 2 | (-) |
| 3 | Throttle Input | dTh/Th for Engine No | 3 | (-) |
| 4 | Throttle Input | dTh/Th for Engine No | 4 | (-) |
| 5 | Throttle Input | dTh/Th for Engine No | 5 | (-) |
| 6 | Throttle Input | dTh/Th for Engine No | 6 | (-) |
| 7 | Throttle Input | dTh/Th for Engine No | 7 | (-) |
| 8 | Throttle Input | dTh/Th for Engine No | 8 | (-) |
| 9 | Double Gimbal CMG Torque | in the X-axis | (ft-lb) | |
| 10 | Double Gimbal CMG Torque | in the Y-axis | (ft-lb) | |
| 11 | Double Gimbal CMG Torque | in the Z-axis | (ft-lb) | |
| 12 | Ext Torque Input 1 | (ft-lb), dir: | 1.00 0.00 0.00 | |
| 13 | Ext Torque Input 2 | (ft-lb), dir: | 0.00 1.00 0.00 | |
| 14 | Ext Torque Input 3 | (ft-lb), dir: | 0.00 0.00 1.00 | |
| 15 | Ext Torque Direct.= | (0.1970 -0.5964 -0.7782) | | |

Outputs = 24

| | | | |
|----|-----------------------------------|------------------------|----------------------|
| 1 | Roll Attitude | (phi-LVLH) | (radians) |
| 2 | Roll Rate | (p-body) | (rad/sec) |
| 3 | Pitch Attitude | (thet-LVLH) | (radians) |
| 4 | Pitch Rate | (q-body) | (rad/sec) |
| 5 | Yaw Attitude | (psi-LVLH) | (radians) |
| 6 | Yaw Rate | (r-body) | (rad/sec) |
| 7 | Angle of attack, alfa, | (radians) | |
| 8 | Angle of sideslip, beta, | (radian) | |
| 9 | Change in Altitude, delta-h, | (feet) | |
| 10 | Forward Acceleration (V-dot) | (ft/sec) | |
| 11 | Cross Range Velocity (Vcr) | (ft/sec) | |
| 12 | Rate-Gyro # 1, Roll Rate | (Body) | (rad/sec) |
| 13 | Rate-Gyro # 2, Pitch Rate | (Body) | (rad/sec) |
| 14 | Rate-Gyro # 3, Yaw Rate | (Body) | (rad/sec) |
| 15 | Gyro # 4, Roll Attitude | (Body) | (radians) |
| 16 | Gyro # 5, Pitch Attitude | (Body) | (radians) |
| 17 | Gyro # 6, Yaw Attitude | (Body) | (radians) |
| 18 | Accelerom # 1, (along X), | (ft/sec ²) | Translat. Accelerat. |
| 19 | Accelerom # 2, (along Z), | (ft/sec ²) | Translat. Accelerat. |
| 20 | Accelerom # 3, (along X), | (ft/sec ²) | Translat. Accelerat. |
| 21 | Accelerom # 4, (along Z), | (ft/sec ²) | Translat. Accelerat. |
| 22 | Double Gimbal CMG Momentum Change | in X-axis | (ft-lb-sec) |
| 23 | Double Gimbal CMG Momentum Change | in Y-axis | (ft-lb-sec) |
| 24 | Double Gimbal CMG Momentum Change | in Z-axis | (ft-lb-sec) |

5.2 Agile Spacecraft with Single-Gimbal CMGs

In our next example we will create a flexible spacecraft that is controlled with an array of 4 SGCMGs, and it implemented as described by the model in Section 2.3. The detailed analysis of this example is in folder “C:\Flixan\Examples\Flex Agile Spacecraft with SGCMG & RCS\CMG Control(g) 4SGCMG Robust_Anal”, and the input file that contains the spacecraft data is in file “FlexSc_4CMG.Inp”. This file contains two almost identical spacecraft datasets. They both include flexibility and the second model includes uncertainties and it is used for analyzing robustness. The file also includes a dataset of 40 selected structural modes to be combined with the flex models. The title of the first vehicle dataset is “Flexible Agile Spacecraft with 4 SG-CMG” and it includes: 4 SG-CMGs, 9 gyros and 6 accelerometer sensors. The data of each SGCMG include: the constant momentum of 1200 (ft-lb-sec), the direction of the gimbal unit vector \underline{m} , the momentum reference direction unit vector \underline{r} (at zero gimbal angle), the pyramid γ and β angles, and the CMG moments of inertia about the CMG axes. At the bottom of the set, it includes the title of the modal data set, and it will use 40 of those modes. The dataset for this configuration is shown below.

```

FLIGHT VEHICLE INPUT DATA .....
Flexible Agile Spacecraft with 4 SG-CMG
! This is a Flex Spacecraft model that includes four 1200 (ft-lb-sec) SGCMGs
Body Axes Output, Attitude=Euler Angles

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft) : 202.05      32.17      0.20896E+08
Moments and products of Inertias Ixx, Iyy, Izz, -Ixy,-Ixz,-Iyz, in (lb-sec^2-ft) : 1.410986E+4  1.30452E+4  0.175972E+4  0.3103E+2,  0.1187E+2,
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet) : 0.0         -0.20975   0.62017
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet) : 0.0         25500.0   0.0         700000.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2) : 0.0         0.0       0.0         0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec) : 0.0         0.0       0.0         0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec) : 0.0000     0.000    0.0000     0.0000     0.0
External Force direction unit vect: (x,y,z), Force application point: (x,y,z) (feet) : Force      1.0       0.0         0.0         -12.1463
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet) : 1.0         1.0       1.0
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} : 0.0         -0.20975  0.62017    0.0
Aero Force Coeff/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}: 0.0         0.0       0.0         0.0         0.0
Aero Force Coeff/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}: 0.0         0.0       0.0000     0.0000     0.0000
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_g,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}: 0.0         0.0       0.0000     0.0000     0.0000
Aero Moment Coeff/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}: 0.0         0.0       0.0000     0.0000     0.0000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}: 0.0         0.0       0.0000     0.0000     0.0000
Aero Moment Coeff/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}: 0.0         0.0       0.0000     0.0000     0.0000

Number of Single Gimbal Control Moment Gyros (SG CMG) ? : 4
CMG: 1 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg) : 1200.0     0.0
Gimbal Direction Vector (m) : 0.92718385  0.0      0.37460659
Momentum Reference Direction Vector (r) : 0.0         1.0       0.0
SGCMG Surface Orientation angles in Pyramid (beta & gamma), (Figure 2.2) : 68.0        90.0
SGCMG Moments of Inertia (Js, Jg, Jo) about: spin, gimbal and output axes : 1.2         0.6       0.8
CMG: 2 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg) : 1200.0     0.0
Gimbal Direction Vector (m) : 0.0         0.92718385  0.37460659
Momentum Reference Direction Vector (r) : -1.0        0.0       0.0
SGCMG Surface Orientation angles in Pyramid (beta & gamma), (Figure 2.2) : 68.0        180.0
SGCMG Moments of Inertia (Js, Jg, Jo) about: spin, gimbal and output axes : 1.2         0.6       0.8
CMG: 3 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg) : 1200.0     0.0
Gimbal Direction Vector (m) : -0.92718385  0.0      0.37460659
Momentum Reference Direction Vector (r) : 0.0         -1.0      0.0
SGCMG Surface Orientation angles in Pyramid (beta & gamma), (Figure 2.2) : 68.0        270.0
SGCMG Moments of Inertia (Js, Jg, Jo) about: spin, gimbal and output axes : 1.2         0.6       0.8
CMG: 4 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg) : 1200.0     0.0
Gimbal Direction Vector (m) : 0.0         -0.92718385  0.37460659
Momentum Reference Direction Vector (r) : 1.0         0.0       0.0
SGCMG Surface Orientation angles in Pyramid (beta & gamma), (Figure 2.2) : 68.0        0.0
SGCMG Moments of Inertia (Js, Jg, Jo) about: spin, gimbal and output axes : 1.2         0.6       0.8

```

```

Number of Gyros, (Attitude and Rate) : 9
Gyro No 1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 2 : Roll Attitude 6.5334 0.0 -1.1892
Gyro No 2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 2 : Pitch Attitude 6.5334 0.0 -1.1892
Gyro No 3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 2 : Yaw Attitude 6.5334 0.0 -1.1892
Gyro No 4 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 2 : Roll Rate 6.5334 0.0 -1.1892
Gyro No 5 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 2 : Pitch Rate 6.5334 0.0 -1.1892
Gyro No 6 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 2 : Yaw Rate 6.5334 0.0 -1.1892
Gyro No 7 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 1 : Roll Rate -2.9379 0.2375 -0.2
Gyro No 8 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 1 : Pitch Rate -2.9379 0.2375 -0.2
Gyro No 9 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 1 : Yaw Rate -2.9379 0.2375 -0.2

```

```

Number of Accelerometers, Along Axes: (x,y,z) : 6
Acceleromet No 1 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2 : X-axis Accelerat. 6.5334 0.0 -1.1892
Acceleromet No 2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2 : Y-axis Accelerat. 6.5334 0.0 -1.1892
Acceleromet No 2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2 : Z-axis Accelerat. 6.5334 0.0 -1.1892
Acceleromet No 4 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4 : X-axis Accelerat. 5.2754 0.0 -2.09
Acceleromet No 5 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4 : Y-axis Accelerat. 5.2754 0.0 -2.09
Acceleromet No 6 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4 : Z-axis Accelerat. 5.2754 0.0 -2.09

```

```

Number of Bending Modes : 40
Flexible Agile Spacecraft with 4 SG-CMG, All Flex Modes

```

```

-----
SELECTED MODAL DATA AND LOCATIONS FOR :
Flexible Agile Spacecraft with 4 SG-CMG, All Flex Modes
! All the flex modes are included except for the first 6 rigid-body modes.
! A total of 40 structural modes.

```

```

MODE# 1/ 7, Frequency (rad/sec), Damping (zeta), Generalized Mass= 3.1905 0.30000E-02 12.000
DEFINITION OF LOCATIONS (NODES) phi along X phi along Y phi along Z sigm about X sigm about Y sigm about Z

Node ID# Modal Data at the 4 Single Gimbal GMGs...
Control Moment Gyros (CMG) Loc 6 0.10717D-02 -0.16639D-01 -0.31137D-01 -0.42835D-01 -0.29132D-02 -0.32023D-03
Control Moment Gyros (CMG) Loc 6 0.10717D-02 -0.16639D-01 -0.31137D-01 -0.42835D-01 -0.29132D-02 -0.32023D-03
Control Moment Gyros (CMG) Loc 6 0.10717D-02 -0.16639D-01 -0.31137D-01 -0.42835D-01 -0.29132D-02 -0.32023D-03
Control Moment Gyros (CMG) Loc 6 0.10717D-02 -0.16639D-01 -0.31137D-01 -0.42835D-01 -0.29132D-02 -0.32023D-03

Node ID# Modal Data at the 9 Gyros ...
Attitude, Rate, Accelerom Sens 2 0.45900D-02 -0.76587D-01 0.46744D-01 -0.42835D-01 -0.29132D-02 -0.32023D-03
Attitude, Rate, Accelerom Sens 2 0.45900D-02 -0.76587D-01 0.46744D-01 -0.42835D-01 -0.29132D-02 -0.32023D-03
Attitude, Rate, Accelerom Sens 2 0.45900D-02 -0.76587D-01 0.46744D-01 -0.42835D-01 -0.29132D-02 -0.32023D-03
Attitude, Rate, Accelerom Sens 2 0.45900D-02 -0.76587D-01 0.46744D-01 -0.42835D-01 -0.29132D-02 -0.32023D-03
Attitude, Rate, Accelerom Sens 2 0.45900D-02 -0.76587D-01 0.46744D-01 -0.42835D-01 -0.29132D-02 -0.32023D-03
Attitude, Rate, Accelerom Sens 2 0.45900D-02 -0.76587D-01 0.46744D-01 -0.42835D-01 -0.29132D-02 -0.32023D-03
Pointing Antena 1 0.19735D-02 -0.34853D-01 0.29482D-01 -0.42912D-01 -0.26934D-02 -0.17005D-03
Pointing Antena 1 0.19735D-02 -0.34853D-01 0.29482D-01 -0.42912D-01 -0.26934D-02 -0.17005D-03
Pointing Antena 1 0.19735D-02 -0.34853D-01 0.29482D-01 -0.42912D-01 -0.26934D-02 -0.17005D-03

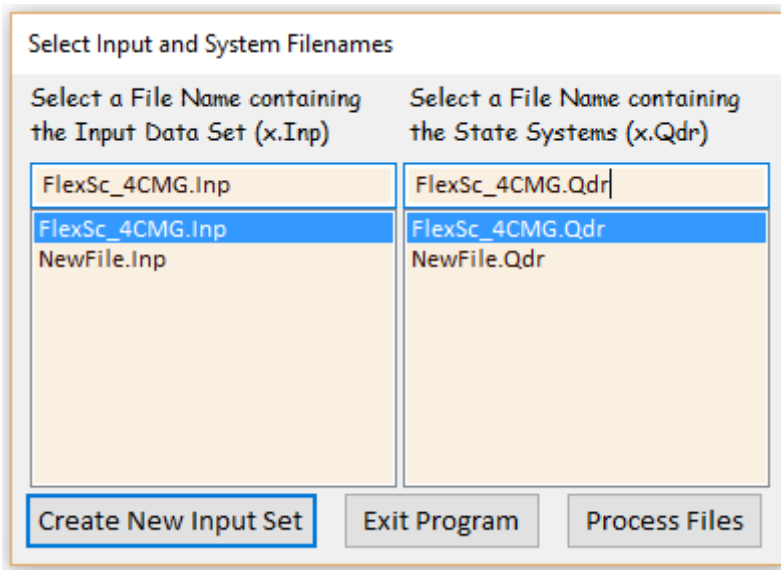
Node ID# Modal Data at the 6 Accelerometers, along (x,y,z)...
Attitude, Rate, Accelerom Sens 2 0.45900D-02 -0.76587D-01 0.46744D-01
Attitude, Rate, Accelerom Sens 2 0.45900D-02 -0.76587D-01 0.46744D-01
Attitude, Rate, Accelerom Sens 2 0.45900D-02 -0.76587D-01 0.46744D-01
Solar Array Hinge 4 -0.28312D-01 0.44163D-02 0.23976D+00
Solar Array Hinge 4 -0.28312D-01 0.44163D-02 0.23976D+00
Solar Array Hinge 4 -0.28312D-01 0.44163D-02 0.23976D+00

Node ID# Modal Data at the Disturbance Point
Reboost Engine Thruster 100 (lb) 5 0.16319D-02 -0.22063D-01 -0.79956D-01 -0.43084D-01 -0.27097D-02 -0.19524D-03

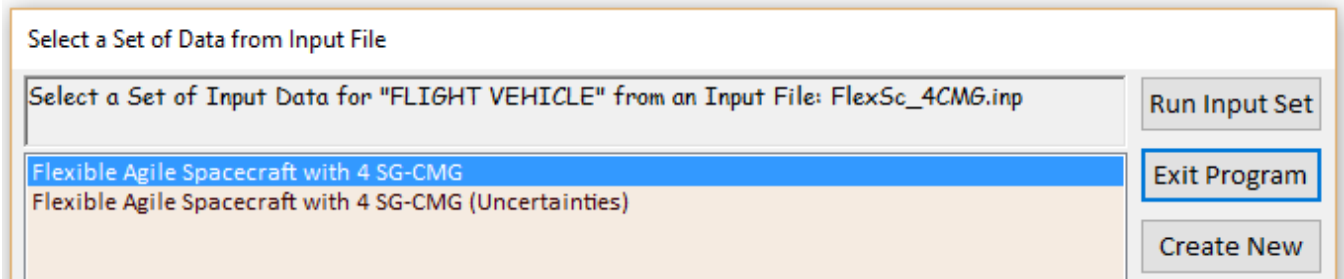
```

The selected modal data set of the flexible spacecraft (first mode is shown above), is also included in the input data file. It includes frequencies (rad/sec), damping coefficients (ζ), modal mass in (slugs), and mode shapes and slopes at specific locations. The locations correspond to the actuator and sensor locations specified in the spacecraft dataset.

To process this dataset using the flight vehicle modeling program start the Flixan program, select the project directory, then go to “*Program Functions*”, “*Flight Vehicle/ Spacecraft Modeling Tools*”, and then “*Flight Vehicle State-Space*”. From the following menu select the input and system filenames and click on “*Process Files*”.



The following menu shows the “*Flight Vehicle*” datasets which are in file “*FlexSc_4CMG.Inp*”. Select the first one and click on “*Run Input Set*” to process it.



The flight vehicle modeling program reads the datasets and displays the following dialog that shows the spacecraft parameters prior to processing. Click on “*Run*” to process the dataset and the program will create the spacecraft state-space system in file “*FlexSc_CMG_FVP.Qdr*”.

Flight Vehicle Parameters

Vehicle System Title
Flexible Agile Spacecraft with 4 SG-CMG

Number of Vehicle Effectors
 Gimballing Engines or Jets. Include Tail-Wags-Dog? WITH TWD / WITHOUT TWD
 Rotating Control Surfaces. Include Tail-Wags-Dog? WITH TWD / WITHOUT TWD
 Reaction Wheels? **Momentum Control Devices**
 Single Gimbal CMGs? Include a 3-axes Stabilized Double Gimbal CMG System? Yes/No

Number of Sensors
 Gyros
 Acceleromet
 Aero Vanes
 External Torques

Modeling Options (Flags)
 Output Rates in: Body Axes / Stability Axes
 Turn Coordination?: Include Turn Coordination / Without Turn Coordination
 Aero-Elasticity Options: Include GAFD, H-param Flex Coupl. data only / Neither Gafd nor Hpar
 Attitude Angles: Euler Angles / Integrals of Rates / LVLH Attitude

Number of Modes
 Structure Bending
 Fuel Sloshing:

Mass Properties | Trajectory Data | Gust/ Aero Paramet. | Aero Force Coeffs | Aero Moment Coeffs | Control Surfaces | Gimbal Engines/ RCS | External Torques
 Reaction Wheels | Single Gimbal CMGs | Double Gimbal CMG System | Slewing Appendages | Gyros | Accelerometer | Aero Sensors | Fuel Slosh | Flex Modes | User Notes

This Vehicle has 4 Single Gimbal CMGs Next SG-CMG

Single Gimbal CMG Parameters for CMG No: 2

| | | | | |
|----------------------------|--------------------------------------|----------------------------------|--------------------------------------|---|
| Gimbal Direction Unit Vect | | Initial Spin Direction Unit Vect | | SG-CMG Momentum (ft-lb-sec) <input type="text" value="1200.0"/> |
| X | <input type="text" value="0.0000"/> | X | <input type="text" value="-1.0000"/> | |
| Y | <input type="text" value="0.92718"/> | Y | <input type="text" value="0.0000"/> | |
| Z | <input type="text" value="0.37461"/> | Z | <input type="text" value="0.0000"/> | Initial Gimbal Angle (deg) <input type="text" value="0.0000"/> |
| CMG Moments of Inertia | | Pyramid Orientation Angles | | |
| Jspin | <input type="text" value="1.2000"/> | beta | <input type="text" value="68.000"/> | (degrees) |
| Jgimb | <input type="text" value="0.60000"/> | gamma | <input type="text" value="180.00"/> | |
| Joutp | <input type="text" value="0.80000"/> | | | |

The table below shows the spacecraft system 5 inputs and 37 outputs. The system has 4 torque inputs that drive the gimbals of the 4 SG-CMGs. The torques are generated by the servo systems that control the gimbal rates. The 5th input is an external disturbance force in the x-direction.

The outputs are the standard flight vehicle model outputs. Some of them are not useful and they can be removed from the model. The set includes 3 gyros, 6 rate gyros, and 6 accelerometers at the locations defined in the input data. The next 8 outputs are the CMG gimbal rates and positions. The last 3 outputs is the combined CMG momentum in spacecraft roll, pitch and yaw axes. Actually they are variations in the CMG momentum from nominal momentum in body axes because this model is linear.

Definition of System Variables

Inputs = 5

```
1   Single Gimbal CMG No 1 Gimbal Torque (ft-lb)
2   Single Gimbal CMG No 2 Gimbal Torque (ft-lb)
3   Single Gimbal CMG No 3 Gimbal Torque (ft-lb)
4   Single Gimbal CMG No 4 Gimbal Torque (ft-lb)
5   Ext Force Direct.=( 1.0000 0.0000 0.0000)
```

Outputs = 37

```
1   Roll Attitude (phi-body) (radians)
2   Roll Rate (p-body) (rad/sec)
3   Pitch Attitude (thet-bdy) (radians)
4   Pitch Rate (q-body) (rad/sec)
5   Yaw Attitude (psi-body) (radians)
6   Yaw Rate (r-body) (rad/sec)
7   Angle of attack, alfa, (radians)
8   Angle of sideslip, beta, (radian)
9   Change in Altitude, delta-h, (feet)
10  Forward Acceleration (V-dot) (ft/sec)
11  Cross Range Velocity (Vcr) (ft/sec)
12  Gyro # 1, Roll Attitude (Body) (radians)
13  Gyro # 2, Pitch Attitude (Body) (radians)
14  Gyro # 3, Yaw Attitude (Body) (radians)
15  Rate-Gyro # 4, Roll Rate (Body) (rad/sec)
16  Rate-Gyro # 5, Pitch Rate (Body) (rad/sec)
17  Rate-Gyro # 6, Yaw Rate (Body) (rad/sec)
18  Rate-Gyro # 7, Roll Rate (Body) (rad/sec)
19  Rate-Gyro # 8, Pitch Rate (Body) (rad/sec)
20  Rate-Gyro # 9, Yaw Rate (Body) (rad/sec)
21  Accelerom # 1, (along X), (ft/sec^2) Translat. Accelerat
22  Accelerom # 2, (along Y), (ft/sec^2) Translat. Accelerat
23  Accelerom # 3, (along Z), (ft/sec^2) Translat. Accelerat
24  Accelerom # 4, (along X), (ft/sec^2) Translat. Accelerat
25  Accelerom # 5, (along Y), (ft/sec^2) Translat. Accelerat
26  Accelerom # 6, (along Z), (ft/sec^2) Translat. Accelerat
27  SGCMG # 1 Gimbal Rate (rad/sec)
28  SGCMG # 2 Gimbal Rate (rad/sec)
29  SGCMG # 3 Gimbal Rate (rad/sec)
30  SGCMG # 4 Gimbal Rate (rad/sec)
31  SGCMG # 1 Gimbal Angle (radian)
32  SGCMG # 2 Gimbal Angle (radian)
33  SGCMG # 3 Gimbal Angle (radian)
34  SGCMG # 4 Gimbal Angle (radian)
35  SGCMG Array Momentum in X-axis (ft-lb-sec)
36  SGCMG Array Momentum in Y-axis (ft-lb-sec)
37  SGCMG Array Momentum in Z-axis (ft-lb-sec)
```

5.3 Flexible Spacecraft with Reaction Wheels and Gimbaling Appendages

In this example we will create a flexible spacecraft model that is controlled with an array of 3 Reaction Wheels and includes four gimbaling appendages which are controlled by servo motors. It is based on the model described in Section 1.1. This example is described in detail in folder “C:\Flixan\ Examples\ Surveillance Satellite React-Wheels”. The input file that contains the spacecraft data is in file “Surv_Sat_RB+Flx.Inp”, and it contains three different data-set models to be processed by the flight vehicle modeling program. It contains also a selected set of 60 flex modes, and Matlab conversion datasets. The vehicle dataset to be analyzed includes 7 reaction control thrusters for attitude control and momentum desaturation, 3 external torques (roll, pitch, and yaw), 3 reaction wheels. For sensors it uses 13 rotational measurements for attitude and rate, and two accelerometers in the X and Y directions.

The reaction wheel data include the direction of the spin unit vectors, the nominal wheel speeds in (rpm), and the wheel moment of inertia about the spin vector. In this case, one of the reaction wheels provides a momentum bias in pitch because its rotor has a constant speed at -4775 (rpm). It is used as a momentum wheel for spin stabilization. The other two wheels have zero momentum bias. They are tilted in the Y-Z plane and they are used for pitch and yaw control. There is no roll control by the RW.

The input file includes a selected set of 60 flex modes. The title of the modal data is included at the bottom of the vehicle dataset. In addition to flexibility this spacecraft includes 4 gimbaling appendages for solar arrays and optical instruments. The dynamic coupling between flexibility and the appendages pivoting is defined by the H-parameters matrix which is in file “Surveillance-Sat.Hpr”. This file also includes the masses, the moments of inertia matrix, hinge directions, hinge coordinates, and the CG coordinates of the 4 appendages. The spacecraft dataset is shown below.

```

FLIGHT VEHICLE INPUT DATA .....
Space Surveillance Satellite with RCS and Reaction Wheels (Gimbals & 60 Flex Modes)
! This Surveillance Satellite model is using the vehicle modeling program that
! includes both, rigid-body LVLH dynamics with momentum bias and flexibility.
! The satellite has (7) RCS jets and (3) Reaction Wheels. One of the wheels
! has a momentum bias of -40 (ft-lb-sec). It has also four gimbaling appendages for
! an optical telescope that rotates in azimuth and elevation, and two solar arrays.
! The coupling coefficients data are read from a separate file Surveillance-Sat.Hpr
! The LVLH Attitude flag defines the spacecraft attitude to be in the LVLH frame
! and also activates the Gravity Gradient dynamics.
!
Body Axes Output, LVLH Attitude

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft) : 50.0      0.0001      0.20896E+08
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft) : 1045.3,    641.02,    505.05,    0.0,      -19.0,
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet) : -0.05,    0.1,      22.625
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet) : 0.0      29000.0    0.0001    800000.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2) : 0.0      0.0      0.0      0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec) : 0.0      0.0      0.0      0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec) : 0.0000    0.000    0.0000    0.0000,  -0.064
W-Gust Azim & Elev angles (deg), or Torque/Force direction (x,y,z), Force Locat (x,y,z) : Torque    0.0      1.0      0.0
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet) : 0.0      1.0      1.0
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} : 0.0      0.0      0.0      -0.0
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}: 0.0      0.0      0.0      0.0      0.0
Aero Force Coeff/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_v}: 0.0      -0.0      0.0000    0.0000    0.00(
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}: 0.0      -0.0      0.0000    0.0000    0.00(
Aero Moment Coeff/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}: 0.0      -0.0      0.0000    0.0000    0.00(
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,FCm/PV,PCm/Ph}: 0.0      -0.0      0.0000    0.0000    0.00(
Aero Moment Coeff/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}: 0.0      0.0      0.0000    0.0000    0.00(

```

```

Number of Thruster Engines, RCS Jets, No Engine Gimbaling : 7

RCS Jet No: 1 +X Back Top Center (0.05 lb jet) : +X Top Jet Throttling Node 21
Engine Nominal Thrust, and Maximum Thrust in (lb) (for throttling) : 0.0 0.05
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): 0.0 0.0 0.0 0.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft): 0.0 0.0 0.0
Thruster location with respect to the Vehicle Reference Axes, Xjet, Yjet, Zjet, (ft) : -2.3625 0.0, 18.075
RCS Jet No: 2 +X Back Bottom center (0.05 lb jet) : +X Bottom Throttling Node 22
Engine Nominal Thrust, and Maximum Thrust in (lb) (for throttling) : 0.0 0.05
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): 0.0 0.0 0.0 0.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft): 0.0 0.0 0.0
Thruster location with respect to the Vehicle Reference Axes, Xjet, Yjet, Zjet, (ft) : -2.3625 0.0, 24.675
RCS Jet No: 3 +Y Front-Left (0.05 lb jet) : +Y Frnt Lft Throttling Node 23
Engine Nominal Thrust, and Maximum Thrust in (lb) (for throttling) : 0.0 0.05
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): 0.0 +90.0 0.0 0.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft): 0.0 0.0 0.0
Thruster location with respect to the Vehicle Reference Axes, Xjet, Yjet, Zjet, (ft) : 1.270833, -1.433333, 24.8025
RCS Jet No: 4 -Y Front-Right (0.05 lb jet) : -Y Frnt Rt Throttling Node 24
Engine Nominal Thrust, and Maximum Thrust in (lb) (for throttling) : 0.0 0.05
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): 0.0 -90.0 0.0 0.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft): 0.0 0.0 0.0
Thruster location with respect to the Vehicle Reference Axes, Xjet, Yjet, Zjet, (ft) : 1.270833, 1.433333, 24.8025
RCS Jet No: 5 -Y Back-Right (0.05 lb jet) : -Y Back Rt Throttling Node 25
Engine Nominal Thrust, and Maximum Thrust in (lb) (for throttling) : 0.0 0.05
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): 0.0 -90.0 0.0 0.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft): 0.0 0.0 0.0
Thruster location with respect to the Vehicle Reference Axes, Xjet, Yjet, Zjet, (ft) : -1.7875, 0.708333, 24.675
RCS Jet No: 6 +Y Back-Left (0.05 lb jet) : +Y Bck Lft Throttling Node 26
Engine Nominal Thrust, and Maximum Thrust in (lb) (for throttling) : 0.0 0.05
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): 0.0 +90.0 0.0 0.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft): 0.0 0.0 0.0
Thruster location with respect to the Vehicle Reference Axes, Xjet, Yjet, Zjet, (ft) : -1.7875, -0.708333, 24.675
RCS Jet No: 7 +X Back Mid-Center (0.05 lb jet) : +X Mid Cntr Throttling Node 27
Engine Nominal Thrust, and Maximum Thrust in (lb) (for throttling) : 0.0 0.05
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): 0.0 0.0 0.0 0.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft): 0.0 0.0 0.0
Thruster location with respect to the Vehicle Reference Axes, Xjet, Yjet, Zjet, (ft) : -2.3625, 0.0, 21.45

Number of External Torques on the Vehicle : 3
Torque No 1 Direction (x, y, z) : 1.0 0.0 0.0
Torque No 2 Direction (x, y, z) : 0.0 1.0 0.0
Torque No 3 Direction (x, y, z) : 0.0 0.0 1.0

Number of Reaction Wheels : 3
RW No: 1, Spin Direct. Unit Vect, Init Rot Speed (rpm), Rotor Mom of Inertia (slg-ft^2) : 0.0 1.0000 0.0000 -4774.6 0.08
RW No: 2, Spin Direct. Unit Vect, Init Rot Speed (rpm), Rotor Mom of Inertia (slg-ft^2) : 0.0 0.93969262 -0.34202014 00.0 0.08
RW No: 3, Spin Direct. Unit Vect, Init Rot Speed (rpm), Rotor Mom of Inertia (slg-ft^2) : 0.0 0.93969262 0.34202014 00.0 0.08

Number of Gyros, (Attitude and Rate) : 13
Gyro No 1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 6 : Roll Rate -0.747 0.114 25.57
Gyro No 2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 6 : Pitch Rate -0.747 0.114 25.57
Gyro No 3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 6 : Yaw Rate -0.747 0.114 25.57
Gyro No 4 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 6 : Roll Attitude -0.747 0.114 25.57
Gyro No 5 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 6 : Pitch Attitude -0.747 0.114 25.57
Gyro No 6 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 6 : Yaw Attitude -0.747 0.114 25.57
Gyro No 7 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 9 : Pitch Attitude 0.0 0.0 0.0
Gyro No 8 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 9 : Yaw Attitude 0.0 0.0 0.0
Gyro No 9 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 10 : Pitch Attitude 0.0 0.0 0.0
Gyro No 10 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 10 : Yaw Attitude 0.0 0.0 0.0
Gyro No 11 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 11 : Roll Attitude 0.0 0.0 0.0
Gyro No 12 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 12 : Pitch Attitude 0.0 0.0 0.0
Gyro No 13 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 13 : Yaw Attitude 0.0 0.0 0.0

Number of Accelerometers, Along Axes: (x,y,z) : 2
Acceleromet No 1 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 7 : X-axis Accelerat. 0.338 0.648 25.57
Acceleromet No 2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 7 : Y-axis Accelerat. 0.338 0.648 25.57

Gimbaling Appendages (Requires Inertial Flex Coupling Coefficients), H-paramet. Filename : Surveillance-Sat.Hpr

Number of Bending Modes : 60
Space Surveillance Satellite with RCS and Reaction Wheels (60 Flex Pre-Selected Modes)
-----

```

The flight vehicle program will process the above dataset and generate the spacecraft state-space system. The modal data set below is also in the input file and contains the selected modal data for the flexible spacecraft model. It includes frequencies (rad/sec), damping coefficients (ζ), modal mass, and mode shapes and slopes. The locations correspond to the locations specified in the spacecraft dataset. The inertial coupling coefficients file includes the H-parameters matrix. Each row corresponds to a mode and only the rows that correspond to the selected modes are included in the calculations.

SELECTED MODAL DATA AND LOCATIONS FOR : Pre-Selected

Space Surveillance Satellite with RCS and Reaction Wheels (60 Flex Pre-Selected Modes)

! The flex modes used in this analysis are the same modes that were selected in the previous flex
 ! surveillance spacecraft analysis. We start from Mode #7 because the first 6 rigid-body modes
 ! are not included in the modal data because the rigid-body dynamics are included in the vehicle.
 ! The Nodes correspond to vehicle locations defined in the vehicle data above.

| MODE# | 1/ 7, Frequency (rad/sec), Damping (zeta), Generalized Mass= | 3.0998 | 0.50000E-02 | 12.000 | | | |
|---------------------------------|--|--|--------------|--------------|--------------|--------------|--------------|
| DEFINITION OF LOCATIONS (NODES) | phi along X | phi along Y | phi along Z | sigm about X | sigm about Y | sigm about Z | |
| | Node ID# | Modal Data at the 7 Engines, (x,y,z)... | | | | | |
| RCS Jet #1 (+X) | 98001 | -0.43556D+00 | -0.25842D-02 | 0.14383D+00 | -0.11015D-02 | 0.64939D-01 | -0.10968D-02 |
| RCS Jet #2 (+X) | 98002 | -0.69445D-02 | 0.46878D-02 | 0.14383D+00 | -0.11019D-02 | 0.64943D-01 | -0.10976D-02 |
| RCS Jet #3 (+Y) | 98003 | -0.25866D-03 | 0.84179D-03 | -0.90543D-01 | -0.11015D-02 | 0.64943D-01 | -0.10969D-02 |
| RCS Jet #4 (-Y) | 98004 | 0.28867D-02 | 0.84191D-03 | -0.93702D-01 | -0.11023D-02 | 0.64944D-01 | -0.10973D-02 |
| RCS Jet #5 (-Y) | 98005 | -0.61695D-02 | 0.40557D-02 | 0.10571D+00 | -0.11017D-02 | 0.64943D-01 | -0.10902D-02 |
| RCS Jet #6 (+Y) | 98006 | -0.77244D-02 | 0.40575D-02 | 0.10727D+00 | -0.11019D-02 | 0.64943D-01 | -0.11047D-02 |
| RCS Jet #7 (+X) | 98007 | -0.21638D+00 | 0.11339D-02 | 0.14383D+00 | -0.11013D-02 | 0.64937D-01 | -0.10964D-02 |
| | Node ID# | Modal Data at the 3 Reaction Wheels... | | | | | |
| Reaction Wheel #1 CG | 58041 | -0.20700D+00 | -0.43105D-03 | 0.41763D-01 | -0.11013D-02 | 0.64943D-01 | -0.10960D-02 |
| Reaction Wheel #2 CG | 58042 | -0.20783D+00 | -0.16859D-02 | -0.31758D-01 | -0.11002D-02 | 0.64942D-01 | -0.11005D-02 |
| Reaction Wheel #3 CG | 58043 | -0.20632D+00 | -0.17810D-02 | -0.38780D-01 | -0.11033D-02 | 0.64942D-01 | -0.10951D-02 |
| | Node ID# | Modal Data at the 3 External Torque Points... | | | | | |
| Cryo Cooler Pump | 40101 | 0.10119D+00 | 0.39271D-02 | -0.98352D-02 | -0.11022D-02 | 0.64950D-01 | -0.10992D-02 |
| Cryo Cooler Pump | 40101 | 0.10119D+00 | 0.39271D-02 | -0.98352D-02 | -0.11022D-02 | 0.64950D-01 | -0.10992D-02 |
| Cryo Cooler Pump | 40101 | 0.10119D+00 | 0.39271D-02 | -0.98352D-02 | -0.11022D-02 | 0.64950D-01 | -0.10992D-02 |
| | Node ID# | Modal Data at the 13 Gyros ... | | | | | |
| Inertial Attitude Sensors | 31001 | 0.51466D-01 | 0.39036D-02 | 0.38773D-01 | -0.11017D-02 | 0.64943D-01 | -0.10969D-02 |
| Inertial Attitude Sensors | 31001 | 0.51466D-01 | 0.39036D-02 | 0.38773D-01 | -0.11017D-02 | 0.64943D-01 | -0.10969D-02 |
| Inertial Attitude Sensors | 31001 | 0.51466D-01 | 0.39036D-02 | 0.38773D-01 | -0.11017D-02 | 0.64943D-01 | -0.10969D-02 |
| Inertial Attitude Sensors | 31001 | 0.51466D-01 | 0.39036D-02 | 0.38773D-01 | -0.11017D-02 | 0.64943D-01 | -0.10969D-02 |
| Inertial Attitude Sensors | 31001 | 0.51466D-01 | 0.39036D-02 | 0.38773D-01 | -0.11017D-02 | 0.64943D-01 | -0.10969D-02 |
| Inertial Attitude Sensors | 31001 | 0.51466D-01 | 0.39036D-02 | 0.38773D-01 | -0.11017D-02 | 0.64943D-01 | -0.10969D-02 |
| Second Mirror | 40102 | 0.17864D+00 | 0.52413D-02 | -0.98558D-02 | -0.11024D-02 | 0.64950D-01 | -0.10944D-02 |
| Second Mirror | 40102 | 0.17864D+00 | 0.52413D-02 | -0.98558D-02 | -0.11024D-02 | 0.64950D-01 | -0.10944D-02 |
| Sensitive Instrument 2 | 40103 | 0.99605D-01 | 0.41431D-02 | 0.46061D-02 | -0.11016D-02 | 0.64944D-01 | -0.10966D-02 |
| Sensitive Instrument 2 | 40103 | 0.99605D-01 | 0.41431D-02 | 0.46061D-02 | -0.11016D-02 | 0.64944D-01 | -0.10966D-02 |
| Sensitive Instrument 3 | 40104 | 0.10548D+00 | 0.36774D-02 | -0.28874D-01 | -0.11017D-02 | 0.64943D-01 | -0.10968D-02 |
| Sensitive Instrument 4 | 40105 | 0.10576D+00 | 0.35477D-02 | -0.36834D-01 | -0.11017D-02 | 0.64943D-01 | -0.10968D-02 |
| Sensitive Instrument 5 | 40106 | 0.96563D-01 | 0.34621D-02 | -0.32667D-01 | -0.11017D-02 | 0.64943D-01 | -0.10968D-02 |
| | Node ID# | Modal Data at the 2 Accelerometers, along (x,y,z)... | | | | | |
| Accelerometers | 31002 | 0.51889D-01 | 0.27108D-02 | -0.32279D-01 | | | |
| Accelerometers | 31002 | 0.51889D-01 | 0.27108D-02 | -0.32279D-01 | | | |

INERTIAL COUPLING COEFFICIENTS (FROM NASTRAN) FOR

Surveillance Satellite with Rotating Solar Arrays and Optical Sensor

NUMBER OF MODES, GIMBALS: 100, 4

Units (ft-lb-sec^2)

| | | | | |
|------------------|------------------|------------------|------------------|------------|
| -2.269347606E-01 | 1.747074520E-01 | -1.915053482E-01 | -2.195309075E-01 | Mode # 1 |
| -1.133417583E+00 | -1.374561104E-02 | 3.669942078E-01 | -2.549929767E-01 | Mode # 2 |
| | | | | |
| -3.350520970E-02 | -3.483536353E-03 | -1.103515707E-02 | -1.353584642E-02 | Mode # 99 |
| -2.108828453E-03 | 4.130124619E-02 | 9.495321928E-03 | -2.236874122E-03 | Mode # 100 |

MOMENTS OF INERTIA MATRIX FOR THE ROTATING BODIES (slug-ft^2)

| | | | | |
|-----------------|-----------------|-----------------|-----------------|---------------------------|
| 6.3954598075 | -0.049079742633 | 0.000000000E+00 | 0.000000000E+00 | Payload Inertia (elevat.) |
| -0.049079742633 | 1.889057024167 | 0.000000000E+00 | 0.000000000E+00 | Payload Inertia (azimuth) |
| 0.000000000E+00 | 0.000000000E+00 | 3.170436278333 | 0.000000000E+00 | Right Solar Array |
| 0.000000000E+00 | 0.000000000E+00 | 0.000000000E+00 | 3.1704239175 | Left Solar Array |

Masses for the 4 Gimbal Appendages (slugs): Payload Elevation, Azimuth, 2 Solar Arrays

2.2 0.88 1.4 1.4

Hinge Direction Unit Vectors for the 4 Gimbal Appendages, Payload Elevat & Azimuth, 2 Sol-Arr pitcl

1.0 0.0 0.0 0.0
 0.0 -1.0 1.0 1.0
 0.0 0.0 0.0 0.0

Appendage Pivot Point Locations (feet) for Payload Elevation, Azimuth, 2 Sol-Arrays

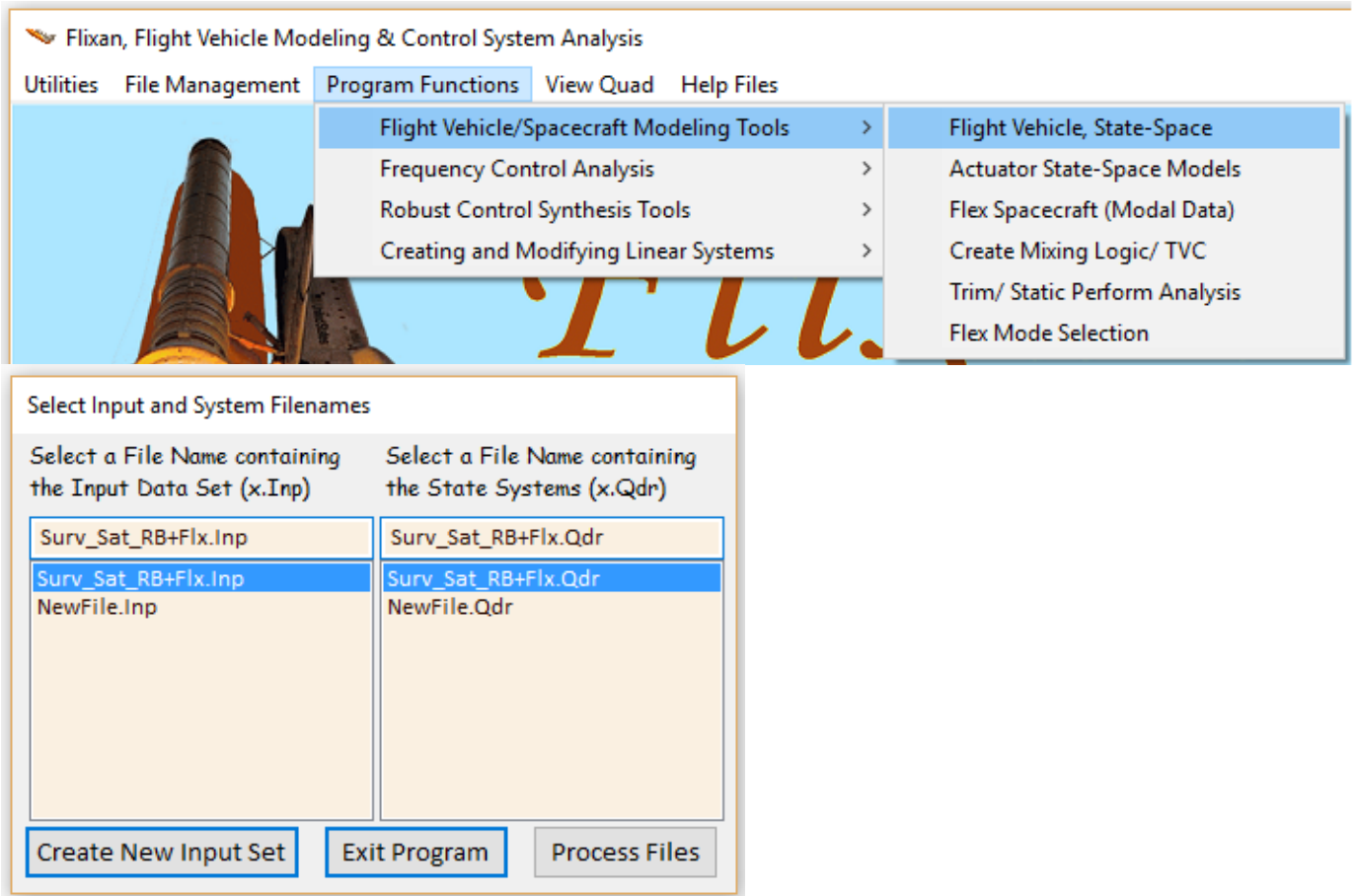
0.06 0.06 -0.372 -0.372
 0.1 0.13 -1.8 +1.8
 25.0 25.2 17.0 17.0

Appendage Center Mass Locations (feet) for Payload Elevation, Azimuth, 2 Sol-Arrays

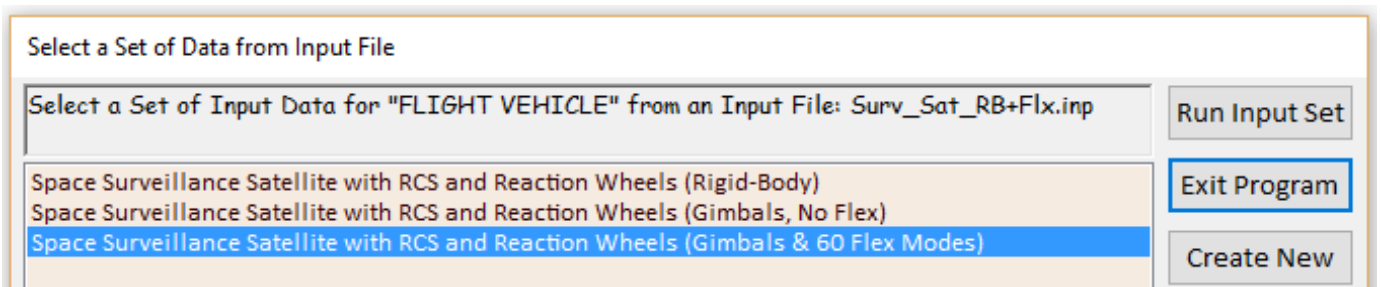
0.06 0.06 -0.372 -0.372
 0.14 0.14 -7.2 +7.2
 25.9 25.9 17.0 17.0

Inertial Coupling Coefficients File "Surveillance-Sat.Hpr"

To process this dataset using the flight vehicle modeling program start the Flixan program, select the project directory, then go to “*Program Functions*”, “*Flight Vehicle/ Spacecraft Modeling Tools*”, and then “*Flight Vehicle State-Space*”. Then from the following menu select the input and system filenames and click on “*Process Files*”.



The menu below shows the three “Flight Vehicle” datasets which are in file “*Surv_Sat_RB+Flx.Inp*”. Select one of them and click on “*Run Input Set*” to process it. The following dialog is presented by the Flight Vehicle Modeling program and it displays the vehicle data in tabs. Click on “*Run*” to process the dataset. The spacecraft system will be saved by the program in file “*Surv_Sat_RB+Flx.Qdr*”.



Flight Vehicle Parameters

Vehicle System Title
 Space Surveillance Satellite with RCS and Reaction Wheels (Gimbals & 60 Flex Modes)

Number of Vehicle Effectors
 Gimballing Engines or Jets. Include Tail-Wags-Dog? 7 WITH TWD WITHOUT TWD
 Rotating Control Surfaces. Include Tail-Wags-Dog? 0 WITH TWD WITHOUT TWD
 Reaction Wheels? 3 **Momentum Control Devices**
 Single Gimbal CMGs? 0 Include a 3-axes Stabilized Double Gimbal CMG System? Yes No

Number of Sensors
 Gyros 13
 Acceleromet 2
 Aero Vanes 0
 External Torques 3

Modeling Options (Flags)
 Output Rates in: Body Axes Stability Axes
 Turn Coordination?: Include Turn Coordin Without Turn Coordin
 Aero-Elasticity Options: Include GAFD, H-param Flex Coupl. data only Neither Gafd nor Hpar
 Attitude Angles: Euler Angles Integrals of Rates LVLH Attitude

Number of Modes
 Structure Bending 60
 Fuel Sloshing 0

Mass Properties | Trajectory Data | Gust/ Aero Paramet. | Aero Force Coeffs | Aero Moment Coeffs | Control Surfaces | Gimbal Engines/ RCS | External Torques
 Reaction Wheels | Single Gimbal CMGs | Double Gimbal CMG System | Slewing Appendages | Gyros | Accelerometer | Aero Sensors | Fuel Slosh | Flex Modes | User Notes

This Vehicle has 3 Reaction/Momentum Wheels
Spin Direction, Rotor Inertia, and Initial rpm Speed of RW No: 1

RW Spin Direction Unit Vector

| | |
|---|--------|
| X | 0.0000 |
| Y | 1.0000 |
| Z | 0.0000 |

Next RW

RW Rotor Mom.Inertia 0.80000E-01
 Initial RW Speed (rpm) -4774.6

The table below shows the spacecraft system 18 inputs and 37 outputs. The system has 7 RCS jet force inputs that provide thrusts in the directions defined in the input data. Actually, the inputs are defined as throttles, rather than forces in the FVMP, which are the ratios of “force divided by max force”. There are 3 RW control torques in the directions defined in the input data. The system also has 4 torques applied at the 4 appendage gimbals. We have also included 3 external torques for disturbance analysis. The last input is an additional disturbance torque in the pitch direction. All torques are in (ft-lb).

The outputs are the standard flight vehicle model outputs. Some of them are not useful for spacecraft applications and they can be removed from the model. The set includes 10 gyros, 3 rate gyros, and 2 accelerometers at the locations defined in the input data. The next 3 outputs are RW rates in (rpm). Actually they are variations in the RW rate from nominal because this model is linear. The last 8 outputs are position angles and rates at the gimbals of the 4 pivoting appendages.

Definition of System Variables

Inputs = 18

```
1 Throttle Input dTh/Th for Engine No 1 (-)
2 Throttle Input dTh/Th for Engine No 2 (-)
3 Throttle Input dTh/Th for Engine No 3 (-)
4 Throttle Input dTh/Th for Engine No 4 (-)
5 Throttle Input dTh/Th for Engine No 5 (-)
6 Throttle Input dTh/Th for Engine No 6 (-)
7 Throttle Input dTh/Th for Engine No 7 (-)
8 React-Wheel No 1 Torque to Spacecraft (ft-lb)
9 React-Wheel No 2 Torque to Spacecraft (ft-lb)
10 React-Wheel No 3 Torque to Spacecraft (ft-lb)
11 Gimbaling Appendage Hinge # 1 Torque (ft-lb)
12 Gimbaling Appendage Hinge # 2 Torque (ft-lb)
13 Gimbaling Appendage Hinge # 3 Torque (ft-lb)
14 Gimbaling Appendage Hinge # 4 Torque (ft-lb)
15 Ext Torque Input 1 (ft-lb), dir: 1.00 0.00 0.00
16 Ext Torque Input 2 (ft-lb), dir: 0.00 1.00 0.00
17 Ext Torque Input 3 (ft-lb), dir: 0.00 0.00 1.00
18 Ext Torque Direct.= ( 0.0000 1.0000 0.0000)
```

Outputs = 37

```
1 Roll Attitude (phi-LVLH) (radians)
2 Roll Rate (p-body) (rad/sec)
3 Pitch Attitude (thet-LVLH) (radians)
4 Pitch Rate (q-body) (rad/sec)
5 Yaw Attitude (psi-LVLH) (radians)
6 Yaw Rate (r-body) (rad/sec)
7 Angle of attack, alfa, (radians)
8 Angle of sideslip, beta, (radian)
9 Change in Altitude, delta-h, (feet)
10 Forward Acceleration (V-dot) (ft/sec)
11 Cross Range Velocity (Vcr) (ft/sec)
12 Rate-Gyro # 1, Roll Rate (Body) (rad/sec)
13 Rate-Gyro # 2, Pitch Rate (Body) (rad/sec)
14 Rate-Gyro # 3, Yaw Rate (Body) (rad/sec)
15 Gyro # 4, Roll Attitude (Body) (radians)
16 Gyro # 5, Pitch Attitude (Body) (radians)
17 Gyro # 6, Yaw Attitude (Body) (radians)
18 Gyro # 7, Pitch Attitude (Body) (radians)
19 Gyro # 8, Yaw Attitude (Body) (radians)
20 Gyro # 9, Pitch Attitude (Body) (radians)
21 Gyro # 10, Yaw Attitude (Body) (radians)
22 Gyro # 11, Roll Attitude (Body) (radians)
23 Gyro # 12, Pitch Attitude (Body) (radians)
24 Gyro # 13, Yaw Attitude (Body) (radians)
25 Accelerom # 1, (along X), (ft/sec^2) Translat. Accelerat.
26 Accelerom # 2, (along Y), (ft/sec^2) Translat. Accelerat.
27 Reaction Wheel # 1 Rate (rpm) relative to vehicle
28 Reaction Wheel # 2 Rate (rpm) relative to vehicle
29 Reaction Wheel # 3 Rate (rpm) relative to vehicle
30 Gimbaling Appendage # 1 Relative Rotation, alfar( 1), (rad)
31 Gimbaling Appendage # 1 Relative Rot.Rate, alfar-dot( 1), (rad/sec)
32 Gimbaling Appendage # 2 Relative Rotation, alfar( 2), (rad)
33 Gimbaling Appendage # 2 Relative Rot.Rate, alfar-dot( 2), (rad/sec)
34 Gimbaling Appendage # 3 Relative Rotation, alfar( 3), (rad)
35 Gimbaling Appendage # 3 Relative Rot.Rate, alfar-dot( 3), (rad/sec)
36 Gimbaling Appendage # 4 Relative Rotation, alfar( 4), (rad)
37 Gimbaling Appendage # 4 Relative Rot.Rate, alfar-dot( 4), (rad/sec)
```