

# 4 Actuator Models

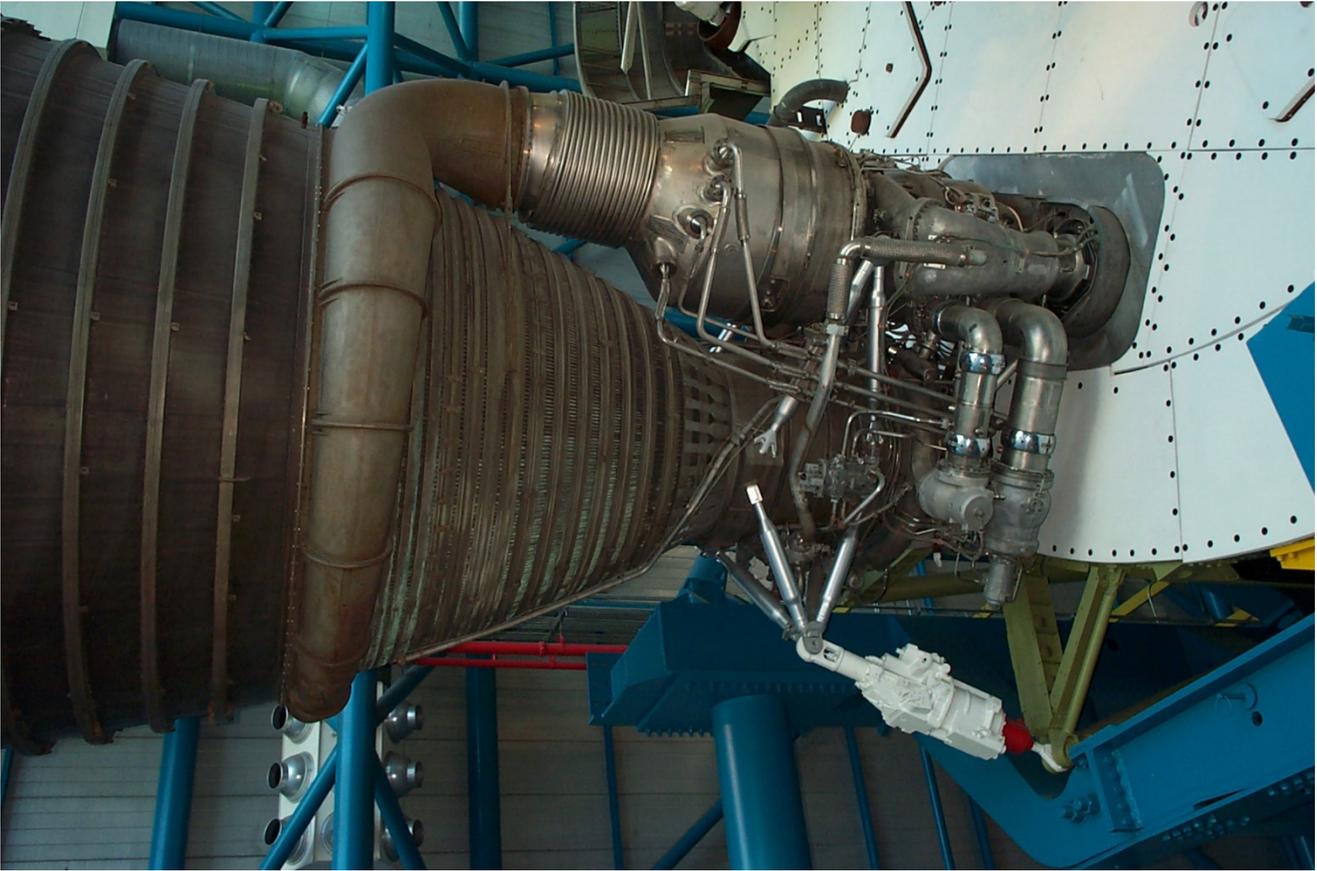
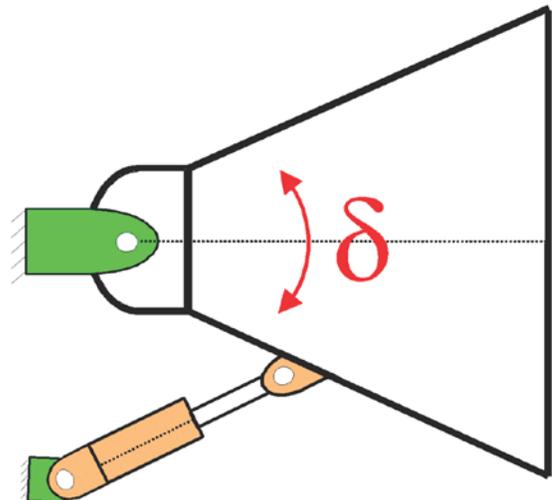


Figure 4.1 TVC engine for a Saturn-5 rocket

Flight vehicles are controlled by Thrust-Vector-Control (TVC) engines that rotate about a gimbal or by control surfaces rotating about a hinge. The purpose of the servo-actuator system is to provide the control force that is needed to rotate an engine or aerosurface in the direction required to maneuver and to stabilize the vehicle. One side of the actuator attaches to the vehicle and the other side which extends is attached to the rotating effector. Since the actuator dynamics greatly affect and couple with the vehicle motion we will dedicate this section to develop actuator models and provide some examples. In launch vehicles, a

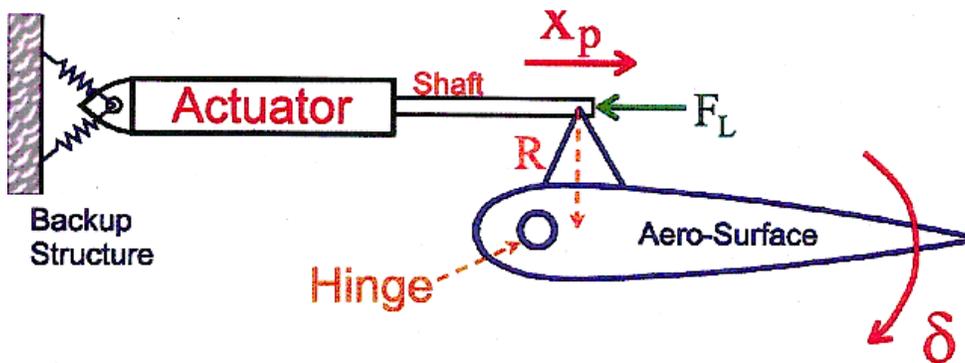
small rotation of the thrust vector angle about the gimbal (typically between  $\pm 5^\circ$  and  $\pm 10^\circ$  in pitch and in yaw) is sufficient to create normal and lateral control forces to stabilize and steer the vehicle, overcome the wind-gust disturbances, and to balance the aerodynamic moments. In an aircraft the control surface deflections are larger, in the order of  $\pm 40^\circ$  relative to the trim position. The control surface is normally the trailing edge of a wing, tail or vertical stabilizer that rotates about a hinge line which is parallel to the



aerosurface and the actuator provides the force to rotate it against the aerodynamic loads. The rotation of the aerosurface changes the airflow around the aircraft and creates the aerodynamic forces and moments needed to trim and to control the flight vehicle. In some cases, it is the leading edge that rotates, and in some aircraft, it is the entire aerosurface that rotates and not just the edge.

Figure 4.1 shows a TVC nozzle of a Saturn-5 rocket connected to the vehicle via the gimbal. The thrust force is transmitted to the vehicle through the gimbal which allows the engine to rotate in pitch and yaw. The actuator is the white hydro-mechanical servo device that provides the force to control the deflection  $\delta$  of the engine in one direction. A TVC engine is typically controlled in two directions by rotating it in pitch and yaw by two orthogonal actuators. One end of the actuator is attached to the vehicle and the other end is an extendable shaft that is attached on the pivoting nozzle via a linkage, as shown in the Saturn-5 Figure 4.1. The extension of the actuator shaft can be varied and it is controlled by a closed-loop servo system that provides hydraulic or electro-mechanical forces on the shaft to rotate and to control the nozzle deflection angle. Each actuator has a sensor that measures the shaft extension and provides feedback to the position control system. The angular rotation of the nozzle is almost proportional to the piston extension, although, not exactly because the measurement is corrupted by the bending and deformation of the support structure, as we shall see. During flight the control system calculates the required changes in the engine's pitch and yaw rotations and the actuators are commanded to either extend or to retract in order to achieve the demanded nozzle deflections and the control forces to guide and stabilize the vehicle.

Figure 4.1b shows an actuator system for an aerosurface. The shaft extension provides the force to rotate the aerosurface about the hinge line. The actuator has to counteract the external forces resisting the motion by providing a load-force  $F_L$  against it. The external forces are mainly due to aerodynamics, but also inertial due to vehicle normal acceleration and vibrations. The actuator extension  $x_p$  is used to calculate the aerosurface rotation. The measurement, however, is slightly corrupted due to deformation of the support structure, shown by the springs.



The selection of an actuator device is determined mainly by the power requirement of the load. The power is determined by the aerodynamic forces and the speed of response. A flight vehicle is easier to be controlled when its static stability is marginal. If the vehicle is passively unstable and diverges too fast then the actuator has to respond fast enough in order to catch up and prevent the instability. If, on the other hand, the vehicle is too stable it becomes less maneuverable and requires a lot of actuator power to steer it. Other factors to be considered for the selection of an actuator include the dynamic characteristics, the power sources available, the reliability of the equipment, and other physical and economic limitations. In this section we will present five actuator models which are typically used in flight control system analysis to pivot the aerosurfaces or TVC nozzles. The actuator models not only

include the shaft motion but also the rotational dynamics of the load and the structural stiffnesses that couple the two dynamic systems together. That is, stiffnesses of the support structure, shaft and load linkage. The shaft position is controlled by a local servo system inside the actuator that receives commands from flight control. The actuator models capture the response of the servo-actuator to commands and the engine or aerosurface rotation at the hinge and the interaction between the two dynamic systems via forces and gear torques.

The actuator models are not only intended to be used alone but also to be coupled with the vehicle flex models and analyze the combined vehicle/ actuator system stability and performance. This is in order to analyze the effects of the "tail-wags-dog" forces on the vehicle at the gimbal produced by the rotational acceleration of the engines and aerosurfaces and also the loading effects against the actuator forces "load-torque" dynamics generated by the vehicle motion. The combined vehicle/ actuator dynamic model must include one actuator per vehicle effector input. The dynamic coupling between vehicle and actuator dynamics must be implemented properly in order to capture the TWD and load-torque dynamic effects. It defines the dynamic interaction between actuators, vehicle dynamics and local structural flexibility at the hinges that affect the local structural stability and actuator performance at the engines or aerosurfaces. The interaction between non-linearities, such as: dead-bands, limits, and Coulomb friction with stiffnesses in the actuator support structure often causes "tail-wag-dog" type of aerosurface oscillations or limit-cycles, if the actuator control system is not properly designed. It is a good practice to keep the engine or aerosurface local bending mode above the TWD zero frequency. We shall demonstrate the dynamic coupling between the vehicle and the actuator systems with examples and will describe the associated stiffnesses. The models are implemented using the Flixan actuator modeling program located under the "*Flight Vehicle Modeling*" group. Each vehicle aerosurface or TVC engine DOF requires a separate actuator model but the actuator models are frequently the same. In order to properly couple the vehicle and actuator dynamics including the TWD/ LT effects the vehicle model must be configured to include the "tail-wags-dog" flag "WITH TWD" that will include the dynamic interaction between local flexibility, actuator dynamics and the flight control system.

The dynamic models require actuator data consisting of parameters such as: piston area, load moment of inertia, amplifier gains, friction coefficients, gear ratios, piston and backup stiffnesses, local geometry, etc. The equations of motion for the five linear actuator types are internally coded in the Flixan actuator program which is similar to the flight vehicle modeling program. It reads the actuator parameters from an input file (.Inp), processes the data, and generates different types of actuator state-space systems in a systems file (.Qdr). The actuator systems have two inputs: (1) engine deflection command  $\delta_c$  in (radians) coming from the flight control system, and (2) load-torque  $T_L$  in (ft-lb), which is an external torque on the actuator generated by aero or vehicle acceleration. The actuator models have three outputs that become inputs to the vehicle dynamic model: (1) effector rotation angle ( $\delta$ ) in (radians), (2) effector rate in (rad/sec), and (3) acceleration in (rad/sec<sup>2</sup>). Definitions of the inputs, states, and output variables are included below the system matrices in (.Qdr) file. In Section (4.3) we will describe the five standard actuator types and demonstrate the operation of the actuator program with examples, but before we get into the actuator details let us first describe how the actuators are combined with the vehicle model.

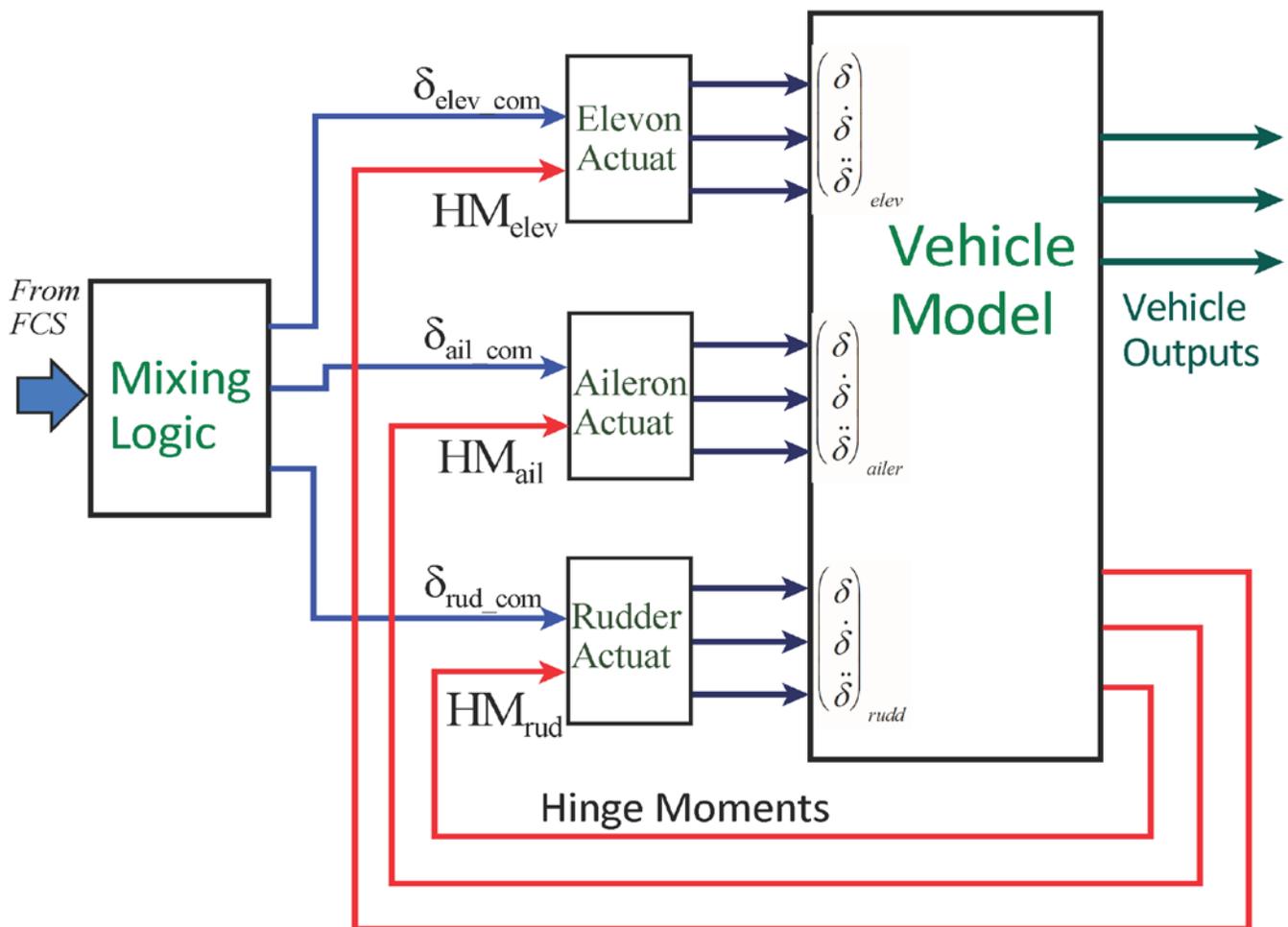
## 4.1 Combining a Flexible Vehicle Model with Actuators

In Flixan the actuator systems are not included in the vehicle equations of motion but they are implemented and analyzed as separate subsystems and are eventually combined with the vehicle dynamics. This offers flexibility in selecting different types of actuators, adjusting the complexity of the model, allows you to include or remove actuator non-linearities in simulations and to analyze the actuator subsystem separately. The actuator models are not just actuator but they include the rotational dynamics of the effectors and the associated stiffnesses of the support structure, actuator/ load linkage, and the compressibility of the shaft. Factors such as, hydraulic fluid compressibility, gimbal friction, nozzle or control surface flexibility, backup structure stiffness, load inertia, non-linearities, and other parameters should be accounted correctly in the actuator model.

The rotational dynamics of the effector load about the hinge is not in the vehicle model but in the actuator. The vehicle model receives the rotational position in (rad), velocity (rad/sec), and acceleration signals from the actuators and calculates the control and reaction forces from the actuator signals. Note, the stiffnesses in the actuator model must be properly accounted depending on the flexibility definition in the vehicle model. If the flexibility of the support structure is already included in the vehicle structural model, it should obviously not be included again in the actuator subsystem. The actuator output signals are not inertial but they represent the relative motion of the effector at the hinge or gimbal, with or without local flexibility.

Figure 4.1.1 shows the interconnection between a flight vehicle and three actuators. The vehicle is controlled by three aerosurfaces (elevon, aileron, and rudder) and each aerosurface is controlled by a separate actuator. The mixing-logic matrix receives roll, pitch and yaw acceleration demands from the flight control system and converts them to elevon, aileron and rudder actuator commands. The aerosurface deflection, rate, and acceleration outputs from the actuators drive the vehicle dynamics. The vehicle, mixing logic matrix, and actuator subsystems are combined together using the Flixan® systems interconnection program or by using Matlab/Simulink®. The aerosurface deflections generate the aero forces (or in the case of a gimbaling engine the TVC forces) that control the vehicle and the accelerations generate the tail-wag-dog forces. The rates create damping forces which are not frequently included.

The actuator subsystems have a second load-torque input to receive external load torques. These are reaction torques generated by external aero forces and vehicle accelerations that close a mechanical feedback loop between vehicle and actuator, see Figure (4.1.1). The load-torques typically oppose the actuator control torques and the actuator must overcome it in order to control the position of the aerosurface or engine. The vehicle model includes load-torque outputs at each hinge. In the case of a TVC engine the vehicle model provides two load-torque outputs for pitch and yaw gimbaling that feed-back to two separate actuators.



**Figure (4.1.1) Vehicle and Actuator Interconnection Diagram**

The analyst must be careful in selecting the stiffness coefficients in the actuator model in order to avoid including them twice, both in the actuator and in the structural models. There are three types of stiffnesses involved in modeling the servo system that controls the position of a control surface or a nozzle relative to the vehicle and they add up together in series to produce a local mode at the hinge:

- (1) The stiffness of the backup structure which is the stiffness of the support structure at the point where the actuator is attached to the vehicle.
- (2) The actuator shaft stiffness consisting of piston plus oil or electrical stiffness, and
- (3) The load linkage stiffness due to structural flexibility at the linkage and the aerosurface itself or the engine nozzle.

The three stiffnesses combine together to a combined stiffness  $K_T$  that in combination with the load moment of inertia produces a local mode. The frequency of this mode must be higher than the actuator bandwidth to avoid exciting it or destabilizing the mode. When analyzing rigid vehicle models, the backup and load stiffnesses are obviously not included in the vehicle and, therefore, all three stiffnesses must be included in the actuator model and in this case the actuator model itself captures the local resonance of the load oscillating at the pivot. The deflection  $\delta$ , rate, and acceleration outputs consist of, not only rigid rotation, but also the effects due to the combined spring constant  $K_T$  from all three stiffnesses.

The position feedback measurement in the actuator control loop is the shaft extension. However, the position sensor measurement includes an error due to the shaft deformation. The measurement does not see the deformation of the backup structure and load. Therefore, all three stiffnesses should be included in the actuator model when: (a) the vehicle model is a rigid body, or (b) vehicle includes only body bending but not local flexibility at the actuator attachments, that is, the back-up structure and the actuator linkage stiffnesses are “rigid” in the finite-elements-model. When the flex vehicle model, however, includes the local structure compliances, they should not be included again in the actuator, otherwise, they are introduced twice. In this case, the actuator model must be stiff and only the shaft or piston stiffness should be included in the model. This type of modeling is important for analyzing the stability and performance of the actuator loop because the flight control system interacts strongly with the actuator dynamics, the load dynamics, and the backup structure, and they often excite each other to instability, mostly due to flexibility and non-linearities, such as, Coulomb friction, backlash, dead-bands, etc.

In the situation where the entire vehicle structure is included in the finite element model, that includes: the aero-surfaces, nozzles, and actuator linkage dynamics. In this case the backup and load stiffnesses are included in the structural model and they must not be included again in the actuator. The hinges or gimbals in the structural model are locked, and it is assumed that there is a rigid link across the two ends of the actuator. In the actuator model the backup and load stiffnesses must be set to infinity and only the actuator piston stiffness should be included. The locked hinges are released in the simulation equations by the actuator models that calculate the rigid rotations, rates, and accelerations for the vehicle equations. The flexing at the hinge due to the effector motion is calculated from the inertial coupling coefficients  $h_s(k,j)$ , according to equations 2.7.4. The actuator models in this case must be “stiff” because they must produce only the “rigid” component of the deflection resulting from the linear extension of the shaft. The deflection due to structural bending at the hinge is already included in the finite element model. Only the actuator shaft stiffness in series with the oil stiffness (for hydraulic actuators), or the shaft stiffness in series with the electrical stiffness (for electro-mechanical actuators), that is  $K_{act}$  instead of  $K_T$ , should be included in the “stiff” actuator model. In the stiff actuator model in Figure 4.3.1b, the backup and load stiffnesses are infinite, because it is assumed that they are included in the structural model. The feedback path via  $(1/K_L)$ , shown in Figure 4.3.1a becomes zero. The shaft stiffness  $K_{act}$  is much stiffer than  $K_T$  which is included in the “soft” actuator model.

## 4.2 Introducing a Compensator

An actuator is a closed-loop position control servo. It is an independent system that controls the position of the effector but also interacting with the flight control system. This local control system must be properly stabilized and perform as expected within the required bandwidth with a satisfactory step response. The dynamic coupling, however, between the actuator, the load and the support structure flexibility introduces a local mode that may degrade the servo system's performance, stability, and it often produces local instability. The actuator servo systems, therefore, often include first or a second order filter compensators  $C(s)$  in the forward or feedback path to enhance the actuator loop stability and improve command following. The actuator modeling program provides the capability to include a first or a second order compensator which is defined in the actuator input data: (1= first order, 2= second order, 0=No compensation). The filters are unit gain, lead-lag, 1<sup>st</sup> or 2<sup>nd</sup> order transfer functions defined as follows:

$$\text{First Order } C(s) = \frac{V_{ef}(s)}{V_e(s)} = \left( \frac{\tau_n s + 1}{\tau_d s + 1} \right)$$

$$\text{Second Order } C(s) = \frac{V_{ef}(s)}{V_e(s)} = \frac{\omega_d^2 \left( s^2 + 2\zeta_n \omega_n s + \omega_n^2 \right)}{\omega_n^2 \left( s^2 + 2\zeta_d \omega_d s + \omega_d^2 \right)}$$

The above transfer functions are implemented in state-space form and are combined with the actuator dynamics. The first order compensator is expressed by the following first order differential equation, where  $V_e$  is the filter input and  $V_{ef}$  is the filter output.

$$\tau_d \dot{x} = -x + \left[ 1 - \frac{\tau_n}{\tau_d} \right] V_e \quad V_{ef} = x + \left( \frac{\tau_n}{\tau_d} \right) V_e$$

The second order filter is expressed by the following state-space representation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -a_2 \\ 1 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_2 \\ b_1 \end{bmatrix} V_e \quad V_{ef} = \begin{pmatrix} \omega_d^2 \\ \omega_n^2 \end{pmatrix} \left\{ \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + V_e \right\}$$

where

$$a_1 = 2\zeta_d \omega_d; \quad a_2 = \omega_d^2; \quad b_1 = 2(\zeta_n \omega_n - \zeta_d \omega_d); \quad b_2 = \omega_n^2 - \omega_d^2$$

- The first order filter is specified in the input data by the numerator and denominator time constants ( $t_n$  and  $t_d$ ).
- The second order filter is specified by the numerator and denominator damping coefficients, and by their frequencies in (rad/sec), ( $\zeta_n, \omega_n, \zeta_d, \omega_d$ ).

### 4.3 Actuator Models

In this section we present equations of motion that describe the dynamics of some commonly used actuator models which are implemented in the Flixan actuator program. Examples of each model are also presented. The nomenclature used between actuator models, such as, moments of inertia, stiffnesses, etc. may be different. The actuators most commonly used in flight vehicles for positioning aerosurfaces and engine nozzles fall into three categories: (a) electro-hydraulic, (b) electro-mechanical, and (c) pneumatic. Pneumatic have limited usage due to low bandwidth and lack of fine positioning accuracy and they are not included here. We begin with a very simple generic model where most of the servo system details are simplified. It captures, however, the main functions which are: actuator shaft positioning servo, load dynamics, and stiffness of the actuator and the supporting structure. This model is used in the early phases of a flight vehicle design when the actuator details are not yet fully defined. It is simple because the servo loop dynamics is reduced to a first order transfer function driving the load dynamics. The next two models are hydraulic actuators taken from two separate sources and the last two are electro-mechanical (EMA). The parameters of these standard models are read by the program from an input data file and they can be adjusted to fit most actuator systems.

Electro-hydraulic actuators are most commonly used for launch vehicle thrust-vector-control, and have gained the widest acceptance in the aerospace industry. They have great power capability and can deliver larger torques than electrical equipment of comparable size and weight. For continuous operation they offer a minimum (equipment/power) ratio. Where intermittent operation is required, a hydraulic system can provide large amount of power from a small volume of accumulator. Their dynamic characteristics are expressed by small time constants and they develop higher peak torque per inertia ratios. The most common form of utilization of hydraulic servos in vehicle control loops consists of high-pressure supply (pump), an electro-hydraulic servo valve, a hydraulic actuator (cylinder), a feedback transducer and a servo-amplifier. The hydraulic power supplies currently used are of two main types. The first type employs a variable displacement pump whose output flow is controlled by means of a servo sensing the high-pressure side of the hydraulic system. A relief valve is also connected from the high-pressure side to the low-pressure side to minimize pressure transients above the operating pressures of the system. For normal operations the valve remains closed, opening only when the pressure exceeds a value overcoming the pre-load on the relief valve. The second type of power supply uses a fixed displacement pump with a relief valve to maintain the supply pressure within set limits, as well as to meet the normal flow requirements. In this system the relief valve is normally open so that supply pressure and valve opening maintain flow through the relief valve equal to the flow output of the fixed displacement pump. When there is a flow demand the relief-valve closes and the supply pressure is reduced. The dynamics of both power supply and relief valve exhibit a fairly flat response with small phase shift within the bandwidth of the overall servo loop, therefore, the supply pressure will be assumed to be constant at the value of zero flow demand. Electro-hydraulic valves are designed for flow or pressure control. These units are highly complex devices and exhibit high order non-linear responses. Still, in the frequency range of interest, they can be represented by a first or second order transfer functions.

The Electro-Mechanical actuators (EMA) are becoming more popular mainly because of simplicity, reliability and reduced need for maintenance in comparison with the hydraulic systems. The electro-mechanical actuator system is powered by a dc motor. The motor is driving a screw gear through a gear reduction that increases the screw torque. The spiral screw is rotating inside the shaft which surrounds the screw like a nut. When the screw mechanism rotates it converts the rotational motion from the motor to translation that extends or retracts the shaft as it pushes against the load. The end of the shaft is connected to the load via a linkage mechanism that can make it pivot about a hinge.

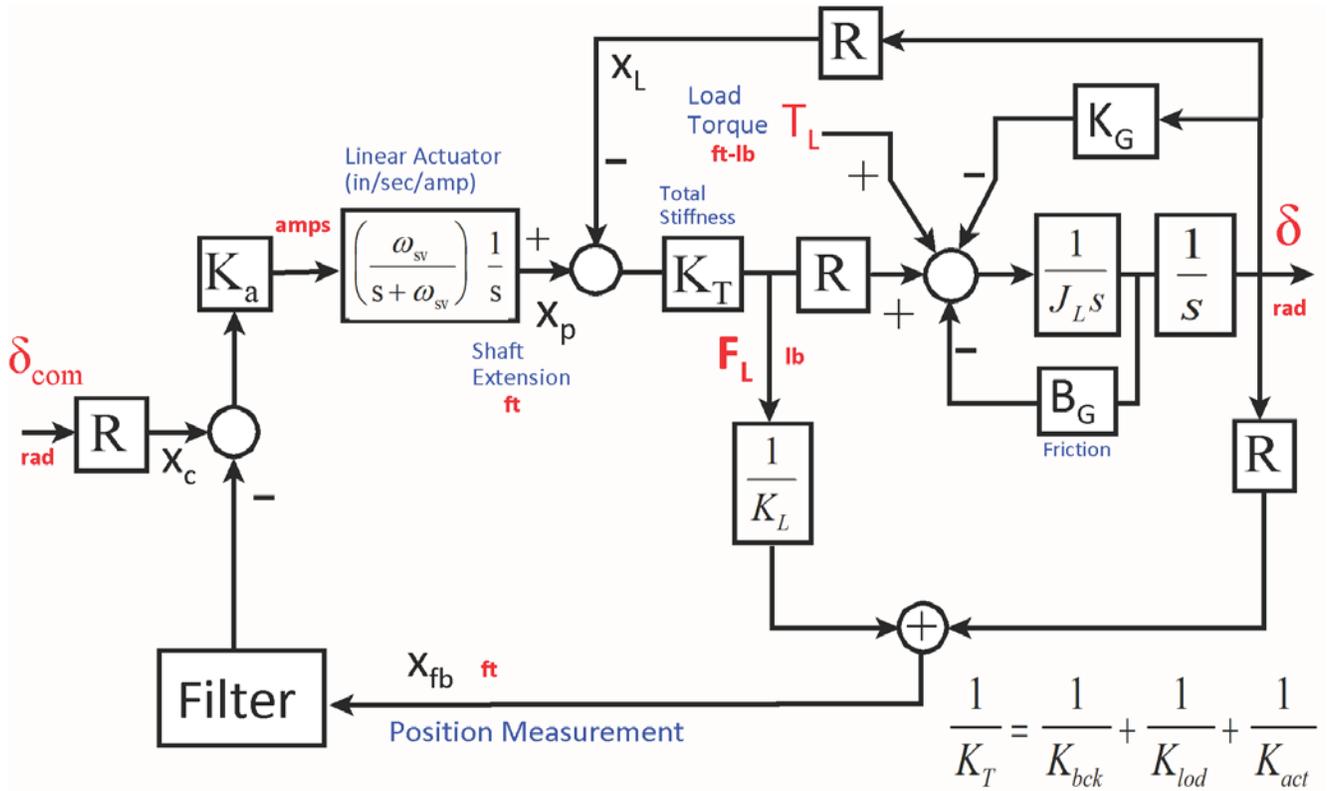
### 4.3.1 Simple Actuator Model

A simple actuator model is shown in figure 4.3.1. The input is gimbal deflection command  $\delta_{com}$  in (rad) which is converted to shaft extension command  $x_c$  in (feet). On the left side we have the actuator device which is a linear velocity control servo that converts the position error to shaft velocity  $v_p$  which is integrated to calculate shaft position  $x_p$ . The bandwidth of the linear actuator is  $\omega_{sv}$  represented by a first order transfer function. The input to the actuator device is amps coming from the feedback error amplifier that has gain  $K_a$  which is converted to shaft velocity  $v_p$  and integrated into shaft position  $X_p$ . The shaft is pushing against the total system stiffness  $K_T$  and creates a shaft force  $F_L$ .  $K_T$  consists of three type of stiffnesses combined in series:

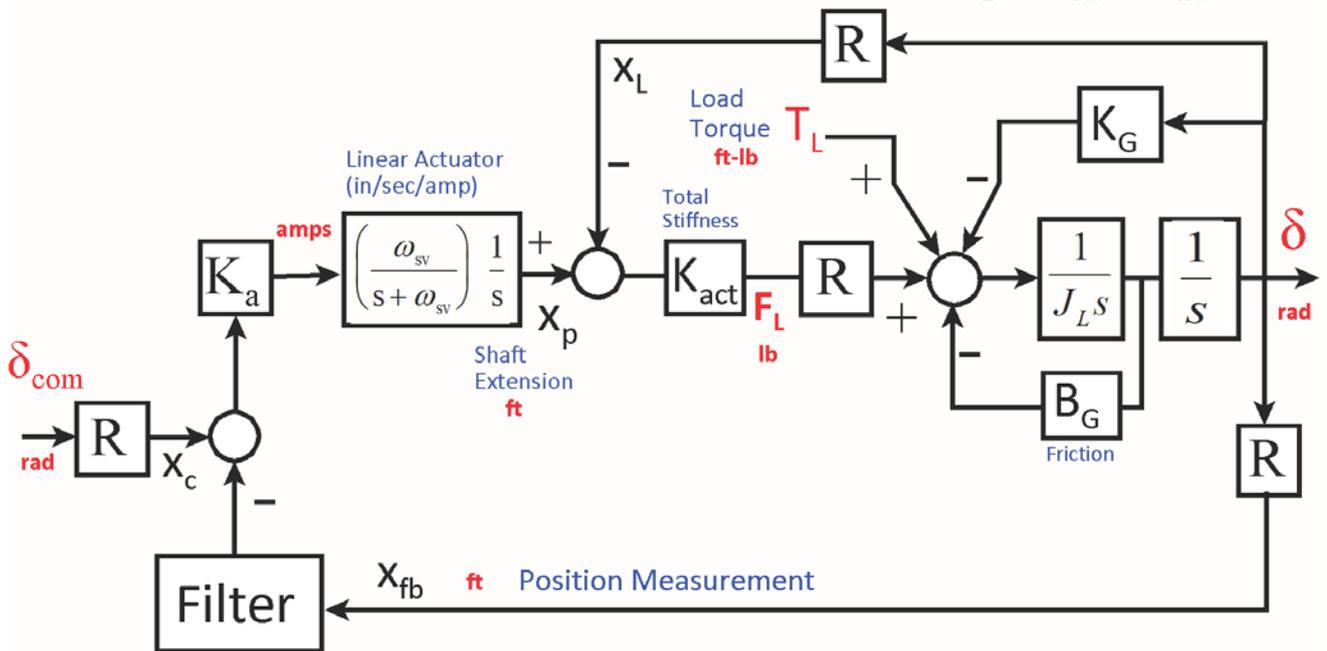
- (1) Stiffness of the backup structure  $K_{bck}$ , that is, where one side of the actuator is attached to the vehicle,
- (2) Actuator stiffness  $K_{act}$  consisting of piston plus oil (or electrical) stiffness, and
- (3) Load stiffness  $K_{lod}$  caused by the flexibility at the linkage where the shaft is attached to the load.

The shaft force is pushing against the load through the moment arm  $R$  and it produces the control torque that rotates the nozzle or aerosurface that has inertia  $J_L$ .  $X_p$  is the position of the shaft's end relative to the actuator and  $X_L$  represents the linear displacement at the nozzle attachment as a result of a load rotation angle  $\delta$ , but this is not necessarily equal to  $X_p$ , because of structural bending at the attachments. In fact, it is the difference between the piston extension and linear nozzle displacement ( $X_p - X_L$ ) that creates the force  $F_L$  pushing against the load via the combined stiffness  $K_T$ .

The control torque due to shaft extension is not the only torque driving the load. There is also an external load torque  $T_L$  that is produced by the vehicle inertial accelerations (both rigid and flex) and by the aerodynamic forces. This external to actuator load torque comes from the vehicle system outputs and creates a mechanical feedback loop. The rotations of the load ( $\delta, \dot{\delta}, \ddot{\delta}$ ) are relative to the vehicle gimbal as a result due to shaft extension  $X_p$  plus external loading torques pushing against the combined structural stiffness  $K_T$ . The gimbal torques also include a component due to viscous damping at the gimbal  $B_G$  and a rotational stiffness at the gimbal  $K_G$  caused by pipes and other mechanical hardware connected to the nozzle. Static and Stiction friction cannot be included in the linear model but only in simulations.



### Simple Actuator Model

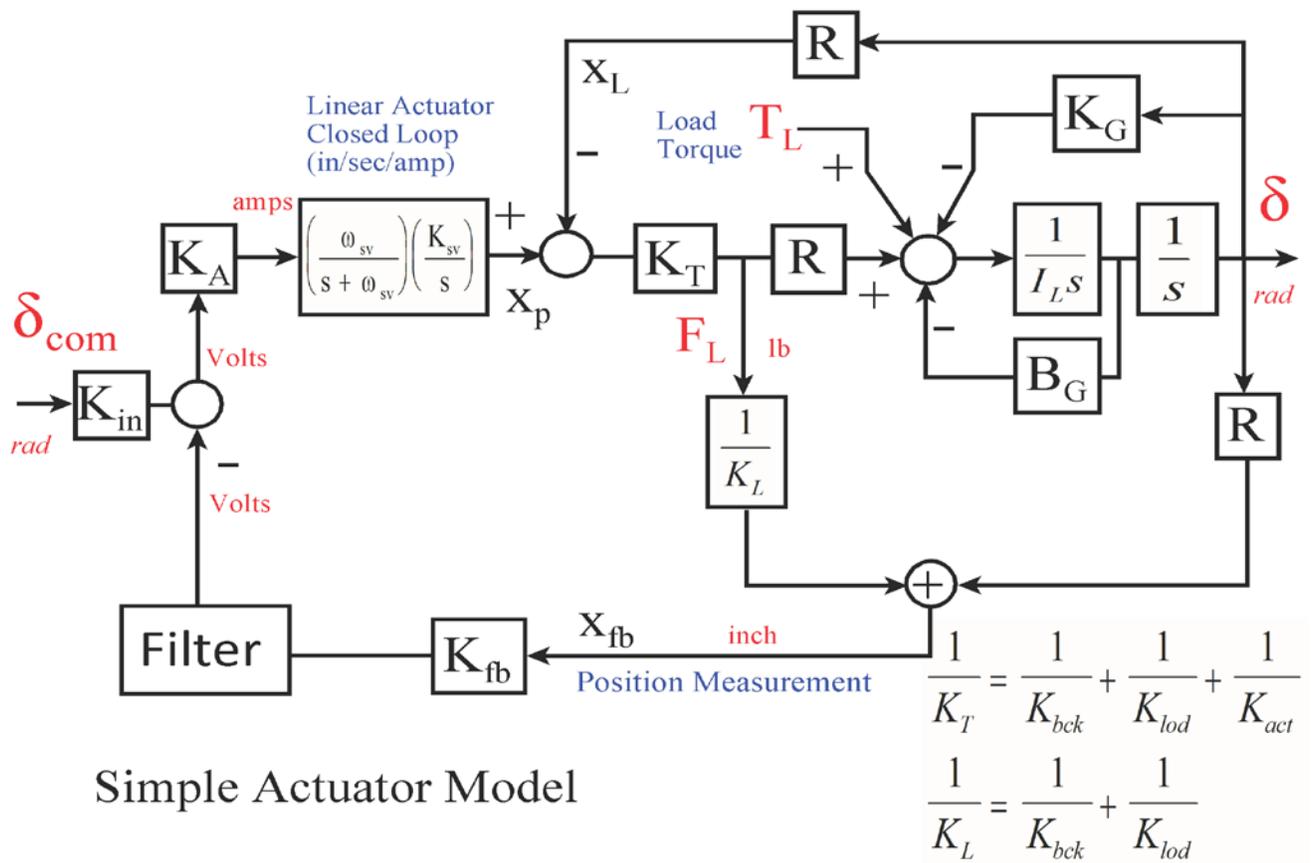


### Stiff Actuator Model

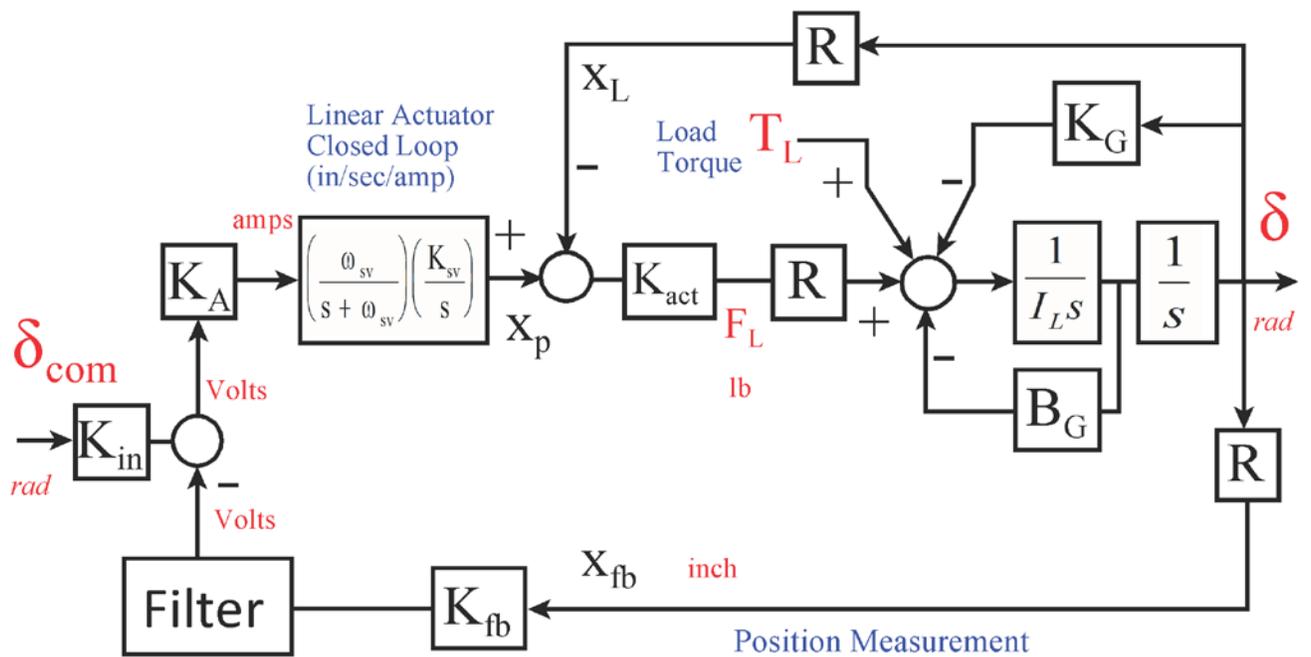
Figure 4.3.1 Simple Actuator Model that includes all 3 Stiffnesses and a “Stiff” version that includes only the Actuator Shaft Stiffness  $K_{act}$ ; Backup and Load Stiffnesses are Infinite.

The actuator position measurement  $X_{fb}$  is located inside the actuator device and it is intended to measure the actuator shaft position. If you ignore the structural stiffnesses, the effector deflection for small angles is nearly proportional to the actuator position. The sensor measures the extension of the rod relative to the actuator device, but this does not necessarily measure the total rotation of the nozzle because the vehicle support structure where the actuator is attached bends as a result of the applied force. The linkage mechanism that connects between the shaft and load also bends, and the actuator rod itself compresses under load. The actual displacement of the effector, therefore, is not only due to the rotation at the hinge but it includes a component due to structural deformations that makes it different from the measurement, and this error must be captured in the sensor model. The position sensor inside the device cannot obviously “see” the deformations at the nozzle and at the backup structure attachments, both stiffnesses combined in  $K_L$ . The measurement, however, is affected by the actuator shaft stiffness  $K_{act}$ . When it compresses, the end of the rod is not exactly where the measurement thinks that it is. In Figure (4.3.1a) the position sensor measurement signal  $X_{fb}$  consists of two components: a component  $(R\delta)$  due to the rotation at the gimbal, and a component,  $(K_T/K_L)(x_p - R\delta)$ , where  $K_T$  is the combined stiffness from all three sources.

Let us assume that the actuator piston position is locked at zero and an external load  $T_L$  is applied to the system. The effector will deflect at an angle  $\delta$  as a result of all three stiffnesses: shaft, load, and backup stiffnesses combined in  $K_T$ . The position sensor in the actuator, instead of measuring zero because the piston does not move, it is measuring a small displacement caused by the compression of the piston, plus some additional compression due to oil or electrical stiffness which are all lumped in the actuator stiffness  $K_{act}$ . The sensor does not see the bending of the backup structure and load. In the measurement we, therefore, subtract from the total flex rotation  $\delta$  the displacement components due to backup and load stiffnesses  $(F_L/K_L)$  and the remaining measurement is the actuator shaft deformation due to stiffness  $K_{act}$ . In the extreme situation, when  $(K_{act} = \infty)$  and  $(K_T = K_L)$ , the feedback measurement becomes equal to the actual shaft position ( $X_{fb} = X_p$ ). In Figure (4.3.1b) we assume that the backup and load stiffnesses are infinite and they have been taken out of the actuator model because they are included in the vehicle structural model, thus avoiding including them twice. Figure (4.3.1b) is used in flexible models.



Simple Actuator Model



Stiff Actuator Model

Figure (4.3.1b) Another Version of Simple actuator model and a “Stiff” alternative that includes only the actuator Shaft Stiffness  $K_{act}$ . Backup and Load Stiffnesses are Infinite.

### 4.3.2 Hydraulic Actuator, Model (A)

The first actuator is an electro-hydraulic servo system shown in Figure (4.3.2). It comprises of a high-pressure hydraulic supply (pump), a servo amplifier, a hydraulic actuator, a servo valve, and a position transducer. The electro-hydraulic servo valve is a flow control type. There is a leakage orifice across the load piston and a differential pressure feedback loop from a load pressure transducer to the servo-valve. The hydraulic cylinder is attached to the vehicle at a support point that has a backup stiffness  $K_{mc}$ . On the other side of the cylinder the piston is attached to the load (a TVC engine). The piston has a spring constant  $K_{mp}$ . This type of electro-hydraulic actuator can be described by the following equations.

The error voltage signal  $V_e$  at the input of the servo-amplifier is:  $V_e = V_c - V_f$ , where  $V_c$  and  $V_f$  are the command and feedback signals respectively. The feedback transducer is integrally built within the cylinder and it measures the nozzle position by measuring the the piston displacement relative to the cylinder. It feeds it back into the servo-amplifier.

$$V_c = K_c \delta_{com} ; \quad V_f = K_f (x_p - x_c) \quad (4.3.2.2 \text{ \& } 3)$$

The error signal  $V_e$  drives a unit gain compensator  $C(s)$ , and the output from the compensator is:

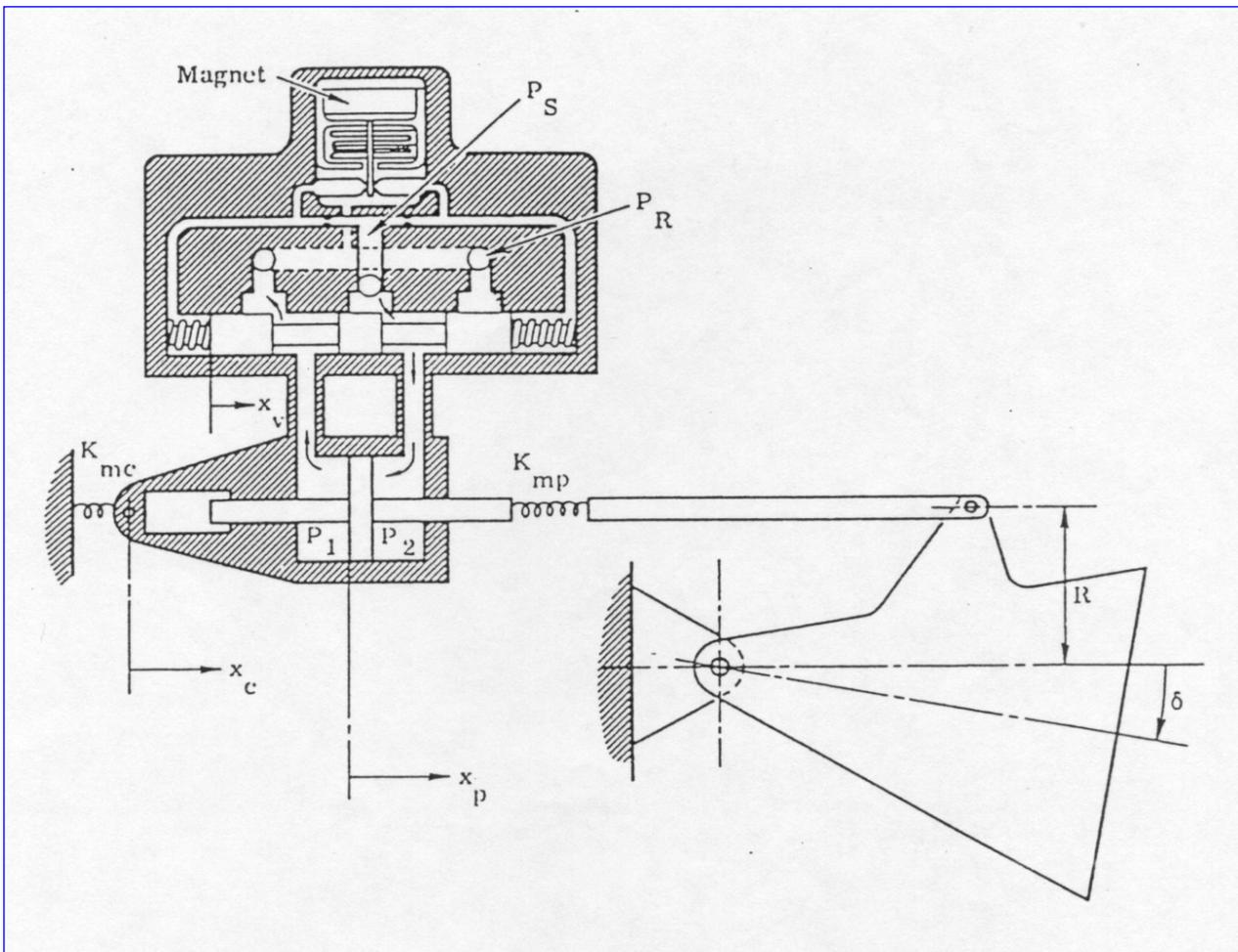
$$V_{ef}(s) = C(s) V_e(s) \quad (4.3.2.4)$$

The servo-valve is a flow-control type coupled to an actuator having a leakage orifice across the load piston to provide damping. The output piston rod is hinged to the engine nozzle that moves about its gimbal point, see figure (4.3.2.1). We can assume that the flow from the servo-valve into one side of the cylinder is equal to the flow out of the other side. For low load pressures, the flow of hydraulic fluid ( $Q_v$ ) from the servo-valve is:

$$Q_v = K_2 x_v \sqrt{P_s - P_r} - l_v P_r \quad (4.3.2.5)$$

Where:

- $x_v$  is the valve spool displacement (ft),
- $P_s$  is the supply pressure to the valve (lb/ft<sup>2</sup>),
- $P_r$  is the reservoir or exhaust pressure (lb/ft<sup>2</sup>),
- $K_2$  is a constant,
- $P_L$  is the differential pressure across the piston,
- $l_v P_L$  is the leakage across the spool (ft<sup>3</sup>/sec).



**Figure (4.3.2) Schematic of a Hydraulic Servo**

The total flow of hydraulic fluid, ( $Q_v$ ) from the servo valve into the actuator cylinder is the sum of the piston displacement flow, leakage, and compressibility flow.

$$Q_v = A(\dot{x}_p - \dot{x}_c) + C_l P_L + \left( \frac{V_t}{4B} \right) \dot{P}_L \quad (4.3.2.6)$$

Where:

- A is the piston cross sectional area ( $\text{ft}^2$ )
- $P_L$  is the load pressure in ( $\text{lb}/\text{ft}^2$ )
- $C_l$  is the orifice leakage coefficient in  $(\text{ft}^3/\text{sec})/(\text{lb}/\text{ft}^2)$
- $\delta_c$  is the commanded engine rotation in (radians)
- $x_p$  is the piston rod displacement, (ft)
- $x_c$  is the cylinder mount displacement with respect to the vehicle, in (ft)
- R is the moment arm in (ft) from the engine pivot to the piston rod, see figure (4.3.2.1).
- $K_f$  is the feedback transducer gain (volt/ft)
- $V_t$  is the volume of fluid under compression ( $\text{ft}^3$ )
- B is the bulk modulus of the hydraulic fluid ( $\text{lb}/\text{ft}^2$ ).

The load pressure ( $P_L$ ) displaces the piston rod and the engine. The displacement ( $x_p$ ) with respect to the cylinder, and the displacement of the cylinder mount relative to the vehicle structure ( $x_c$ ) is obtained from the following equations

$$A P_L = K_{mp} (x_p - R \delta); \quad \text{and} \quad x_c = -\frac{A P_L}{K_{mc}} \quad (4.3.2.7 \ \& \ 8)$$

Where:

- $K_{mp}$  is the spring constant of the piston rod (lb/ft),
- $K_{mc}$  is the spring constant of the cylinder mount (lb/ft),
- $R$  is the moment arm of the actuator from engine gimbal (ft)
- $\delta$  is the engine rotation in radians.

From equations (4.3.2.6) to (4.3.2.8) we obtain the following equation, where  $Q_v$  represents the valve flow rate in (ft<sup>3</sup>/sec) which is proportional to the spool displacement.

$$\dot{P}_L = \frac{(-C_l P_L + Q_v - A R \dot{\delta})}{\left[ \frac{V_t}{4B} + A^2 \left( \frac{1}{K_{mp}} + \frac{1}{K_{mc}} \right) \right]} \quad \text{where :} \quad (4.3.2.9), (4.3.2.10)$$

$$Q_v = K_v \left\{ K_a V_{ef} - K_{dpf} \left( \frac{\tau_c s}{\tau_c s + 1} \right) P_L \right\}$$

The second term in equation (4.3.2.10) is a dynamic pressure feedback term from the load pressure to the servo valve and it is used to control system damping, where:

- $K_v$  is the servo-valve flow gain in (ft<sup>3</sup>/sec/ma)
- $K_a$  is the servo-amplifier gain in (ma/volt)
- $K_{dpf}$  is the dynamic pressure feedback gain in (ma/lb/ft<sup>2</sup>)
- $\tau_c$  is the dynamic pressure feedback time constant in (sec)
- $A$  is the piston cross sectional area (ft<sup>2</sup>)

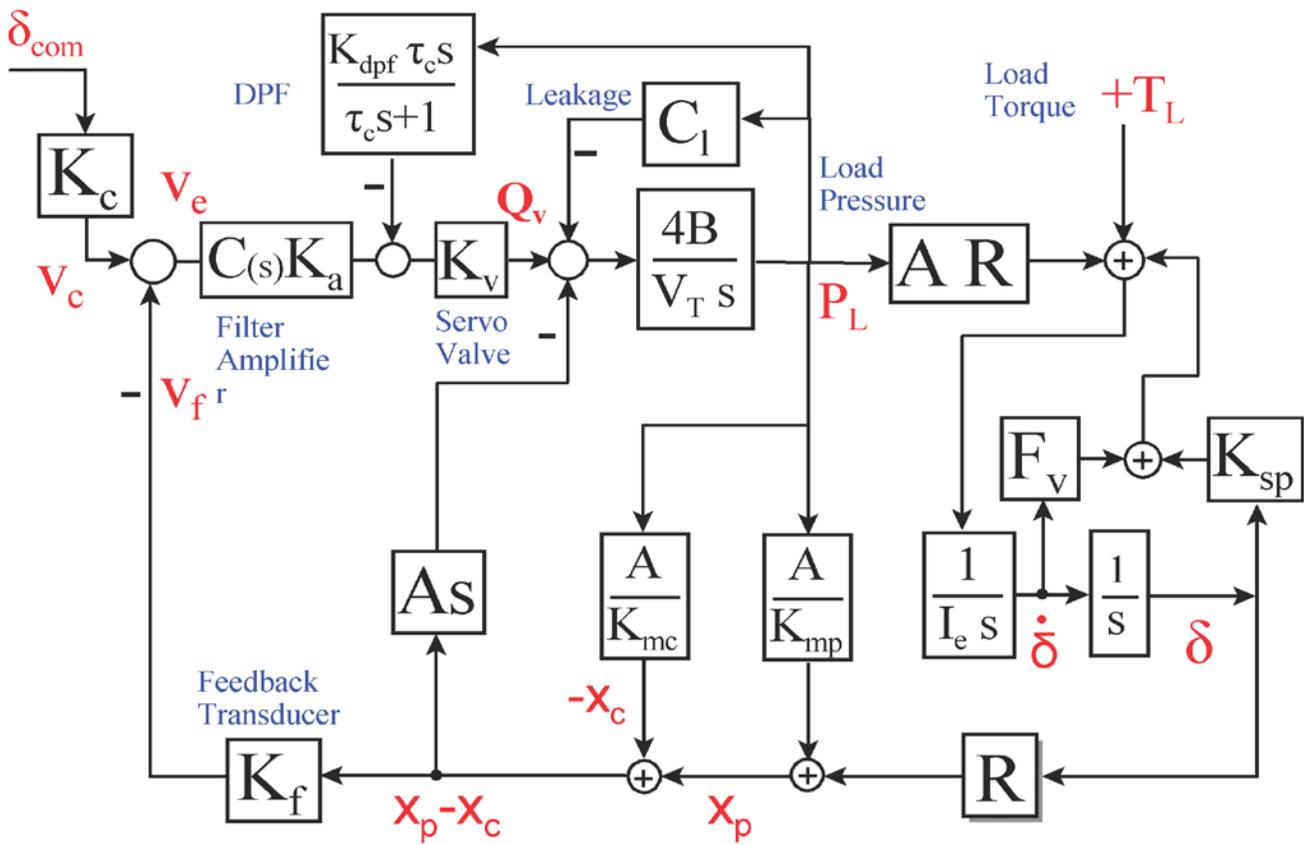
The moment equation of the engine nozzle rotation about its pivot is as following:

$$I_e \ddot{\delta} = (R A) P_L + T_L - F_v \dot{\delta} - K_{sp} \delta \quad (4.3.2.11)$$

Where:

- $I_e$  is the engine inertia in (slug-ft<sup>2</sup>),
- $F_v$  is the viscous friction coefficient (ft-lb)/(rad/sec),
- $K_{sp}$  is the engine bearing spring constant (ft-lb/rad),
- $\delta$  is the engine rotation in radians.
- $T_L$  is the load torque due to the vehicle motion (ft-lb)

The first term on the right-hand side of equation (4.3.2.11) represents the torque on the engine nozzle due to the actuator piston pressure. The second term is the external load torque created by the vehicle motion (rigid body and flexibility). The third term is a damping torque due to friction at the gimbal, and the fourth term is the torque due to spring stiffness at the pivot.



Electro-Hydraulic Actuator Model (A)

### 4.3.3 Hydraulic Actuator, Model (B)

This actuator model is also an electro-hydraulic type and it was used to model the Space Shuttle actuators for linear analysis. Its block diagram is shown in Figure (4.3.3) and the equations of motion are shown below. The model is relatively simple because a lot of the details are left out or combined into gains. The piston force is calculated from the following equation:

$$F_L = K_T (x_r - R \delta) \tag{4.3.3.1}$$

Where:

- $F_L$  is the piston load force in (lb)
- $K_t$  is the piston rod stiffness in (lb/ft)
- $x_r$  is the actuator rod displacement in (ft)
- $R$  is the distance between the engine pivot and the actuator rod (ft), see Figure (4.3.2)
- $\delta$  is the nozzle or surface rotation about the gimbal (radians).

The nozzle deflection measurement is calculated from a displacement sensor located in the actuator and not at the gimbal. The measurement in Equation (4.3.3.2) is corrected to account for the backup structure deformation, as it was already explained in the simple actuator model, in Section (4.3.1).

$$\delta_{fb} = K_{fb} \left( R \delta + \frac{F_L}{K_L} \right) \quad (4.3.3.2)$$

Where:

- $K_{fb}$  is the position feedback gain in (rad/ft)
- $K_L$  is the engine mount structural stiffness in (lb/ft)

The error signal  $\delta_e$  is the difference between the commanded and the sensed engine rotation. This signal drives the unit gain compensator  $C(s)$ , which is optional. It may be used to improve the actuator loop stability or performance.

$$\delta_e = \delta_c - \delta_{fb} \quad \text{and} \quad \delta_{ef} = C(s) \delta_e \quad (4.3.3.3)$$

The Differential Pressure Feedback (DPF) loop is an acceleration feedback. It uses a primary pressure feedback loop and it is intended to smooth out the actuator response by increasing the system damping and to minimize the steady-state error. A sensor measures the pressure differential across the piston, the signal is shaped with a high pass filter in the electronics and it is fed back in the actuator command. The gain  $K_{dp}$  is a non-linear function that relates the rate feedback signal as a function differential force. The DPF loop can be approximated with the following first order transfer function

$$\frac{T_D(s)}{F_L(s)} = K_{dp} \frac{\tau_c s}{\tau_c s + 1} \quad (4.3.3.4)$$

Where:

- $K_{dp}$  is the DPF loop linearization gain in (ft-lb/lb)
- $\tau_c$  is the DPF loop time constant

The actuator piston displacement is:

$$\dot{x}_r = K_{act} \left( K_{av} \delta_{ef} - T_D \right) \quad (4.3.3.5)$$

Where:

- $K_{act}$  is the power valve and actuator gain, (ft/sec)/(ft-lb)
- $K_{av}$  is the amplifier & torque motor gain, (ft-lb/rad)

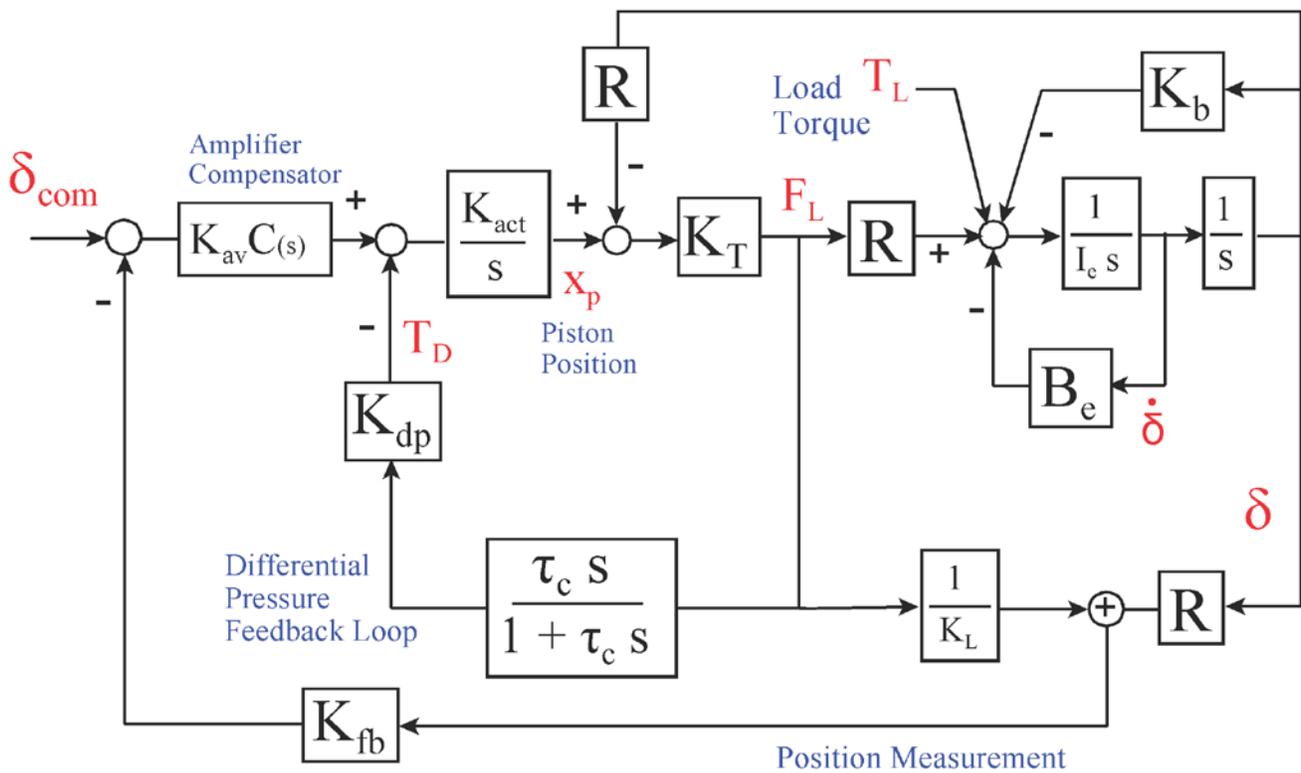
The effector deflection in (radians) is obtained from the following equation

$$I_e \ddot{\delta} = -B_e \dot{\delta} - (K_b + R^2 K_T) \delta + T_L + (R K_T) x_r \quad (4.3.3.7)$$

Where:

- $B_e$  is the engine viscous damping in (ft-lb)/(rad/sec)
- $K_b$  is the engine gimbal spring constant in (ft-lb/rad)
- $T_L$  is the load torque in (ft-lb)
- $K_T$  is the combined stiffness in (ft-lb/rad)

The first term on the right side of equation (4.3.3.7) represents the engine torque due to viscous damping, the second term is engine torque due to the combined (gimbal plus piston) stiffness, the third term is the external load torque generated by the normal acceleration of the vehicle, (rigid body plus bending). The last term represents the torque on the engine due to the actuator piston displacement ( $x_r$ ) pushing against the combined system stiffness  $K_T$ , consisting of: backup structure, load linkage, and actuator piston stiffness. They combine in series by adding their inverses, as calculated in Figure (4.3.1).



### Hydraulic Actuator Model (B)

Figure (4.3.3) Shuttle Hydraulic Actuator Block Diagram

### 4.3.4 Electro-Mechanical Actuator (EMA), Model (A)

The block diagram of an EMA servo actuator is shown in Figure (4.3.4). It consists of an electric motor that provides the torque to move the actuator rod through a system of gears that convert the rotational motion to linear motion of the rod. The actuator is attached to the load (which is either an engine nozzle or a control surface) via a stiff mounting mechanism, and the load is free to rotate about a gimbal. The actuator motor “sees” the engine load via a large gear reduction ratio (N). The motor control system uses rotation angle position feedback ( $\Theta$ ) to control the motor position, in combination with motor speed for damping. An optional filter C(s) may be included to compensate the error signal ( $V_e$ ) and to improve performance, if necessary. The filter output drives the amplifier which provides current to the motor. The following equations describe the EMA servo system dynamics starting with the Motor Inertia Equation.

$$J_m \ddot{\Theta} = T_m - K_{md} \dot{\Theta} - \left( \frac{G_t}{N} K_m R \right) x_f \quad (4.3.4.1)$$

Where  $T_m$  is the torque generated by the electric motor in (ft-lb), the second term on the right is the torque due to viscous damping, and the third term is the torque due to the actuator displacement via the stiff mounting mechanism.

- $J_m$  is the motor shaft inertia in (ft-lb-sec<sup>2</sup>)
- $\Theta$  is the motor rotation angle in (radians)
- $R$  is the actuator moment arm from the engine pivot in (ft).
- $K_m$  is the spring constant of the actuator/engine flexion attachment (lb/ft)
- $N$  is the Gear Ratio
- $G_t$  is the efficiency factor for the motor mechanical conversion, (typical value 1 - 1.5)
- $x_f$  is the actuator rod displacement in (ft)
- $K_{md}$  is the motor viscous damping coefficient (ft-lb/rad/sec).

The following equation couples the motor rotation angle to actuator displacement  $x_f$  (ft).

$$x_f = R \left( \frac{\Theta}{N} - \delta \right) \quad (4.3.4.2)$$

Where: the first right term of equation (4.3.4.2) is due to the motor shaft rotation ( $\Theta$ ), and the second is due to the engine rotation ( $\delta$ ). The engine nozzle rotation is obtained from the following equation.

$$J_e \ddot{\delta} = -F_v \dot{\delta} - K_{sp} \delta + T_L + K_m R x_f \quad (4.3.4.3)$$

Where the first term on the right hand side is the torque due to engine bearing viscous friction ( $F_v$ ), the second term is due to engine bearing spring constant ( $K_{sp}$ ), the third is load torque due to vehicle acceleration (coming from the vehicle model), and the last term is the torque due to the actuator motion ( $x_f$ ) coupled to the engine via the flexion spring constant  $K_m$ . The position error signal  $V_e$  in (volt) is described by the following equation:

$$V_e = G_v \{G_p \delta_c - K_{pl} (\delta + B_i x_f) - K_{pm} \Theta\} \quad (4.3.4.4)$$

Where:

- $J_e$  is the engine load inertia in (ft-lb-sec<sup>2</sup>)
- $\delta$  is the engine rotation angle in (radians)
- $G_v$  is a position to voltage conversion constant (volt/count)
- $G_p$  is an input angle command conversion constant (count/rad)
- $K_{pl}$  is position feedback gain from the load (counts/rad)
- $B_i$  is the sensor bias (rad/rad)
- $K_{pm}$  is position feedback gain from the motor in (counts/rad)
- $K_v$  is the rate feedback gain volts/(rad/sec)

The signal ( $V_i$ ) that drives the servo-motor amplifier and the servo-amplifier transfer function are as follows, where  $C(s)$  is the compensator

$$V_i = C(s)V_e - K_v \dot{\Theta} \quad \frac{V_o(s)}{V_i(s)} = K_a \left[ \frac{I}{\tau_a s + I} \right] \quad (4.3.4.5) \text{ \& } (4.3.4.6)$$

The Motor Torque Equation is

$$T_m(s) = \frac{K_t}{R_m} \left( \frac{I}{\tau_m s + I} \right) (V_o - K_{emf} \dot{\Theta}) \quad (4.3.4.7)$$

Where:

- $K_a$  is the amplifier gain in (volts/volt)
- $\tau_a$  is the amplifier time constant in seconds
- $T_m$  is the motor torque in (ft-lb)
- $K_t$  is the motor current to torque constant (ft-lb/amps)
- $R_m$  is the motor winding resistance in (ohms)
- $\tau_m$  is the motor time constant in (seconds)
- $K_{emf}$  is the motor back-emf gain (volt/rad/sec)

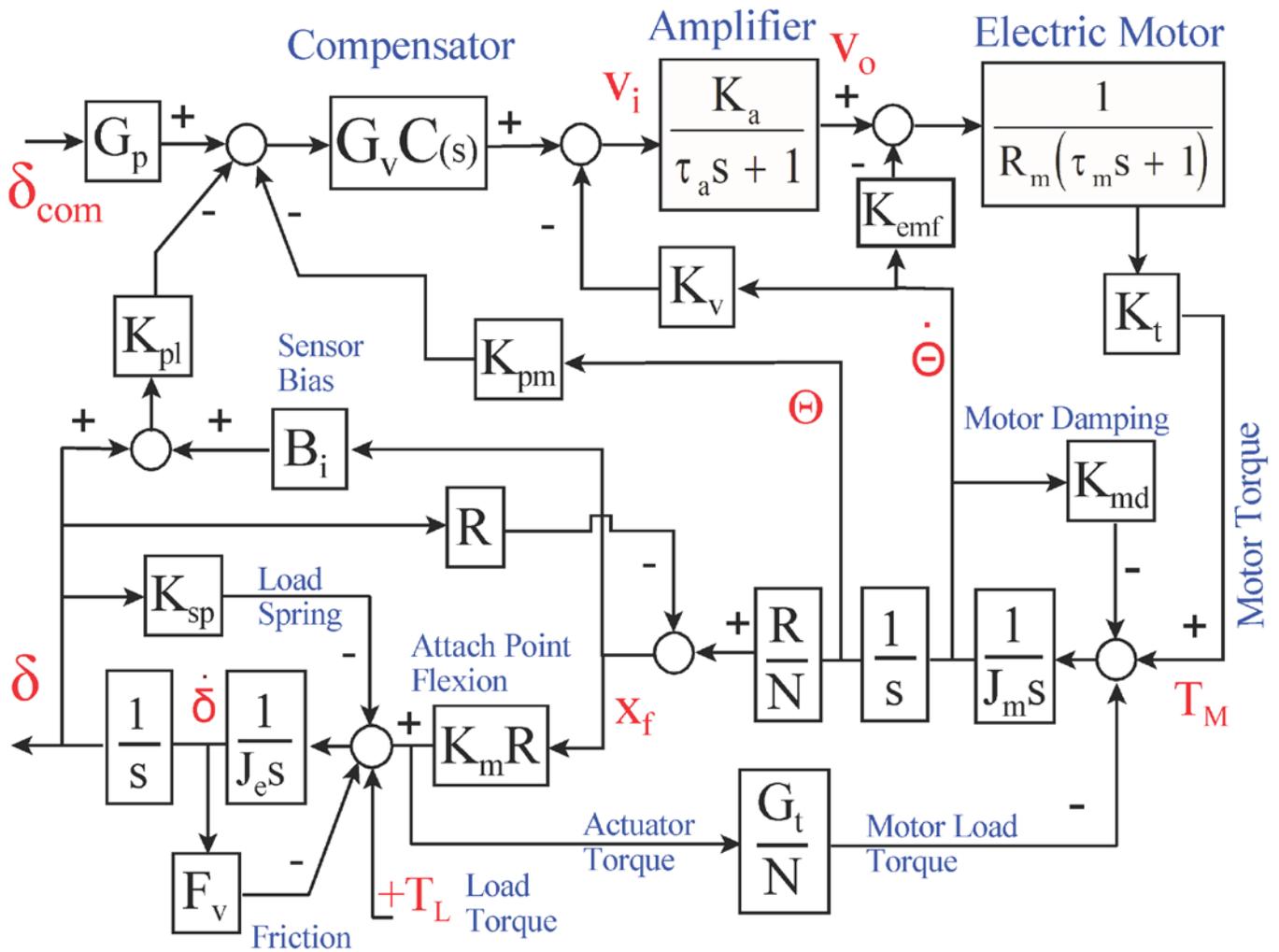


Figure (4.3.4) Electro-Mechanical Actuator Model (A) Block Diagram

### 4.3.5 Electro-Mechanical Actuator (EMA), Model (B)

The EMA model in Figure 4.3.5.1 is different from the previous EMA model (A). It uses a spinning motor that is controlled by dc voltage via a current amplifier and it provides the power to drive the load. The motor torque is transferred from the rotor to a spiral screw that is rotating via a gear mechanism, a small gear driving a bigger gear, and it produces a higher torque. The rotation of the screw converts the rotational motion to translational that extends or retracts the shaft. The end of the shaft is connected to an engine or aerosurface via a linkage mechanism and it is pushing against the load that rotates about a hinge in order to control the vehicle. The other side of the actuator is attached to a stiff point on the vehicle structure. The EMA system uses two gear ratios. The first gear ratio  $N_{gear}$  defines the number of motor spins for one rotation of the screw gear and the second gear ratio  $N_{screw}$  defines the number of screw rotations per unit length extension of the shaft.

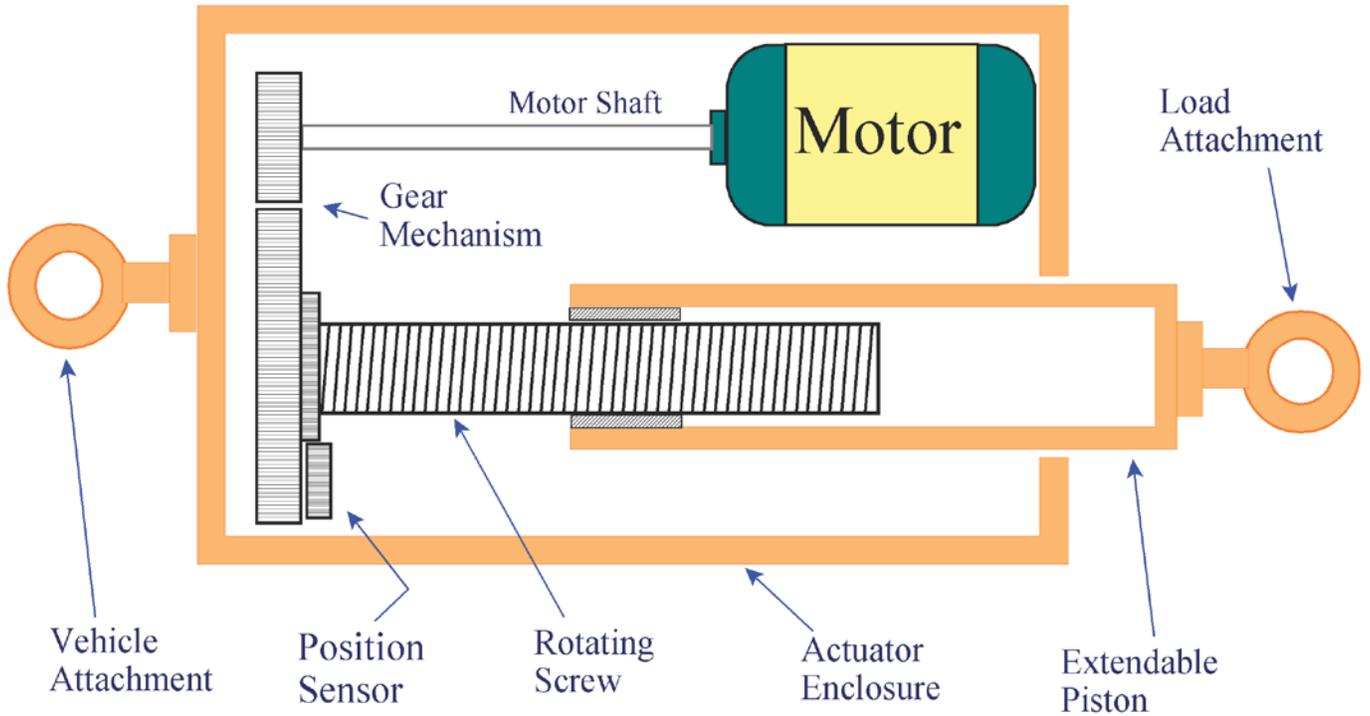


Figure (4.3.5.1) Electro-Mechanical Actuator System (B)

A current amplifier supplies the dc current to the motor armature that drives the dc motor and the current is converted to torque that spins the rotor. The voltage output of the PID controller is converted to torque  $T_m$  that spins the rotor  $\dot{\mu}$ .

$$i = \left( \frac{1}{Ls + R} \right) [v_{in} - K_{emf} \dot{\mu}] \quad (\text{motor current})$$

$$T_c = K_T i - K \dot{\mu} \quad (\text{electric torque})$$

$$I_m \dot{\mu} = T_c - \frac{K_{gs} \varepsilon}{N_{gear}} - K_{mfr} \dot{\varepsilon} - K_{visc} \dot{\mu} - K_c \text{sign}(\dot{\mu}) \quad (4.3.5.1)$$

$$T_m = K_{eff} K_{gs} \varepsilon \quad (\text{motor output torque})$$

$$P = Ri^2 \quad (\text{dissipated power})$$

The EMA system block diagram is shown in Figure (4.3.5.2). It includes the actuator, the rotational dynamics of the load and also the mechanical stiffnesses which are coupling them together. The motor speed and the shaft extension rate are limited by the amplifier current supply. A resolver is measuring the shaft position by counting the number of screw rotations and it is used for position feedback  $x_{fb}$ . A rate sensor from the motor speed provides the necessary damping and stability characteristics in the actuator control loop. The gimbals deflection command  $\delta_{com}$  is in (radians). It is converted to actuator position command in (feet) which is subtracted from the measured shaft position to become a position error commanding the PID control system. The controller generates the voltage  $v_i$  and current as a

function of error that drive the dc motor. Integral and rate feedback from the motor rate are included in the PID control system to provide the desired closed-loop response. A low-pass 1<sup>st</sup> or 2<sup>nd</sup> order compensator C(s) may be also included.

The motor dynamics Equation 4.3.5.1 is simplified by approximating the electrical characteristics and the back-emf with a first order lag of bandwidth  $\omega_m$  and the motor torque gain  $K_v$  in (ft-lb/volt) which includes the current amplifier, control electronics and it converts the input voltage  $v_i$  to motor torque. The inputs to the equation are torques and the output is rotor rate ( $d\mu/dt$ ) in (rad/sec). There are four torques that affect the motor speed: an electrical control torque that drives the motor, a reaction torque due to loading and two friction torques.

$$\dot{\mu} = \frac{1/I_m}{(s + \omega_m)} \left[ K_v v_i - \frac{T_m}{N_{gear}} - K_{mfr} \dot{\varepsilon} - K_C \text{sign}(\dot{\mu}) \right] \quad (4.3.5.1)$$

The torques opposing the motor speed are:

- the torque applied to the screw via the gear ratio  $N_{gear}$  but reduced in size by the same gear reduction ratio  $N_{gear}$ ,
- a coulomb friction torque  $K_C$  always in the opposite direction to the rotor speed, and
- a damping torque ( $K_{mfr}$ ) due to the relative speed, (rotor-load) rates.

The motor drives a gear mechanism that rotates the screw at an angle ( $\theta$ ). The torque on the screw ( $T_m$ ) generated by the motor is.

$$T_m = K_{gs} \left( \frac{\mu}{N_{gear}} - \theta \right); \quad \varepsilon = \left( \frac{\mu}{N_{gear}} - \theta \right) \quad (4.3.5.2)$$

Where:

- $\omega_m$  is the motor bandwidth in (rad/sec),
- $I_m$  is the motor inertia (slug-ft<sup>2</sup>)
- $\varepsilon$  is the relative position (rotor-load)
- $\mu$  is the motor rotation angle in (radians)
- $\theta$  is the screw rotation angle in (radians)
- $N_{gear}$  is the motor to screw gear ratio
- $N_{screw}$  is the shaft screw ratio (ft/rad)
- $T_m$  is the torque on the screw
- $K_{gs}$  is the gear stiffness in (ft-lb/rad).
- $K_{mfr}$  is the motor gear damping friction, (ft-lb/rad/sec) due to relative speed
- $K_C$  is the motor Coulomb Friction (ft-lb)

The direction of the screw rotation causes the shaft to either extend or to retract. Equation (4.3.5.3) calculates the screw position angle  $\theta$  as a function of the supplied torque  $T_m$  and the load force  $F_L$ . The shaft position  $x_p$  is linearly related to the screw angle by the screw gear ratio.

$$\begin{aligned} J_a \ddot{\theta} &= K_{eff} T_m - N_{screw} F_L - K_{dmp} \dot{\theta} - K_C \text{sign}(\theta) \\ F_L &= K_T [x_p - x_L] \quad x_L = R\delta \\ x_p &= N_{screw} \theta \end{aligned} \quad (4.3.5.3)$$

Where:

- $N_{screw}$  is the screw gear ratio in (ft/radian). It represents the length of the piston extension when the screw mechanism rotates one radian.
- $F_L$  is the force across the actuator shaft in (lb) due to the load.
- $J_a$  is the combined inertia of the gears plus the screw
- $R$  is the moment arm, shaft to hinge in (ft)
- $K_{dmp}$  is the shaft friction (ft-lb/rad/sec)
- $x_p$  is the piston extension and it is related to the spiral shaft rotation ( $\theta$ ) by the screw gear ratio

Equation (4.3.5.4) calculates the load angular acceleration as a function of the applied torques.

$$J_e \ddot{\delta} = F_L R - B_e \dot{\delta} - T_{stat} \text{sign}(\dot{\delta}) - K_g \delta + T_{Ext\_Load} \quad (4.3.5.4)$$

Where:

- $J_e$  is the load moment of inertia of the engine or the control surface about the hinge
- $\delta$  is the load deflection in (rad)
- $K_g$  is the stiffness at the hinge (ft-lb/rad)
- $B_e$  is the Viscous Friction (ft-lb/rad/sec)
- $T_{static}$  is the Static Friction in (ft-lb)
- $T_{ext\_load}$  is the external torque due to aerodynamics and vehicle acceleration forces.

The variable ( $x_L=R\delta$ ) represents the distance traveled at the end of the shaft (the point where it is attached to the load) when it rotates at an angle  $\delta$ . It is not only the piston extension  $x_p$  that creates the force  $F_L$  against the load, but it is the difference ( $x_p-x_L$ ) that is pushing against the total system stiffness  $K_T$ . The rotation angle  $\delta$  represents the combined rotation of the load at the gimbal consisting of two components: a component due to the piston extension  $x_p$ , plus a component due to structural bending  $K_T$ . Equation (4.3.5.5) calculates the resolver feedback measurement  $x_{fb}$ .

$$x_{fb} = x_p \frac{K_T}{K_L} + x_L \left( 1 - \frac{K_T}{K_L} \right) = x_p \frac{K_T}{K_L} + \frac{F_L}{K_{act}} \quad (4.3.5.5)$$

It represents what the position sensor measures inside the actuator. It consists of two components, a component due to actual shaft position ( $x_p$ ), and a component due to compression inside the actuator caused by the actuator internal stiffness ( $K_{act}$ ). The system has a total stiffness ( $K_T$ ). Part of the total stiffness is due to backup structure and load stiffnesses ( $K_L$ ), and the other part is due to the shaft and electrical compression ( $K_{act}$ ) inside the actuator. The sensor measures shaft rotations due to compression ( $K_{act}$ ) but it doesn't see the deformation due to ( $K_L$ ). The stiffnesses are related by the following equation

$$\frac{1}{K_T} = \frac{1}{K_L} + \frac{1}{K_{act}} \quad (4.3.5.6)$$

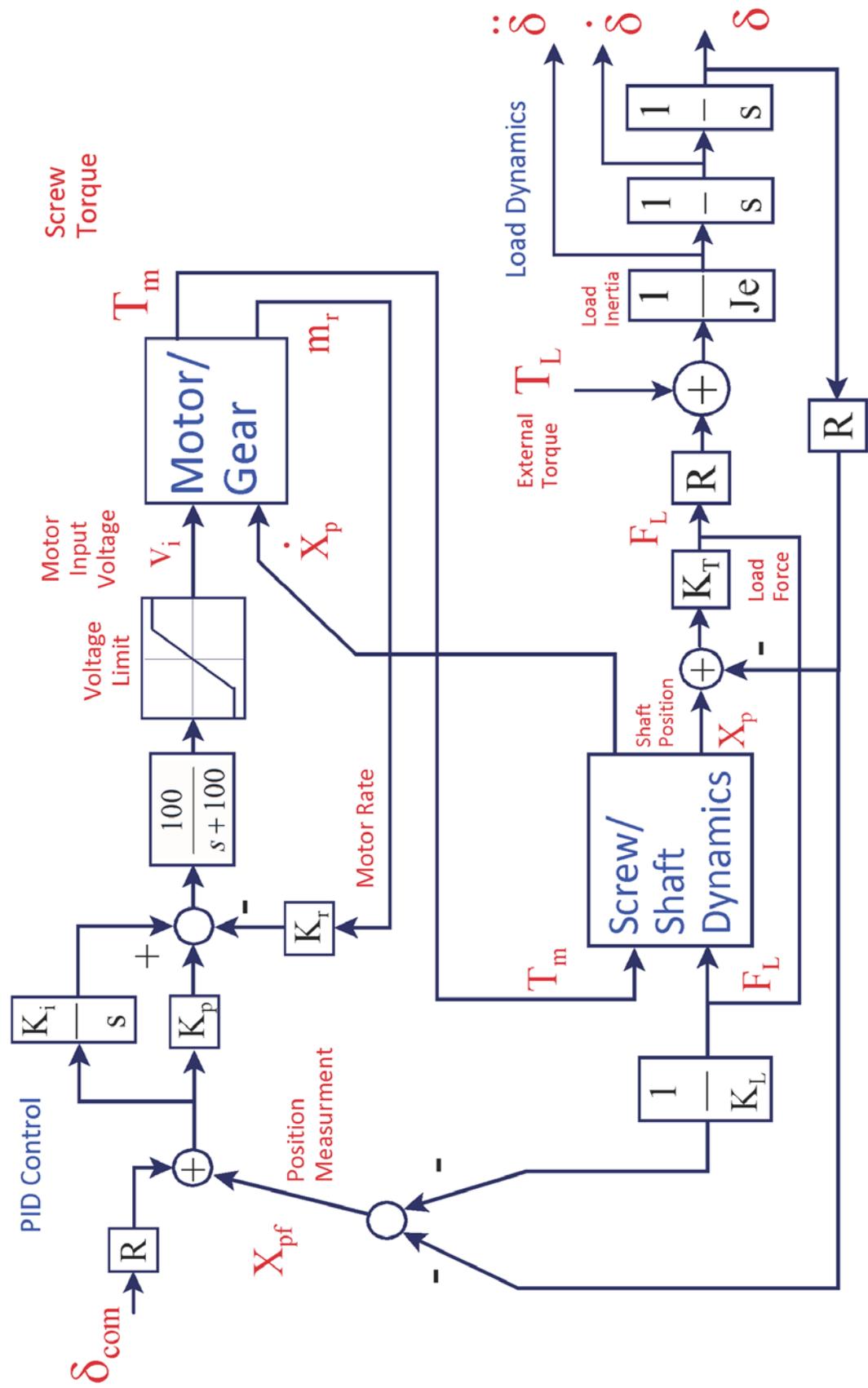
Consider the two extreme situations. First, when the load is very light, there is no friction, and the load does not resist the motion of the shaft ( $x_p$ ). The shaft force ( $F_L$ ) is reduced to zero. The extension due to ( $\delta$ ), ( $x_L = x_p$ ), and the feedback position measurement at the output of the summing junction becomes  $x_{fb} = x_p$ . This is exactly what you would expect for the position sensor to measure. Let us now assume that there is infinite stiffness in the actuator, and  $K_{act}$  is infinite. The total stiffness now reduces to backup plus load flexure ( $K_T = K_L$ ). Assume also that the shaft is locked at zero position ( $x_p=0$ ). If we apply an external load torque to the load ( $T_L$ ), the piston force is: ( $F_L = -T_L/R$ ). The feedback sensor measurement  $x_{fb}$  will be zero as you would expect because the sensor cannot see the deformation due to ( $K_L$ ) outside the actuator box.

$$x_{fb} = x_L - \frac{K_T}{K_L} x_L = 0$$

In actuality, however, when you are holding ( $x_p$ ) to zero and are applying an external load ( $T_L$ ) the sensor at the shaft will measure the small compression of the motor and shaft. The sensor measurement  $x_{fb}$  will be:

$$x_{fb} = x_L \left( 1 - \frac{K_T}{K_L} \right) = x_L \frac{K_T}{K_{act}} = \frac{T_L/R}{K_{act}}$$

This is exactly what you would expect from the sensor to see; an extension measurement due to the external load acting against ( $K_{act}$ ) alone.



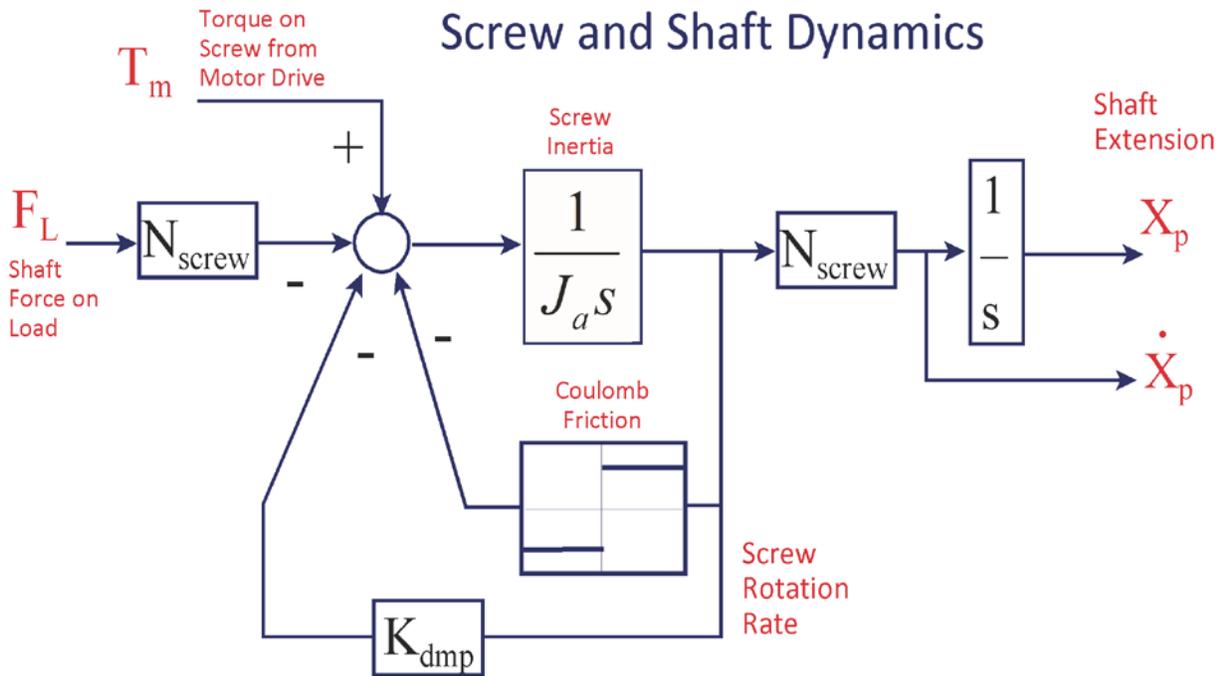
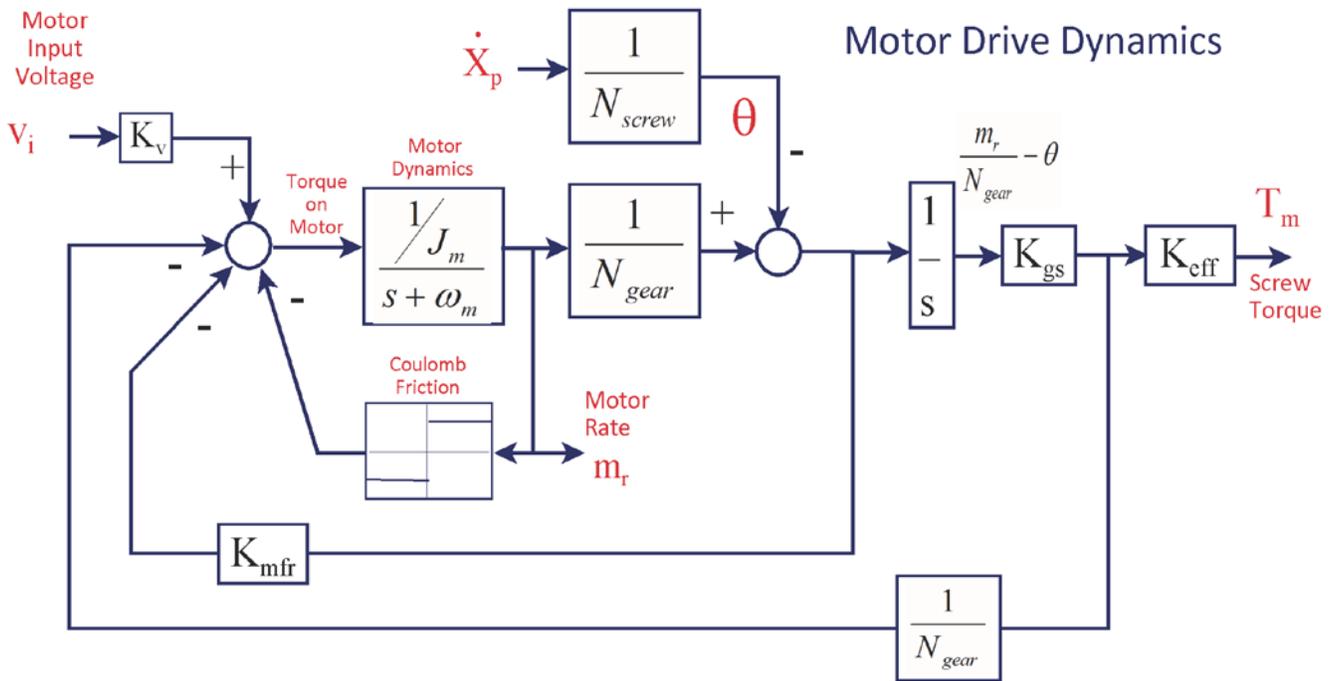
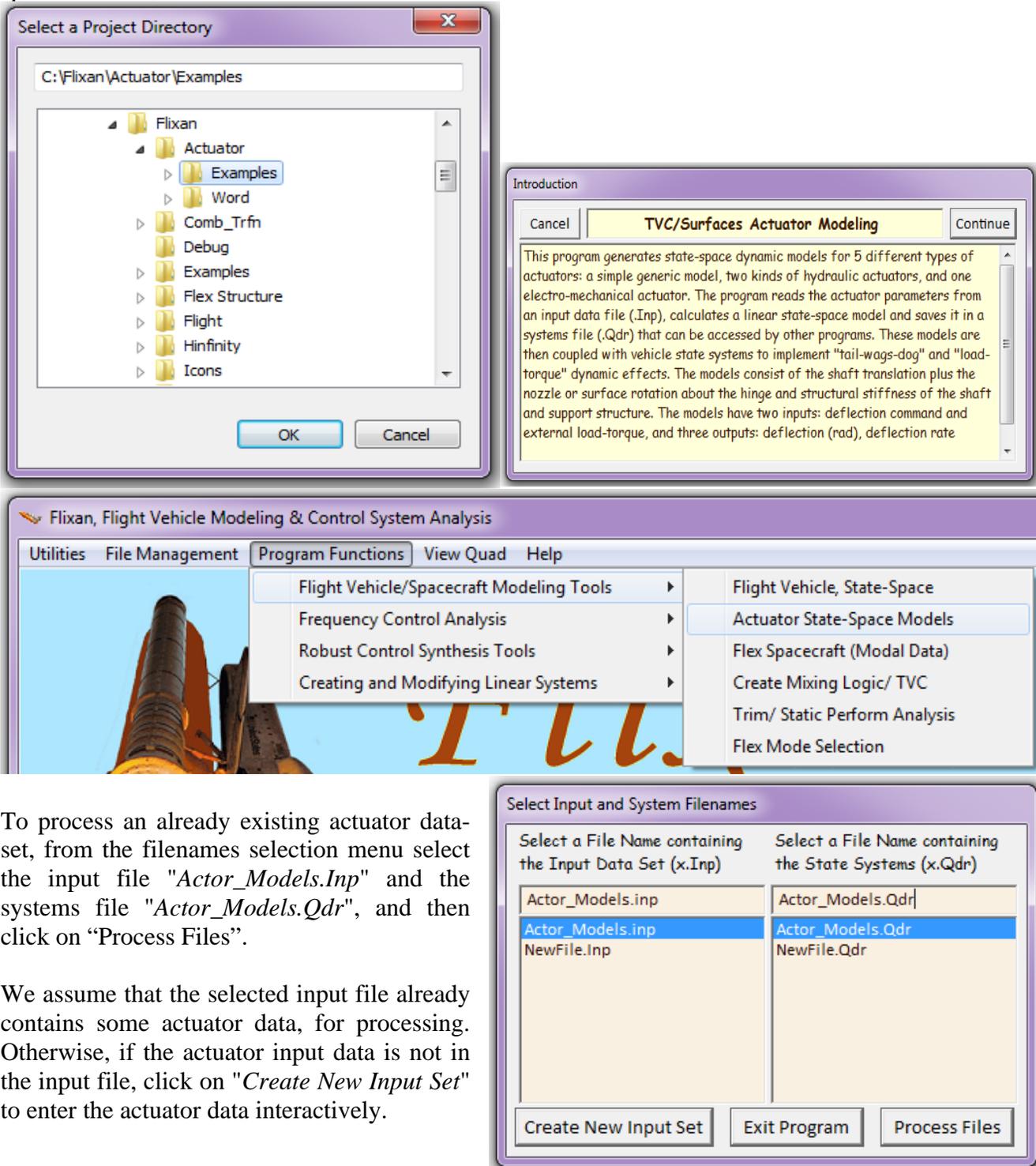


Figure 4.3.5.2 Electro-Mechanical Actuator (B) Block Diagram with Subsystems

## 4.4 Running the Actuator Modeling Program

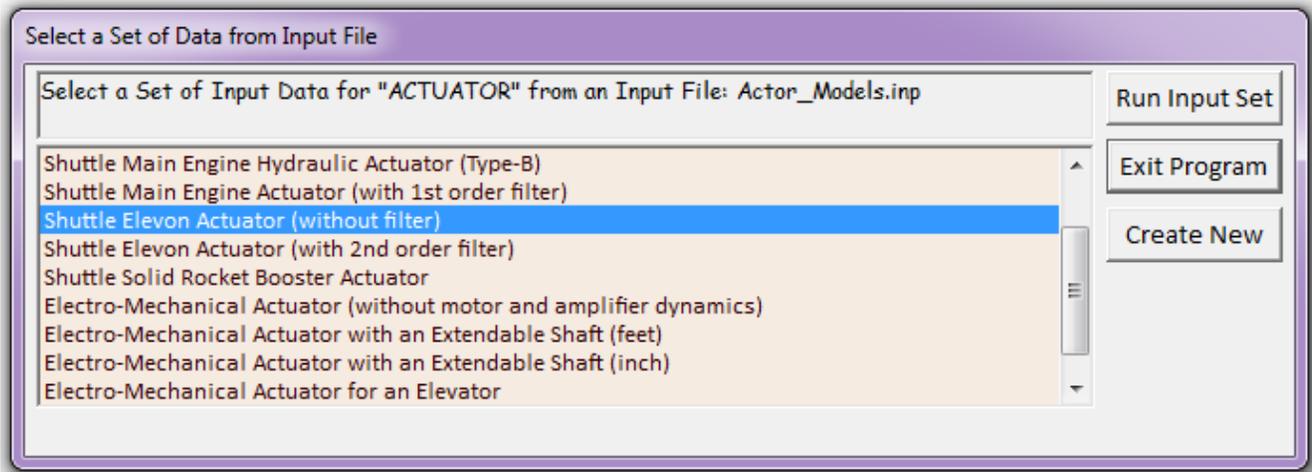
The actuator modeling program includes the five different types of actuator models described. It runs either interactively or in batch mode, similar to the vehicle modeling program. To run it interactively, start the Flixan program, select the project directory which in this example it is "C:\Flixan\Actuator\Examples" and contains the input data and the system files (.Inp) and (.Qdr). Then, from the Flixan main menu select "Program Functions", "Flight Vehicle/Spacecraft Modeling Tools", and "Actuator State-Space Models", as shown below.



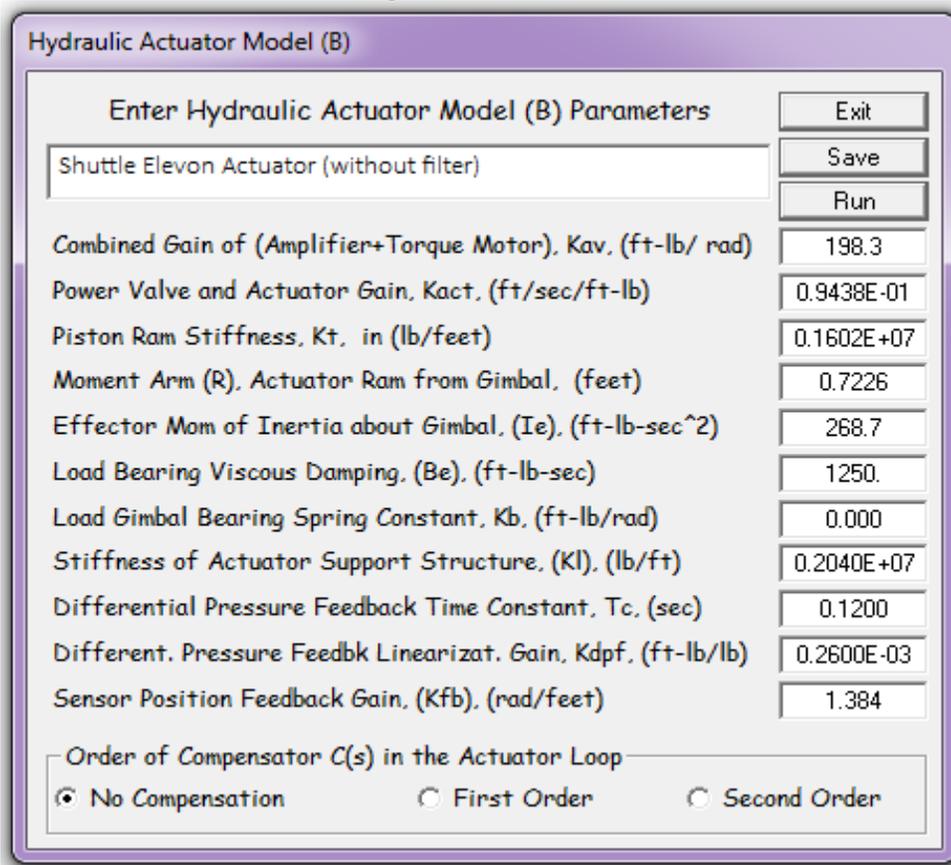
To process an already existing actuator data-set, from the filenames selection menu select the input file "Actor\_Models.Inp" and the systems file "Actor\_Models.Qdr", and then click on "Process Files".

We assume that the selected input file already contains some actuator data, for processing. Otherwise, if the actuator input data is not in the input file, click on "Create New Input Set" to enter the actuator data interactively.

The following menu shows the titles of the actuator data-sets which are included in the input file "Actor\_Models.Inp". Select the title "Shuttle Elevon Actuator (without filter)" and click on "Run Input Set" to process the actuator data. You may also click on "Create New" to create a new set or "Exit the Program".



The program reads the input data and opens the following dialog that shows the actuator parameters to be processed. The user can modify some of the parameters using this dialog and save the new data-set under a different title. Click on "Run" and the actuator program will process the data-set and create the actuator state-space system in file "Actor\_Models.Qdr" using the same title "Shuttle Elevon Actuator (without filter)". The title of the system is the same as the title in the input data. The comment lines from the data-set are also transferred to the system.



The following data-set is the actuator data that we just processed from file "Actor\_Models.Inp". The first line: "ACTUATOR INPUT DATA ... HYDRAULIC TYPE B" is the actuator id line which identifies the type of data. The Flxan program recognizes it, and also the actuator type which is (Hydraulic type B in this case), and uses the proper utility to process the data. The second line is the actuator title: "Shuttle Elevon Actuator (without filter)". The green lines below the title beginning with (!) are comments that further describe the actuator.

```

ACTUATOR INPUT DATA ..... HYDRAULIC TYPE B
Shuttle Elevon Actuator (without filter)
! Hydraulic Actuator Type (B) for the Space Shuttle Elevon,
! Without a Filter Compensator C(s)
Symbol      Parameter Description      (Units)      Value
C(s)      Order of Compensat:(0,1,2), Coefficients      (---)      0
Kav      Total Gain of Amplifier + Torque Motor      (ft-lb/rad)      198.329
Kact      Actuator Gain .....(ft/s/ft-lb)      0.09438
Kt      Piston Ram Stiffness ..... (lb/ft)      0.16020E+07
R      Moment Arm of Actuator Ram from Gimbal      (feet)      0.7226
Ie      Elevon Inertia about Gimbal .....(ft-lb-s^2)      268.67
Be      Elevon Viscous Damping .....(ft-lb-sec)      1250.0
Kb      Elevon Gimbal Bearing Spring Constant      (ft-lb/rad)      0.0
Kl      Elevon Mount Structural Stiffness ..... (lb/ft)      0.204E+07
Tc      Different. Pressure Feedbk Time Constant      (seconds)      0.12
Kdpf      Differ Pressure Feedbk Linearizat. Gain      (ft-lb/lb)      0.26E-03
Kfb      Position Feedback Gain ..... (rad/feet)      1.38396

```

The following state-space system is created by the program in file "Actor\_Models.Qdr" and corresponds to the above actuator data. It is identified with the same title and uses the same comments as the input data.

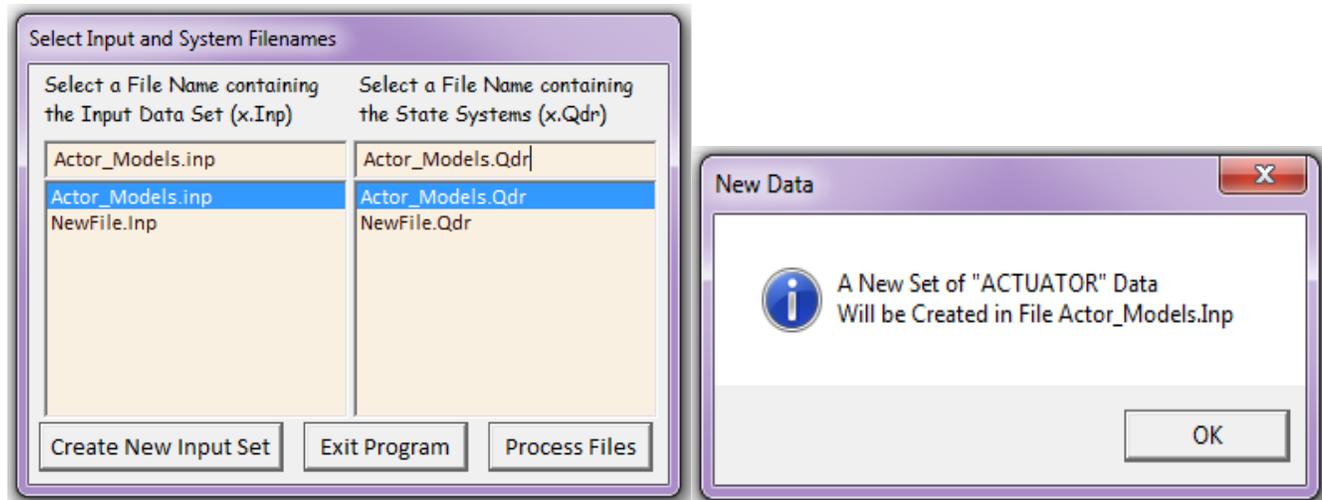
```

STATE-SPACE SYSTEM ...
Shuttle Elevon Actuator (without filter)
! Hydraulic Actuator Type (B) for the Space Shuttle Elevon, Without Filter
Number of Inputs, States, Outputs, Sample Time dT (for discrete)= 2 4 3 0.0000
Matrices: (A,B,C,D)
Matrix A      Size = 4 X 4
      1-Column      2-Column      3-Column      4-Column
1-Row -0.476444909333E+02 -0.847340446418E+04 -0.167404640751E+04 -0.300977352000E+03
2-Row -0.943800000000E-01 -0.203433315667E+02 -0.401912611041E+01 0.000000000000E+00
3-Row 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01
4-Row 0.000000000000E+00 0.430865076116E+04 -0.311343104001E+04 -0.465254773514E+01
-----
Matrix B      Size = 4 X 2
      1-Column      2-Column
1-Row 0.779654257565E+04 0.000000000000E+00
2-Row 0.187182910200E+02 0.000000000000E+00
3-Row 0.000000000000E+00 0.000000000000E+00
4-Row 0.000000000000E+00 0.372203818811E-02
-----
Matrix C      Size = 3 X 4
      1-Column      2-Column      3-Column      4-Column
1-Row 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00
2-Row 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01
3-Row 0.000000000000E+00 0.430865076116E+04 -0.311343104001E+04 -0.465254773514E+01
-----
Matrix D      Size = 3 X 2
      1-Column      2-Column
1-Row 0.000000000000E+00 0.000000000000E+00
2-Row 0.000000000000E+00 0.000000000000E+00
3-Row 0.000000000000E+00 0.372203818811E-02

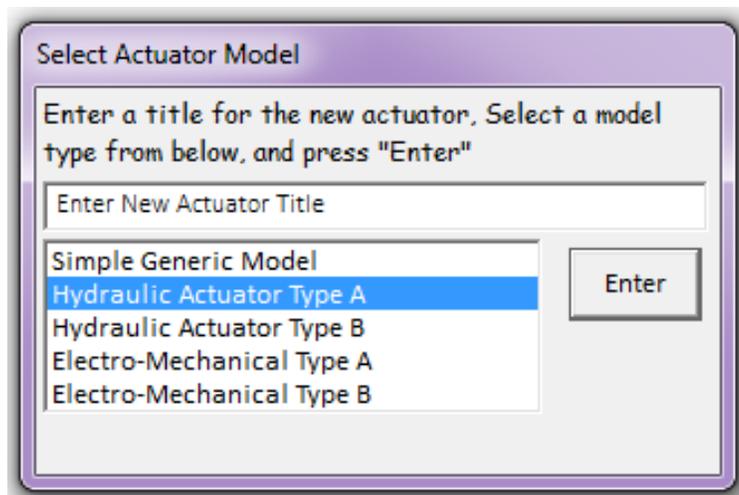
```

## Creating a new set of actuator data in file

When the actuator data are not in the input data file, they can be entered interactively, saved, and then processed, as follows. Begin the actuator program and use the filenames selection menu to select the input and system files, as before. This time click on "Create New Input Set" instead of processing an already existing dataset.



In the next dialog, click OK to confirm a new actuator. From the following menu you must select the type of actuator that you want to create. There are 5 existing types to choose from. In this case we select the "Hydraulic Actuator Type A", and click on "Enter".



The program presents the following data entry dialog for entering the parameters of the selected actuator type. Start by entering the title "Missile Actuator-A", which should be less than 100 characters long and different from any other titles already saved in the input file. Enter also the actuator parameters in the corresponding fields. After entering the data, do not run it yet, but click on "Save", and the program will save the new actuator dataset in the input file "Actor-Models.Inp". Then you may click on "Run" to process it, and to create the new actuator state-space system in file "Actor-Models.Qdr". The reader can find more actuator modeling examples in file "\\Flixan\Actuator\Examples\Actor-Models.Inp".

Hydraulic Actuator Model (A)

Enter Hydraulic Actuator Model (A) Parameters

Missile Actuator-A

Input Command Gain, ( $K_c$ ), (Volt/Rad)	46.12
Gain of Servo-Amplifier, $K_a$ , (ma/volt)	20.40
Power Valve Flow Gain, $K_v$ , ( $\text{ft}^3/\text{sec}/\text{ma}$ )	0.5960E-04
Different Pressure Feedback Time Const. $T_c$ , (sec)	0.1000
Different. Pressure Feedback Gain, $K_{dpf}$ , ( $\text{ma}/\text{lb}/\text{ft}^2$ )	0.1000E-04
Volume of Compressed Fluid, ( $V_t$ ), ( $\text{feet}^3$ )	0.1160E-02
Bulk Modulus of Hydraulic Fluid, (BM), ( $\text{lb}/\text{ft}^2$ )	0.2280E+08
Stiffness of Piston plus Mount Backup, $K_m$ , ( $\text{lb}/\text{ft}$ )	0.1330E+08
Leakage Coeffic. (Orifice+Spool), ( $\text{ft}^3/\text{sec}/\text{lb}/\text{ft}^2$ )	0.2000E-10
Moment Arm, Actuator Rod from Gimbal, ( $R$ ) in (ft)	0.3000
Piston Cross-Sectional Area, ( $A$ ), ( $\text{feet}^2$ )	0.1500E-01
Load Inertia about Gimbal, ( $I_e$ ), ( $\text{ft}\text{-lb}\text{-sec}^2$ )	0.5600
Gimbal Bearing Viscous Damping, ( $F_v$ ), ( $\text{ft}\text{-lb}\text{-sec}$ )	410.0
Engine Gimbal Bearing Spring Constant, $K_{sp}$ , ( $\text{ft}\text{-lb}/\text{rad}$ )	0.1110E+06
Actuator Position Feedback Gain, $K_f$ , (volts/ft)	132.0

Order of Compensator  $C(s)$  in the Forward Loop

No Compensation
  First Order
  Second Order

Exit  
Save  
Run

## Compensator Options

As we already discussed the actuator is an inner local loop that must be stable with an acceptable response to commands within the required bandwidth and step response requirements. The dynamic coupling between actuator, vehicle, and support structure flexibility introduces local resonances and TWD dynamics that may degrade the servo system's performance and stability, and the actuator servo systems loop often includes a compensators in order to improve the actuator loop stability and improve the command following. The actuator modeling program provides the capability to include a first or a second order compensator which is defined in the actuator input data. After entering the actuator data interactively using the data entry dialog the user may decide to include either: a first order or a second order compensator, or no compensator (default), using the 3-option buttons located at the bottom of the dialog. If you select to include a compensator a new dialog comes up where the user enters the transfer function coefficients. The compensator coefficients are saved in the input file data-set together with the actuator parameters. In fact, the first line in the input data defines the compensator transfer function  $C(s)$  (if any) which is inserted in the actuator position control loop as shown in the actuator block diagrams.

For a first order compensator the input data line consists of 3 numbers: “**Order of Compensator: (0,1,2), Coefficients: 1,  $\tau_n$ ,  $\tau_d$** ” where:  $\tau_n$  and  $\tau_d$  are the numerator and denominator time constants of the compensator transfer function:

$$C(s) = \frac{\tau_n s + 1}{\tau_d s + 1}$$

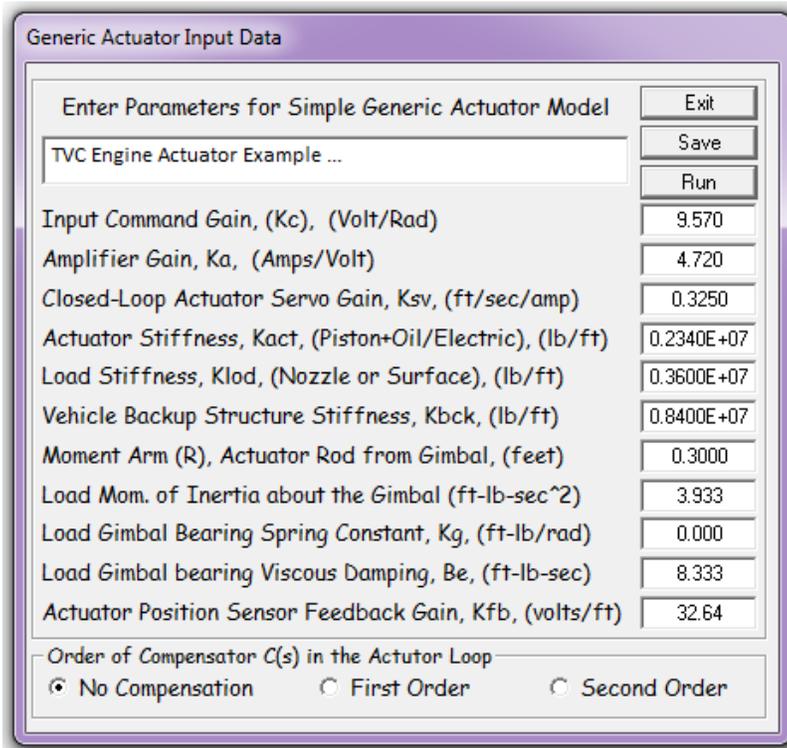
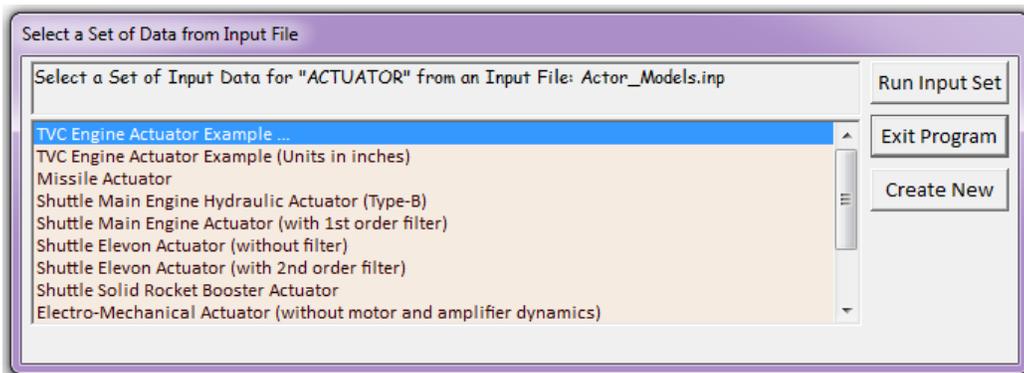
For a second order compensator the input data line consists of 5 numbers: “**Order of Compensator: (0,1,2), Coefficients: 2,  $\zeta_n$ ,  $\omega_n$ ,  $\zeta_d$ ,  $\omega_d$** ” where  $\zeta_n$ , and  $\omega_n$  are the numerator damping coefficient and frequency in (rad/sec), and  $\zeta_d$ , and  $\omega_d$  are the denominator damping and frequency in (rad/sec) of the compensator transfer function.

$$C(s) = \frac{\omega_d^2 \left( s^2 + 2\zeta_n \omega_n s + \omega_n^2 \right)}{\omega_n^2 \left( s^2 + 2\zeta_d \omega_d s + \omega_d^2 \right)}$$

# Actuator Examples

The following examples further illustrate the creation of different actuator models using the actuator modeling program. The actuator program implements five different types of actuators from very simple to more complex. The example datasets are included in the input data file “*Actor-Models.Inp*” which is located in directory “*Flixan\ Actuator\ Examples*”. It includes data for several actuators of all 5 types. A batch set at the top of the file can be used to process the entire file in batch. The state-space systems are saved in the systems file: “*Actor\_Models.Qdr*”.

## 4.5.1 Simple Generic Actuator Model Example



The title of the first actuator set is “*TVC Engine Actuator Example ...*” and its type is “*Simple Generic Model*”. The simple actuator model was described in Section (4.3.1). To run this actuator in Flixan you must first select the project directory “*Flixan\ Actuator\ Examples*”, select the filenames, and from the following menu select the first title, as shown, and click on “*Run Input Set*”.

The program identifies the actuator type, reads the selected actuator data, and displays the actuator parameters in the corresponding dialog type, shown below. Click on “*Run*” to process the data and to create the state-space system, which has the same title: “*TVC Engine Actuator Example ...*” and it is saved in the selected systems file “*Actor-Models.Qdr*”.

## 4.5.2 Electro-Hydraulic Model (A) Actuator for a Small Missile

The next example is a hydraulic actuator for a small missile. It uses the Hydraulic Type (A) model that was described in Section (4.3.2). Its title is “*Missile Actuator*” and the input data is also in file “*Actor\_Models.Inp*”. The program reads the actuator data, recognizes that it is a hydraulic type-A model, and opens-up the corresponding Hydraulic Actuator Type (A) dialog form. Click on “Run” button and the program will process the data and create the state-space system “*Missile Actuator*” which will be saved in the systems file “*Actor\_Models.Qdr*”.

Parameter	Value
Input Command Gain, $K_c$ , (Volt/Rad)	46.12
Gain of Servo-Amplifier, $K_a$ , (ma/volt)	20.40
Power Valve Flow Gain, $K_v$ , (ft <sup>3</sup> /sec/ma)	0.5960E-04
Different Pressure Feedback Time Const. $T_c$ , (sec)	0.1000
Different. Pressure Feedback Gain, $K_{dpf}$ , (ma/lb/ft <sup>2</sup> )	0.1000E-04
Volume of Compressed Fluid, $V_t$ , (feet <sup>3</sup> )	0.1160E-02
Bulk Modulus of Hydraulic Fluid, $B_M$ , (lb/ft <sup>2</sup> )	0.2880E+08
Stiffness of Piston plus Mount Backup, $K_m$ , (lb/ft)	0.1333E+08
Leakage Coeff. (Orifice+Spool), (ft <sup>3</sup> /sec/lb/ft <sup>2</sup> )	0.2000E-10
Moment Arm, Actuator Rod from Gimbal, $R$ in (ft)	0.3000
Piston Cross-Sectional Area, $A$ , (feet <sup>2</sup> )	0.1500E-01
Load Inertia about Gimbal, $I_e$ , (ft-lb-sec <sup>2</sup> )	0.5600
Gimbal Bearing Viscous Damping, $F_v$ , (ft-lb-sec)	410.0
Engine Gimbal Bearing Spring Constant, $K_{sp}$ , (ft-lb/rad)	0.1110E+06
Actuator Position Feedback Gain, $K_f$ , (volts/ft)	132.0

Order of Compensator  $C(s)$  in the Forward Loop

No Compensation     First Order     Second Order

### 4.5.3 Electro-Hydraulic Model (B) Actuator for a Shuttle Main Engine

In this example we are using the Hydraulic Model (B) actuator type that was described in Section (4.3.3) to create a Shuttle main engine actuator system. The input data is also in “*Actor\_Models.Inp*”, and its title is “*Shuttle Main Engine Actuator (with 1st order filter)*”. We will also include a first order lag filter  $C(s)$  in the actuator control loop. We run the actuator program as before, select the actuator title from the menu, and the program will display the following actuator dialog form. Notice that in the actuator data the order of the compensator is set to (1) followed by the numerator and denominator time-constants ( $\tau_n$  and  $\tau_d$ ) of the 1<sup>st</sup> order transfer function. The presence of a first order lag  $10/(s+10)$  in the actuator loop is also evident by the next dialog showing the filter parameters. Click on “OK”, and the program calculates the state-space system and saves it in the systems file “*Actor\_Models.Qdr*”, using the title “*Shuttle Main Engine Actuator (with 1st order filter)*”. The plot in Figure (4.5.3.1) compares the actuator's closed loop response with (green) and without (blue) the filter.

Hydraulic Actuator Model (B)

Enter Hydraulic Actuator Model (B) Parameters

Shuttle Main Engine Actuator (with 1st order filter)

Combined Gain of (Amplifier+Torque Motor), $K_{av}$ , (ft-lb/ rad)	0.9100
Power Valve and Actuator Gain, $K_{act}$ , (ft/sec/ft-lb)	53.95
Piston Ram Stiffness, $K_t$ , in (lb/feet)	0.2028E+07
Moment Arm (R), Actuator Ram from Gimbal, (feet)	2.480
Effector Mom of Inertia about Gimbal, ( $I_e$ ), (ft-lb-sec <sup>2</sup> )	4516.
Load Bearing Viscous Damping, ( $B_e$ ), (ft-lb-sec)	0.1650E+05
Load Gimbal Bearing Spring Constant, $K_b$ , (ft-lb/rad)	0.2716E+05
Stiffness of Actuator Support Structure, ( $K_l$ ), (lb/ft)	0.2796E+07
Differential Pressure Feedback Time Constant, $T_c$ , (sec)	0.1266
Different. Pressure Feedbk Linearizat. Gain, $K_{dpf}$ , (ft-lb/lb)	0.4040E-06
Sensor Position Feedback Gain, ( $K_{fb}$ ), (rad/feet)	0.4000

Order of Compensator  $C(s)$  in the Actuator Loop

No Compensation     First Order     Second Order

Enter Filter Coefficients

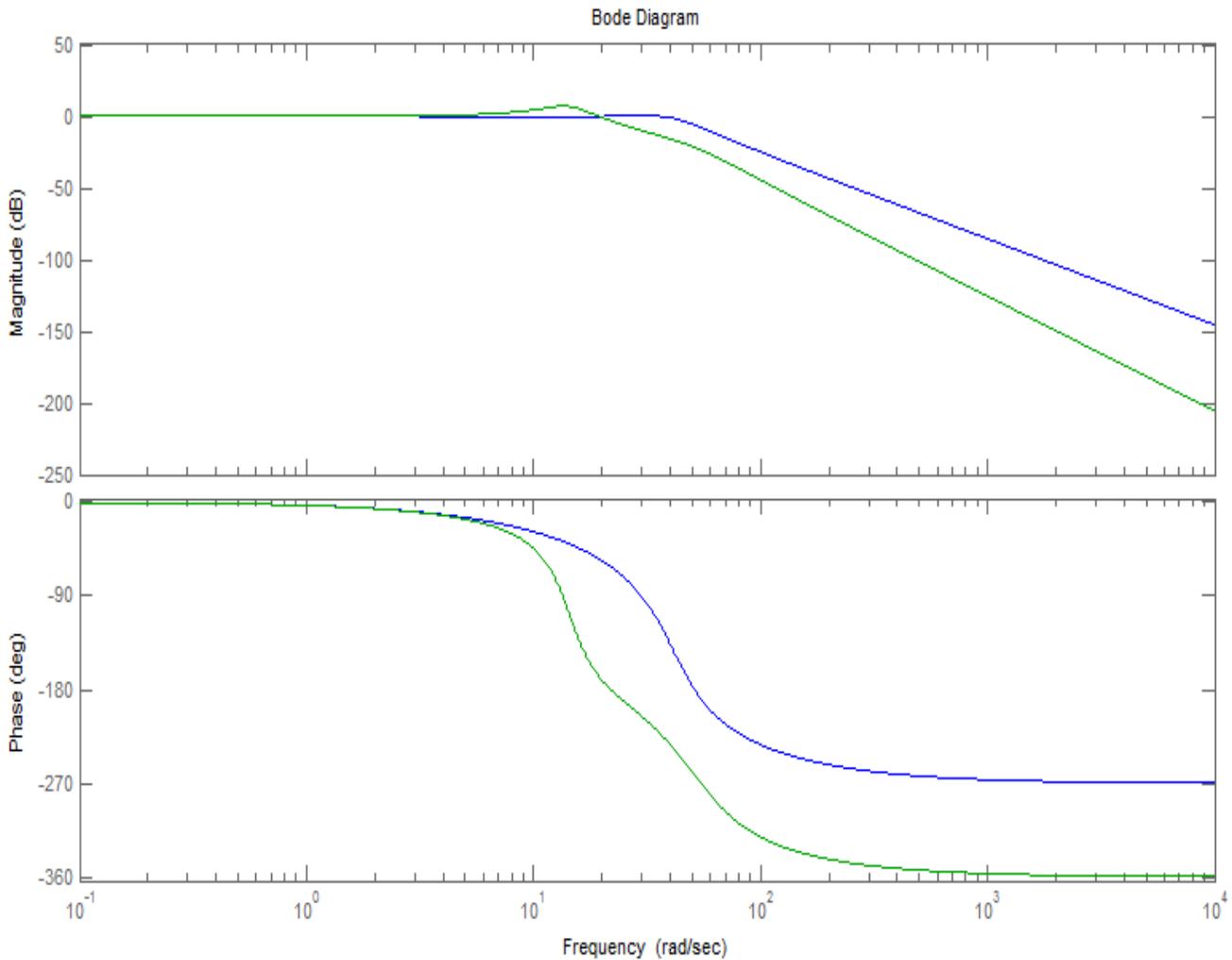
Define a First Order Filter Transfer Function:

$$\frac{(T_n s + 1)}{(T_d s + 1)}$$

OK

Numerator Time Constant ( $T_n$ ) in (sec)    0.000

Denominator Time Constant ( $T_d$ ) in (sec)    0.1000



**Figure (4.5.3) Shuttle Main Engine Actuator Frequency Response With (Green), and Without the First Order Lag Filter (Blue)**

#### 4.5.4 Electro-Hydraulic Model (B) Actuator for the Shuttle Elevon

In this example we will use the same "Electro-Hydraulic Model (B)" actuator type described in Section (4.3.3) to create a Space Shuttle Elevon actuator system that includes a second order Notch filter in the control loop. Its title is "Shuttle Elevon Actuator (with 2nd order filter)". Run the Flixan actuator program and from the filename selection menu select the same input and system files "Actor-Models.Inp" and "Actor\_Models.Qdr", as before, and from the actuator title selection menu choose "Shuttle Elevon Actuator (with 2nd order filter)". The following dialog shows the actuator parameters. Notice that a second order compensator C(s) is selected. In the actuator data the order of the compensator is set to (2) followed by the 2<sup>nd</sup> order transfer function's numerator and denominator damping coefficients and frequencies. The second order compensator parameters are shown in the next dialog. The filter is a notch intended to attenuate a local structural resonance at 65 (rad/sec). Click "OK" and the program will process the data to generate the state-space model of the actuator including the notch filter. It will save it in the systems file with the same title "Shuttle Elevon Actuator (with 2nd order filter)".

Notice that in the input data file includes a similar actuator data-set: “*Shuttle Elevon Actuator (without filter)*” that uses the same actuator parameters but without the notch filter. We used both systems to calculate the Elevon actuator frequency response with and without the notch filter, as shown in figure (4.5.4). They both have a control bandwidth of approximately 40 (rad/sec) and the notch is visible at 65 (rad/sec).

Hydraulic Actuator Model (B)

Enter Hydraulic Actuator Model (B) Parameters

Shuttle Elevon Actuator (with 2nd order filter)

Exit

Save

Run

Combined Gain of (Amplifier+Torque Motor), $K_{av}$ , (ft-lb/ rad)	198.3
Power Valve and Actuator Gain, $K_{act}$ , (ft/sec/ft-lb)	0.9438E-01
Piston Ram Stiffness, $K_t$ , in (lb/feet)	0.1602E+07
Moment Arm (R), Actuator Ram from Gimbal, (feet)	0.7226
Effector Mom of Inertia about Gimbal, ( $I_e$ ), (ft-lb-sec <sup>2</sup> )	268.7
Load Bearing Viscous Damping, ( $B_e$ ), (ft-lb-sec)	1250.
Load Gimbal Bearing Spring Constant, $K_b$ , (ft-lb/rad)	0.000
Stiffness of Actuator Support Structure, ( $K_I$ ), (lb/ft)	0.2040E+07
Differential Pressure Feedback Time Constant, $T_c$ , (sec)	0.1200
Different. Pressure Feedbk Linearizat. Gain, $K_{dpf}$ , (ft-lb/lb)	0.2600E-03
Sensor Position Feedback Gain, ( $K_{fb}$ ), (rad/feet)	1.384

Order of Compensator  $C(s)$  in the Actuator Loop

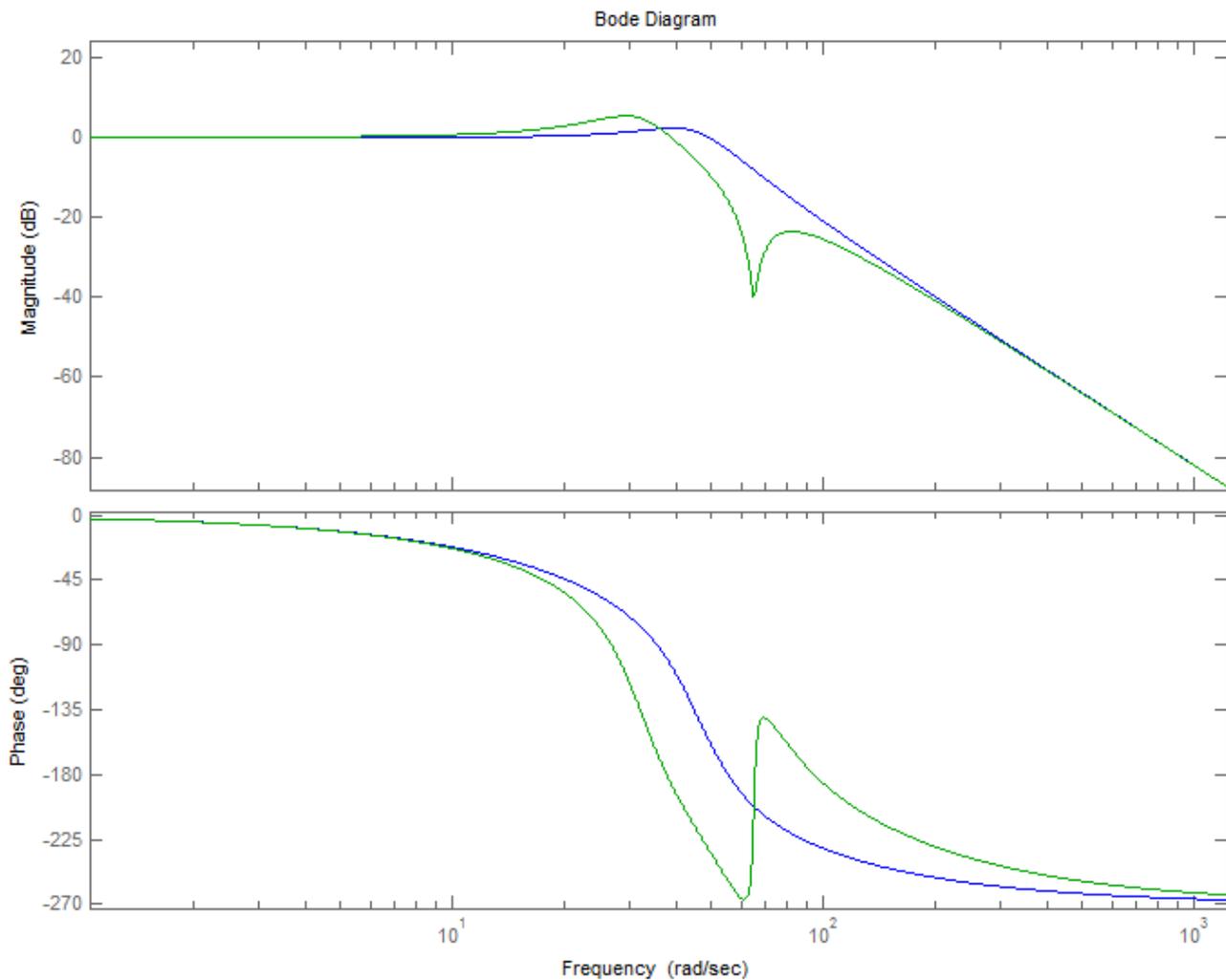
No Compensation       First Order       Second Order

Enter Filter Coefficients

Define a second order numerator, second order denominator transfer function in terms of damping and frequency

OK

Numerator Damping (zeta), and Frequency (omega) in (rad/sec)	0.1500E-01	65.00
Denominator Damping (zeta), and Frequency (omega) in (rad/sec)	0.6000	65.00



**Figure 4.5.4 Frequency Response of the Shuttle Elevon Actuator With and Without a Notch Filter at 65 (rad/sec)**

#### 4.5.5 Electro-Mechanical Model (A) Actuator

In this example we are creating an "Electro-Mechanical Model (A)" type of actuator which is described in Section (4.3.4). The input data for this actuator are also in file "Actor\_Models.Inp" and its title is "Electro-Mechanical Actuator (without motor and amplifier dynamics)". The following dialog shows its parameters. Notice at the bottom of the dialog that there is no filter included. It also provides the option to include the servo motor and amplifier dynamics which are approximated with a first order lag. However, in this example they are well above the actuator bandwidth and they are not included. In the input data line: "Include Motor and Amplifier Dynamics, (0=no,1=yes): 0", the answer is "0", indicating that they are not included. This option of excluding the servo-motor and amplifier dynamics is used only in the electro-mechanical actuator models because the servo-motor and amplifier dynamics are usually at a higher bandwidth in comparison with the load dynamics and sometimes it is not necessary to include them in the model.

Electro-Mechanical Actuator, Model (A) Data

Enter Electro-Mechanical Actuator Model (A) Parameters

Electro-Mechanical Actuator (without motor and amplifier dy

Exit

Save

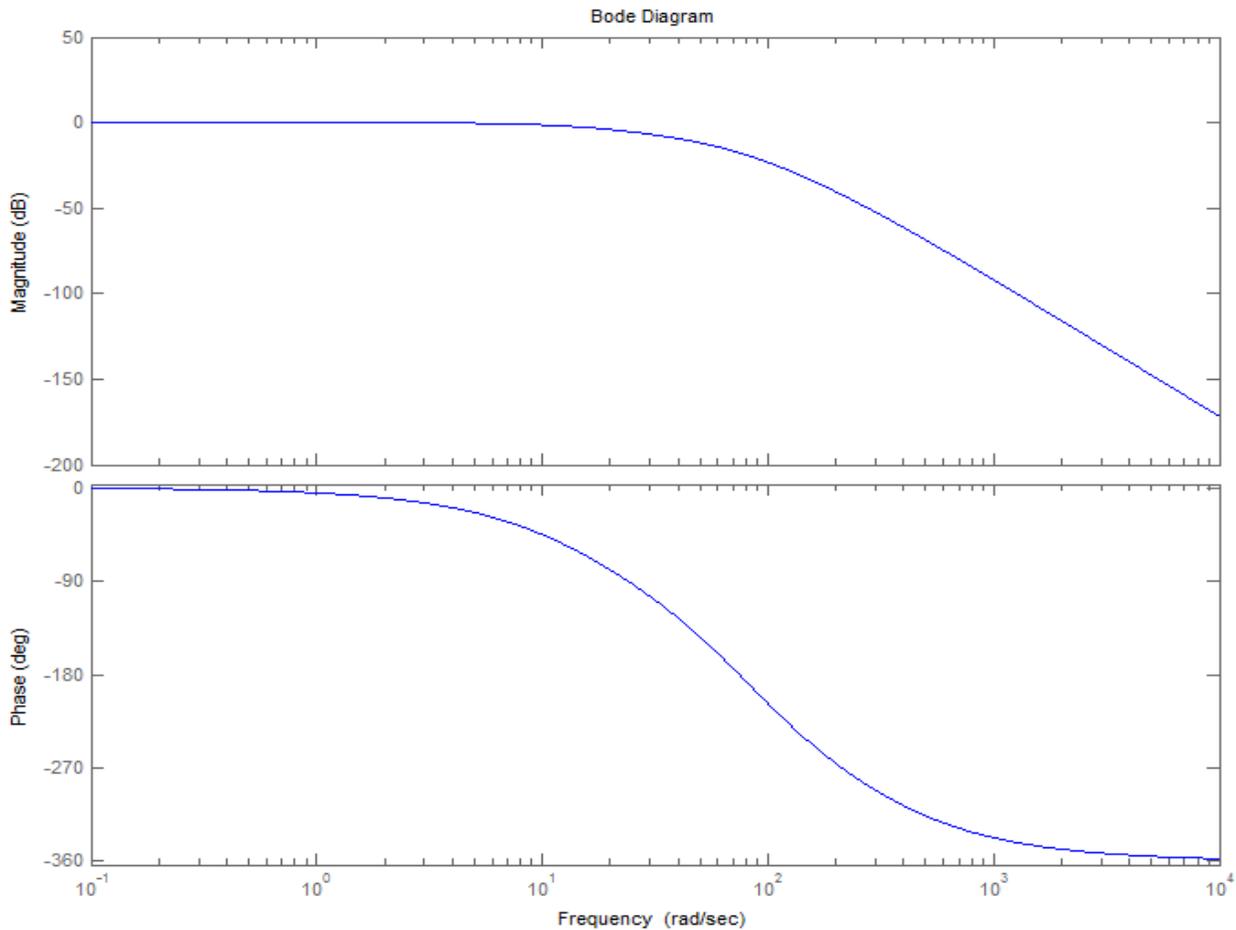
Run

Input Command Angle Conversion ( $G_p$ ), (counts/radian)	0.1200E+05
Position Feedback Gain from Load ( $K_{pl}$ ), (counts/rad)	0.000
Position Feedback Gain from Motor ( $K_{pm}$ ), (counts/rad)	2.540
Position to Voltage Conversion ( $G_v$ ), (volts/count)	0.3930
Rate Feedback Gain from Motor ( $K_v$ ), (volts/rad/sec)	0.1600E-01
Amplifier Gain ( $K_a$ ), (volts/volt)	1.000
Amplifier Time Constant ( $T_a$ ), (seconds)	0.1000E-02
Motor Winding Resistance ( $R_m$ ), (Ohms)	0.1000
Motor Time Constant ( $T_m$ ), (seconds)	0.9000E-03
Motor Back EMF Gain ( $K_{emf}$ ), (volts/rad/sec)	0.4350E-01
Motor Current to Torque Gain ( $K_t$ ), (ft-lb/amp)	0.3217E-01
Motor Moment of Inertia ( $J_m$ ), (ft-lb-sec <sup>2</sup> )	0.7166E-04
Motor Viscous Damping ( $K_{md}$ ), (ft-lb/rad/sec)	0.000
Motor to Load Gear Ratio ( $N$ ), (rad/rad)	4723.
Load Mom of Inertia about Gimbal ( $J_e$ ), (ft-lb-sec <sup>2</sup> )	80.60
Moment Arm ( $R$ ) Actuator Shaft from Gimbal (feet)	1.160
Load Viscous Damping Coefficient ( $F_v$ ), (ft-lb/rad/sec)	0.1077E+05
Gimbal Bearing Spring Constant ( $K_{sp}$ ), (ft-lb/rad)	0.000
Attach Point Flexion Spring Constant ( $K_m$ ), (lb/ft)	0.3600E+06
Mechanical Conversion Efficiency Factor ( $G_t$ )	1.100
Sensor Bias Coefficient ( $B_i$ ), (rad/rad)	0.000
Include the Torque Motor and Amplifier Dynamics ?	<input type="radio"/> Yes <input checked="" type="radio"/> No

Order of Compensator  $C(s)$  in the Actuator Loop

No Compensation     First Order     Second Order

The program reads the actuator input parameters from file "Actor\_Models.Inp", generates the state-space model of the electro-mechanical type (A) actuator, and saves it in the systems file "Actor\_Models.Qdr", using the same title "Electro-Mechanical Actuator (without motor and amplifier dynamics)" above the matrices.



**Figure (4.5.5) Frequency Response of the Electro-Mechanical Model (A) Actuator**

#### 4.5.6 Electro-Mechanical Type (B) Actuator Model

In this example we will model and analyze an electro-mechanical type (B) actuator for a rudder surface using the EMA type (B) model that was described in Section (4.3.5). We will use Flixan to create the linear model and we will also create a non-linear version of the same model in Simulink. We will simulate its non-linear response to deflection commands and also to external loads. Finally we will calculate and compare the frequency responses of the two models. The actuator parameters for this model are also in file "*Actor\_Models.Inp*" and its title is "*Electro-Mechanical Rudder Actuator, Type-B (feet)*".

The title of the actuator data-set is selected from the following menu and the actuator parameters are shown in the EMA type (B) dialog below. The actuator also includes a first order compensator  $[100/(s+100)]$  in the control loop. Click on "Run", and a smaller dialog shows up for browsing or entering the 1<sup>st</sup> order compensator data. Click "OK" to accept the filter, process the data, and to save the system in file "*Actor\_Models.Qdr*" under the same title "*Electro-Mechanical Rudder Actuator, Type-B (inch)*". This system is also saved as a Matlab function in file "*ema\_b.m*" to calculate its frequency response.

Select a Set of Data from Input File

Select a Set of Input Data for "ACTUATOR" from an Input File: Actor\_Models.inp

Run Input Set

Exit Program

Create New

- Shuttle Main Engine Actuator (with 1st order filter)
- Shuttle Elevon Actuator (without filter)
- Shuttle Elevon Actuator (with 2nd order filter)
- Shuttle Solid Rocket Booster Actuator
- Electro-Mechanical Actuator (without motor and amplifier dynamics)
- Electro-Mechanical Rudder Actuator, Type-B (feet)**
- Electro-Mechanical Rudder Actuator, Type-B (inch)
- Electro-Mechanical Actuator for an Elevator
- Electro-Mechanical Actuator with an Extendable Push Rod Simplified

Electro-Mechanical Actuator, Model (B) Data

Enter Electro-Mechanical Actuator Model (B) Parameters

Electro-Mechanical Rudder Actuator, Type-B (feet)

Exit

Save

Run

Position Error Loop Gain, (Kpl) in (Volt/ft) 40.00

Motor Rate Feedback Gain, (Kmr). (Volt/ft/sec) 0.000

Position Error Integral Gain, (Ki). (Volt/ft-sec) 0.000

Servo Motor Bandwidth, Wm. (rad/sec) 3000.

Motor Control Torque Gain, (Kv) in (ft-lb/ volt) 490.2

Motor Rotor Mom of Inertia, (Jm) in (ft-lb-sec^2) 0.6536E-03

Gear Ratio (Ngear). (Motor Turns/ Screw Turns) 17.00

Screw Ratio (Nscrew). (Piston Extens/Screw Turn). (ft/rad) 0.3180E-01

Motor Gear Stiffness (Kgs). (ft-lb/ rad) 8376.

Motor Gear Friction (Kmfrr). (ft-lb/ rad/sec) 0.000

Shaft Viscous Friction, (Kdmp) (ft-lb/ rad/sec) 0.1000E-02

Motor/ Screw Gear Effectiveness Coeffic. (Keff) 0.8500

Moment of Inertia of Gear plus Screw, (Ja). (ft-lb-sec^2) 0.3500E-02

Actuator (Piston+Electrical) Stiffness, (Kact) in (lbf/ft) 0.1400E+06

Load Stiffness, (Klod). (Surface or Nozzle). (lbf/ft) 0.1810E+06

Vehicle Backup Structure Stiffness, (Kbck) in (lb/ft) 0.3500E+06

Moment Arm (R). Actuator Rod to Gimbal dist. in (feet) 3.600

Effectr Load Mom of Inertia about Hinge, (Je). (ft-lb-sec^2) 52.20

Effector Hinge Viscous Damping (Be) in (ft-lb-sec) 1600.

Effector Hinge Bearing Spring Constant, (Kg) in (ft-lb/rad) 0.000

Order of Compensator C(s) in the Forward Loop

No Compensation  First Order

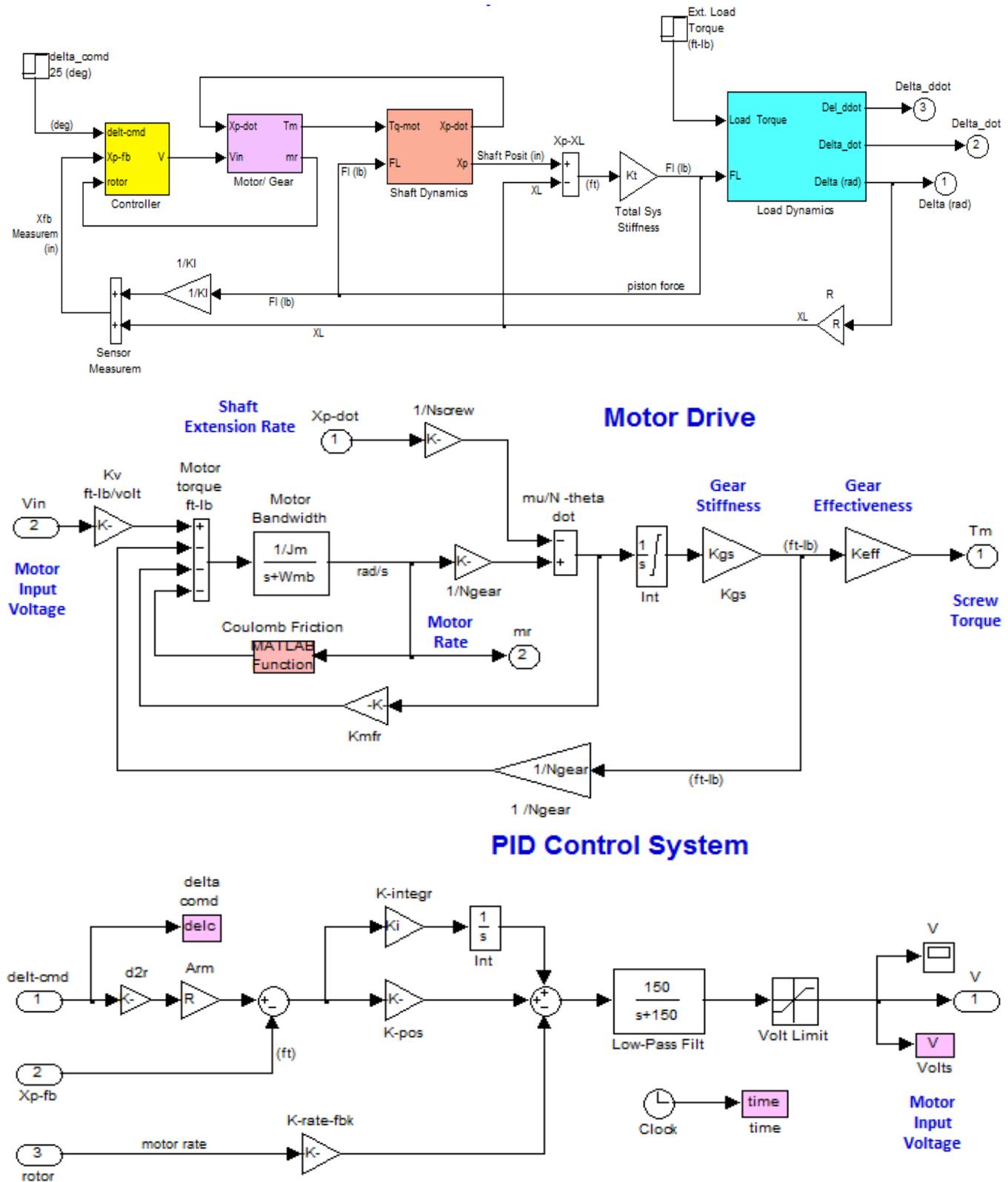
Enter Filter Coefficients

Define a First Order Filter Transfer Function:  
 $(Tn s + 1) / (Td s + 1)$  OK

Numerator Time Constant (Tn) in (sec) 0.000

Denominator Time Constant (Td) in (sec) 0.1000E-01

This actuator system is also implemented using Matlab/ Simulink in directory "C:\Flixan\Actuator\Examples\EM-Actuators\Rudder". The Simulink model is "Rudder\_EMAB.Mdl", shown in Figure (4.5.6.1), and it includes the actuator non-linearities. The parameters are initialized from file "run.m".



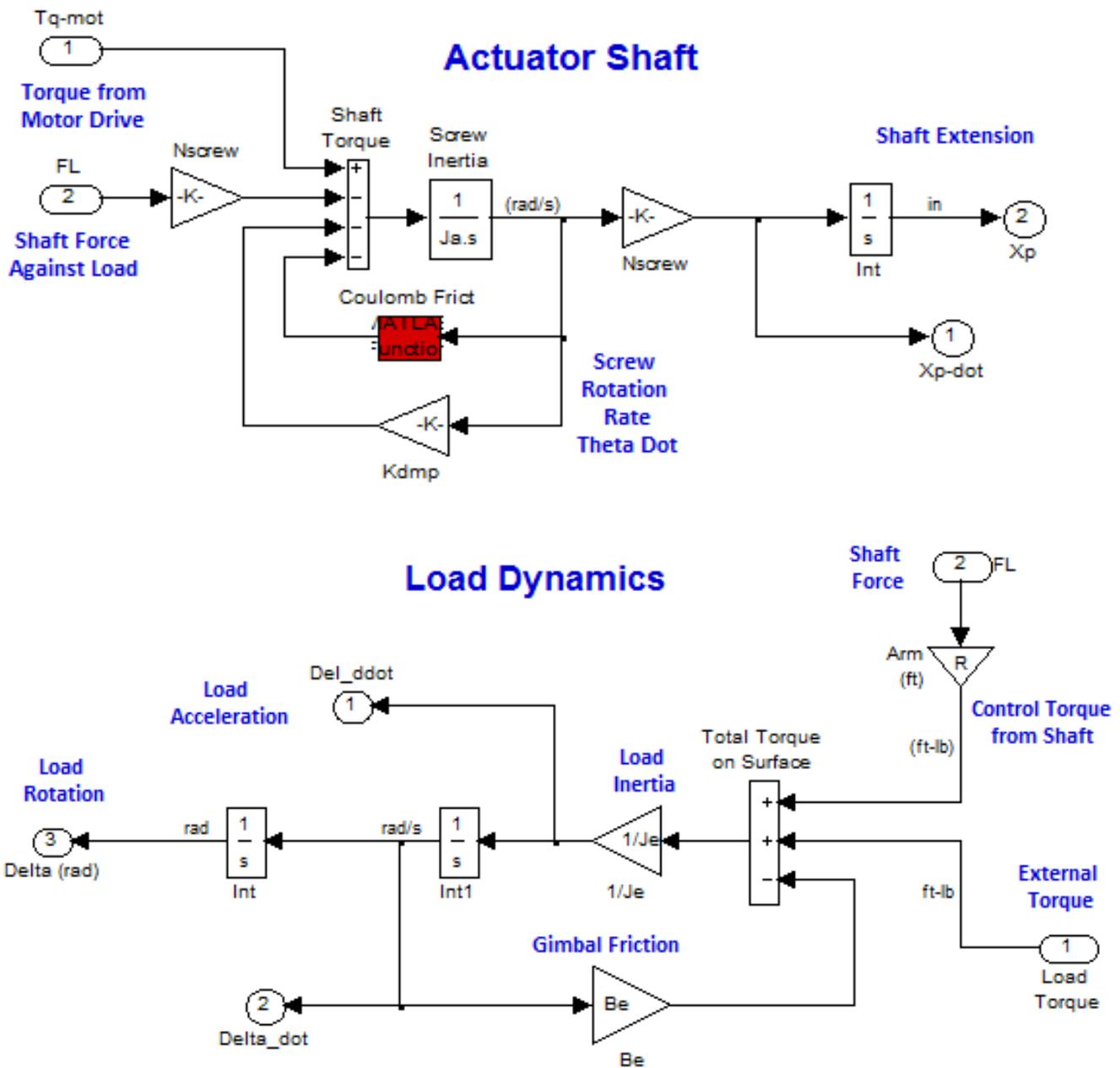
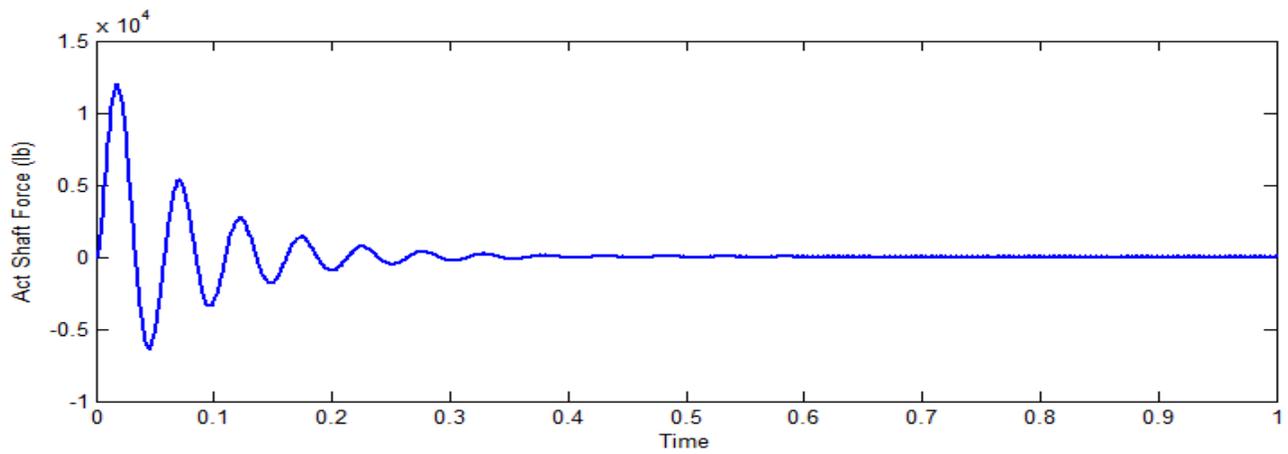
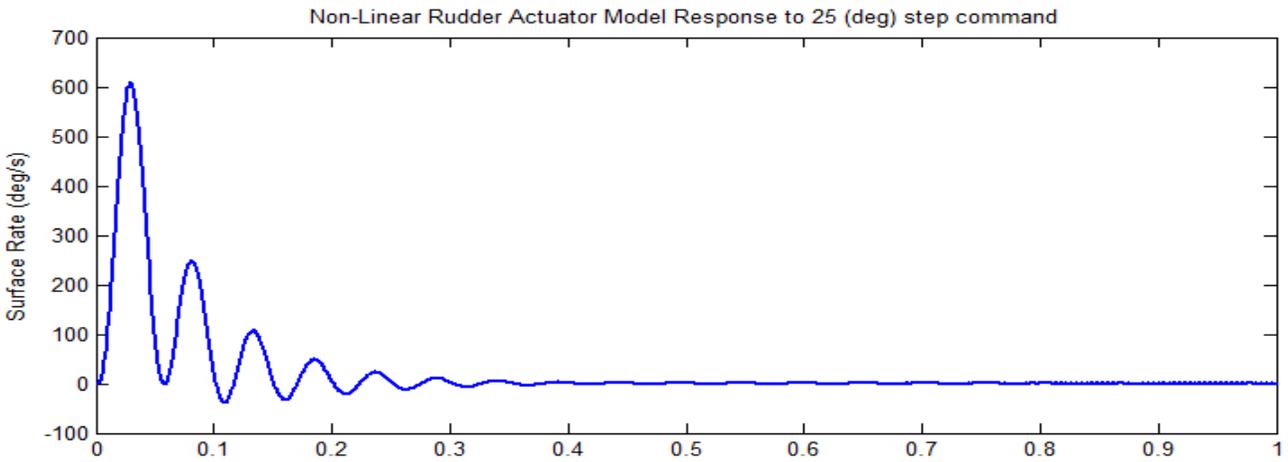
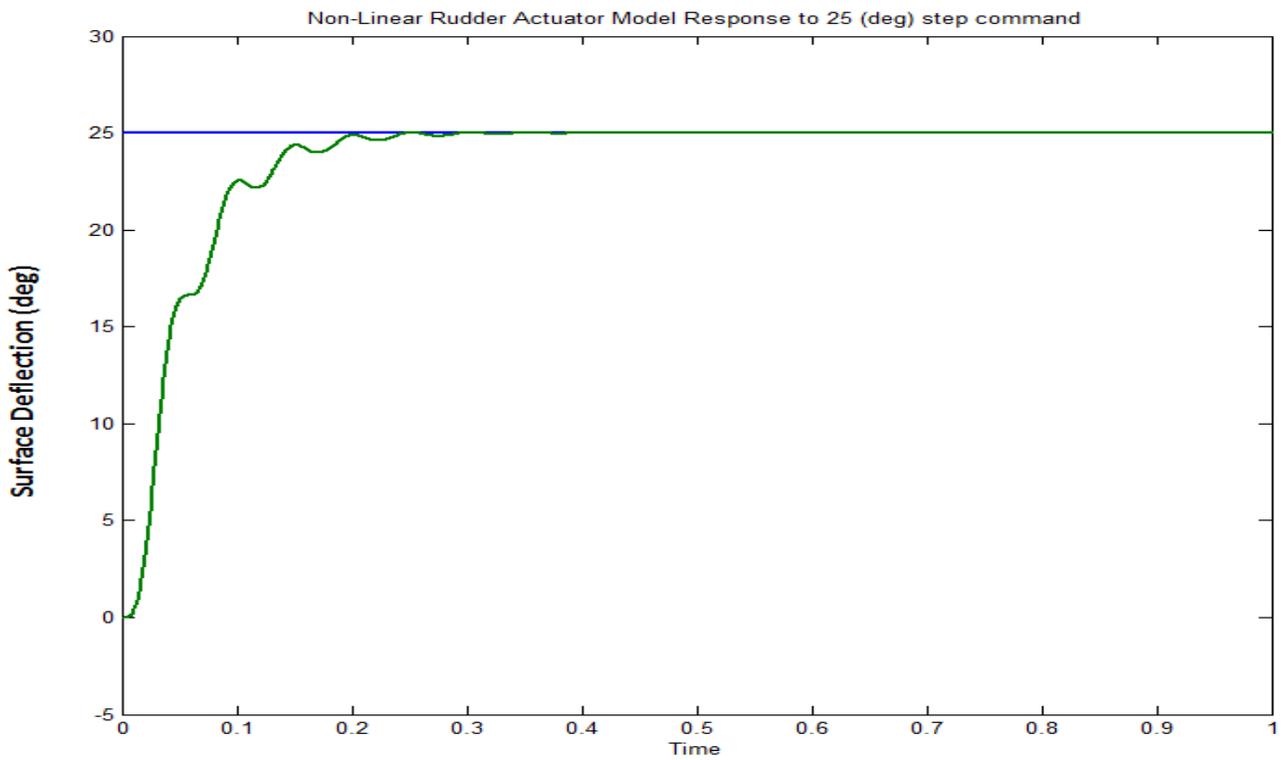
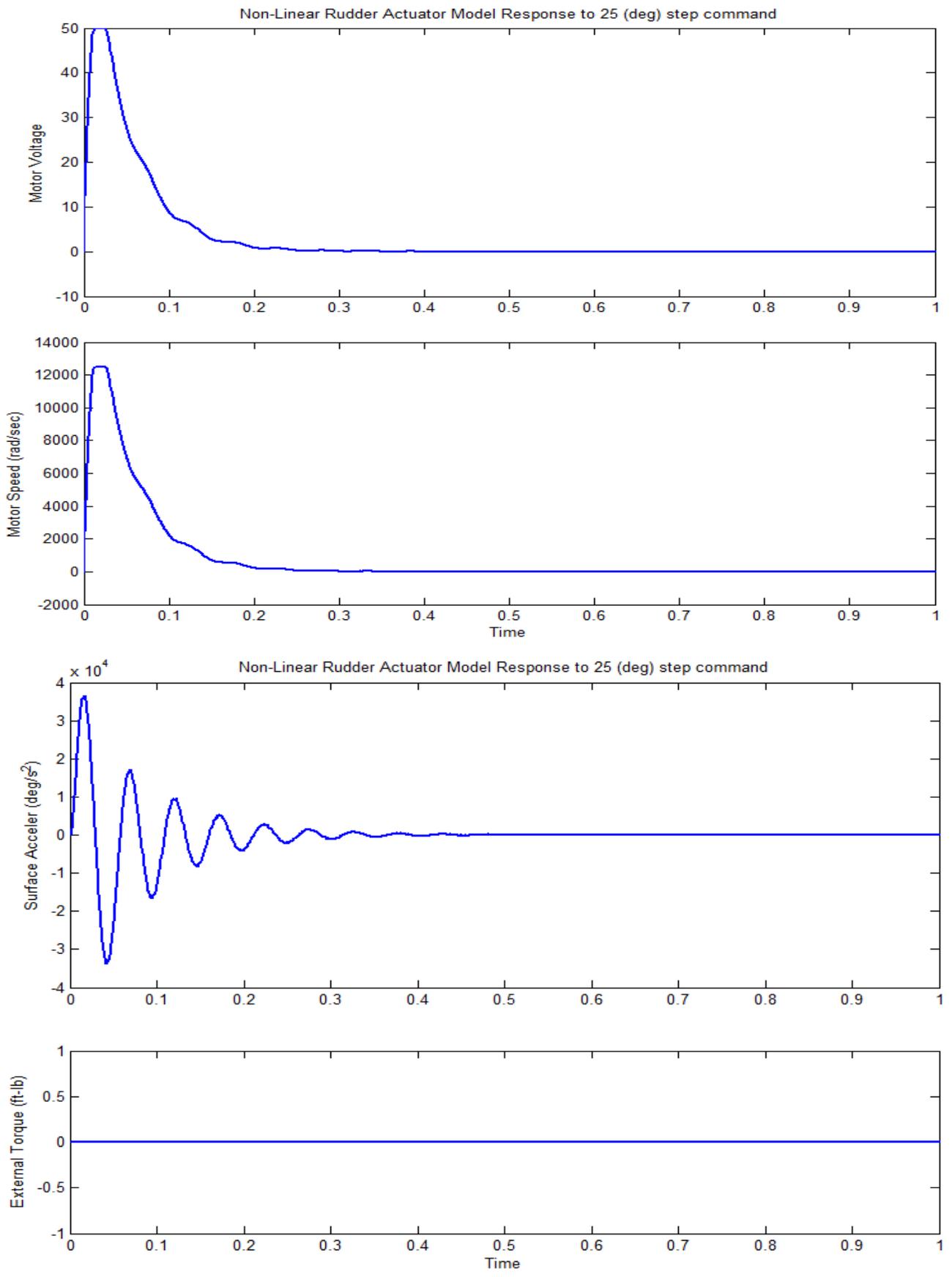


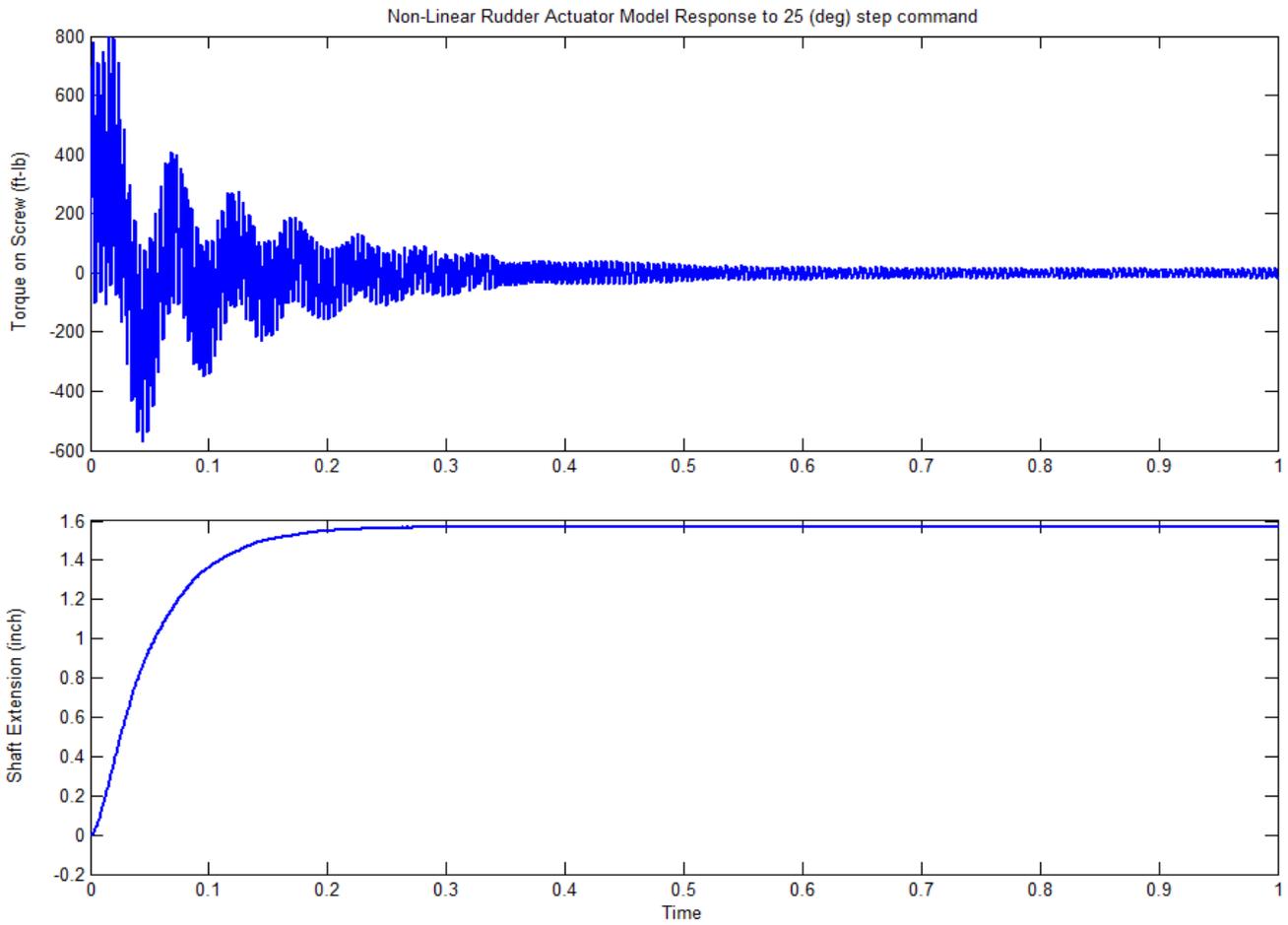
Figure (4.5.6.1) Non-Linear Simulink Model "Rudder\_EMAB.Mdl" for simulating the Electro-Mechanical actuator system response to external loads and rudder deflection commands

### Simulations

Figure (4.5.6.2) shows the actuator's response to a  $25^\circ$  deflection command. The system's response time is 0.15 (sec) and there is no steady-state error. The motor voltage momentarily peaks to the 50 volts saturation level and the motor speed is proportional to the voltage. The combined stiffnesses due to the support structure, shaft, and rudder stiffness create a resonance at approx 22 Hz which damps out reasonably fast. The actuator shaft displacement is about 1.5 (in). The screw torque from the motor is noisy because of the non-linearities and the gear stiffness.

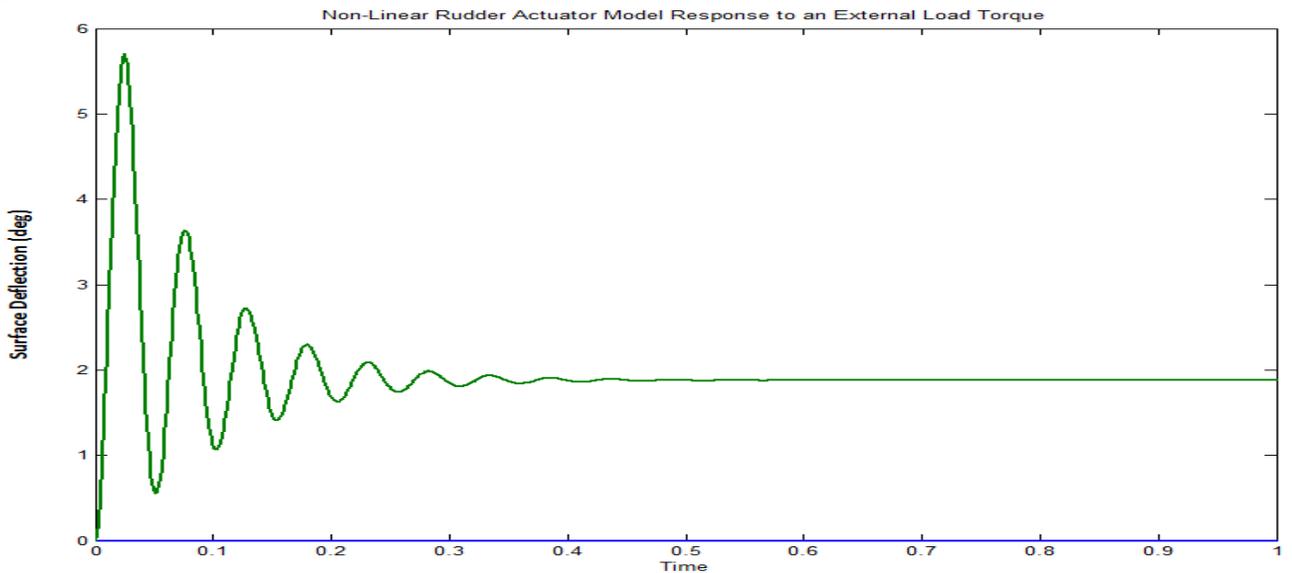


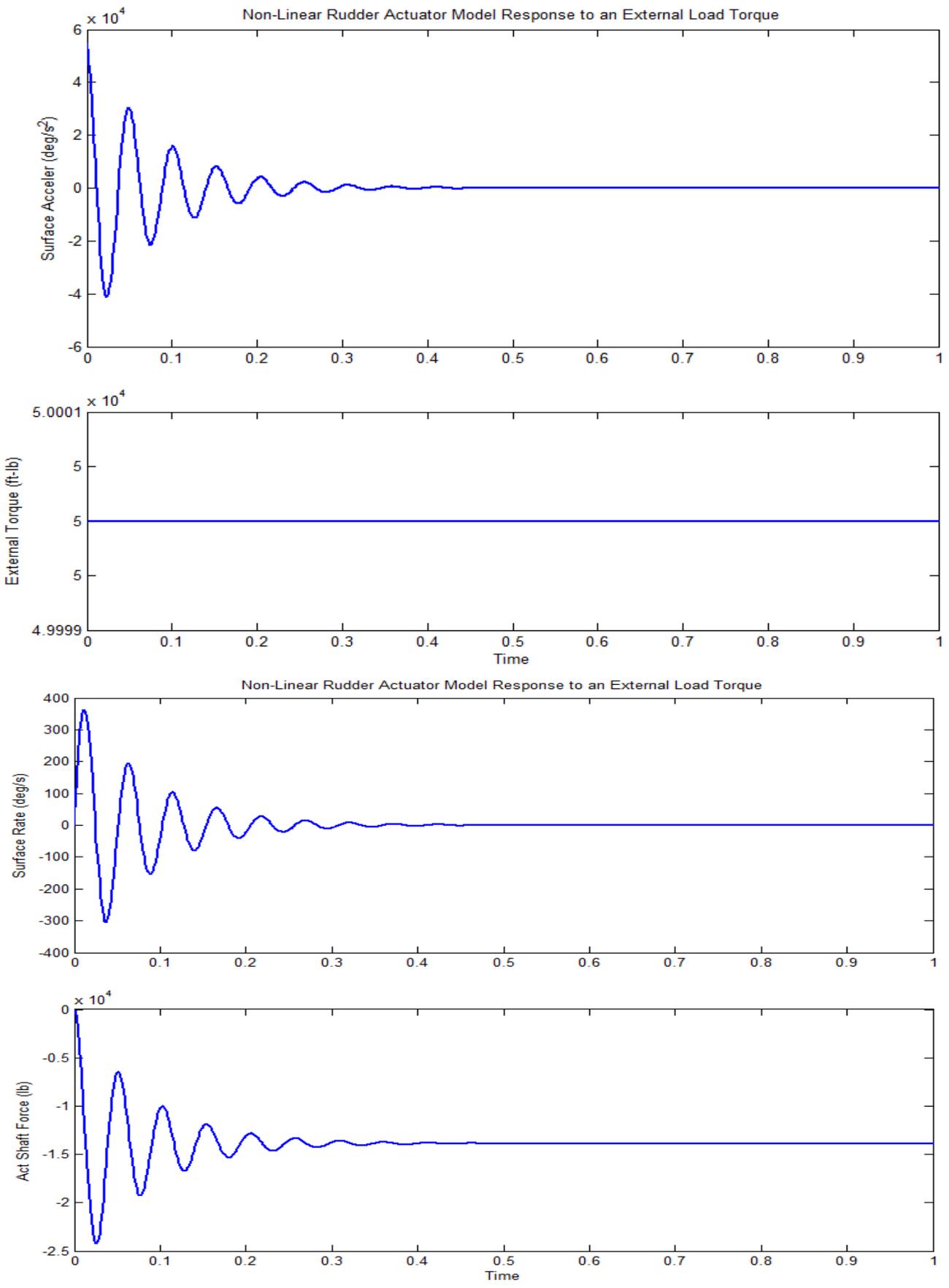




**Figure (4.5.6.2) Non-Linear Actuator System's response to a 25° deflection command**

The next set of plots in Figure (4.5.6.3) shows the actuator system response to a steady 5,000 (lb) external load-torque with zero deflection command. The load causes a steady shaft displacement of 0.1 (in) and 2° of surface deflection due to structural stiffness. The voltage goes negative to counteract the disturbance but it only takes a very small voltage at steady-state to supply 400 (ft-lb) of torque to the screw when the motor speed becomes zero.





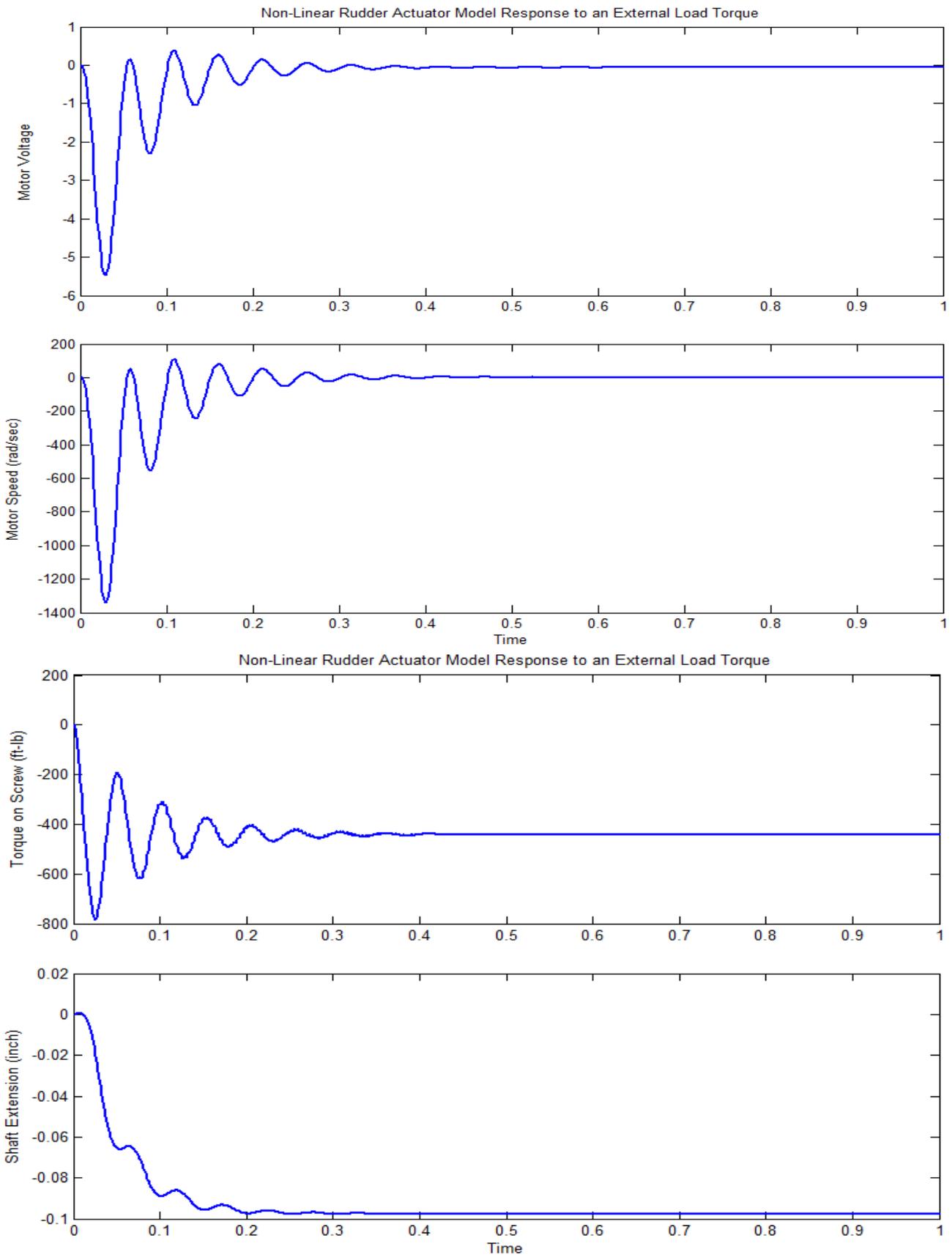


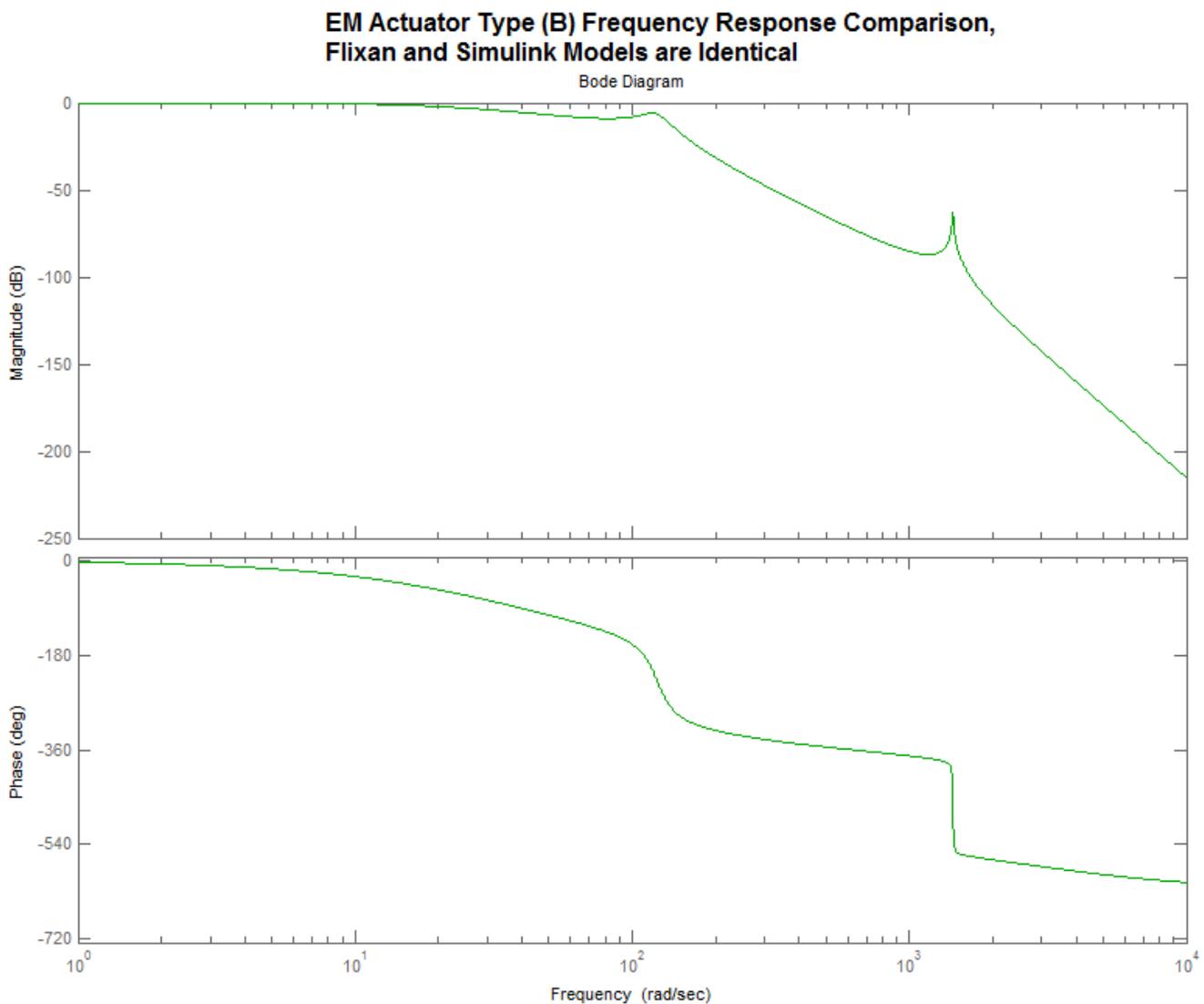
Figure (4.5.6.3) Non-Linear Actuator System's response to an External Load-Torque

## Linear Analysis

The following Matlab script "frequ.m" calculates and plots the frequency responses of the two EMA type (B) models. That is, the Flixan generated model from file "ema\_b.m" and the Simulink model "Rudder-EMAB-Anal.Mdl" shown in Figure (4.5.6.1). Figure (4.5.6.4) compares the responses and they are identical. The structural resonance is clearly visible at 22 Hz.

```
run
[a1,b1,c1,d1]= ema_b;
[a2,b2,c2,d2]= linmod('Rudder_EMAB_Anal');
w=logspace(0,4,10000);

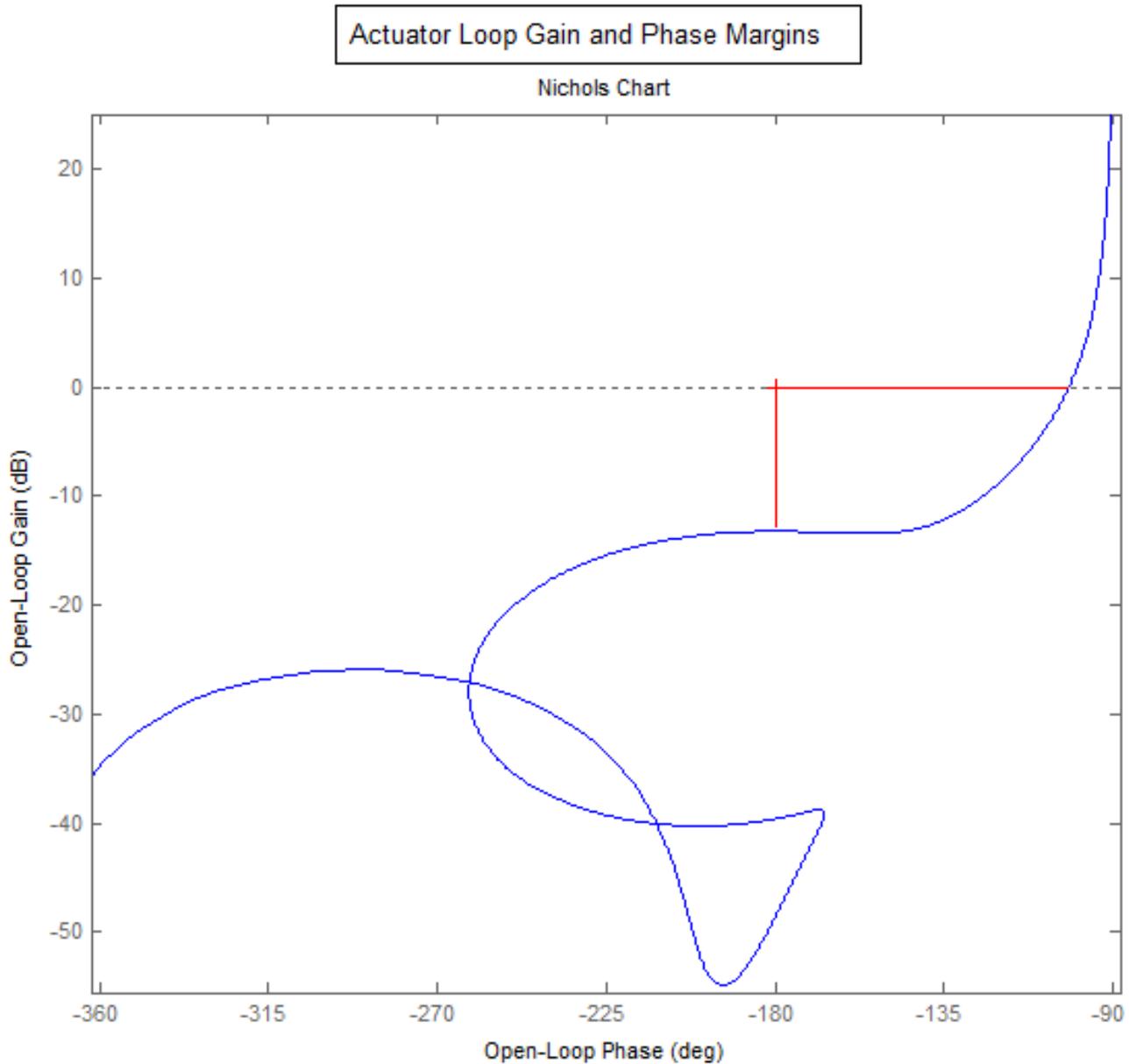
sys1=ss(a1, b1(:,1), c1(1,:), d1(1,1));
sys2=ss(a2, b2(:,1), c2(1,:), d2(1,1));
w=logspace(0,4,8000);
figure(1); Bode(sys1,sys2,w)
```



**Figure (4.5.6.4) Frequency response comparison between the Flixan and Simulink implementations of the closed-loop EM actuator model. The responses are identical.**

The same script file also generates the Nichols plot using a similar open-loop Simulink model to show the actuator loop gain and phase margins.

```
[a3,b3,c3,d3]= linmod('Rudder_EMAB_OpenLp');  
sys3=ss(a3, b3(:,1), c3(1,:), d3(1,1));  
figure(3); Nichols(sys3,w)
```

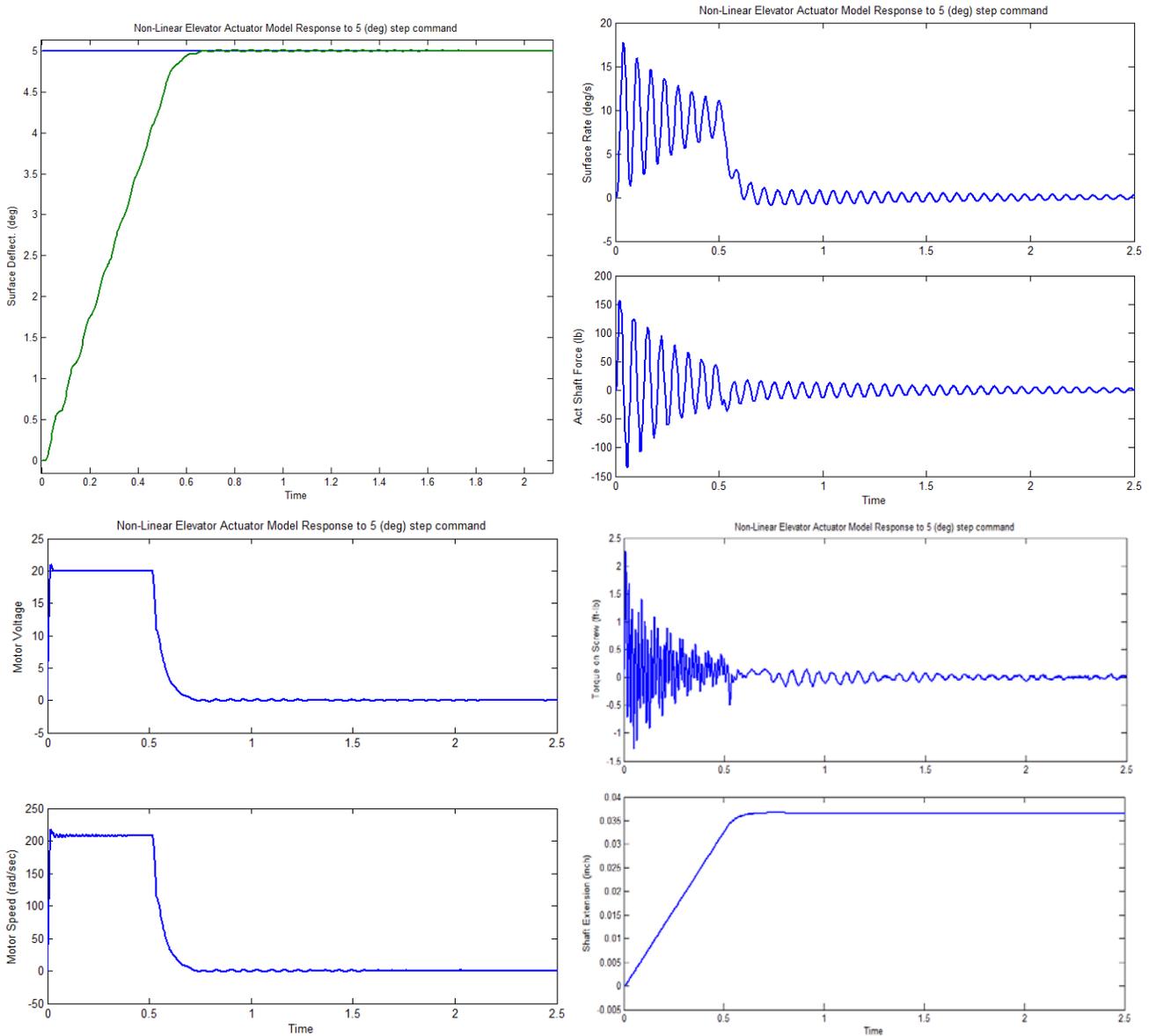


**Figure (4.5.6.5) Nichols Plot showing the Gain and Phase margins of the Rudder EMA type (B) system**

There are more actuator models included in files "Actor\_Models.Inp" and "Actor\_Models.Qdr" which are shown below.

### 4.5.7 Electro-Mechanical Type (B) Actuator Model for an Elevon

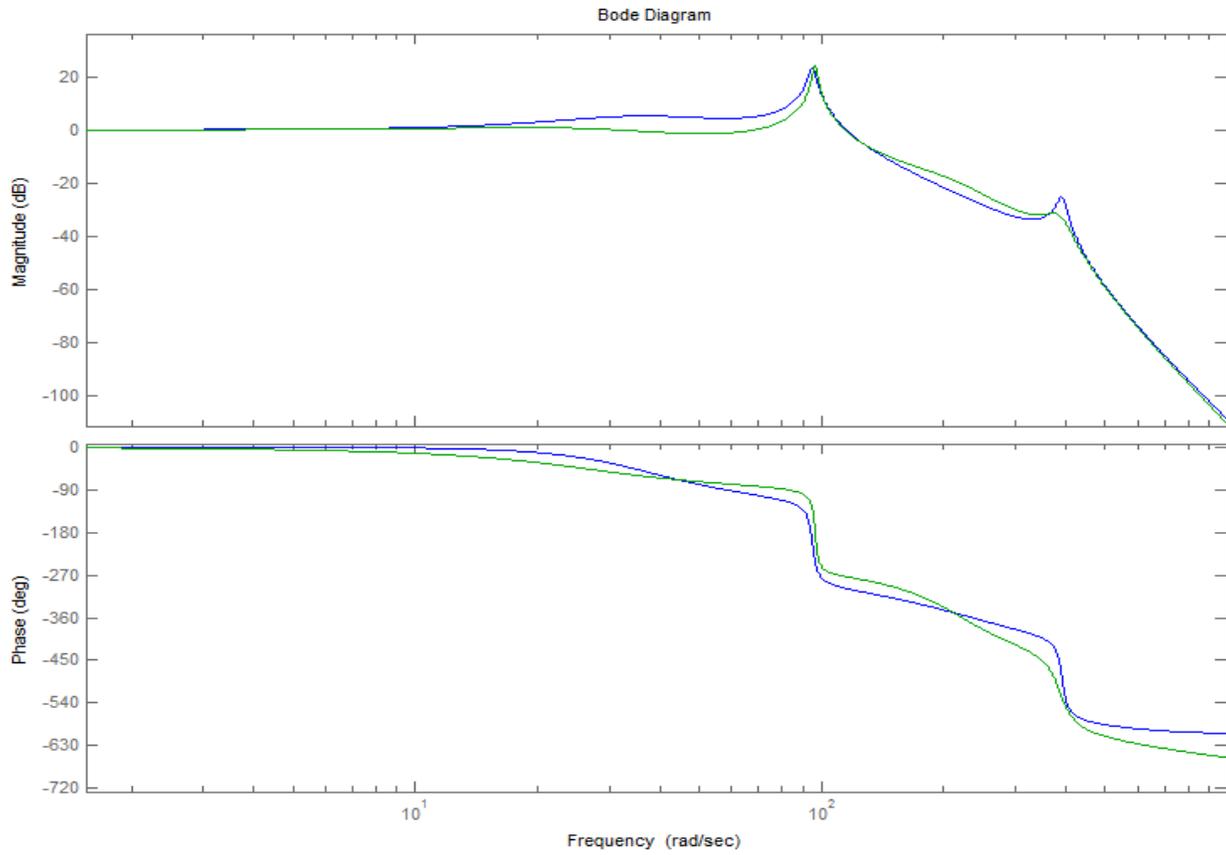
The input file "Actor\_Models.Inp" includes also the EMA parameters for an Elevator actuator. Its title is "Electro-Mechanical Actuator for an Elevator" and its Matlab system function is "elev\_ema\_b.m". This EMA system is also implemented in a Simulink model "Elevator\_EMAB.mdl" in folder "C:\Flixan\Actuator\Examples\EM-Actuators\Elevator" that includes all actuator non-linearities. Its block diagram is similar to Figure (4.5.6.1), but it includes a 2<sup>nd</sup> order low-pass filter and a lead-lag in the feedback path. Figure (4.5.7.1) shows its response to a 5° deflection command.



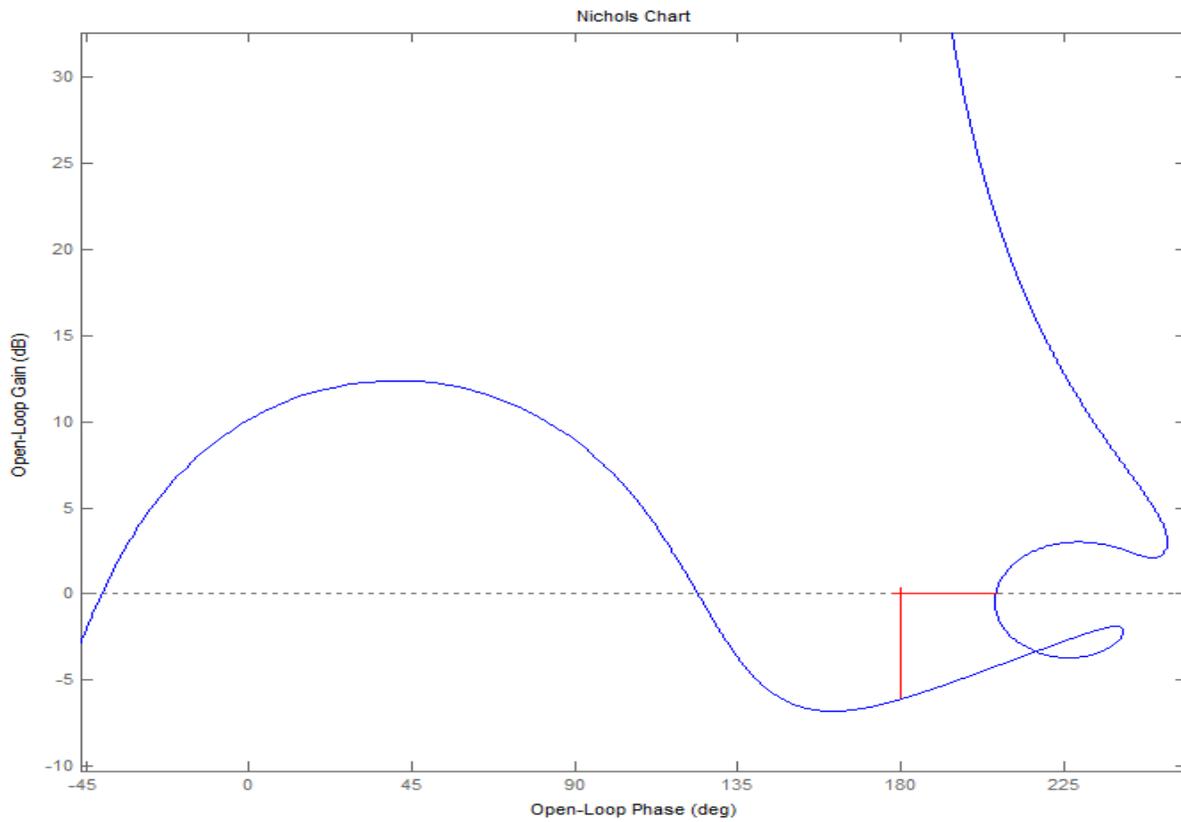
**Figure (4.5.7.1) Elevon Actuator Response to a 5 (deg) step deflection command**

Figure (4.5.7.2) shows the actuator frequency response calculated from the two models: the Flixan model and the Simulink model. They are a little different because the Simulink model includes a lead-lag filter in the position measurement feedback. The responses show two resonances due to backup and gear stiffnesses. Figure (4.5.7.3) is a Nichols plot that shows the actuator loop phase and gain margins.

### Elevator EMA Actuator Frequency Response Using Different Compensators



### Stability Margins for the Elevator Actuator



## 4.6 The Input Data File "Actor\_Models.Inp"

### BATCH MODE INSTRUCTIONS .....

Batch set for creating a bunch of actuator models

! This Batch generates several types of actuator models. It also saves them as Matlab

! functions for further analysis

```
!
Actuator Model : TVC Engine Actuator Example ...
Actuator Model : TVC Engine Actuator Example (Units in inches)
Actuator Model : Missile Actuator
Actuator Model : Shuttle Main Engine Hydraulic Actuator (Type-B)
Actuator Model : Shuttle Main Engine Actuator (with 1st order filter)
Actuator Model : Shuttle Elevon Actuator (without filter)
Actuator Model : Shuttle Elevon Actuator (with 2nd order filter)
Actuator Model : Shuttle Solid Rocket Booster Actuator
Actuator Model : Electro-Mechanical Actuator (without motor and amplifier dynamics)
Actuator Model : Electro-Mechanical Rudder Actuator, Type-B (feet)
Actuator Model : Electro-Mechanical Rudder Actuator, Type-B (inch)
Actuator Model : Electro-Mechanical Actuator for an Elevator
Actuator Model : Electro-Mechanical Actuator with an Extendable Push Rod Simplified
!
! Matlab Models ...
To Matlab Format : TVC Engine Actuator Example ...
To Matlab Format : Missile Actuator
To Matlab Format : Shuttle Main Engine Hydraulic Actuator (Type-B)
To Matlab Format : Shuttle Main Engine Actuator (with 1st order filter)
To Matlab Format : Shuttle Elevon Actuator (without filter)
To Matlab Format : Shuttle Elevon Actuator (with 2nd order filter)
To Matlab Format : Electro-Mechanical Actuator (without motor and amplifier dynamics)
To Matlab Format : Electro-Mechanical Rudder Actuator, Type-B (feet)
To Matlab Format : Electro-Mechanical Rudder Actuator, Type-B (inch)
To Matlab Format : Electro-Mechanical Actuator with an Extendable Push Rod Simplified
-----
```

### ACTUATOR INPUT DATA ..... SIMPLE GENERIC MODEL

TVC Engine Actuator Example ...

Symbol	Parameter Description	(Units)	Value
C(s)	Order of Compensat:(0,1,2), Coefficients	(---)	0
Kc	Input Command Gain	(volt/rad)	9.5700
Ka	Gain of Amplifier	(amps/volt)	4.7200
Ksv	Closed-Loop Actuator Servo Gain	(ft/sec/amp)	0.325
Kact	Actuator Stiffness (Piston+Oil+Electric)	(lb/ft)	2340000.0
Klod	Load Stiffness,(Con. Surface or Nozzle)	(lb/ft)	3600000.0
Kbck	Vehicle Backup Structure Stiffness	(lb/ft)	8400000.0
R	Moment Arm of Actuator Rod from Gimbal..	(feet)	0.3
Jl	Load Inertia about the Gimbal	(ft-lb-s^2)	3.9333
Kg	Load Gimbal Bearing Spring Constant	(ft-lb/rad)	0.0000
Bg	Load Gimbal Bearing Viscous Damping	(ft-lb-sec)	8.3333
Kfb	Actuator Position Feedback Gain	(volts/ft)	32.64

### ACTUATOR INPUT DATA ..... HYDRAULIC TYPE A

Missile Actuator

Symbol	Parameter Description	(Units)	Value
C(s)	Order of Compensat:(0,1,2), Coefficients	(---)	0
Kc	Input Command Gain	(volt/rad)	46.12
Ka	Gain of Servo-Amplifier	(ma/volt)	20.4
Kv	Power Valve Flow Gain	(ft^3/sec/ma)	0.596E-04
Kdpf	Differential Pressure Feedbk Gain	(ma/lb/ft^2)	0.1E-4
Tc	Differential Pressure Feedback Time Constant	(sec)	0.1
Cl	Leakage Coeffic. (Orifice+Spool),	(ft^3/sec/lb/ft^2)	0.2E-10
Vt	Volume of Compressed Fluid	(ft^3)	0.11600E-02
BM	Bulk Modulus of Hydraulic Fluid	(lb/ft^2)	0.28800E+08
Km	Stiffness of Piston Rod plus Mount Backup	(lb/ft)	0.13333E+08
R	Moment Arm of Actuator Rod from Gimbal..	(feet)	0.30000
A	Piston Cross-Sectional Area	(feet^2)	0.015
Ie	Engine Inertia about Gimbal	(ft-lb-s^2)	0.56
Fv	Engine Viscous Damping	(ft-lb-sec)	410.00
Ksp	Engine Gimbal Bearing Spring Constant	(ft-lb/rad)	0.11100e+06
Kf	Position Transducer Feedback Gain	(volts/ft)	132.0

**ACTUATOR INPUT DATA ..... HYDRAULIC TYPE B**

**Shuttle Main Engine Hydraulic Actuator (Type-B)**  
**! Shuttle Main Engine Actuator Without Compensator**  
**! Using the Hydraulic Actuator Model Type (B)**

Symbol	Parameter Description	(Units)	Value
C(s)	Order of Compensat:(0,1,2), Coefficients	(---)	0
Kav	Total Gain of Amplifier + Torque Motor	(ft-lb/rad)	0.91000
Kact	Power Valve and Actuator Gain	(ft/s/ft-lb)	53.950
Kt	Piston Ram Stiffness	(lb/ft)	0.20280E+07
R	Moment Arm of Actuator Ram from Gimbal	(feet)	2.4800
Ie	Engine Inertia about Gimbal	(ft-lb-s <sup>2</sup> )	4516.0
Be	Engine Viscous Damping	(ft-lb-sec)	16500.
Kb	Engine Gimbal Bearing Spring Constant	(ft-lb/rad)	27160.0
Kl	Engine Mount Structural Stiffness	(lb/ft)	0.27960E+07
Tc	Different. Pressure Feedbk Time Constant	(seconds)	0.12660
Kdpf	Differ Pressure Feedbk Linearizat. Gain	(ft-lb/lb)	0.40400E-06
Kfb	Position Feedback Gain	(rad/feet)	0.4

**ACTUATOR INPUT DATA ..... HYDRAULIC TYPE B**

**Shuttle Main Engine Actuator (with 1st order filter)**  
**! Shuttle Main Engine Actuator, same as above, with 10/(s+10) Compensator**  
**! in the control loop Using the Hydraulic Actuator Model Type (B)**

Symbol	Parameter Description	(Units)	Value
C(s)	Order of Compensat:(0,1,2), Coefficients	(---)	1, 0.0, 0.1
Kav	Total Gain of Amplifier + Torque Motor	(ft-lb/rad)	0.91000
Kact	Power Valve and Actuator Gain	(ft/s/ft-lb)	53.950
Kt	Piston Ram Stiffness	(lb/ft)	0.20280E+07
R	Moment Arm of Actuator Ram from Gimbal	(feet)	2.4800
Ie	Engine Inertia about Gimbal	(ft-lb-s <sup>2</sup> )	4516.0
Be	Engine Viscous Damping	(ft-lb-sec)	16500.
Kb	Engine Gimbal Bearing Spring Constant	(ft-lb/rad)	27160.0
Kl	Engine Mount Structural Stiffness	(lb/ft)	0.27960E+07
Tc	Different. Pressure Feedbk Time Constant	(seconds)	0.12660
Kdpf	Differ Pressure Feedbk Linearizat. Gain	(ft-lb/lb)	0.40400E-06
Kfb	Position Feedback Gain	(rad/feet)	0.4

**ACTUATOR INPUT DATA ..... HYDRAULIC TYPE B**

**Shuttle Elevon Actuator (without filter)**  
**! Hydraulic Actuator Type (B) for the Space Shuttle Elevon, Without Filter**

Symbol	Parameter Description	(Units)	Value
C(s)	Order of Compensat:(0,1,2), Coefficients	(---)	0
Kav	Total Gain of Amplifier + Torque Motor	(ft-lb/rad)	198.329
Kact	Actuator Gain	(ft/s/ft-lb)	0.09438
Kt	Piston Ram Stiffness	(lb/ft)	0.16020E+07
R	Moment Arm of Actuator Ram from Gimbal	(feet)	0.7226
Ie	Elevon Inertia about Gimbal	(ft-lb-s <sup>2</sup> )	268.67
Be	Elevon Viscous Damping	(ft-lb-sec)	1250.0
Kb	Elevon Gimbal Bearing Spring Constant	(ft-lb/rad)	0.0
Kl	Elevon Mount Structural Stiffness	(lb/ft)	0.204E+07
Tc	Different. Pressure Feedbk Time Constant	(seconds)	0.12
Kdpf	Differ Pressure Feedbk Linearizat. Gain	(ft-lb/lb)	0.26E-03
Kfb	Position Feedback Gain	(rad/feet)	1.38396

**ACTUATOR INPUT DATA ..... HYDRAULIC TYPE B**

**Shuttle Elevon Actuator (with 2nd order filter)**  
**! Hydraulic Actuator Type (B) for the Space Shuttle Elevon, Including**  
**! a second order notch filter in the control loop at 65 (rad/sec)**

Symbol	Parameter Description	(Units)	Value
C(s)	Order of Compensat:(0,1,2), Coefficients	(---)	2, 0.015, 65.0, 0.6, 65.0
Kav	Total Gain of Amplifier + Torque Motor	(ft-lb/rad)	198.329
Kact	Actuator Gain	(ft/s/ft-lb)	0.09438
Kt	Piston Ram Stiffness	(lb/ft)	0.16020E+07
R	Moment Arm of Actuator Ram from Gimbal	(feet)	0.7226
Ie	Elevon Inertia about Gimbal	(ft-lb-s <sup>2</sup> )	268.67
Be	Elevon Viscous Damping	(ft-lb-sec)	1250.0
Kb	Elevon Gimbal Bearing Spring Constant	(ft-lb/rad)	0.0
Kl	Elevon Mount Structural Stiffness	(lb/ft)	0.204E+07
Tc	Different. Pressure Feedbk Time Constant	(seconds)	0.12
Kdpf	Differ Pressure Feedbk Linearizat. Gain	(ft-lb/lb)	0.26E-03
Kfb	Position Feedback Gain	(rad/feet)	1.38396

**ACTUATOR INPUT DATA ..... HYDRAULIC TYPE B**

**Shuttle Solid Rocket Booster Actuator**

Symbol	Parameter Description	(Units)	Value
C(s)	Order of Compensat:(0,1,2), Coefficients	(---)	0
Kav	Total Gain of Amplifier + Torque Motor	(ft-lb/rad)	1.8152
Kact	Power Valve and Actuator Gain	(ft/s/ft-lb)	75.4
Kt	Piston Ram Stiffness	(lb/ft)	0.45000E+07
R	Moment Arm of Actuator Ram from Gimbal	(feet)	5.8
Ie	Engine Inertia about Gimbal	(ft-lb-s <sup>2</sup> )	11000.0
Be	Engine Viscous Damping	(ft-lb-sec)	17166.0
Kb	Engine Gimbal Bearing Spring Constant	(ft-lb/rad)	0.19166E+06
Kl	Engine Mount Structural Stiffness	(lb/ft)	0.60000E+07
Tc	Different. Pressure Feedbk Time Constant	(seconds)	0.1252
Kdpf	Differ Pressure Feedbk Linearizat. Gain	(ft-lb/lb)	0.40000E-06
Kfb	Position Feedback Gain	(rad/feet)	0.172

**ACTUATOR INPUT DATA ..... ELECTROMECHANICAL TYPE A**

**Electro-Mechanical Actuator (without motor and amplifier dynamics)**

!  
! This EMA actuator was developed for an Advanced Launch Booster Vehicle  
!

Symbol	Parameter Description	(Units)	Value
C(s)	Order of Compensat:(0,1,2), Coefficients	(---)	0
	Include Motor and Amplif. Dynamics	(0=no,1=yes)	0
Gp	Input Command Angle Conversion	(counts/rad)	12000.
Kpl	Position Feedback Gain from Load	(counts/rad)	0.0
Kpm	Position Feedback Gain from Motor	(counts/rad)	2.54
Gv	Position to Voltage Conversion Gain	(volt/count)	0.393
Kv	Rate Feedback Gain from Motor	(volt/rad/s)	0.016
Ka	Amplifier Gain	(volt/volt)	1.0
Ta	Amplifier Time Constant	(seconds)	0.001
Rm	Motor Winding Resistance	(ohms)	0.1
Tm	Motor Time Constant	(seconds)	0.9e-3
Kemf	Motor Back EMF Gain	(volt/rad/s)	0.435e-01
Kt	Motor Current to Torque Gain	(ft-lb/amp)	0.3217e-01
Jm	Motor Moment of Inertia	(ft-lb-s <sup>2</sup> )	0.71660e-04
Kmd	Motor Viscous Damping	(ft-lb/rd/s)	0.00000
N	Motor to Load Gear Ratio	(rad/rad)	4723.
Je	Engine Inertia about Gimbal	(ft-lb-s <sup>2</sup> )	80.6
R	Moment Arm of Actuator from Gimbal	(feet)	1.16
Fv	Engine Viscous Damping Coefficient	(ft-lb/rad/s)	10770.
Ksp	Engine Gimbal Bearing Spring Constant	(ft-lb/rad)	0.00000
Km	Attach Point Flexion Spring Constant	(lb/ft)	0.36e+06
Gt	Mechanical Conversion Efficiency Factor	(---)	1.1
Bi	Sensor Bias Coefficient	(rad/rad)	0.0

**ACTUATOR INPUT DATA ..... ELECTROMECHANICAL TYPE B**

**Electro-Mechanical Rudder Actuator, Type-B (feet)**

! This EMA actuator Uses an Extendable Push Rod to Drive a Load.  
! The end of the rod is attached to a lever arm that rotates a control Surface.  
!

Symbol	Parameter Description	(Units)	Value
C(s)	Order of Compensat:(0,1,2), Coefficients	(---)	1, 0.0, 0.01
Kpl	Position Error Loop Gain	(Volt/ft)	40.0
Krf	Rate Feedback Gain	(Volt/in/sec)	0.0
Ki	Position Error Integral Gain	(Volt/in-sec)	0.0
Wm	Servo Motor Bandwidth,	(rad/sec)	3000.0
Kv	Servo Motor Torque Gain	(ft-lb/volt)	490.196
Jm	Motor Rotor Moment of Inertia	(in-lb-sec <sup>2</sup> )	6.536e-4
Ngr	Gear Ratio (Motor Turns/ Screw Turns)	( - )	17.0
Nscr	Screw Ratio (Piston Extension/Screw Turns)	(in/rad)	0.0318
Kgs	Motor Gear Stiffness	(in-lb/rad)	8376.0
Kmfr	Motor/ Gear Friction	(ft-lb/rad/sec)	0.0
Kdmp	Shaft Damping Friction	(ft-lb/rad/sec)	0.001
Keff	Gear Effectiveness Coefficient	( - )	0.85
Ja	Gear plus Screw Inertia	(in-lb-sec <sup>2</sup> )	0.0035
Kact	Actuator Stiffness (Piston+Electric)....	(lb/in)	140000.0
Klod	Load Stiffness,(Con. Surface or Nozzle)	(lb/in)	181000.0
Kbck	Vehicle Backup Structure Stiffness	(lb/in)	350000.0
R	Moment Arm of Actuator Rod from Gimbal..	(in)	3.6
Je	Load Inertia about the Gimbal	(in-lb-sec <sup>2</sup> )	52.2
Be	Nozzle Bearing Viscous Damping	(in-lb-sec)	1600.0
Kg	Gimbal Bearing Spring Constant	(in-lb/rad)	0.0000

**ACTUATOR INPUT DATA ..... ELECTROMECHANICAL TYPE B**

**Electro-Mechanical Actuator for an Elevator**

**! This EMA system has a full PID controller with a second order compensator  
!**

Symbol	Parameter Description	(Units)	Value
C(s)	Order of Compensat:(0,1,2), Coefficients	(---)	2, 1.,2100., 0.7,240.0
Kpl	Position Error Loop Gain	(Volt/ft)	12600.0
Krf	Rate Feedback Gain	(Volt/ft/sec)	0.0126
Ki	Position Error Integral Gain	(Volt/ft-sec)	378000.0
Wm	Servo Motor Bandwidth,	(rad/sec)	1000.0
Kv	Servo Motor Torque Gain	(ft-lb/volt)	208.0
Jm	Motor Rotor Moment of Inertia	(ft-lb-sec^2)	0.02
Ngr	Gear Ratio (Motor Turns/ Screw Turns)	( - )	8.195
Nscr	Screw Ratio (Piston Extension/Screw Turns)	(ft/rad)	0.002605
Kgs	Motor Gear Stiffness	(ft-lb/rad)	80.0
Kmfr	Motor/ Gear Friction	(ft-lb/rad/sec)	8.0
Kdmp	Shaft Damping Friction	(ft-lb/rad/sec)	0.001
Keff	Gear Effectiveness Coefficient	( - )	0.85
Ja	Gear plus Screw Inertia	(ft-lb-sec^2)	0.000423
Kact	Actuator Stiffness (Piston+Electric)...	(lb/ft)	1.0e7
Klod	Load Stiffness,(Con. Surface or Nozzle)	(lb/ft)	456000.0
Kbck	Vehicle Backup Structure Stiffness .....	(lb/ft)	456000.0
R	Moment Arm of Actuator Shaft from Gimbal..	(feet)	0.42
Je	Load Inertia about the Gimbal .....	(ft-lb-sec^2)	4.2
Be	Aero-Surface Bearing Viscous Damping	(ft-lb-sec)	26.
Kg	Aero-Surface Bearing Spring Constant	(ft-lb/rad)	0.0

**ACTUATOR INPUT DATA ..... SIMPLE GENERIC MODEL**

**Electro-Mechanical Actuator with an Extendable Push Rod Simplified**

**! This EMA actuator is a Simplified version of the above EMA system  
! The end of the rod is attached to a lever arm that rotates a control Surface.  
! A first order compensator (1000/s+1000) is used to model the motor dynamics**

Symbol	Parameter Description	(Units)	Value
C(s)	Order of Compensat:(0,1,2), Coefficients	(---)	1, 0.0, 0.001
Kc	Input Command Gain .....	(volt/rad)	0.3
Ka	Gain of Amplifier .....	(amps/volt)	120.0
Ksv	Closed-Loop Actuator Servo Gain .....	(ft/sec/amp)	0.039
Kact	Actuator Stiffness (Piston+Oil+Electric)	(lb/ft)	2340000.0
Klod	Load Stiffness,(Con. Surface or Nozzle)	(lb/ft)	3600000.0
Kbck	Vehicle Backup Structure Stiffness ....	(lb/ft)	8400000.0
R	Moment Arm of Actuator Rod from Gimbal..	(feet)	0.3
Jl	Load Inertia about the Gimbal .....	(ft-lb-s^2)	3.93
Kg	Load Gimbal Bearing Spring Constant	(ft-lb/rad)	0.0000
Bg	Load Gimbal Bearing Viscous Damping ....	(ft-lb-sec)	84.0
Kfb	Actuator Position Feedback Gain .....	(volts/ft)	1.0

END-

## 4.7 The Systems File "Actor\_Models.Qdr"

STATE-SPACE SYSTEM ...

TVC Engine Actuator Example ...

Number of Inputs, States, Outputs, Sample Time dT (for discrete)= 2 3 3 0.0000

Matrices: (A,B,C,D)

Matrix A Size = 3 X 3

	1-Column	2-Column	3-Column
1-Row	-0.241076622222E+02	-0.778862933333E+01	0.000000000000E+00
2-Row	0.000000000000E+00	0.000000000000E+00	0.100000000000E+01
3-Row	0.925431571454E+05	-0.277629471436E+05	-0.211865354791E+01

-----Matrix

B Size = 3 X 2

	1-Column	2-Column
1-Row	0.146803800000E+02	0.000000000000E+00
2-Row	0.000000000000E+00	0.000000000000E+00
3-Row	0.000000000000E+00	0.254239442707E+00

-----Matrix

C Size = 3 X 3

	1-Column	2-Column	3-Column
1-Row	0.000000000000E+00	0.100000000000E+01	0.000000000000E+00
2-Row	0.000000000000E+00	0.000000000000E+00	0.100000000000E+01
3-Row	0.925431571454E+05	-0.277629471436E+05	-0.211865354791E+01

-----Matrix

D Size = 3 X 2

	1-Column	2-Column
1-Row	0.000000000000E+00	0.000000000000E+00
2-Row	0.000000000000E+00	0.000000000000E+00
3-Row	0.000000000000E+00	0.254239442707E+00

-----  
Definition of System Variables

Inputs = 2

Load Deflection Command (Delta\_com), (rad)  
External Load Torque (Tl), (ft-lb)

Outputs = 3

Load Rotation at Gimbal (Delta) (rad)  
Load Rate at Gimbal (Delta-dot) (rad/sec)  
Load Accelerat. (Delta-dot-dot) (rad/sec^2)

States = 3

Actuator Piston Position (Xp) (feet)  
Load Rotation, (Delta) (radians)  
Load Rate, (Delta-dot) (rad/sec)

-----

STATE-SPACE SYSTEM ...

Missile Actuator

Number of Inputs, States, Outputs, Sample Time dT (for discrete)= 2 4 3 0.0000

Matrices: (A,B,C,D)

Matrix A Size = 4 X 4

Table with 4 columns (1-Column to 4-Column) and 4 rows (1-Row to 4-Row) of numerical values for Matrix A.

-----Matrix

B Size = 4 X 2

Table with 2 columns (1-Column to 2-Column) and 4 rows (1-Row to 4-Row) of numerical values for Matrix B.

-----Matrix

C Size = 3 X 4

Table with 4 columns (1-Column to 4-Column) and 3 rows (1-Row to 3-Row) of numerical values for Matrix C.

-----Matrix

D Size = 3 X 2

Table with 2 columns (1-Column to 2-Column) and 3 rows (1-Row to 3-Row) of numerical values for Matrix D.

Definition of System Variables

Inputs = 2

- Engine Deflection Command (Delta\_com) (rad)
External Load-Torque (Tl), (ft-lb)

Outputs = 3

- Engine Gimbal Rotation (Delta) (rad)
Engine Gimbal Rate (Del-dot) (rad/sec)
Gimbal Accelerat. (Delta-dot-dot) (rad/sec^2)

States = 4

- Load Pressure (PL) (lb/ft^2)
Engine Gimbal Rotation (Delta) (rad)
Engine Gimbal Rate, (Delta-dot) (rad/sec)
Differential Pressure Feedback (DPF) (ft-lb)

STATE-SPACE SYSTEM ...

Shuttle Main Engine Actuator (with 1st order filter)

! Shuttle Main Engine Actuator, same as above, with 10/(s+10) Compensator in the control loop
! Using the Hydraulic Actuator Model Type (B)

Number of Inputs, States, Outputs, Sample Time dT (for discrete)= 2 5 3 0.0000

Matrices: (A,B,C,D)

Matrix A Size = 5 X 5
1-Column 2-Column 3-Column 4-Column 5-Column
1-Row -0.521007765548E+02 0.000000000000E+00 0.000000000000E+00 -0.203189376000E+01 0.402237129840E+02
2-Row -0.539500000000E+02 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.490945000000E+02
3-Row 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00
4-Row 0.000000000000E+00 0.111369353410E+04 -0.276797413640E+04 -0.365367581931E+01 0.000000000000E+00
5-Row 0.000000000000E+00 -0.290128755365E+01 -0.272480686695E+01 0.000000000000E+00 -0.100000000000E+02

Matrix B Size = 5 X 2
1-Column 2-Column
1-Row 0.000000000000E+00 0.000000000000E+00
2-Row 0.000000000000E+00 0.000000000000E+00
3-Row 0.000000000000E+00 0.000000000000E+00
4-Row 0.000000000000E+00 0.221434898140E-03
5-Row 0.100000000000E+02 0.000000000000E+00

Matrix C Size = 3 X 5
1-Column 2-Column 3-Column 4-Column 5-Column
1-Row 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00 0.000000000000E+00
2-Row 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00
3-Row 0.000000000000E+00 0.111369353410E+04 -0.276797413640E+04 -0.365367581931E+01 0.000000000000E+00

Matrix D Size = 3 X 2
1-Column 2-Column
1-Row 0.000000000000E+00 0.000000000000E+00
2-Row 0.000000000000E+00 0.000000000000E+00
3-Row 0.000000000000E+00 0.221434898140E-03

Definition of System Variables

Inputs = 2
Nozzle Deflection Command (Delta\_com) (rad)
Load-Torque at the Gimbal, (Tl) (ft-lb)

Outputs = 3
Nozzle Gimbal Rotation (Delta) (rad)
Nozzle Gimbal Rate (Delta-dot) (rad/sec)
Gimbal Accelerat. (Delta-dot-dot) (rad/sec^2)

States = 5
Differ. Pressure Feedback (DPF) (ft-lb)
Piston Displacement (Xr) (feet)
Engine Gimbal Rotation (Delta) (rad)
Engine Gimbal Rate (Delta-dot) (rad/sec)
First Order Compensator State

STATE-SPACE SYSTEM ...

Shuttle Elevon Actuator (with 2nd order filter)

! Hydraulic Actuator Type (B) for the Space Shuttle Elevon, Including a second order notch filter
! in the control loop at 65 (rad/sec)

Number of Inputs, States, Outputs, Sample Time dT (for discrete)= 2 6 3 0.0000

Matrices: (A,B,C,D)

Matrix A Size = 6 X 6
1-Column 2-Column 3-Column 4-Column 5-Column 6-Column
1-Row -0.476444909333E+02 -0.847340446418E+04 -0.167404640751E+04 -0.300977352000E+03 0.000000000000E+00 0.779654257565E+04
2-Row -0.943800000000E-01 -0.203433315667E+02 -0.401912611041E+01 0.000000000000E+00 0.000000000000E+00 0.187182910200E+02
3-Row 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00 0.000000000000E+00
4-Row 0.000000000000E+00 0.430865076116E+04 -0.311343104001E+04 -0.465254773514E+01 0.000000000000E+00 0.000000000000E+00
5-Row 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 -0.422500000000E+04
6-Row 0.000000000000E+00 0.826523299588E+02 0.163291905426E+02 0.000000000000E+00 0.100000000000E+01 -0.780000000000E+02

Matrix B Size = 6 X 2
1-Column 2-Column
1-Row 0.779654257565E+04 0.000000000000E+00
2-Row 0.187182910200E+02 0.000000000000E+00
3-Row 0.000000000000E+00 0.000000000000E+00
4-Row 0.000000000000E+00 0.372203818811E-02
5-Row 0.000000000000E+00 0.000000000000E+00
6-Row -0.760500000000E+02 0.000000000000E+00

Matrix C Size = 3 X 6
1-Column 2-Column 3-Column 4-Column 5-Column 6-Column
1-Row 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00
2-Row 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00 0.000000000000E+00
3-Row 0.000000000000E+00 0.430865076116E+04 -0.311343104001E+04 -0.465254773514E+01 0.000000000000E+00 0.000000000000E+00

Matrix D Size = 3 X 2
1-Column 2-Column
1-Row 0.000000000000E+00 0.000000000000E+00
2-Row 0.000000000000E+00 0.000000000000E+00
3-Row 0.000000000000E+00 0.372203818811E-02

-----Definition
of System Variables

Inputs = 2
Nozzle Deflection Command (Delta\_com) (rad)
Load-Torque at the Gimbal, (Tl) (ft-lb)

Outputs = 3
Nozzle Gimbal Rotation (Delta) (rad)
Nozzle Gimbal Rate (Delta-dot) (rad/sec)
Gimbal Accelerat. (Delta-dot-dot) (rad/sec^2)

States = 6
Differ. Pressure Feedback (DPF) (ft-lb)
Piston Displacement (Xr) (feet)
Engine Gimbal Rotation (Delta) (rad)
Engine Gimbal Rate (Delta-dot) (rad/sec)
Second Order Compens. State # 1
Second Order Compens. State # 2

STATE-SPACE SYSTEM ...

Electro-Mechanical Actuator (without motor and amplifier dynamics)

! This EMA actuator was developed for an Advanced Launch Booster Vehicle

Number of Inputs, States, Outputs, Sample Time dT (for discrete)= 2 4 3 0.0000

Matrices: (A,B,C,D)

Matrix A Size = 4 X 4

	1-Column	2-Column	3-Column	4-Column
1-Row	0.000000000000E+00	0.100000000000E+01	0.000000000000E+00	0.000000000000E+00
2-Row	-0.481461241333E+04	-0.267110661457E+03	0.157440483560E+07	0.000000000000E+00
3-Row	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.100000000000E+01
4-Row	0.127252256394E+01	0.000000000000E+00	-0.601012406948E+04	-0.133622828784E+03

-----Matrix

B Size = 4 X 2

	1-Column	2-Column
1-Row	0.000000000000E+00	0.000000000000E+00
2-Row	0.211713257047E+08	0.000000000000E+00
3-Row	0.000000000000E+00	0.000000000000E+00
4-Row	0.000000000000E+00	0.124069478908E-01

-----Matrix

C Size = 3 X 4

	1-Column	2-Column	3-Column	4-Column
1-Row	0.000000000000E+00	0.000000000000E+00	0.100000000000E+01	0.000000000000E+00
2-Row	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.100000000000E+01
3-Row	0.127252256394E+01	0.000000000000E+00	-0.601012406948E+04	-0.133622828784E+03

-----Matrix

D Size = 3 X 2

	1-Column	2-Column
1-Row	0.000000000000E+00	0.000000000000E+00
2-Row	0.000000000000E+00	0.000000000000E+00
3-Row	0.000000000000E+00	0.124069478908E-01

Definition of System Variables

Inputs = 2

- Engine Deflection Command (Del\_com) (rad)
- External Load-Torque (Tl), (ft-lb)

Outputs = 3

- Engine Gimbal Rotation (Delta) (rad)
- Engine Gimbal Rate (Del-dot) (rad/sec)
- Engine Gimbal Acceler. (Del-d-dot) (rad/sec^2)

States = 4

- Torque Motor Angle (theta), (radians)
- Torque Motor Rate (theta-dot), (rad/sec)
- Engine Gimbal Rotation (Delta), (radians)
- Engine Gimbal Rate (Delta-dot), (rad/sec)

STATE-SPACE SYSTEM ...

Electro-Mechanical Rudder Actuator, Type-B (feet)

! This EMA actuator Uses an Extendable Push Rod to Drive a Load. The end of the rod is attached to a lever arm that rotates a control Surface.

Number of Inputs, States, Outputs, Sample Time dT (for discrete)= 2 8 3 0.0000

Matrices: (A,B,C,D)

Matrix A Size = 8 X 8
1-Column 2-Column 3-Column 4-Column 5-Column 6-Column
1-Row 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00
2-Row -0.205278194611E+07 -0.285714285714E+00 0.210685104220E+07 0.000000000000E+00 0.119657142857E+06 0.000000000000E+00
3-Row 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00 0.000000000000E+00
4-Row 0.141263958768E+03 0.000000000000E+00 -0.159921462756E+05 -0.306513409962E+02 0.000000000000E+00 0.000000000000E+00
5-Row 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01
6-Row 0.753833969328E+06 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 -0.443431746664E+05 -0.300000000000E+04
7-Row -0.171690899847E-01 0.000000000000E+00 -0.165632943569E+01 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00
8-Row -0.686763599390E+02 0.000000000000E+00 -0.662531774276E+04 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00

Matrix B Size = 8 X 2
1-Column 2-Column
1-Row 0.000000000000E+00 0.000000000000E+00
2-Row 0.000000000000E+00 0.000000000000E+00
3-Row 0.000000000000E+00 0.000000000000E+00
4-Row 0.000000000000E+00 0.191570881226E-01
5-Row 0.000000000000E+00 0.000000000000E+00
6-Row 0.000000000000E+00 0.000000000000E+00
7-Row 0.360000000000E+01 0.000000000000E+00
8-Row 0.144000000000E+05 0.000000000000E+00

Matrix C Size = 3 X 8
1-Column 2-Column 3-Column 4-Column 5-Column 6-Column
1-Row 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00
2-Row 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00 0.000000000000E+00
3-Row 0.141263958768E+03 0.000000000000E+00 -0.159921462756E+05 -0.306513409962E+02 0.000000000000E+00 0.000000000000E+00

Matrix D Size = 3 X 2
1-Column 2-Column
1-Row 0.000000000000E+00 0.000000000000E+00
2-Row 0.000000000000E+00 0.000000000000E+00
3-Row 0.000000000000E+00 0.191570881226E-01

Definition of System Variables

Inputs = 2
Deflection Command (Delta\_com) (radians)
External Load-Torque (Tl), (ft-lb)

Outputs = 3
Gimbal Deflection (Delta) (rad)
Gimbal Rate (Delta-dot) (rad/sec)
Gimbal Accelerat (Delta-dot-dot) (rad/sec^2)

States = 8
Screw Gear Rotation Angle (theta), (rad)
Screw Gear Rate (theta-dot) (rad/sec)
Gimbal Deflection (Delta) (rad)
Gimbal Rate (Delta-dot) (rad/sec)
Torque Motor Rot Angle (mu), (radians)
Motor Rot Rate (mu-dot), (radian/sec)
Position Error Integral, (ft-sec)
First Order Compensator State

STATE-SPACE SYSTEM ...

Electro-Mechanical Actuator for an Elevator

! This EMA system has a full PID controller with a second order compensator

Number of Inputs, States, Outputs, Sample Time dT (for discrete)= 2 9 3 0.0000

Matrices: (A,B,C,D)

Matrix A Size = 9 X 9
1-Column 2-Column 3-Column 4-Column 5-Column 6-Column
1-Row 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00
2-Row -0.164332679910E+06 -0.236406619385E+01 0.576581599115E+06 0.000000000000E+00 0.196164125909E+05 0.000000000000E+00
3-Row 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00 0.000000000000E+00
4-Row 0.580700039108E+02 0.000000000000E+00 -0.936253421979E+04 -0.619047619048E+01 0.000000000000E+00 0.000000000000E+00
5-Row 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01
6-Row -0.387107734092E+04 0.400000000000E+03 -0.160243785686E+05 0.000000000000E+00 -0.595610129988E+02 -0.105052179301E+04
7-Row -0.254692999609E-02 0.000000000000E+00 -0.936253421979E-02 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00
8-Row -0.139674252249E+09 0.000000000000E+00 -0.513443623621E+09 0.000000000000E+00 0.000000000000E+00 -0.548402400000E+05
9-Row -0.12400852562E+06 0.000000000000E+00 -0.455828086038E+06 0.000000000000E+00 0.000000000000E+00 -0.486864000000E+02
9-Column
1-Row 0.000000000000E+00
2-Row 0.000000000000E+00
3-Row 0.000000000000E+00
4-Row 0.000000000000E+00
5-Row 0.000000000000E+00
6-Row 0.135836734694E+03
7-Row 0.000000000000E+00
8-Row -0.576000000000E+05
9-Row -0.336000000000E+03

Matrix B Size = 9 X 2
1-Column 2-Column
1-Row 0.000000000000E+00 0.000000000000E+00
2-Row 0.000000000000E+00 0.000000000000E+00
3-Row 0.000000000000E+00 0.000000000000E+00
4-Row 0.000000000000E+00 0.238095238095E+00
5-Row 0.000000000000E+00 0.000000000000E+00
6-Row 0.718848000000E+06 0.000000000000E+00
7-Row 0.420000000000E+00 0.000000000000E+00
8-Row 0.230329008000E+11 0.000000000000E+00
9-Row 0.204482880000E+08 0.000000000000E+00

Matrix C Size = 3 X 9
1-Column 2-Column 3-Column 4-Column 5-Column 6-Column
1-Row 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00
2-Row 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00 0.000000000000E+00
3-Row 0.580700039108E+02 0.000000000000E+00 -0.936253421979E+04 -0.619047619048E+01 0.000000000000E+00 0.000000000000E+00
9-Column
1-Row 0.000000000000E+00
2-Row 0.000000000000E+00
3-Row 0.000000000000E+00

Matrix D Size = 3 X 2
1-Column 2-Column
1-Row 0.000000000000E+00 0.000000000000E+00
2-Row 0.000000000000E+00 0.000000000000E+00
3-Row 0.000000000000E+00 0.238095238095E+00

Definition of System Variables

Inputs = 2
Deflection Command (Delta\_com) (radians)
External Load-Torque (Tl), (ft-lb)

Outputs = 3
Gimbal Deflection (Delta) (rad)
Gimbal Rate (Delta-dot) (rad/sec)
Gimbal Accelerat (Delta-dot-dot) (rad/sec^2)

States = 9
Screw Gear Rotation Angle (theta), (rad)
Screw Gear Rate (theta-dot) (rad/sec)
Gimbal Deflection (Delta) (rad)
Gimbal Rate (Delta-dot) (rad/sec)
Torque Motor Rot Angle (mu), (radians)
Motor Rot Rate (mu-dot), (radian/sec)
Position Error Integral, (ft-sec)
Second Order Compens. State # 1
Second Order Compens. State # 2

STATE-SPACE SYSTEM ...

Electro-Mechanical Actuator with an Extendable Push Rod Simplified

! This EMA actuator is a Simplified version of the above EMA system The end of the rod is
! attached to a lever arm that rotates a control Surface. A first order compensator (1000/s+1000)
! is used to model the motor dynamics

Number of Inputs, States, Outputs, Sample Time dT (for discrete)= 2 4 3 0.0000

Matrices: (A,B,C,D)

Matrix A Size = 4 X 4
1-Column 2-Column 3-Column 4-Column
1-Row 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.390000000000E-01
2-Row 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00
3-Row 0.926208651399E+05 -0.277862595420E+05 -0.213740458015E+02 0.000000000000E+00
4-Row -0.577777777778E+05 -0.186666666667E+05 0.000000000000E+00 -0.100000000000E+04

Matrix B Size = 4 X 2
1-Column 2-Column
1-Row 0.000000000000E+00 0.000000000000E+00
2-Row 0.000000000000E+00 0.000000000000E+00
3-Row 0.000000000000E+00 0.254452926209E+00
4-Row 0.360000000000E+05 0.000000000000E+00

Matrix C Size = 3 X 4
1-Column 2-Column 3-Column 4-Column
1-Row 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00 0.000000000000E+00
2-Row 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00
3-Row 0.926208651399E+05 -0.277862595420E+05 -0.213740458015E+02 0.000000000000E+00

Matrix D Size = 3 X 2
1-Column 2-Column
1-Row 0.000000000000E+00 0.000000000000E+00
2-Row 0.000000000000E+00 0.000000000000E+00
3-Row 0.000000000000E+00 0.254452926209E+00

Definition of System Variables

Inputs = 2
Load Deflection Command (Delta\_com), (rad)
External Load Torque (Tl), (ft-lb)

Outputs = 3
Load Rotation at Gimbal (Delta) (rad)
Load Rate at Gimbal (Delta-dot) (rad/sec)
Load Accelerat. (Delta-dot-dot) (rad/sec^2)

States = 4
Actuator Piston Position (Xp) (feet)
Load Rotation, (Delta) (radians)
Load Rate, (Delta-dot) (rad/sec)
First Order Compensator State

END-

## EM Actuator of a Launch Vehicle

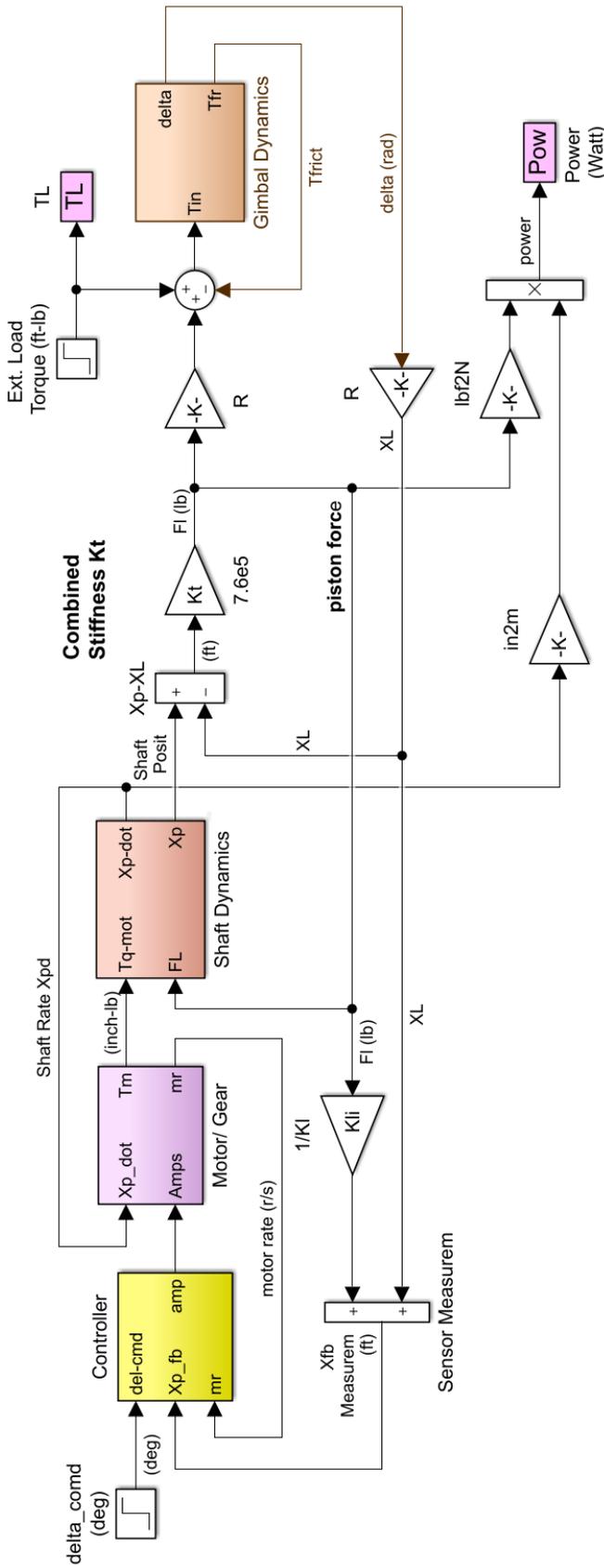
In this example we analyze an electro-mechanical TVC actuator used to gimbal the engine of a launch vehicle. We will develop a non-linear simulation model using Simulink, a linear model for control analysis, and will recreate the linear model using the Flixan actuator modeling program, option: “EMA, type B” to compare results. The analysis files for this example are in: “*Actuator\Examples\EM-Actuators\Launch Vehicle*”.

The Simulink file for the non-linear model is “*NonLin\_EMA.slx*”, shown in Figure 1a. It consists of a PID control system receiving shaft position and motor rate from resolvers and generates the current driving the motor through a current amplifier. The current in the amplifier is limited to  $\pm 2$  (Amps). The motor generates the torque that rotates the screw, shown in Figure 2a. There is a gear reduction between the rotor and screw rotations that increases the screw torque. Figure 2b shows the screw/shaft dynamics. The screw is driven through the shaft (which is like a nut around the screw) by the motor torque and generates the shaft extension. The shaft position output  $x_p$  generates a force against the load that rotates the engine. The load force (FL) opposes the motor torque. There is also a static friction feedback resisting the rotation of the screw. Figure 2c is the engine dynamics as it rotates about the gimbal and it includes 200 (ft-lb) static friction at the gimbal. It is coded as a Matlab function “Gimbal” and calculates friction force and acceleration which is integrated twice for gimbal position.

### Actuator Response to a 2° Step Command

Figures 3 show the EMA actuator system’s response to 2° gimbal command. Static friction of 200 (ft-lb) is included at the engine gimbal, and static friction of 2 (ft-lb) is also included at the screw/shaft mechanism. Figure 3a shows the gimbal response as it catches up with the command. The 20 Hz oscillation is due to the combined stiffness  $K_T$  at the gimbal. It is shown more emphatically in the acceleration that also shows a high frequency oscillatory squeak, caused by the static friction, as the nozzle slowly approaches its commanded value. Figure 3b shows the shaft position and velocity. The shaft extends 0.35 (inch) to produce the 2° gimbal angle. The shaft velocity saturates at 2.9 (in/sec) due to the motor current limits. The high frequency oscillations in the velocity is caused by the gear stiffness mode. The actuator power is calculated by multiplying shaft velocity times the load force. Figure 3c shows the motor spin rate is limited to 670 (rad/sec) because the current supplying the motor is also limited by the amplifier to  $\pm 2$  Amps. The load force oscillates at the 20 Hz gimbal mode. The gear stiffness between the motor and screw causes the torque and shaft velocity to oscillate at high frequency 1200 (rad/sec).

# Non-Linear Actuator Model



# PID Control System

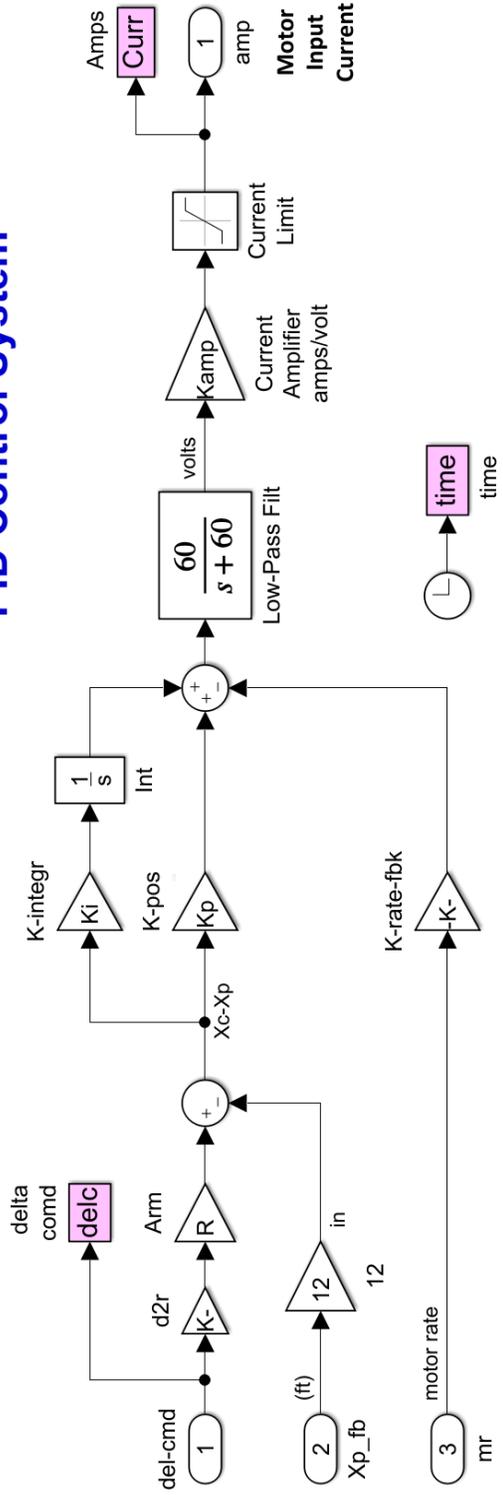


Figure 1 EMA System Block Diagram

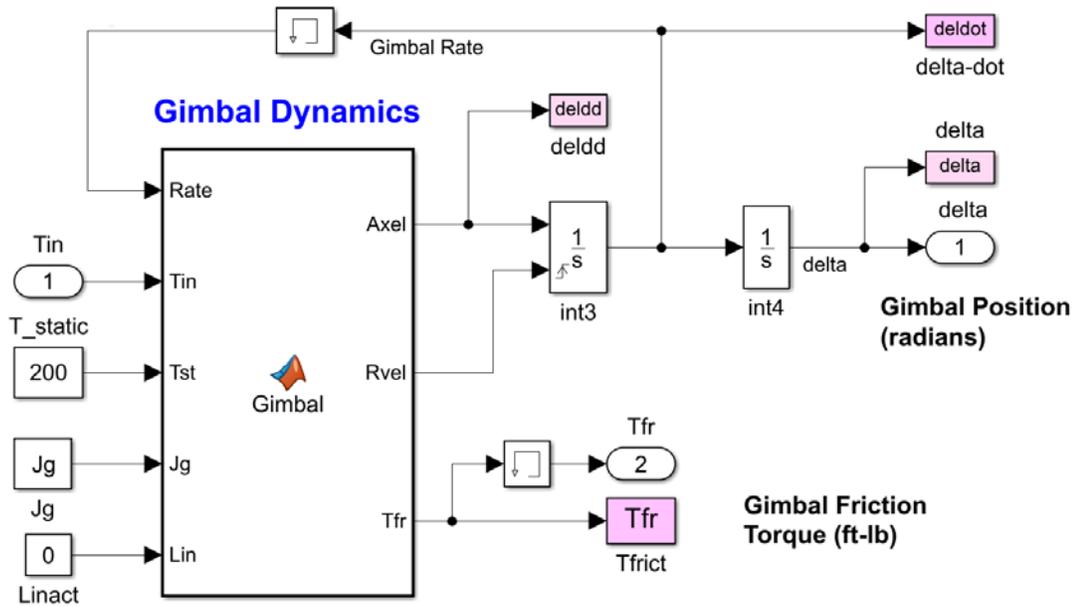
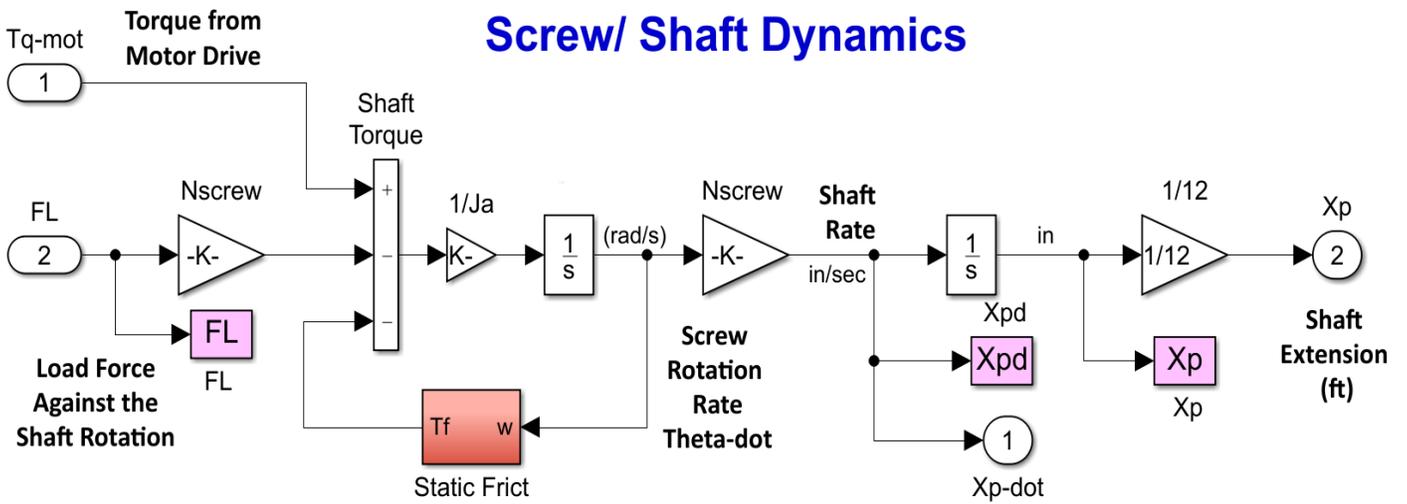
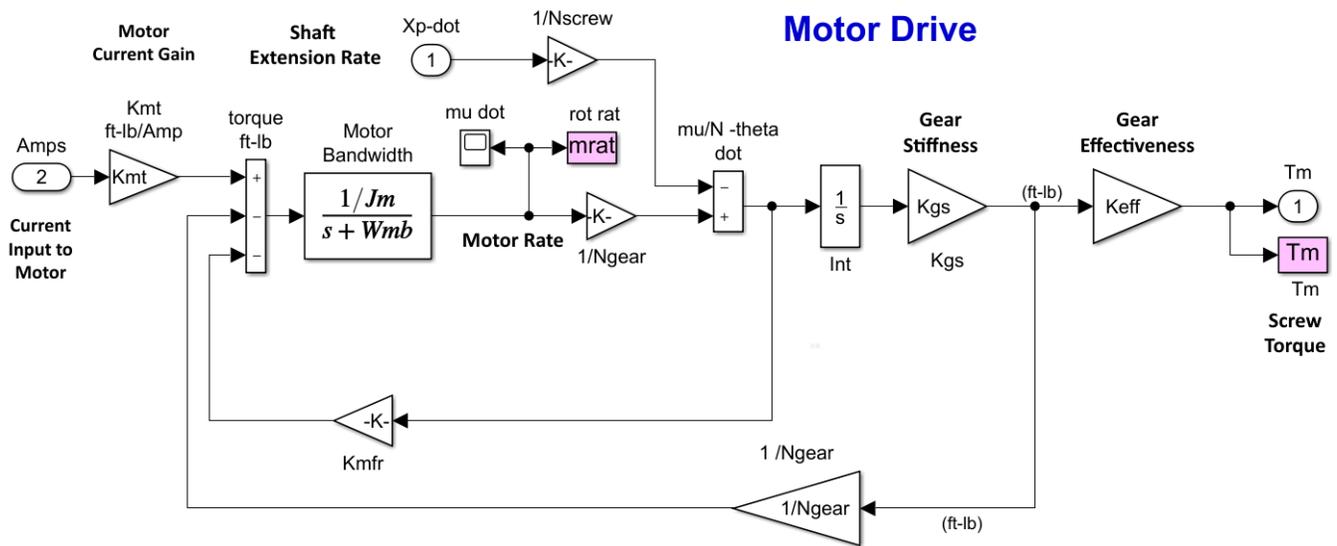


Figure 2 EM Actuator Subsystems

```

function [Axel,Rvel,Tfr] = Gimbal(Rate,Tin,Tst,Jg,Bvf,Lin)
% Gimbal Friction Dynamics
% Tin: Input Torque (ft-lb)
% Rate: Gimbal Rate (rad/sec)
% Bvf: Viscous Friction
% Tfr: Gimbal Friction Torque (ft-lb)
% axel: Gimbal Acceleration (ft/sec2)
% Tst: Gimbal Static Friction Torq (ft-lb)
% Jg: Gimbal Inertia (slg-ft^2)

if Lin>0.5 % Linear Actuator (1)
    Tfr=Bvf*Rate; % Viscous Friction Torque
    Axel=(Tin-Tfr)/Jg; % Gimbal Acceleration
    Rvel=0; % Dont Reset
else % Non-Linear Actuator (0)
    if abs(Rate)<0.01 % If Not Moving Much
        if abs(Tin)>Tst*1.05 % Exceeds Stiction, Start Moving
            Tfr=Tst*sign(Tin); % Friction Torque resist motion
            Axel=(Tin-Tfr)/Jg; % Gimbal Acceleration
            Rvel=0;
        else % Stop the Motion
            Axel=0;
            Tfr=0; % No Frict when not moving
            Rvel=1; % Reset Velocity to Zero
        end
    else % Gimbal is Fast Enough
        Tfr=Tst*sign(Rate); % Friction Torque resist motion
        Axel=(Tin-Tfr)/Jg; % Gimbal Acceleration
        Rvel=0;
    end
end
end
end

% EMA Actuator Model
r2d= 180/pi; d2r=pi/180;
Kload=1.2e9; % Load Stiffness
Kback=1.122e6; % Backup Structure
Kact= 2.652e6; % Shaft Stiffness Kact= 221,000 (lb/in)
Kti=(1/Kload)+(1/Kback)+(1/Kact); Kt=1/Kti; % Total Stiffness
Kli=(1/Kload)+(1/Kback); Kl=1/Kli; % Stiffness of Load + Backup

Kp= 2.3; % PID gain
Ki= 0.4; % PID gain
Krfb= 0.014; % Rate Feedback Gain
Alim= 2.0; % Controller Output Current Limit 2 Amp;
Kamp=5.0; % Current Amplifier Gain (amps/volt)
R= 10; % Moment Arm (in) R= 3.6
Jg= 33.45; % Engine Inertia Jg= 33.45
Ja= 0.0026562; % Gear plus Screw Inertia
Kmt= 404; % Motor Gain (ft-lb/Amps)
Jm= 1.2e-2; % Motor Rotor Mom Inertia
Wmb= 100; % Motor Bandwidth (rad/sec)
Kmfr= 0.02; % Motor Friction 0.4
Keff= 0.85; % Motor Gear Effectiveness 0.85
Kgs= 4000.0; % Gear Stiffness Kgs=8376.0
Nscrew= 0.0313; % Screw Ratio Nscrew (inch/rad)
Ngear= 7.11; % Gear Ratio Ngear= 17.0
Kdmp= 0.02; % Shaft Damping Kdmp= 0.285
Be=1000; % Viscous Damping Coefficient at Gimbal

```

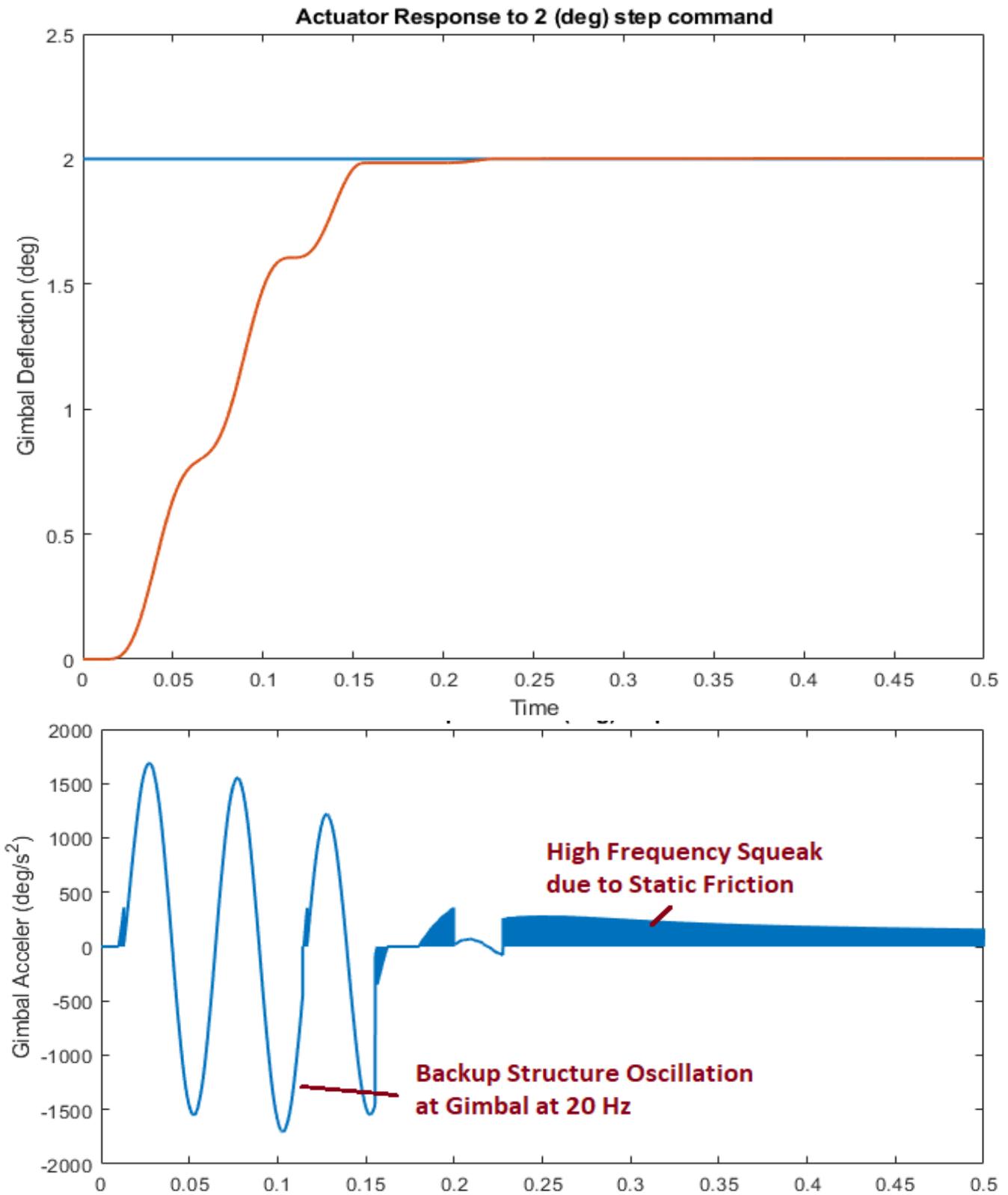


Figure 3a Gimbal Response to 2<sup>nd</sup> Step Command

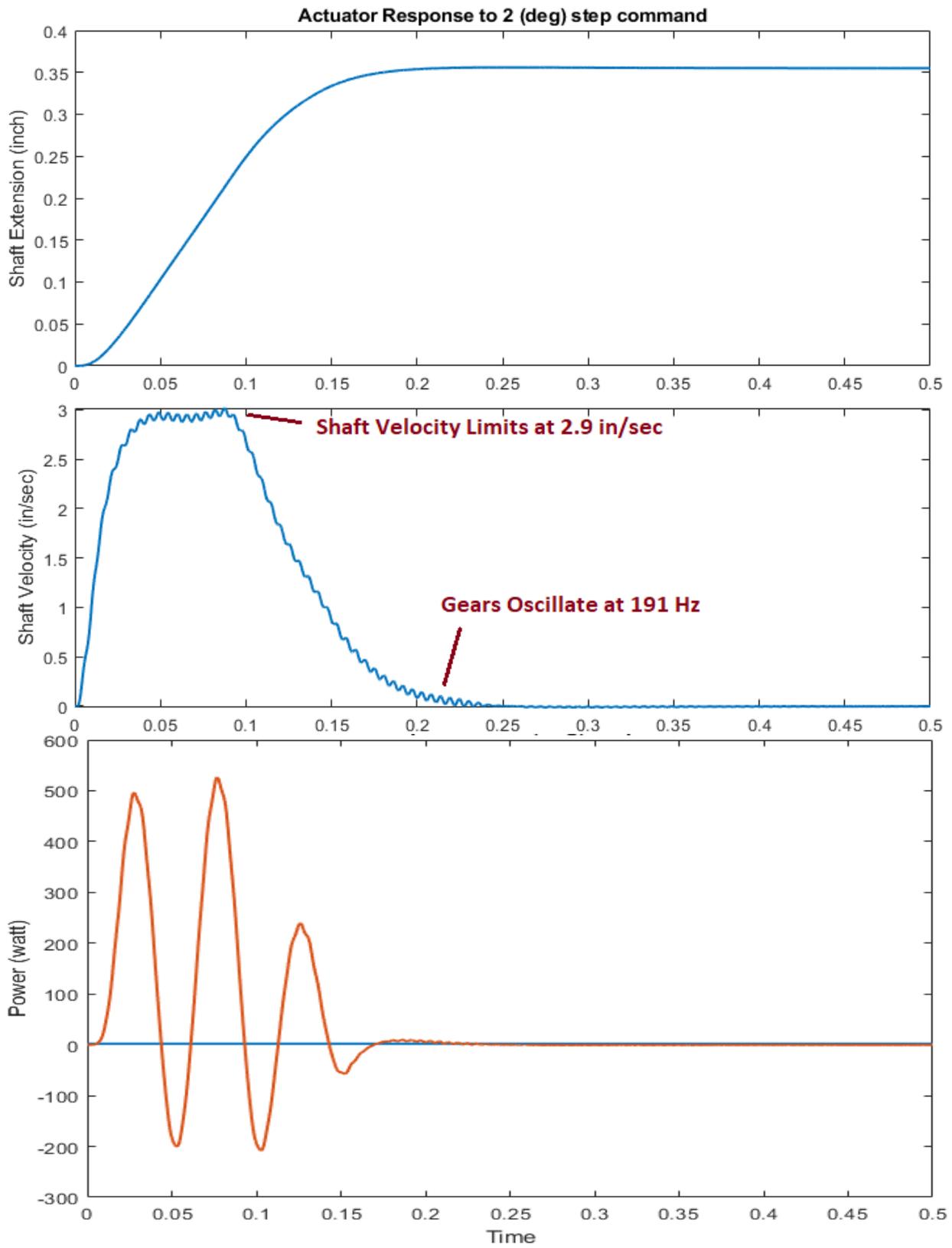


Figure 3b Shaft Position, Velocity, and Actuator Power

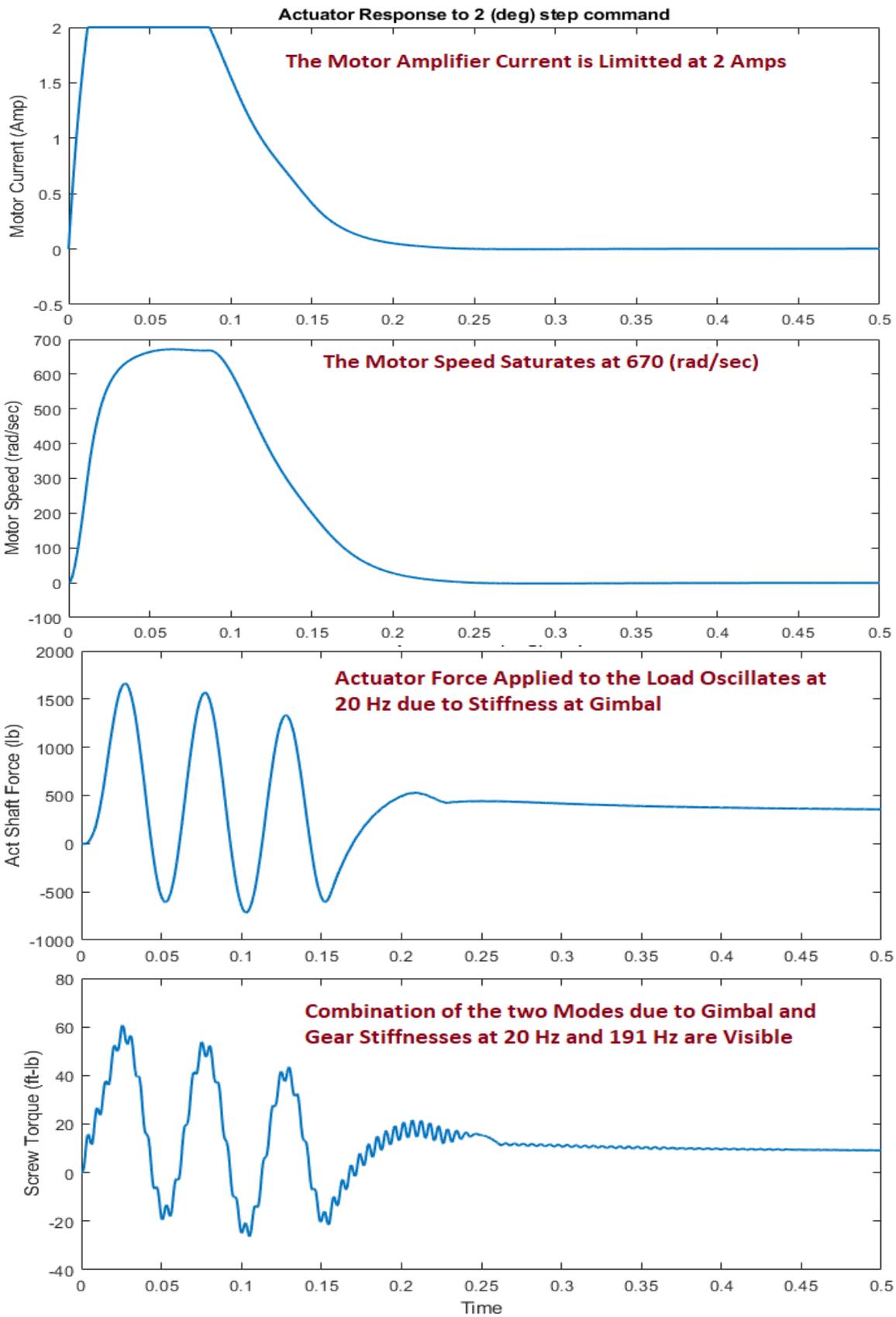


Figure 3c Motor Current and Speed, Shaft Force and Torque on Screw

## Frequency Response Analysis

The input to the Flixan program that generates the linear actuator is in file “*EMA\_Actuat.Inp*”, and is shown below. The actuator system is saved in file “*EMA\_Actuat.Qdr*”. It is also saved as a function “*ema\_stg1.m*” that can be loaded into Matlab.

Figure 4a shows the frequency response of the linear actuator/ gimbal system, gimbal response over gimbal command. The Coulomb frictions are replaced with viscous values that have similar responses. The control bandwidth is set at 15 (rad/sec). The gimbal TWD mode is at 120 (rad/sec), and the gear mode due to gear stiffness is at 1200 (rad/sec). The Nichols plot in Figure 4b shows the actuator control loop stability characteristics. The loop is opened at the control system output.

```

BATCH MODE INSTRUCTIONS .....
Batch set for creating an EMA actuator model
!
Actuator Model   : Electro-Mechanical Stage-1 Actuator
To Matlab Format : Electro-Mechanical Stage-1 Actuator
-----
ACTUATOR INPUT DATA ..... ELECTROMECHANICAL TYPE B
Electro-Mechanical Stage-1 Actuator
! This is 1st Stage EMA actuator. Uses an Extendable Push Rod to Drive the Engine.
!
Symbol          Parameter Description                (Units)      Value
C(s)            Order of Compensat:(0,1,2), Coefficients  (---)        1, 0.0, 0.0166
Kpl             Position Error Loop Gain                        (Volt/ft)    27.6
Krf            Rate Feedback Gain                    (Volt/ft/sec) 6.1947e-5
Ki             Position Error Integral Gain                (Volt/ft-sec) 4.8
Wm            Servo Motor Bandwidth,                          (rad/sec)    100.0
Kv            Servo Motor Torque Gain                      (ft-lb/volt) 2020.0
Jm            Motor Rotor Moment of Inertia                (in-lb-sec^2) 1.2e-2
Ngr           Gear Ratio (Motor Turns/ Screw Turns)      ( - )        7.11
Nscr          Screw Ratio (Piston Extension/Screw Turns) (ft/rad)     0.00261
Kgs           Motor Gear Stiffness                            (in-lb/rad)  4000.0
Kmf           Motor/ Gear Friction                            (ft-lb/rad/sec) 0.02
Kdmp          Shaft Damping Friction                            (ft-lb/rad/sec) 0.02
Keff          Gear Effectiveness Coefficient                ( - )        0.85
Ja           Gear plus Screw Inertia                            (in-lb-sec^2) 0.002656
Kact          Actuator Stiffness (Piston+Electric).... (lb/in)      2.652e6
Klod          Load Stiffness, (Con. Surface or Nozzle) (lb/in)      1.2e+9
Kbck          Vehicle Backup Structure Stiffness ..... (lb/in)      1.122e6
R            Moment Arm of Actuator Rod from Gimbal.. (in)         0.8333
Je           Load Inertia about the Gimbal ..... (in-lb-sec^2) 33.45
Be           Nozzle Bearing Viscous Damping ..... (ft-lb-sec)  1000.0
Kg           Gimbal Bearing Spring Constant                (ft-lb/rad)  0.0000
-----
MATLAB CONVERSIONS
-----
CONVERT TO MATLAB FORMAT ..... (Title, System/Matrix, m-filename)
Electro-Mechanical Stage-1 Actuator
System
EMA_Stg1
-----

```

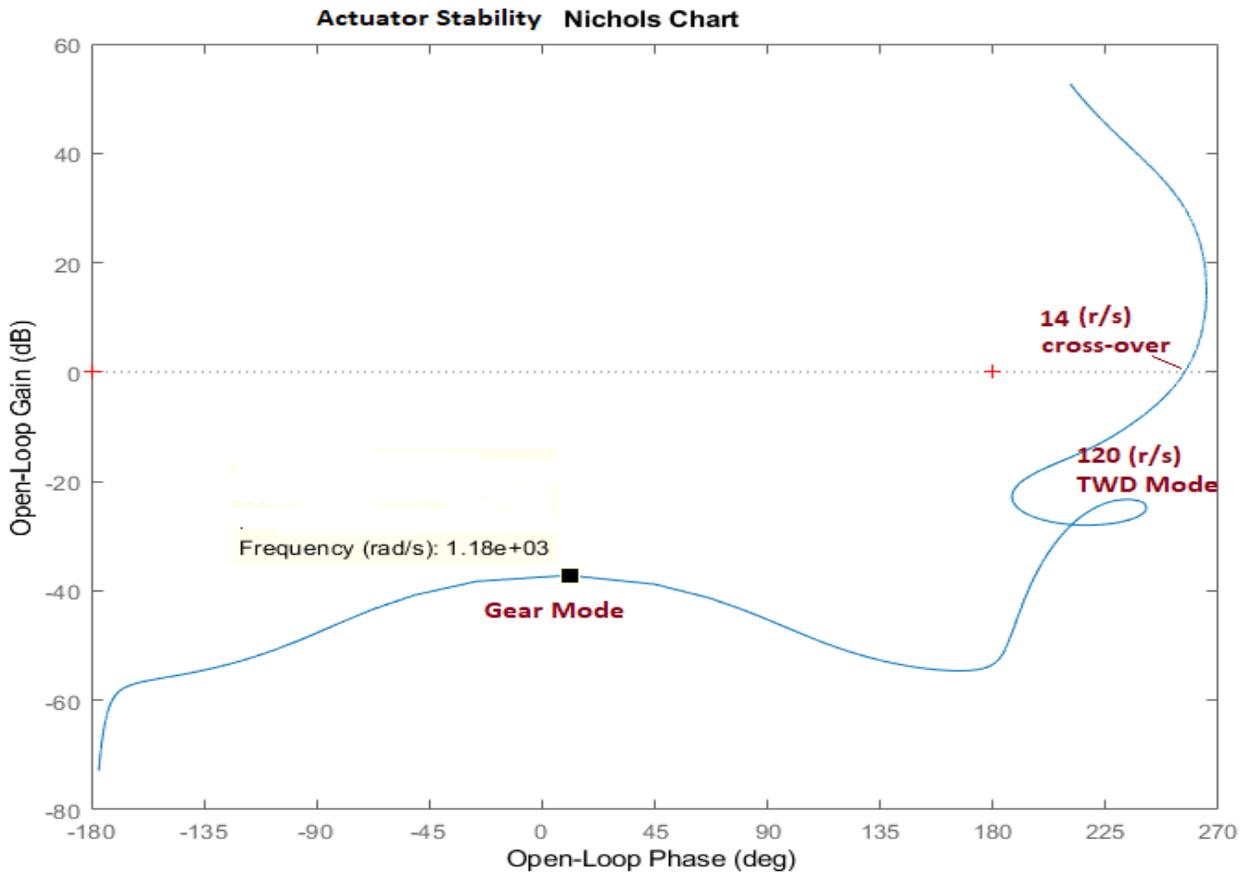
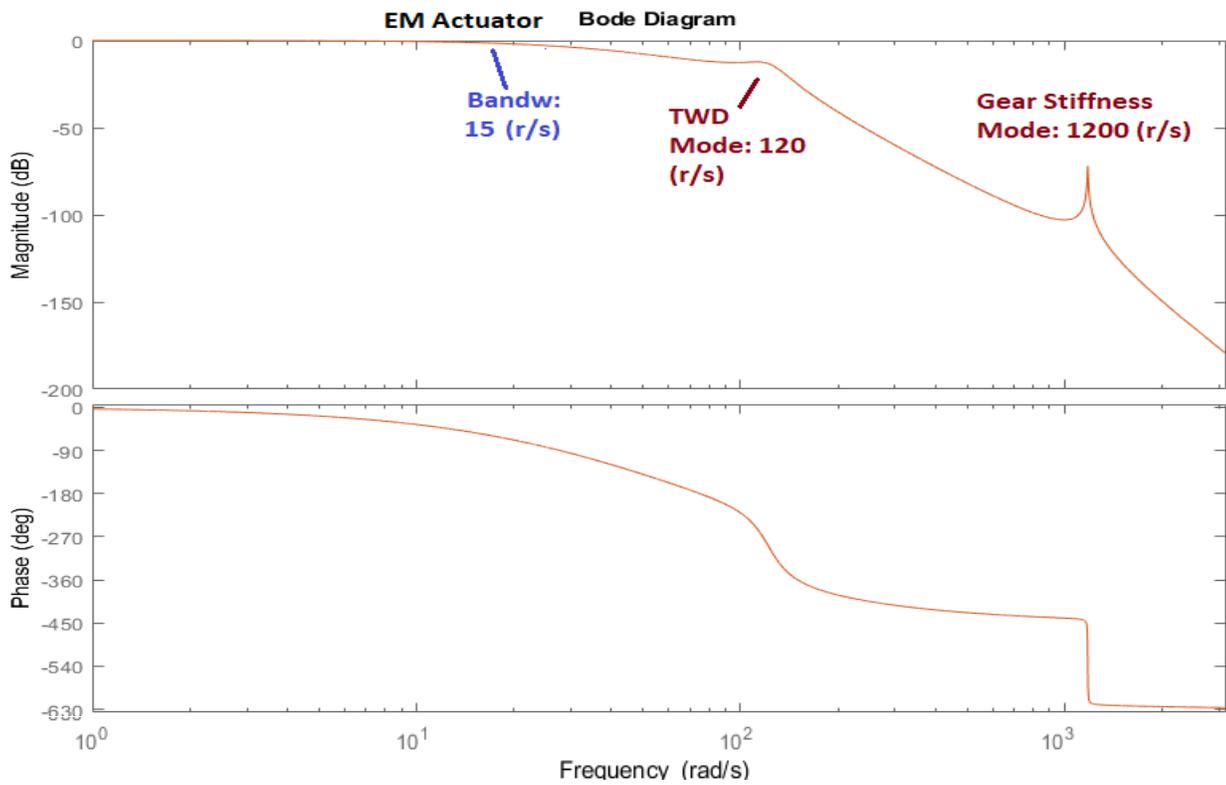


Figure 4 Nichols Plot Showing the Actuator Control Loop Gain and Phase Margins