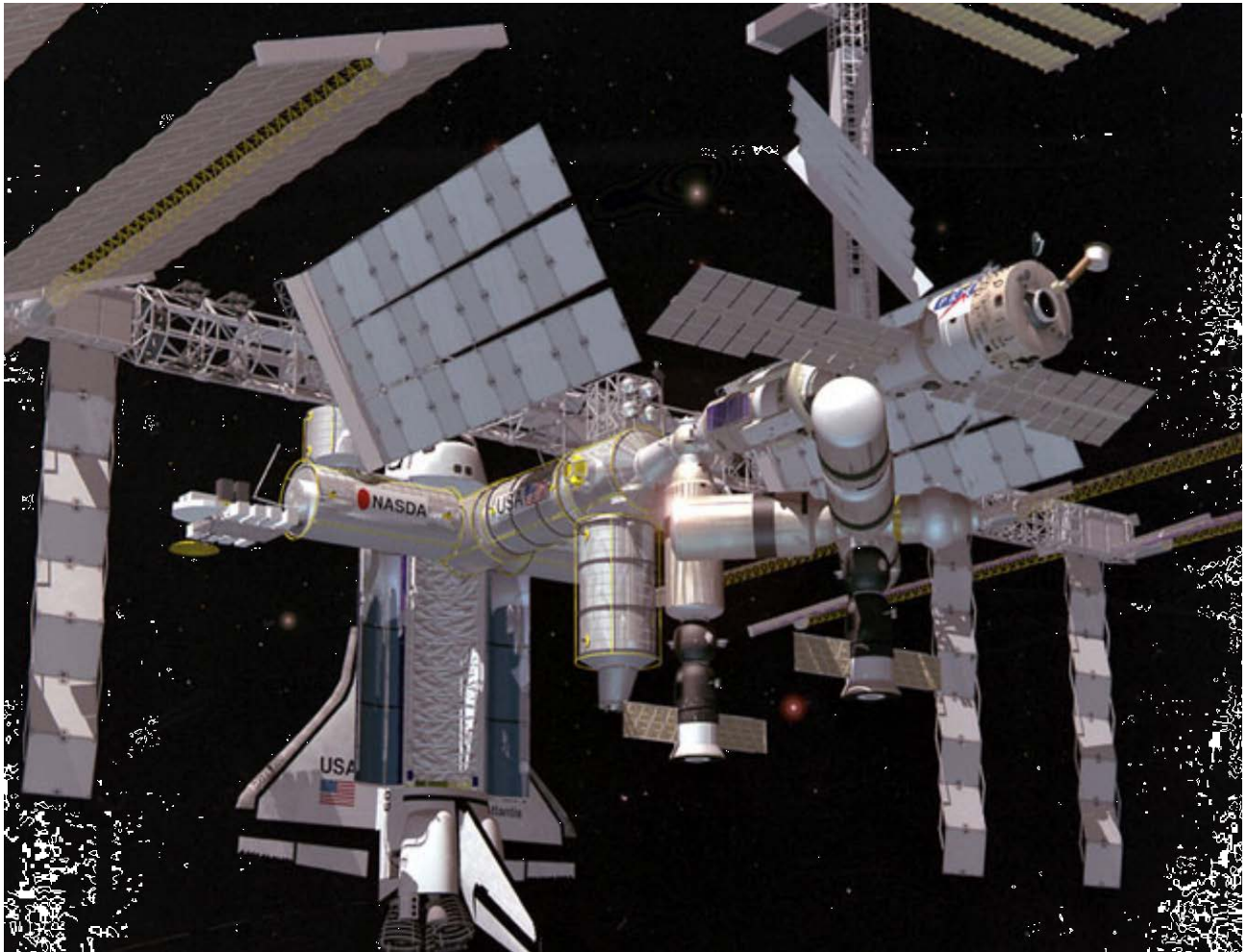# Flexible Spacecraft Modeling Program

Introduction

This program is used for creating state-space models of flexible space structures, such as a satellite or a large flexible space station with solar arrays and rotating optical sensors. It does not create multi-body non-linear simulation models, but instead, it uses modal data generated from a finite elements program (FEM), such as NASTRAN, and creates the state-space models for a selected set of inputs, outputs and flex modes. Linear models are used for analyzing the spacecraft at a fixed configuration, and not for large angle maneuvering or multi-body payload slewing. They are easier to generate and analyze than non-linear multi-body models because the dynamic modeling has already been done in the FEM. They are used in applications that do not require large angle slewing, such as attitude control and payload pointing servo system design, stability, jitter, and sensitivity analysis. They are also used for analyzing the interaction between the spacecraft structure and the various control loops and for small angle time-domain simulations. A linear state-space model can be used,

for example, to analyze stability of the various control loops, and also to evaluate the effect of spacecraft disturbances to an optical or a micro-gravity sensor as a result of the interaction between the structure and the control loops. The inputs to the linear models are either external forces and moments applied to the structure such as reaction control thrusters, control moment gyros, and disturbances, or they can be control torques at the hinges of pivoting appendages such as solar arrays and payloads. The system outputs are either translation or rotational sensors measuring position, velocity, or acceleration at certain locations on the spacecraft, or rotational measurements of the gimbaling appendages relative to the spacecraft. The models created by the flex spacecraft modeling program are simpler in the sense that the rigid-body, flex, and gimbaling appendage dynamics are all combined in the modal data. The rigid-body modes are disguised as flex modes by the structure analyst. The vehicle modeling program, on the other hand, is more complex because it captures detailed models of the rigid-body dynamics and the modal data consist of only structural modes.

Data Files

The program uses several input and output data files, such as modal data files, input files that define the spacecraft actuator and sensor locations, and output files where the linear state-space models are saved by the program. These files are described in more detail below.

Modal Data File (Mod): The modal data file contains the modal data of the flexible structure which is generated by a finite elements modeling program. It has a filename extension "*.Mod*". Typically, it contains several hundred modes, starting with the first six rigid-body modes which are at zero frequency and characterize the rigid-body behavior of the spacecraft including the attached appendages (if any). The remaining modes are structural modes associated with resonant frequencies. The modal data file is obtained by post-processing the FEM program output. The data must be converted to a standard Flixan format that can be read by the mode selection program. This reformatting process prior to mode selection is also needed for reducing the excessive amount of data provided by the FEM program. During reformatting we read a smaller number of spacecraft locations. That is, we read only the locations (nodes) which are possible candidates to be included in the state-space models (typically 15 to 45 nodes are sufficient). The first six modes in the modal data file are the rigid body modes that have zero natural frequency and damping coefficients. The remaining modes, starting from mode 7, are the flexible modes. Each mode in the modal data file consist of the following parameters: the mode frequency in (rad/sec), the damping coefficient (ζ), the generalized mass (m_g), the generalized modal displacements (φ), and the modal slopes (σ) in (rad/inch), consisting of 3 translations and 3 rotations with respect to the body axes, at each node. The sample modal data file below contains the modal data for a flexible space station structure that consists of 55 modes and 28 nodes (the shapes and slopes for only the first mode are shown). The nodes are listed by the node id number. The node id numbers are big numbers used in the original FEM model to identify structure locations which are in the order of several thousands.

```
STRUCTURAL MODAL DATA
FLAXIBLE SPACE STRUCTURE WITH SOLAR ARRAYS
NUMBER OF MODES = 55   NODES= 28

Mode #  1, Omega= 0.0, Zeta=0.0, General.Mass=  1.0000
Node ID:  Along X      Along Y      Along Z      About X      About Y      About Z
  1060    0.10643D+00  0.74926D-04 -0.64510D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
  3000    0.97502D-01  0.74926D-04 -0.63436D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
  2060    0.88572D-01  0.74926D-04 -0.62363D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
  3100    0.00000D+00  0.00000D+00  0.00000D+00  0.00000D+00  0.00000D+00  0.00000D+00
  6070    0.41841D-01  0.10176D-03 -0.67460D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
   505    0.82939D-01  0.86029D-04 -0.66119D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
  6045    0.27333D-01  0.10176D-03 -0.65716D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
  6039    0.25100D-01  0.10176D-03 -0.65448D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
  6001    0.19520D-01  0.10176D-03 -0.64777D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
  1000    0.17290D-01  0.10176D-03 -0.64510D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
  7001    0.12824D-01  0.10176D-03 -0.63973D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
  7010    0.16632D-02  0.10176D-03 -0.62631D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
  2000   -0.57120D-03  0.10176D-03 -0.62363D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
  5001   -0.28039D-02  0.10176D-03 -0.62095D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
  5039   -0.83810D-02  0.10176D-03 -0.61424D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
  5045   -0.10614D-01  0.10176D-03 -0.61156D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
   410   -0.68540D-01  0.11749D-03 -0.60475D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
  5070   -0.25122D-01  0.10176D-03 -0.59412D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
  9017   -0.12152D-02  0.98025D-02 -0.39355D-01 -0.13631D-07 -0.45285D-04 -0.11342D-04
  9018   -0.45271D-02 -0.56588D-02  0.22381D-01 -0.13631D-07 -0.45285D-04 -0.11342D-04
  9007   -0.12152D-02  0.29089D-02 -0.11831D-01 -0.13631D-07 -0.45285D-04 -0.11342D-04
  1160   -0.71853D-01  0.12859D-03 -0.64510D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
  4000   -0.80783D-01  0.12859D-03 -0.63436D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
  2160   -0.89714D-01  0.12859D-03 -0.62363D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
  1050    0.97519D-01  0.77610D-04 -0.64510D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
  2050    0.79657D-01  0.77610D-04 -0.62363D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
  1150   -0.62938D-01  0.12591D-03 -0.64510D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
  2150   -0.80800D-01  0.12591D-03 -0.62363D-03 -0.13631D-07 -0.45285D-04 -0.11342D-04
--------------------------------------------------------------------------------
```

Note, that not all the flex modes have to be included in the linear analysis model. Some of the modes are negligible and they can be excluded from the state-space model by means of a mode selection process to be discussed later.

Nodes Look-Up Table File (.Nod): The nodes file contains information about the locations (nodes) which have been included in the modal data file. The filename extension is (.Nod), and it includes four columns of data. The first column has short labels (40 characters long) describing the vehicle locations which are included in the modal data file. The nodes are listed in the same order as they are listed in the modal data file. In the second column are the node numbers, starting from 1 to the max number of nodes which are included in the modal data file (.Mod). The node numbers are used to identify the actuator and sensor locations in file. The third column contains the Node Ids, which are very big numbers identifying the nodes in the FEM. The last column, which is optional and not used by the program, includes the coordinates of the node locations along the vehicle x, y, and z axes. The nodes file is used as a look-up table by the mode selection program to help the user identify interactively the actuator and sensor locations. A typical nodes file is shown below.

```
NODE IDENTIFICATION TABLE FOR THE LARGE SPACE STATION WITH SOLAR ARRAYS AND CMG
Node Description                      Node No   FEM ID    Node Location X,Y,Z (feet)
-----------------------------------------------------------------------------------
Payload boom, (top-right)                1      1060
Payload boom, (center)                   2      3000
Payload boom, (top-left)                 3      2060
Unknown                                  4      3100
Right Solar Array Boom                   5      6070      0.0    232.35    18.0
Right Solar Array (Connect)              6       505
Right SA Boom, Outboard the Hinge        7      6045
Right SA Boom, Inboard the Hinge Servo   8      6039
Right Boom (Sensor Assembly)             9      6001
Right Keel/Boom Intrsect, CMG Location  10      1000
CG Center of Structure                  11      7001
Left Keel/Boom Intersection             12      7010
Unknown                                 13      2000
Left Boom                               14      5001
Left Boom, Inboard the Hinge Servo      15      5039
Left SA Boom, Outboard the Hinge Servo  16      5045
Left Solar Array (Connect)              17       410
Left SA Boom, Extreme end               18      5070      0.0   -224.63    16.0
Right Habitat Module                    19      9017
Module, (Front Dock)                    20      9018
Center Habitat Module                   21      9007
Bottom Payload Boom (Right)             22      1160
Bottom Payload Boom (Center)            23      4000
Bottom Payload Boom (Left) Antenna      24      2160
Top Right RCS Thruster Assembly         25      1050      0.0     73.7   -185.0
Top Left RCS Thrusters Assembly         26      2050      0.0    -65.58  -185.0
Bottom Right RCS Thruster Assembly      27      1150      0.0     73.7    144.0
Bottom Left RCS Thrusters Assembly      28      2150      0.0    -65.58   144.0
```

Input Data File (.Inp): This file contains the input data used by the flexible spacecraft modeling program to compute a linear state-space model of the spacecraft. It consists of a set of flex spacecraft configuration data described below.

1. The data set starts with a label "FLEXIBLE SPACECRAFT FE MODEL ..." that defines the identity of the data, that is, the flex spacecraft Flixan subprogram that will process the data.
2. It is followed by the spacecraft title, one line (100 characters long), and some comment lines that briefly describe the spacecraft configuration. The same title and comment lines will also appear in the state-space model created.
3. A set of data that defines the input forces and torques. The corresponding node location and the direction of the force or torque. The direction is defined by a unit vector.
4. A set of data that defines the output sensors. Translation or rotational sensors, the sensor location (node number), the sensor direction (x, y, or z for translation) or (roll, pitch, and yaw for rotation). Also the type of sensor (position, rate, or acceleration), either translation or rotation.
5. A reduced set of modal data selected from the original modal data file. The reduced set of modes is obtained during the input data file preparation process, see examples. The modal data consist of: mode frequencies, damping, generalized mass, modal translations and modal slopes.
6. An optional reference to a file name that contains the inertial coupling coefficients (H-parameters matrix). It is only required when the space vehicle has gimbaling appendages. This file includes the coupling dynamics between the gimbaling appendages and structural flexibility. It includes also the moments of inertia matrix of the gimbaling appendages, and the rotational directions. The rotational directions of the appendages is not needed in this program because this information is already captured in the rigid-body section of the

H-parameters matrix (first six rows). It is used only by the flight vehicle modeling program when gimbaling appendages are included.

The input file may also contain input data used by other programs, such as the flight vehicle modeling program, interconnection and transfer function utilities, etc. A sample input data file for a flex spacecraft can be found in file "Surveillance-Sat.Inp" in folder "...\Examples\ Surveillance Satellite React-Wheels".

Coupling Coefficients File (.Hpr): The inertial coupling coefficients (or H-parameters matrix) are optional and they are only needed when the flex spacecraft has gimbaling bodies. They are used for modeling the coupling interaction between the spacecraft bending equations and the equations that gimbal the appendages relative to the spacecraft as a result of the applied hinge torques. They are also obtained from the FEM, extracted from the mass matrix, that is, the upper and lower right quadrants of the mass matrix. The original modal data were calculated assuming that the hinges are locked. The hinges are later released in the simulation equations by the coupling coefficients.

The inertial coupling coefficients matrix must be saved in a file *".Hpr"* using the Flixan standard format shown in the example table below. The rows correspond to the modes. The first six rows correspond to the 6 rigid-body modes. The columns correspond the hinges. In the following example we have an H-parameters matrix of 100 rows by 4 columns corresponding to a structure of 100 modes and 4 gimbaling appendages. Each element in the coupling coefficients matrix defines an amount of flex excitation of a bending mode (determined by the row) excited by the rotational acceleration of an appendage (determined by the corresponding column). Below the H-parameters matrix, the moments of inertia matrix of the appendages (4 x 4 in this case) is also included in the same file. They are in the order that corresponds to the H-parameter columns order. The last set of data in the file includes the unit vector directions of the hinges (four hinges in this case). Note, the hinge vector directions are not used by the flex spacecraft modeling program.

```
INERTIAL COUPLING COEFFICIENTS (FROM NASTRAN) FOR
Survellance Satellite with Rotating Solar Arrays and Optical Sensor
NUMBER OF MODES, GIMBALS:         100, 4
Units (ft-lb-sec^2)

-2.269347606E-01  1.747074520E-01 -1.915053482E-01 -2.195309075E-01        Mode # 1
-1.133417583E+00 -1.374561104E-02  3.669942078E-01 -2.549929767E-01        Mode # 2
 4.682515329E-02  7.409226449E-01 -2.031611937E-01 -2.588066122E-01
  -----------------------------------------------------------------
  -----------------------------------------------------------------
 7.590326391E-02  1.119420489E-01 -5.917569875E-04 -3.510433226E-03
-3.350520970E-02 -3.483536353E-03 -1.103515707E-02 -1.353584642E-02        Mode # 99
-2.108828453E-03  4.130124619E-02  9.495321928E-03 -2.236874122E-03        Mode # 100

 MOMENTS OF INERTIA MATRIX OF THE ROTATING BODIES (slug-ft^2)
 6.3954598075     -0.049079742633   0.000000000E+00  0.00000000    Payload Inertia (elevat)
-0.049079742633   1.889057024167   0.000000000E+00  0.00000000    Payload Inertia (azimutth)
 0.000000000E+00  0.000000000E+00  3.170436278333   0.00000000       Right Solar Array
 0.000000000E+00  0.000000000E+00  0.000000000E+00  3.1704239175      Left Solar Array

 Hinge Direction Unit Vectors for the 4 Gimbal Appendages, Payload Elevat & Azimuth, 2 Sol-Arr
 1.0              0.0               0.0               0.0
 0.0             -1.0               1.0               1.0
 0.0              0.0               0.0               0.0
```

When the spacecraft has gimbaling bodies and we plan to use an H-parameters file to model the attached bodies interaction with flexibility, the following line containing the coefficients filename must be included as a last statement in the spacecraft input data set in file (.Inp).

```
Inertial Coupling Coefficients (H-Parameters) File Name for the 4 gimbaling bodies:
Surveillance-Sat.Hpr
```

It lets the program know the (.Hpr) filename where to read the coupling coefficients from. In the example above the H-parameters filename is (`Surveillance-Sat.Hpr`). If this line is not included at the end, but the spacecraft data set ends with the selected modal data the program will assume that the flexible spacecraft has its gimbals "locked " and the model will not allow for relative motion at the hinges. The state-space model will lack the additional gimbal torque inputs, gimbal rotation states and outputs associated with the gimbaling bodies.

State-Space Systems File (.Qdr): The spacecraft state-space model output created by the flexible spacecraft modeling program is saved in a standard Flixan systems file (.Qdr). The system title is the same as the title of the input data set. The definitions of the state-space system variables are listed below the quadruple matrices. This system is usually combined together with the attitude control system and other subsystems to form a closed-loop or open-loop system models that can be used for further analysis. For more details read the Surveillance Satellite example.

Flexible Spacecraft Equations

In the Flexible Spacecraft Modeling program the spacecraft dynamics is defined strictly in terms of mode shapes and frequencies, including also the rigid-body modes which are usually the first six modes. The gimbaling appendages are optional and they are defined by the H-parameters matrix (H). The mode shapes and frequencies are created by the FEM program assuming that all gimbaling hinges are "locked" at a fixed orientation angles. The system consists of three sets of equations. The first set of equations describe how the structure modes are excited by the external forces and torques. The second set of equations describe the rotation of the appendages (α) with respect to the vehicle as a result of the torques (T_α) at the joints, which control the orientation angles of the appendages. The third set is the measurement equations for the translation and rotation sensors mounted to the spacecraft. The interaction between the modes and the rotating appendages is defined in the coupling coefficients matrix (H). This linear multi-body model is only useful for small angle rotations in the order of 5. For large angle slewing, a non-linear multi-body simulation is needed. The linear model is useful for control design and stability analysis purposes. To evaluate, for example, the performance of the attitude control system with respect to commands, sensitivity to disturbances (jitter analysis), and to evaluate the interaction between the flexible modes, the ACS, and the various gimbaling control loops.

The following equation written in matrix form describes the excitation of the bending modes vector ($\underline{\eta}$) by the external forces and torques, and also by the gimbal accelerations (α) via the coupling coefficients matrix.

$$M_G\left(\ddot{\eta} + 2\varsigma\Omega\dot{\eta} + \Omega^2\eta\right) + H\underline{\ddot{\alpha}} = \Phi^T\begin{bmatrix} F \\ T \end{bmatrix}$$

The following dynamic equation describes the rotations of the gimbaling bodies ($\underline{\alpha}$) relative to the vehicle as a result of the torques at the hinges, and also due to structure flexibility ($\underline{\eta}$).

$$I_\alpha\underline{\ddot{\alpha}} + H^T\underline{\ddot{\eta}} = \underline{T}_\alpha$$

Where:
$\underline{\eta}$ is the modal displacement vector for (n) modes
M_G is a (n x n) diagonal matrix whose elements are the modal masses
Ω is a (n x n) diagonal matrix whose elements are the mode frequencies in (rad/sec)
ς is the modal damping coefficient of each mode, typically (ς=0.005)
$\underline{\alpha}$ is a vector of dimension (m) representing the rotation angles of the payloads with respect to the spacecraft, where (m) is the number of gimbaling bodies
H   is the Inertial Coupling Coefficients matrix of dimension (n x m). It couples the motion of the gimbaling bodies with modal displacements vector q
Φ^T is the mode shapes matrix of size (n x 6) containing the modal data at the points where control forces, moments, and disturbances are applied to the s/c structure.
$\underline{F}$ is a vector of the externally applied forces along x, y, and z
$\underline{T}$ is a vector of the externally applied moments about axes x, y, and z
I_α is an (m x m) moments of inertia matrix of the (m) gimbaling bodies about their axis of rotation in (ft-lf-sec^2)
$\underline{T}\alpha$ is a vector of size (l) representing the control torques in (ft-lb) at the payload gimbals.

The moments of inertia matrix (I_α) and the coupling coefficients matrix (H) are extracted from the mass matrix of the finite elements model. The measurements vector $\underline{X}_s$ consists of displacements or rotations at various sensor points on the structure. Each measurement is a linear combination of the generalized mode displacements (η_i), that is both, rigid body plus flexible mode displacements.

$$\underline{X}_S = \Phi_S\underline{\eta} = \sum_{i=1}^{n}\phi_{si}\eta_i$$

Where:
$\underline{X}_s$ is a sensor measurement vector of dimension (6) representing three translations and three rotations at point (s)
Φ_s is a (6 x n) modal matrix containing the mode shapes and slopes for (n) modes at the sensor location (s)

Model Preparation and Reduction

During model preparation we create a set of data in an input file "*.Inp*" that characterizes the input/ output structure of the spacecraft, parameters such as input forces, torques, sensors, directions, etc. The input data set also contains a reduced set of flex modes. The data file is then processed by the program to produce the flex spacecraft state-space model. During data preparation the program searches through the modal data file and selects a smaller number of modes that faithfully represent the spacecraft dynamics between the inputs and outputs defined. A typical finite element model consists of a large number of masses and degrees of freedom used for modeling a complex vehicle structure. This results in a very large number of flexible modes. Several hundreds of the FEM modes (usually below a certain cut-off frequency) are saved in the modal data file (.Mod). It is not practical to include all the modes from the modal data file in the control analysis model, because, (a) it will slow down the analysis, (b) it will complicate the control design, (c) a much smaller number of modes, (20-40 modes), is usually sufficient to approximate the spacecraft dynamics within the control bandwidth and a decade or two higher, between a small set of inputs and outputs. Those inputs and outputs represent the controls, disturbances, measurements, and sensitive locations on the spacecraft.

The modes are selected based on their modal strength, which is computed by the amount of mode controllability at the points of excitation, and the mode observability at the sensors. The dominant modes may vary depending on the locations and directions of the actuators and sensors. The selection of modes in the analysis model strongly depends on the type of analysis that the user is planning to perform. If the model is to be used for attitude control stability analysis and it has attitude sensors for stabilization and thrusters for control, the modes must be selected from the reaction jets to the attitude control sensors. If sensitivity analysis is performed, on the other hand, on a space structure to determine, for example, the effect of crew disturbances to an optical instrument on board, the modes in the analysis model must be selected based on the modal strengths between the crew compartment and the locations of the optical instruments.

The original FEM output contains also a very large number of nodes which are locations in the structural model. Each node in the FEM is identified with a long 5 to 7 digits number. For control analysis, however, a much smaller number of modes and nodes are needed in the analysis model. A typical flex model used for control analysis has between 15 and 40 nodes at locations such as, actuators, sensors, disturbance forces, fuel tanks, etc. Prior to model preparation the analyst must create a nodes mapping file *".Nod"*, which is used by the program in look-up table displays. The flex spacecraft modeling program during model preparation activates a mode selection process where the user selects a set of dominant modes from the modal data file (.Mod). The modes are selected interactively by the user from a display, based on their relative mode strength between a user defined set of inputs and outputs. The extracted set of modal data is also scaled in the process, if needed, in order to match the spacecraft parameters because they may have been created in different units and directions. The selected modal data are also included in the input data file.

When the spacecraft has gimbaling appendages with coupling coefficients defined in the H-parameters file (.Hpr), the following line must be included as a last statement in the spacecraft input data set which defines the coefficients filename, as described earlier.

```
Inertial Coupling Coefficients (H-Parameters) File Name for the 4 gimbaling bodies:
Surveillance-Sat.Hpr
```

If the filename line is not included but the spacecraft data set ends with the selected modal data, the program will assume that the flexible spacecraft has no gimbals. The block diagram in Figure 1 shows the model preparation process and the data files involved.
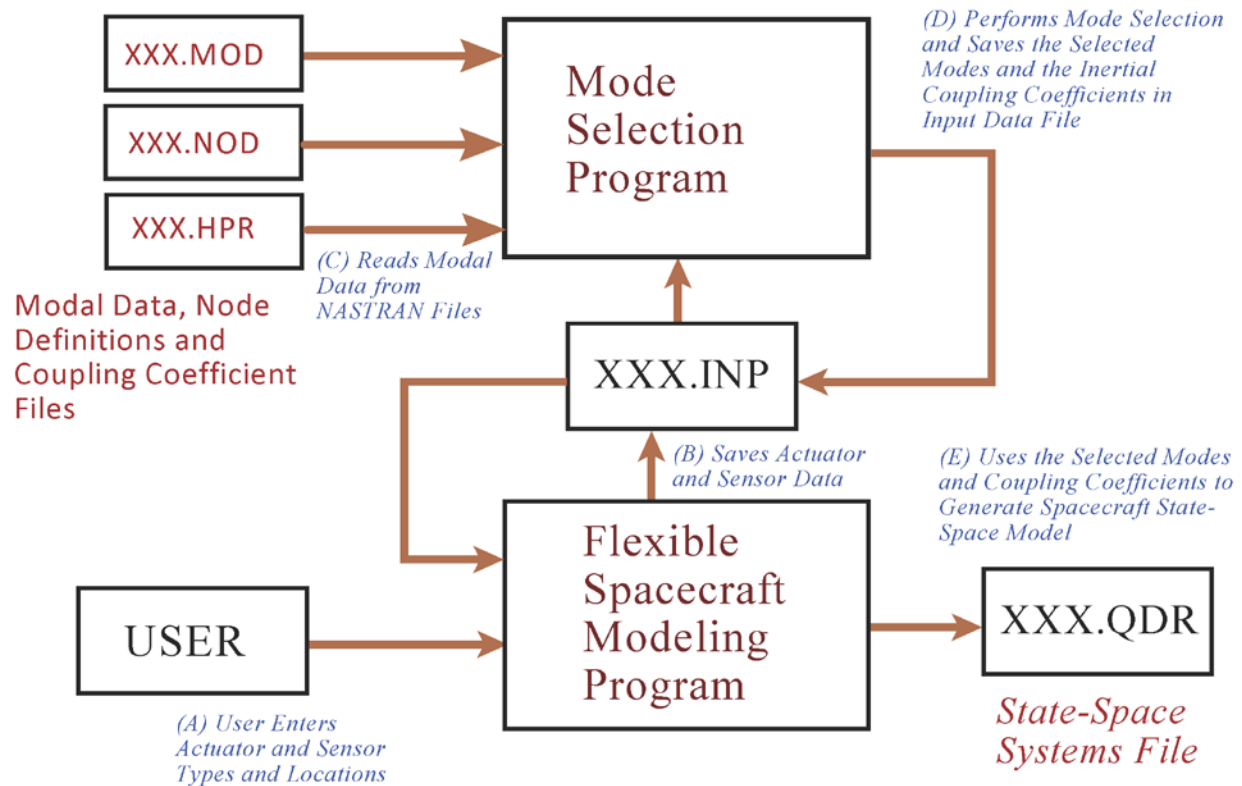

Figure 1 Flexible Spacecraft Modeling Process with Input/ Output Data Files